Mathematics

Planning Guide

Grade 2

Addition and Subtraction Facts to 18

Number

Specific Outcome 10

This Planning Guide can be accessed online at:
http://www.learnalberta.ca/content/mepg2/html/pg2_additionandsubtractionfactsto18/index.html

Table of Contents

Planning Guide: *Grade 2 Addition and Subtraction Facts to 18*

Strand: Number
Specific Outcome: 10

This *Planning Guide* addresses the following outcomes from the Program of Studies:

Strand: Number	
Specific Outcome:	10. Apply mental mathematics strategies, such as: • making doubles • making 10 • one more, one less • two more, two less • building on a known double • thinking addition for subtraction for basic addition facts and related subtraction facts to 18.

Curriculum Focus

This sample targets the following changes to the curriculum:

- The specific outcome focuses on applying mental mathematics strategies for the basic addition and subtraction facts to 18; whereas the previous mathematics curriculum specified that Grade 2 students should recall these facts to ten. The previous curriculum specified that students should connect manipulatives, diagrams and symbols, as well as apply a variety of estimation and mental mathematics strategies to addition and subtraction problems. It did not explicitly state what these mental mathematics strategies might be or that these strategies needed to be developed with the basic facts as groundwork for their application to larger numbers. The new curriculum sets out in the specific outcome several of these mental mathematics strategies, so it is clear what is expected: "using doubles; making ten; one more, one less; two more, two less; building on a known double and thinking addition for subtraction" (Alberta Education 2007, p. 65).

What Is a Planning Guide?

Planning Guides are a tool for teachers to use in designing instruction and assessment that focuses on developing and deepening students' understanding of mathematical concepts. This tool is based on the process outlined in *Understanding by Design* by Grant Wiggins and Jay McTighe.

Planning Steps

The following steps will help you through the Planning Guide:

Step 1: Identify Outcomes to Address

Guiding Questions

- What do I want my students to learn?
- What can my students currently understand and do?
- What do I want my students to understand and be able to do based on the Big Ideas and specific outcomes in the program of studies?

Big Ideas

- Numbers are related to each other in numerous ways. For example, 8 is 2 more than 6, 2 less than 10, 1 more than 7, 1 less than 9, double 4, composed of 3 + 5 or 1 + 7, and can be recognized in several dot arrangements, such as two sets of 4 or a set of 5 and a set of three. Eight is also related to 18, 28, 38 and many other numbers.
- The order of adding addends does not change the sum. This is the commutative property of addition. Knowing this reduces the number of addition facts a person needs to learn in half. For example, if you know 3 + 5 = 8, then you also know 5 + 3 = 8. The commutative property does not apply to subtraction.
- Many of the basic fact sums and differences can be generated by using a small number of known facts, such as doubles and combinations that make ten, freeing up your memory and reducing effort. It is much easier to access information that is interconnected to other information than to recall isolated, discreet facts.
- The relationship to 5 and 10 of other numbers is very useful, as these two numbers form benchmarks during mental calculations of math facts.
- Addition and subtraction are inverse operations, so you can do subtraction by thinking addition. This reduces the number of basic facts students must learn. Students generally find addition facts much easier to learn than subtraction.

This is an even more powerful outcome than just learning the facts with these strategies. If students can apply the mental math strategies to problems with larger numbers, they will have demonstrated their growing number sense and have a good foundation for addition and subtraction using larger numbers.

Sequence of Outcomes from the Program of Studies

See http://education.alberta.ca/teachers/core/math/programs.aspx for the complete program of studies.

Grade 1

Specific Outcomes

10. Describe and use mental mathematics strategies (memorization not intended), such as:
    - counting on and counting back
    - making 10
    - using doubles
    - thinking addition for subtraction

    for basic addition facts and related subtraction facts to 18.

Grade 2

Specific Outcomes

10. Apply mental mathematics strategies, such as:
    - using doubles
    - making 10
    - one more, one less
    - two more, two less
    - building on a known double
    - thinking addition for subtraction

    for basic addition facts and related subtraction facts to 18.

Grade 3

Specific Outcomes

10. Apply mental mathematics strategies and number properties, such as:
    - using doubles
    - making 10
    - using the commutative property
    - using the property of zero
    - thinking addition for subtraction

    for basic addition facts and related subtraction facts to 18.

Step 2: Determine Evidence of Student Learning

Guiding Questions

- What evidence will I look for to know that learning has occurred?
- What should students demonstrate to show their understanding of the mathematical concepts, skills and Big Ideas?

Using Achievement Indicators

As you begin planning lessons and learning activities, keep in mind ongoing ways to monitor and assess student learning. One starting point for this planning is to consider the achievement indicators listed in the *Mathematics Kindergarten to Grade 9 Program of Studies with Achievement Indicators*. You may also generate your own indicators and use them to guide your observation of the students.

The following indicators may be used to determine whether or not students have met this specific outcome. Can students:

- explain how a mental mathematics strategy could be used to determine a basic fact?
- demonstrate a mental mathematics strategy that could be used to determine a basic fact?
- use and describe a personal strategy for determining a sum to 18?
- increase efficiency by adopting a new strategy or changing personal strategies?
- explain reasoning for a personal strategy selection that points to efficiency?
- analyze a personal strategy created by another person to determine if it suits the given problem and is efficient?
- demonstrate in solving problems the application of a personal strategy?
- add to find the answer to a subtraction problem?
- identify the related subtraction fact for any addition fact?

Sample behaviours to look for related to these indicators are suggested for some of the activities listed in **Step 3, Section C: Choosing Learning Activities** (p. 14).

Step 3: Plan for Instruction

Guiding Questions

- What learning opportunities and experiences should I provide to promote learning of the outcomes and to permit students to demonstrate their learning?
- What teaching strategies and resources should I use?
- How will I meet the diverse learning needs of my students?

A. Assessing Prior Knowledge and Skills

Before introducing new material, consider ways to assess and build on students' knowledge and skills related to counting.

Questions to Consider

- Do students have instant recognition of spatial arrangements? Most will instantly recognize the number of dots on a standard die without counting.
- Do they also recognize immediately the complete number of dots on various dominoes?
- Do they have instant recognition of the number of dots in configurations in other settings?
- Do they recognize them on ten frames?

