Equivalent Fractions

Identifying equivalent fractions is useful to:
- compare quantities or amounts that the fractions represent
- reduce fractions to their lowest form
- add and subtract fractions.

**Equivalent fractions** are fractions that represent the same amount but have different numerators and denominators.

**Examples**

A) In both of the shapes below, half of the shape is shaded.

This shape has two parts.
One part is shaded.
This equals \( \frac{1}{2} \).

This shape has 6 parts.
3 parts are shaded.
This equals \( \frac{3}{6} \), which is the same amount as \( \frac{1}{2} \).

\( \frac{3}{6} \) and \( \frac{1}{2} \) are equivalent fractions.

B) \( \frac{1}{2} \) and \( \frac{2}{4} \) have different numerators and denominators, but they represent the same amount. They are equivalent fractions.

\[ \frac{1}{2} \quad \frac{2}{4} \]
C) Pumpkin pie is Dave’s favourite pie. Would he rather have \( \frac{1}{3} \) or \( \frac{2}{6} \) of the pie?

\[
\frac{1}{3} = \frac{2}{6}
\]

Answer: \( \frac{1}{3} \) and \( \frac{2}{6} \) are equivalent fractions. They represent the same amount of pie.

**Identifying Equivalent Fractions Using Multiplication**

Multiply the numerator and the denominator of a fraction by the same number to get an equivalent fraction.

### Examples

A) Are \( \frac{2}{6} \) and \( \frac{6}{18} \) equivalent fractions?

\[
\frac{2 \times 3}{6 \times 3} = \frac{6}{18}
\]

\( \frac{2}{6} \) and \( \frac{6}{18} \) are equivalent fractions.

B) Are \( \frac{4}{5} \) and \( \frac{16}{20} \) equivalent fractions?

\[
\frac{4 \times 4}{5 \times 4} = \frac{16}{20}
\]

\( \frac{4}{5} \) and \( \frac{16}{20} \) are equivalent fractions.

C) Are \( \frac{3}{4} \) and \( \frac{21}{28} \) equivalent fractions?

\[
\frac{3 \times 7}{4 \times 7} = \frac{21}{28}
\]

\( \frac{3}{4} \) and \( \frac{21}{28} \) are equivalent fractions.
Identifying Equivalent Fractions Using Division

Divide the numerator and denominator by the same number to get equivalent fractions and to reduce fractions to their lowest form.

**Examples**

A) Are \(\frac{6}{10}\) and \(\frac{3}{5}\) equivalent fractions?

\[
\frac{6}{10} \div 2 = \frac{3}{5}
\]

\(\frac{6}{10}\) and \(\frac{3}{5}\) are equivalent fractions.

B) Are \(\frac{9}{24}\) and \(\frac{3}{8}\) equivalent fractions?

\[
\frac{9}{24} \div 3 = \frac{3}{8}
\]

C) Are \(\frac{6}{36}\) and \(\frac{1}{6}\) equivalent fractions?

\[
\frac{6}{36} \div 6 = \frac{1}{6}
\]

Identifying Equivalent Fractions Using LCM

Lowest Common Multiples (LCM) can be used to find a common denominator when adding and subtracting fractions and mixed numbers.

To find equivalent fractions of two or more fractions, list multiples of each denominator. Identify the smallest (lowest) multiple that is present in both sets.

**Examples**

A) Find LCMs to add \(\frac{2}{3} + \frac{1}{4}\).

Identify the multiples of 3 and 4.

The multiples of 3: 3, 6, 9, 12, 15, …
The multiples of 4: 4, 8, 12, 16, …
The smallest multiple that is the same in both sets is 12.
The lowest common multiple, also called **lowest common denominator (LCD)** for 3 and 4 is 12.

Identify equivalent fractions of $\frac{2}{3}$ and $\frac{1}{4}$ using 12 (LCM) as the denominator.

$$\frac{2}{3} = \frac{?}{12}$$

What number multiplied by 3 equals 12? The number is 4.
Multiply the numerator 2 by 4 = 8.

$$\frac{2 \times 4}{3 \times 4} = \frac{8}{12}$$

$$\frac{1}{4} = \frac{?}{12}$$

What number multiplied by 4 equals 12? The number is 3.
Multiply the numerator 1 by 3 = 3.

$$\frac{1 \times 3}{4 \times 3} = \frac{3}{12}$$

Now, add the equivalent fractions.
$$\frac{8}{12} + \frac{3}{12} = \frac{11}{12}$$

Remember, to find equivalent fractions, multiply or divide the numerator and denominator by the same number.

**B) Find LCMs to add $\frac{3}{8} + \frac{4}{6}$.**

The multiples of 8: 8, 16, **24**, 32, …
The multiples of 6: 6, 12, 18, **24**, …

The LCD or LCM is 24.

$$\frac{3 \times 3}{8 \times 3} = \frac{9}{24} \quad \frac{4 \times 4}{6 \times 4} = \frac{16}{24}$$

$$\frac{9}{24} + \frac{16}{24} = \frac{25}{24} = 1 \frac{1}{24}$$
1. Use fraction circles or other strategies to show that the following fractions are equivalent; that is, they represent the same amount.

\[
\begin{align*}
\frac{2}{4} \text{ and } \frac{4}{8} & \quad \frac{4}{6} \text{ and } \frac{12}{18} \\
\frac{1}{3} \text{ and } \frac{3}{9} & \quad \frac{6}{7} \text{ and } \frac{12}{14} \\
\frac{2}{5} \text{ and } \frac{6}{15} & \quad \frac{1}{2} \text{ and } \frac{9}{18}
\end{align*}
\]

2. Use multiplication to find 3 equivalent fractions for each of the following. Prepare to explain your strategy.

a) \( \frac{1}{4} \)  
   b) \( \frac{4}{9} \)  
   c) \( \frac{3}{8} \)  
   d) \( \frac{7}{12} \)

3. Use division to find 2 equivalent fractions for each of the following. Prepare to explain your strategy.

a) \( \frac{40}{60} \)  
   b) \( \frac{30}{100} \)  
   c) \( \frac{24}{36} \)  
   d) \( \frac{100}{1000} \)

4. Four friends all work on the school newspaper. For this month’s edition, Garth contributed \( \frac{4}{11} \) pages, Graham contributed \( \frac{4}{32} \) pages, Cody contributed \( \frac{3}{13} \) pages and Carrie contributed \( \frac{2}{1} \) page. The principal wrote a full page that was also added to the paper.

To make the paper a full 9 pages, how many more pages must be added?

How many pages does the paper have?
5. Selma is a cook at a busy diner. The lunch-hour rush is about to begin and she has misplaced some of her measuring utensils. Look at the following chart and fill in the missing numerator or denominator. Supply the correct answer so that Selma’s meals remain tasty! The first one has been done for you.

<table>
<thead>
<tr>
<th>Amount of ingredient necessary</th>
<th>Multiply by</th>
<th>Equivalent measuring amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{2}{3} )</td>
<td>( \frac{2}{3} \times 2 )</td>
<td>( \frac{4}{6} )</td>
</tr>
<tr>
<td>( \frac{3}{4} )</td>
<td></td>
<td>( \frac{3}{8} )</td>
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<td>( \frac{1}{2} )</td>
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<td>( \frac{8}{15} )</td>
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<td>( \frac{4}{6} )</td>
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<td>( \frac{2}{8} )</td>
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<tr>
<td>( \frac{5}{7} )</td>
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<td>( \frac{15}{2} )</td>
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