Specific Outcome 8 from Grade 1 mentions in the indicators representation on a ten frame of numbers, so it is likely that the majority of Grade 2 students have some experience with ten frames.

- Show the student dot plate flash cards for one to three seconds.
- Students are asked to construct the pattern they viewed on their mats with counters.
- Questioning follows about how many dots they saw and how they saw them.
  This is done for a few configurations each day until the students learn to recognize the number in the configurations without counting (Van de Walle and Lovin 2006, p. 44). The students' competency at recognizing spatial arrangements can be done individually with a minimal investment of time.

Have the students learned what 1 and 2 more or less than each number is without counting on or back (Specific Outcome 8 from Grade 1)?

- A whole class assessment can be done by giving the students a sheet with random numbers written on it in the centre and columns on either side to write the numbers 1 or 2 less and 1 or 2 more, respectively.
- Careful observation of the students while they complete this activity will give you an indication of which students are already proficient—those who do this very speedily and without using fingers or subvocalizing counting on or back.
- Students who are not yet accomplished can use the calculator to give plus or minus 2 numbers by entering +2 = or 2 – 2 = and then entering numbers of their choice or from a deck of cards provided.
- Students should enter a number, answering the question, then press the = key to confirm their answers.

If you have number machines (see directions for making these in Patterns and Relations Outcome 2)

- Make a set of cards for plus 2 and another for minus 2.
- Students try to "beat" the number machine at giving the correct number, which the machine will spit out for each card.
- Students will hold the card with the beginning number in the top slot and not let go until they have the solution so that they can "beat" the machine.
- Any hesitation the students have in dropping the card through the slot will indicate that students do not have automatic knowledge of these relationships yet.

It is also likely that students will be proficient first with the 2 more and the 2 less will follow, as is typical of students' mastery of addition facts first and then subtraction facts.

You also need to know if your students are using 5 and 10 as benchmarks.

- Give each student a five frame printed in the centre of a page and some counters to check their knowledge of its use.
- Any counters beyond 5 may be placed on the page, but not in the frame.
- Tell them the rule that only one counter may be placed in a square.
- Then ask them to show various numbers.
- Ask them to describe their numbers on the frame.
- Based on the rule given, no configuration is wrong.
- The goal is to see if everyone in the class is trained to use frames in a consistent manner, filling them from top left to right.
- Students may describe, for example, that 4 is 2 and 2 and place 2 counters on one side and 2 on the other.

Questioning should focus on how many more counters students would need to place on the frames to have 5 or how many more than 5 a number is.

Similarly, focus attention on the relation to ten with ten frames. If practice using ten frames is needed

- Read numbers from a list of random numbers and have the students show each number on the ten frame.
- When the next number is called, have the students determine what they need to do to alter the past number to create the new number.

Van de Walle and Lovin describe a ten-frame flash card game in which students quickly tell how many dots are on the ten frame or alternatively tell how many white spaces are left on it. Further variations such as having the students say the number 2 more or say the number 2 less provide practice for students who need work on those relationships.

Students also need to have part–part–whole relationships for numbers at least to 10 in place. This means that they know and understand that 5 can be represented as a set of 1 and a set of 4 or a set of 2 and a set of 3.

- Students can be shown a number that you think they should be successful with and then hide part of it.
- This can easily be done with Unifix trains.
- The whole train of nine, for example, is shown to the students and it is confirmed that the students know it is a train of nine.
- Then hide the nine train and take away some of the cars.
- If the portion of the train that you bring forward then has only 5 cars, what number of cars were removed?

Repeat for other combinations of nine. If the student is able to tell you without counting the units, then the part–part–whole relationships are known for nine and they can move on to other numbers.

This activity can be done in a variety of ways if practice is needed.

- Students can pretend that some counters are left in a cave by placing a margarine tub over a number of the counters on the mat.
- They can do the "What's in the tent?" activity found on page 129 of the *Alberta Diagnostic Mathematics Program, Division I, Operations and Properties*).
- Students enjoy making up all the combinations for each number with manipulatives. For example, with Unifix they use two colours to show the combinations.
- A discussion could follow about whether 1 + 4 is different from 4 + 1.
- Students may note that 1 yellow and 4 red is not the same group as 4 yellow and 1 red; there is an even more pronounced difference if the yellow represents dollars and the reds pennies.
- To add challenge and mathematical interest for the more advanced students, ask them to record the number of possible combinations for each number from 1 to 10 and note the patterns.
- After they have recorded the number of combinations for the first few numbers, can they predict how many combinations there will be for the next numbers? Have them build them and check their predictions.

Do the students have knowledge of the doubles from their work in Grade 1?

- Inquire what they did or could visualize for doubles, such as: 3—a bug with three legs on each side
- 4—a spider with four legs on each side
- 5—the fingers on two hands
- 6—an egg carton or two dice
- 7—two weeks on the calendar
- 8—a box of crayons with two rows of eight
- 9—an 18-wheeler with nine wheels on each side (Van de Walle and Lovin 2006, p. 56).

Students generally find doubles easy to learn and will likely be relatively proficient with these; however, you may still want them to draw some illustrations of their aids for recalling doubles or do some activities to review doubles, since many of the strategies used in Grade 2 to generate number facts are dependent upon doubles.

- Students can use a calculator as a double maker by entering 2 X =.
- Then when they enter any number it will be doubled. They can play "heads versus calculator," in which one student has a calculator that has been converted to a double maker and the other student responds using only brain power.
- Number cards are turned up and the student relying on brain power may say the answer as soon as it is known.
- The student using the calculator must see the answer on the display screen before saying it.
- The person whose says the correct answer first wins a point.
- Points can be tracked by taking a Unifix for each point earned, and then the students can compare the height of their towers at the game's end.
- The game should eventually show that for basic facts, heads beat calculators and that is why we bother to learn them.

Making 10 was specified in the corresponding Grade 1 specific outcome. It can be tested by giving students a page of addition equations. Direct the students to answer or circle only those equations whose sums are ten. An observation of how quickly and accurately this is done will tell you if the student knows the combinations for 10 well and, if not, which ones need work. If the students' knowledge of combinations of 10 is weak, they can reinforce this skill through the following.

- Pairs of students who each have two sets of shuffled cards 1 to 9 each lay down a card at the same time.
- If a pair of cards are a combination of 10, the first student to say "makes 10" wins the pair of cards.
- The student who has collected the most cards once they run out of cards is the winner of that round.

Adapted with permission from Trevor Calkins, *Power of Ten: Brain Compatible Strategies for Learning to Add, Subtract and Regroup* (revised ed.) (Victoria, BC: Power of Ten Educational Consulting, 2003), p. C-7.

A game of fish can be played in which two to four students have 36 or 45 sets of shuffled cards labelled 1 to 9.

- Each player is dealt five cards.
- Players take turns asking the player to their left for a number that would go with one of the cards in their hand to make a combination of ten.
- If the player is given the card with the number requested, those two cards are laid down and become points for that player.
- The player should say the combination as it is laid down, for example, 6 + 4 = 10.
- Now that player is able to ask for another number.
- When the player to the left cannot supply the number requested, the player tells the asker, "Go fish."

- That student then selects a card from the deck on the table.
- The next person can now ask for a number needed to make a ten combination.
- The game ends when one player is out of cards.

Grade 1 students were also expected to describe and use the "think addition for subtraction" strategy. The development of this can be checked by giving students a page of subtraction equations and asking them to write the addition fact that could help them with each one. In a structured interview or an activity for students who need practice, provide about 10 cards, of which five have a subtraction equation and the other five the corresponding addition equation. Spread these mixed up cards out on a table and ask the student to match the subtraction equations with the corresponding helpful addition equations.

Make sure that students can think of numbers from 11 to 20 as 10 and some more. Trevor Calkins (2003) notes that being able to visualize these numbers is a prerequisite of subtraction (p. 28). Van de Walle and Lovin (2006) point out that students are not ready to use the making 10 strategy until they have learned to think of these numbers as 10 and some more (p. 103). It is easy to assess the students' ability to visualize these numbers as 10 and some more. Ask the students how they see fifteen. If the students say 10 and 5, they are likely ready to move onto subtraction and use the make 10 strategy. If students state they see 1 and 5, you know they need more work on activities with numbers 10 to 20 and place value (Calkins 2003, p. 28). Thinking of the teen numbers as 10 and some more is the first step in understanding the base ten system.

If a student appears to have difficulty with these tasks, consider further individual assessment, such a structured interview, to determine the student’s level of skill and understanding. See **Sample Structured Interview: Assessing Prior Knowledge and Skills** (p. 12).

Sample Structured Interview: Assessing Prior Knowledge and Skills

Directions	Date:	
Instant Recognition	**Not Quite There**	**Ready to Apply**
"I am going to give you a quick look at some dot cards. I want you to tell me how many dots there are on them and how you see them grouped." Show dot cards that have a variety of dot combinations for numbers up to ten for 5–6 seconds and observe the accuracy of the responses, such as, "I saw 3 and 4 makes 7."	• Cannot recognize any of the numbers without counting. • Recognizes the first number, but does not get to the second on each dot combination. • Recognizes only four or fewer or only certain configurations for some numbers. • Recognizes both small numbers up to five in configurations, but is unable to give you the sum instantly in some cases.	• Recognizes the subsets within each configuration and the sum they make instantly.
Readiness for Subtraction		
Ask the student to visualize 15 and tell you what they see. "In your mind I want you to make a picture of 15. What do you see?"	• The student sees a 1 and a 5.	• The student sees a 10 and a 1.
Knowing One More and One Less		
Give a student the deck of cards for one more and a number machine. Ask the student to beat the number machine at adding one more to each number. Repeat with a deck of cards for one less, then two more and two less, if the student is ready.	• Makes errors in naming the number one more. • Holds the card in the upper slot while solving for one more.	• Drops in the cards quite quickly and beats the number machine regularly.

Ten Frame Competency		
"I am going to give you a quick peek at some ten frames with counters on them. Please tell me how many counters are on each frame." Show the student some ten frame flashcards for about five to six seconds each. Note the degree of accuracy.	• Does not recognize any amounts correctly – is unlikely familiar with a ten frame. • Does not recognize most numbers over 5 correctly – is unlikely to be using 5 or 10 as referents. • Has trouble recognizing 8 – may find it difficult due to its distance from 5 and 10.	• Accurately and promptly identifies the number of counters shown on each frame.
Recognizing Parts that Can Make Up the Whole of a Number with Trains		
Show students a Unifix cube train of a number such as 7 that you think he or she should know the combinations of and establish that the student knows there are 7 cubes in the train. Then put the train behind your back and put 4 of the cubes into your other hand. Bring out the 3 cars remaining and ask, "How many train cars are in my other hand?" After the student answers, bring forward the hand behind your back and show the number of cars in it. Do this for other combinations of 7. If the student is successful, move up to check for combinations of 8, 9 and 10.	• Identifies the wrong numbers of missing cars. • Hesitates before answering and may use fingers to count before responding correctly.	• Child responds promptly and accurately for all combinations of numbers presented up to ten.
Think Addition for Subtraction		
Place ten or twelve cards on the table, five or six that have addition facts and the other half the corresponding subtraction facts. Direct the student to match the subtraction facts with the addition facts that would help solve them.	• Does not correctly match all the addition and subtraction facts.	• Correctly matches all the addition facts with the corresponding subtraction facts.

B. Choosing Instructional Strategies

Consider the following instructional strategies for teaching addition and subtraction facts:

- Introduce strategies with problems that relate to the students' lives. Allow the students time to create their personal strategies and then share them. Encourage critical thinking about relative strategy efficiency. Have the students critique their personal strategies as well as those of their classmates to decide which strategy works best for them and why.
- Provide a variety of problems representing the different addition and subtraction situations with varying degrees of difficulty to differentiate instruction. Wording and using higher numbers impact the level of difficulty. Work with the whole group initially and help the students recognize which numbers in a problem refer to a part or to a whole. Insure they understand that there are comparison problems and part-part-whole problems that do not involve taking any away, but are still subtraction situations.
- Demonstrate mental math strategies with manipulatives and in written formats so students can see how they work or could be communicated on paper. Model using mental math strategies by thinking aloud.
- Guide discussions by asking questions to encourage thinking about number relationships, the connection between addition and subtraction, and the students' personal strategies.
- There are many strategies for figuring out facts and students will have personal preferences. Generally students will select an efficient strategy if exposed to a variety. Students need only to understand the strategies that others elect to use.
- Teaching mental math strategies needs to be explicit.
- Drill comes after a strategy is in place, but is not automatic. Drill done too soon is ineffective. As Van de Walle and Lovin (2006) state, "If drill is undertaken when counting is the only strategy available, all you get is faster counting" (p. 95).
- Time is not a factor in learning mental math strategies. Timed tests are often detrimental to student's learning strategies. The use of timed fact tests is not recommended.

C. Choosing Learning Activities

The following learning activities are examples that could be used to develop student understanding of the concepts identified in Step 1.

Sample Activities:

1. **Strategies Built on Doubles** (p. 15)
2. **Making Ten** (p. 19)
3. **Adding with Zeros** (p. 21)
4. **One or Two More or Less** (p. 22)
5. **Addition for Subtraction** (p. 23)
6. **Use and Describe a Personal Strategy for Determining a Sum to 18 and the Corresponding Subtraction** (p. 26)
7. **Refining Personal Strategies to Increase Their Efficiency** (p. 27)

Sample Activity 1: Strategies Built on Doubles

1. Doubles Plus One or Minus One

For many students, the doubles become anchors or referents for most of their other addition facts. So before moving onto the following strategies, which are built on doubles, insure students have an automatic knowledge of the ten doubles facts. Doubles plus one gives students another twenty facts from the hundred they need to learn. Some strategies have several names. In the achievement indicators doubles plus or minus one is referred to as "doubles plus one," "doubles take away one" and "building on a known double." It does not matter what you call the strategies, as long as the names clearly relate to how they work. Use the names you choose consistently. Smooth transitions from grade to grade would be enhanced if staff within your school agreed on the names to be used for various strategies.

Look For …

Do students:

- ☐ use their fingers to count or touch their chin or desk as they count ?
- ☐ subvocalize as they count?
- ☐ tap with their pencil or make dots as they count?
- ☐ recognize the quantities to six in different configurations?
- ☐ respond with the doubles readily only to 10, 12 or 18?
- ☐ know the numbers one more and one less without counting on or back?

The following two strategies collectively are sometimes called "neighbours" or "near doubles." If the question they are trying to solve is 6 + 7, they may have elected to think of the double 6 + 6 = 12 and so will add one more to twelve to provide the answer 13. If instead they had first thought of 7 + 7 = 14 as their double, they would realize that house number is one more than the house they are searching for and so take one from fourteen, resulting in the answer thirteen.

Since this strategy was not in the Grade 1 curriculum, it will need to be developed carefully and practised repeatedly if students are to make it their own. The general methods of explicitly developing student awareness and usage of a new strategy follows and can be applied to all the strategies that follow.

Begin by presenting some story problems that are interesting and meaningful to your students. Allow them the time to solve them by creating their own personal strategies. Discuss how students solved the problems. Make the numbers to be added in the examples high enough that students for the most part do not know the sum automatically; they are more likely to be aware of strategies they are employing then. As students share their ideas, highlight the variety amongst the students. Record the shared strategies and pay tribute to the originators so that students see that their strategies are valued and variations are acceptable. It will also give students a place to look to review as they try to employ the strategies and make them their own.

After several days of short periods (five to ten minutes) of work on problems with equations that were well suited to the particular strategy of focus, ask the students to consider which pairs of numbers being added are suited to this strategy. Record a collection of the numbers.

Is there a particular relationship that all these pairs of numbers have? Have the students show or tell how they would use the strategy for each pair they offer.

Next give the students a sheet of addition facts to be solved and asked them to only do the ones that fit the doubles plus or minus one strategy. Or if you want them to do all of them, ask them to circle the ones for which the strategy was useful. In these early practice sheets, it is most effective to only include problems that lend themselves to two strategies and then gradually build up to three. The first sheet might only have doubles and neighbours. The next day students might search out the neighbours from doubles, as well as equations in which one of the addends is one or two.

Some activities for practising doubles plus or minus one follow:

- Have the students roll a die and say the complete fact for doubling the number on the die and adding one. Use either ten-sided dice or blank dice that you have labelled with numbers that you want the students to practise with, such as 4, 5, 6, 7, 8 and 9. So if a student rolled 8, the equation stated would be 8 + 9 = 17. You can ask for the two equations to be written, if that suits your objectives better. In this case, that would be 8 + 9 = 17 or 9 + 8 = 17.
- Have the students do the same activity as above, but make the equation that uses a number one less than the double. For example, if the student rolled 8, the equation would be 8 + 7 = 15.
- Give the students a page divided into nine blocks with a double fact written in each block. Then give the students nine facts that are corresponding doubles plus one that have been produced in the same sized blocks and cut apart, and ask the students to put or glue the upper centimetre of each double plus one onto the double that was helpful. Van de Walle (2006) and Lovin go so far as to suggest flashcards be made with varying levels of support for students as they learn to employ the strategy. For example, in this case, one set of flashcards could actually show the double without its sum in a thinking cloud beside the near double. A second set of cards could show the near double fact and say double plus 1 with an arrow pointing to the smaller number, so the student is clear which number to double. Another set of flashcards could show the fact and have a cloud that states "double plus 1," but without the arrow to the smaller addend (p. 102). Sets of cards could also be made for doubles minus one.

It is very motivating for the students to see how these strategies help them conquer so many of the sums on the addition table. Have a class addition table posted or give one to each student. First shade in the sums of the doubles. Students may be anxious to colour in those facts they think they know already, such as adding on one or two. You may decide whether you want them to do this yet or not. These facts are purposely being left towards the end of these lessons to discourage students from employing counting on as their main strategy. Especially if students have been trained to add with dot numbers, touching numerals in the number of places that the number represents, it is a hard habit to break and a very inefficient strategy for students in Grade 2 and beyond.

2. Two Aparts or Sharing to Make Doubles, Equalizing, Evening Out; Balancing

This strategy is commonly used, but does not seem to have a name that is widely used. Since students are already good at doubles and have even learned to use this knowledge to access the sums of many more facts, it does not take them long to add this additional strategy to their repertoire once they become aware of it. For numbers that are two apart, such as 5 + 7, students can understand that if the larger number would give one to the smaller number, the numbers would be balanced, shared equally, the piles even and the numbers are converted into doubles.

As described above, go through the process of presenting problems with numbers two apart to allow students to contribute their personal strategies. If students do not come up with this strategy, you can suggest it. The students are also likely to come up with the extension of doubles plus and minus one; that is, doubles plus or minus two described in Section c that follows. It applies to the same facts that are two apart.

- Ask the students to look at a series of flashcards or equations presented on the board or overhead and ask them to explain which strategy they would use, why and how it works.
- Have them identify the problems from among those given that suit this strategy or have them offer equations that go with the strategy.
- Have them practise these equations on flash cards that have cues about using the strategy as outlined above.
- Give the students a number of equations on a page to cut apart, and ask small groups of students to sort them based on the strategy that best suits them. Then the class can discuss their results and see if there were any discrepancies. This is an opportunity to point out that students may select the strategies that work best for them, as long as they are efficient; it is a matter of personal preference.
- Throw a die and have the students make number sentences with the numbers shown.
- Invite the students to play concentration with sets of cards that have the sums that are two apart and the matching double sums.

3. Doubles Plus and Minus Two

This strategy works for the same facts as the previous strategy, Two Aparts, but approaches the solutions differently. This strategy is just like doubles plus or minus one except that you add or subtract two after using the double for either the lowest or highest addend. For example, if the fact is 6 + 8, it could be solved as 6 + 6 + 2= 14 or as 8 + 8 –2 = 14.

Presenting this strategy can be done in two ways. One is to give the students a few word problems with addends that are two apart and ask them to share their strategies for solving the equations. It is quite possible that among students' strategies shared for "Two Aparts" will be both the balancing strategy and the doubles plus or minus two strategy. A second way to introduce it, if it has not already come up with the previous strategy, is to present to students a set of equations in which the numerals are two apart and ask for other possible strategies to solve them. If your students are already very proficient with the addition facts, ask them how they could solve the equations, if they didn't know these facts automatically or if they

suddenly could not think of the sum. Once the strategy is discovered, have the students practise it in any of the ways that are listed above. Once several strategies are known by your students, it is wise to have students deal with them in mixed practice. If once the strategy is introduced it is left to the students' discretion to employ it, for many students this will not be enough for them to claim it as their own. When faced with equations that are best suited to a number of different strategies, students may revert to using only one strategy, rather than pausing to consider which strategy is the best for solving each one. Unfortunately, too often the fall-back strategy is counting on or back.

- An additional activity for practising this particular strategy, along with doubles plus or minus one, is to have the students spin a spinner divided in half with +1 on one half and +2 on the other. Have the students throw a die that has been marked with numbers such as 3 through 8. They are to double the number shown on the die and add either one or two more depending upon the outcome of the spinner. This can be either an oral or written activity.
- Similarly, ask the students to roll a die and double the number shown and then do only one of the following four possibilities: plus one, minus one, plus two or minus two.

Sample Activity 2: Making Ten

In this strategy, students need to decompose one number to find one part that will go with the other number to make ten and then add to the ten the left over number to arrive at the sum. For example, given the equation 9 + 6, students need to know that 6 can be broken into the two parts 5 and 1, and that 10 is 9 + 1. Then they join the 1 with the 9 to make 10. Finally, the 10 is added to the 5 for a total of 15. This strategy is particularly useful for equations in which one of the addends is 8 or 9. If a student is solving 9 + 6, he or she may consider the related question of 10 + 6, whose sum is easier to solve. Then the student recognizes that this related sum will be one greater than the sought sum and so mentally subtracts the one from 16 to find the sum of 15. These students see the relationship of these nearby facts to the ones they are solving.

If the problem were 8 + 6, they might loan 2 to 8 to make 10 and think 10 + 6 = 16; however, since they loaned 2, they reclaim it by subtracting 2 from the sum.

- One of the easiest ways for students to discover this strategy is to see it. Give students two ten frames and have them make the two numbers given in the equation each on a different frame and with different coloured counters. Then when asked what they could do to find the sum, students will usually suggest moving one or two counters as appropriate to the nine or eight and then combine the ten with the leftover counters on the other frame (Van de Walle and Lovin 2006, p. 103).
- Do the same activity with Unifix cubes in two colours on two strips the height of ten cubes that are marked off into a template for the ten Unifix positions. If students build the towers of the two addends and then need to calculate, they can move one or two cubes from the shortest tower to complete the nine or ten tower and then quickly arrive at the total sum. A coloured line across each template at the top of five cube spaces will serve as a five benchmark for students so they can avoid counting for numbers six and up.
- Ask the students to practise these sums that include an addend of nine or ten with flashcards that are marked to remind students to use this strategy and how it works (Van de Walle and Lovin 2006, p. 103).
- To wean students from making each pair of addends with manipulatives, they can be shown ten frames on the overhead that have eight or nine represented on them. Then place on the overhead a frame with another number represented on it. Ask the students to mentally "take" the one or two dots from the frame of the smaller number and visualize them being added to the nine or eight to make ten on the first frame. As students give the answer, they should first tell what they did to figure it out.
- Trevor Calkins (2003) has students playing a card game to learn to add to nine with visualization and then the same game is later played to add to eight.

Another making tens strategy that is useful for students to know is the "ten hunt." Students look for the numbers that will make ten when adding more than two addends. Many students will always add the addends in the sequence presented, slowing them down. When students realize that adding in any order produces the same sum and they are proficient at recognizing the numbers eleven and up as ten and some more, hunting for combinations to make ten and adding them first and then other numbers is most efficient. One way to check if your students are using this strategy is to lay out three or more cards with numbers of which two or more, not in

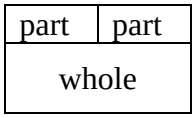
sequence, can be paired to make the sum of ten. Ask the students to find the sum and observe whether they do so by rearranging the number sequence mentally or manually. It may require that you ask the students to think aloud as they do this one-on-one with you.

It should be noted that although the commutative property of addition is in the Grade 3 curriculum, it is helpful to students as they learn their addition and subtraction facts to know that they have the choice of their favourite way of recalling a fact and its turnaround. If a student finds it easy to answer 9 + 7, but difficult to think what 7 + 9 is, he or she can reorder the equation mentally to solve it. A discussion about this may make it easier for some of your students. The fact that this property has a mathematical name can certainly wait until Grade 3. The discussion might centre around a group of problems that students are asked to solve in which pairs of facts are used. The stories of the problems may vary. Students can compare the problems to note the sums are the same. Students might like to share which facts they find easier in one direction than the other. For example, a student may say, "I always say to myself 7 + 8, even when the equation says 8 + 7."

Sample Activity 3: Adding with Zeros

Create a number of problems to solve that have zero as an addend. Ask the students if there is a pattern in the sums. Students tend to quickly recognize that adding zero to any number yields the same number. If a student has difficulty with this property of zero, it is likely because he or she has over-generalized. The student may think that when adding the sum will always be larger and when subtracting the difference is always smaller than the minuend. Correcting this misconception should allow the student to add and subtract with zero accurately.

Suggested Activities
Some work with concrete materials helps students who have not already independently acquired this concept. Use an addition mat that is set up to show addition as two parts and a resulting whole such as shown below. With manipulatives, solve a series of equations with zero either as the first or second addend. This will help students visualize and understand the effect of adding zero to any number.



It is helpful for the students to have a quick and easy way to check how zero affects a sum or difference.

A little story about something they can visualize is helpful, particularly when the number to be added is only one. Being able to think, "If I have one penny in this hand and none in this hand and I put my hands together, how many will I have?" Likewise, "If I have a number in this hand and I take none away, how many would I have left?" These little stories to check on the zero property seem rudimentary; however, the same stories are required by many junior high students doing algebra when they need to check on the effect of adding n + 2n, as they have to adjust to the fact the n really means 1n and the sum in this case is 3n. So it is understandable that your Grade 2 students do need some practical ways of thinking about what zero means and how it does not change the outcome in addition and subtraction.

Practising with problems that have three or more addends, one of which is zero, is useful to secure the concept. Some students may not even write the zero into the equation when solving story problems including zero amongst the three or more addends because they know the sum will be unchanged by a zero.

If the students have been keeping track on an addition table the sums that they know, it is likely they have been anxious to fill in the row and column for adding with zero. This knowledge adds nineteen facts to their repertoire.

Sample Activity 4: One or Two More or Less

Likely the students have been anxious to fill in all the facts they know by adding on one or two on the addition table. These are approached after the other strategies to prevent students from adopting counting on as their only strategy and applying it to the many equations for which it is inefficient. If students have been taught to add in Grade 1 by touching numerals the number of times it represents, it is very hard to break them of the habit of counting on as a strategy to apply to all facts. Knowledge of the relationship to numbers that are related by one or two more gives the students thirty-six facts. To introduce this strategy, a series of stories to solve in which one of the addends is either 1 or 2 can be presented. As the students share their strategies, you will see at what stage they are. Hopefully those who are still counting on begin with the larger number and count on the one or two. If you have left this strategy until the others have been taught, most of your students are likely to explain that they just know the sums and that tells you they have established the relationships between those numbers that are one or two more without counting. It is at that stage that you can have them practise these facts in a variety of activities.

- Label a die with +1 and +2 and have the students roll it with a ten-sided die. The addition facts created may be done orally or written down.
- Have the students use a spinner with the two halves labelled 1 more and 2 more in conjunction with a ten-sided die or one on which you have placed the numbers of your choice, such as three to nine.
- Have the student use two ten frames to see the numbers as they add in their heads. Instead of placing counters on them, it would be handier to have shaded in boxes of varying numbers on ten frame cards and have the student add one or two to each number that is turned up. These cards will allow the student to visualize the additional box or two shaded in and recognize immediately the number of boxes based on his or her knowledge of the configuration of the ten frame.

The same process may be used to introduce subtraction problems in which a subtrahend of one or two is subtracted from the minuend. The following are ways to practise this.

- Label a die with -1 and -2 and have the students shake it with a ten-sided die. The subtraction facts created may be done orally or written down. When mixed practice is desired, the die can be labelled +1, +2, +2, -1, -2, -2. Insure that the second die used has no number lower than 2. You can place a small sticker with a different number over numbers that are too low or ask the students to roll again if they get a 0 or 1 on the die.
- A spinner with the two halves labelled 1 less and 2 less can be used in conjunction with a ten-sided die (adapted as above) or one on which you have placed the numbers of your choice, such as three to nine. Mixed practice can be created by using a spinner with four equal sections marked: 1 more, 1 less, 2 more, two less.
- Have the student use a ten frame or two to visualize the numbers as they add or subtract in their heads. Instead of placing counters on them, it would be handier to have shaded in boxes of varying numbers from two up on ten frame cards and the student subtracts one or two from each number turned up. These cards will allow the student to visualize the additional box or two shaded in or the removal of shading and recognize immediately the number of boxes based on his or her knowledge of the configuration of the ten frame.

Sample Activity 5: Addition for Subtraction

Addition fact mastery is essential to the success of this strategy. This strategy is easiest for students to access if they have had practice finding missing addends in addition problems. In so they are used to asking themselves what number plus the given number will produce the given sum or outcome. Since subtraction is the inverse of addition, what students are essentially learning is how to read the subtraction equation backwards to make it an addition situation. For example, if the subtraction equation is 11 – 5 = ____, you want the students to be able to start at the right and interpret this as asking what number plus five would give me eleven. Students who have mastered the subtraction facts have not done so through counting back. Problems that encourage students to think addition for subtraction are those that sound like they have a missing addend.

For example, "David had 7 toy cars. For his birthday he received more. Now he has 12 toy cars. How many cards did David get for his birthday?" In this case the structure of the problem is that the change is unknown. The student knows what amount David had to start with, so the starting or initial amount is known. The resulting amount or outcome is known. The amount by which the start changed is what the student has to figure out.

In a similar question, the initial part might have been the unknown. For example, "Susan had some toy ponies and at a garage sale found and bought three more. Now Susan has 11 toy ponies. How many did Susan have before her purchase at the garage sale?"

Both of the above examples are considered to be adding to or joining problems based upon an analysis of their structure. When the items in a problem are not necessarily physically brought together, they are considered part–part–whole problems.

For example, "SunLee has 15 coins. Nine of them are nickels and the rest are dimes. How many dimes does she have?" The structure of these problems tends to lead students to think addition for subtraction. Once the students share this as a strategy for finding differences, it can be encouraged and its use expanded to any subtraction situations. Some students use "think addition" for all subtraction facts. For a further look at the types of problems based upon structure, see the section on addition and subtraction.

One of the best ways to enhance the students' ability to use this strategy is to reinforce the number families. This can be done in a variety of ways. Students can be asked to write the subtraction sentence that is the inverse of an addition equation. Sometimes students are asked to write all four equations that belong to a family. This can be done with tiles, bingo chips or little cards that the students manipulate.

- Students have three tiles of one colour on which the addition, subtraction and equal signs are shown.
- They have another three cards/tiles/chips on which the three related numbers are written.
- It is the students' job to move the tiles around to form the four related number stories and record them. This prevents students from adding one of the smaller numbers onto the largest number and creating a fourth number that is not in the same family.

Van de Walle and Lovin suggest making practice activities for the three numbers that belong together. They introduce them by placing three numbers from a family on a circle with the largest number circled. Students are presented with a number of these family grouping circles. They are asked why the numbers belong together and why one is circled. Then a follow-up activity can be circles with two of the numbers shown. Students are to figure out the missing number. Similarly, strips are made on which numbers from one family are positioned at the top, the bottom and in the middle. One number is left off these strips and the students are asked to figure out the missing number. The largest number is always circled. Work sheets can be made that have only the facts for two number families that need practice or from facts that lend themselves to practising a particular strategy that needs reinforcing.

Look For …

Do students:

- ☐ put the smaller number at the beginning of a subtraction sentence?
- ☐ ever put the larger number as an addend in the addition equations?

Another way for students to associate the three numbers that belong in the same family more closely is to use triangular flashcards as addition and subtraction cards.

- Each card has the largest number at the apex of the triangle and the two smaller numbers at the two base vertices.
- By covering with your fingers the top number and moving either left to right or right to left, the two addition equations in the family are revealed. For example, if the numbers are 4 and 5 on the base corners, the equations are 4 + 5 = ? and 5 + 4 = ? Under your fingers would be the 9 at the apex. By covering alternately the 4 and 5, the card would ask 9 – 5 = ? and 9 – 4 = ? These cards can be made or are available commercially.

Although some students will use "think addition" for all subtraction facts, be aware that other students will be using alternatives, particularly for facts with minuends greater than ten. For example, when solving 15 – 9, a student may elect to think 1 more added to 9 makes ten and ten and five make fifteen. So add the 1 and the 5 and the total difference is six. This is "building up through ten." Another student may alter the numbers to make the solution easier and then compensate, as in 15 – 9 would be easier if it were 15 – 10 = 5 and then add one more to the difference because one was added to the subtrahend. This is an example of playing fair. If the student added one to a number on the left of the equal sign, to be fair the student needs to do the same on the right. Students can relate to the concept of fairness. Just as if a parent gave one child an extra dollar allowance, it would only be fair if the other child were also given an extra dollar. Some consider this last strategy to be "building down through ten." This strategy is the only one that is really done as take away. Both building up through ten and back through ten are most commonly used for problems with 8 or 9 as the subtrahend. Although some students will use it for other numbers because they are so proficient with the combinations of ten and see numbers as components of ten.

This strategy can be brought to the students' awareness through subtraction problems to be solved that have 8 or 9 as the subtrahends. Then the students can share their strategies. An alternative is to present the students with a ten frame on the overhead that has nine dots shown. Discuss how numbers from eleven to eighteen could be built starting with this nine. Similarly, you can show a ten frame with eight squares filled in and ask the students how they could build it to numbers from eleven through seventeen. Then with one of the frames in view, show numbers from eleven to eighteen and ask the students to explain how they figured out the difference between the number shown on the frame and the one on the card.

Activities to encourage students to subtract using building up or down through ten may include the following:

- Use flash cards to practise facts with eight or nine in the subtrahend that include cues to use and how this strategy works.
- Have the students lay down a ten frame with either eight or nine on it. Give the students a die labelled from twelve to seventeen, and have them subtract the eight or nine from the number thrown on the die.
- Play concentration or "addition subtraction match up" with cards that have the subtraction and corresponding addition sentences on them.
- Make a sheet with ten or twelve rectangles, each bearing a subtraction equation. Also make a second sheet divided up the same way as the first with the corresponding addition equations needed to solve the subtraction facts by thinking addition. Have the students cut the addition facts on the second sheet into rectangular cards. These are then placed over the subtraction facts that each one helps to solve.

Sample Activity 6: Use and Describe a Personal Strategy for Determining a Sum to 18 and the Corresponding Subtraction

This achievement indicator moves students from explaining and demonstrating the various strategies to the application level. Are the students selecting an appropriate strategy to arrive at a sum and then able to use it to solve the corresponding subtraction fact? Once all the previously mentioned strategies have been discovered and practised, students can be given some subtraction problems to solve in a format as in the example below. Be sure to model some examples of problems using this format before assigning such problems. After modelling in a think aloud example, give some problems for the students to demonstrate how to solve by orally giving you the information required to complete all four boxes. Then the class should be ready to handle similar problems independently. If you think they need more support, they can work in pairs followed by pairs sharing. Then one spokesperson from each of the groups of four would share all the parts of his or her group's solution. The italic print in this example shows how a student might complete it.

Sadie had read part of her new book before the class left the library. That night she read another 8 pages. Now she is on page 17. How many pages of her book had she read in the library?

Subtraction Equation *17 – 8 = 9*	Helpful Addition Fact *9 + 8 = 17*
Sentence Answer to the Problem *Sadie read 9 pages of her new book during library period.*	Personal Strategy Used for Addition Fact *Doubles Plus One* *8 + 8 + 1 =* *16 + 1 = 17*

This is a very structured way of insuring students are using the strategies. Practice that is set up to require all these steps be written is one way of developing the habit in students, so that they mentally proceed through these steps on their own. Habits take a while to form, so a few problems done like this daily for at least six weeks should secure this procedure as a habit. Such practice is needed for students to become adept at recognizing all the different types of subtraction situations. The posted names of strategies in the classroom with an example under each and a brief description will support students, if they need to review their options before selecting a strategy. More importantly, for most students these posted strategies will give them a reference for the language and spelling needed when explaining a strategy.

Sample Activity 7: Refining Personal Strategies to Increase Their Efficiency

Observing students as they share and work on addition or subtraction facts and related problems will give you the greatest opportunity to ask questions that allow students to reflect upon their strategies and refine them as necessary.

One way to get students to think about adopting more efficient strategies is to give them an example of what a student was doing to solve a problem using a less than efficient method, such as counting on for a problem such as 8 + 6. Students will share better methods, such as double 6 plus 2 or two parts made doubles by sharing so that 8 gives 1 to the 6, making it 7 + 7.

Students can explain how hard it is to keep track of how many you have counted on when the number to be added is more than the fingers on one hand and how slow this process is in comparison to either of the strategies first suggested. You are more likely to have students voluntarily debate which of the first two is the better strategy. That is an opportunity to explain to students that we all have our preferences.

Similarly, ask students to share what they would say to someone who was solving 9 + 7 by counting on. What would they show and tell that person to help them be faster and more accurate, in other words, more efficient? This can be done in individual interviews with students about whom you have concerns.

Step 4: Assess Student Learning

Guiding Questions

- Look back at what you determined as acceptable evidence in Step 2.
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

In addition to ongoing assessment throughout the lessons, consider the following sample activities to evaluate students' learning at key milestones. Suggestions are given for assessing all students as a class or in groups, individual students in need of further evaluation, and individual or groups of students in a variety of contexts.

A. Whole Class/Group Assessment

The following is a performance assessment that could be used with a whole group or class. Be sure to read the problem to the class to minimize any difficulties based upon reading skills rather than mathematics skills.

Performance Assessment: Fitness Tests in Grade 2 Gym

The gym teacher did a fitness test with your class this fall and then repeated these tests with your class in the spring to see how much your fitness levels have improved. You did each activity in two gym classes and each time your partner recorded your score. After the second gym class, you totaled your two scores for every station. Your test scores for the two tests are shown below. Find your total score for each activity and record what strategy you used to find the total.

Station Activities	Fall test one	Fall test two	Sum of two scores	Strategy used to get the answer
Curl Ups	4	6		
Step Ups	6	7		
Jumping rope without a miss	7	9		
Push Ups	3	8		

In May the students repeated the tests twice and the totals for both times were figured out by each student. Then the teacher asked the students to work out how much improvement they had made in each activity. If this is your score card for October and May, figure out how much you improved by and show the strategies you used.

Station Activities	October Sum of Scores	May Sum of Scores	Improved By	Strategy used to get the answer
Curl Ups	9	13		
Step Ups	4	9		
Jumping rope without a miss	6	15		
Push Ups	6	12		

Performance Assessment Answer Key: Fitness Tests in Grade 2 Gym

The gym teacher did a fitness test with your class this fall and then repeated these tests with your class in the spring to see how much your fitness levels have improved. At each station you had to do a different activity. You and your group moved together from station to station. You did each activity in two gym classes and each time your partner recorded your score. After the second gym class in which you did fitness tests, you totaled your two scores for every station. The teacher wanted to be sure that students added carefully and accurately, so asked that all students show which strategies they used to figure out their total scores. This is the chart of a student who was absent. The teacher asked you to do your best to complete this student's record card.

Station Activities	Fall test one	Fall test two	Sum of two scores	Strategy used to get the answer
Curl Ups	4	6	10	two apart or equalizing to make a double, 4 + 6 = 5 + 5
Step Ups	6	7	13	double plus or minus one 6 + 6 + 1 or 7 + 7 – 1 = 13
Jumping rope without a miss	7	9	16	making ten or playing fair 9 + 1 + 6 or 8 + 8 = 16
Push Ups	3	8	11	counting on or making ten 8 + 1 +1 + 1 or 8 + 2 +1 = 11

In May the students repeated the tests twice and the totals for both times were figured out by each student. Then the teacher asked the students to work out how much improvement they had made in each activity. If this is your score card for October and May, figure out how much you improved by and show the strategies you used.

Station Activities	October Sum of Scores	May Sum of Scores	Improved By	Strategy used to get the answer
Curl Ups	9	13	4	subtraction through ten 13 – 10 = 3 + 1 more = 4 or 9 + 1 = 10 and 3 more so 3 + 1 = 4
Step Ups	4	9	5	think addition 4 + ? = 9 since 4 + 5 = 9 then 9 – 4 = 5
Jumping rope without a miss	6	15	9	subtract or add through 10 15 – 5 = 10 – 1 more = 9 or 6 + 4 = 10 + 5 so 4 + 5 = 9
Push Ups	6	12	6	think addition 6 + ? = 12 doubles 6 + 6 = 12, so 12 – 6 = 6

Rubric for the Performance Assessment: Fitness Tests in Grade 2 Gym

	Needs Instruction	Nearly There	Proficient
Selecting, Using and Explaining Strategies for Addition Facts	Sums may be correct, but no strategies supplied or those supplied are limited to counting on and possibly some reference to doubles, but not doubles plus or minus one.	Sums may be correct, but the strategies shown refer only to doubles plus and minus one and counting on. If making ten or building through ten is noted, the application of it is unclear.	All strategies used apply well to the addends and it is apparent the student is selecting judiciously from the complete repertoire of strategies.

	Needs Instruction	Nearly There	Proficient
Selecting, Using and Explaining Strategies for Subtraction Facts	Differences may be correct, but either no strategies are supplied or "think addition" is named, but it is unclear how it was employed.	"Think addition" is the only strategy employed. This may be used with all subtraction facts with success.	All differences are correct and the strategies chosen are suitable, clearly understood and show some flexibility to use more than just "think addition."

B. One-on-one Assessment

See the structured interview examples. (p. 12–13)

C. Applied Learning

Provide opportunities for students to use addition and subtraction in practical situations and observe whether or not the strategies transfer. For example, ask the students to figure out how many days they have left to finish their boat project, if there are 8 school days left in this month of January and 9 before Family Day in February?

Does the student explain that you could double eight and add one or alternatively double nine and subtract one?

In parent–teacher–student conferences, ask parents if students use these strategies when doing calculations at home.

If you have student-led conferences, have the students show parents how they can match a list of strategies with problems or equations that have numbers suited to these strategies. Students can tell their parents their preferred strategies and solve the problems or equation using them. Does the student explain the strategies clearly to their parents?

Step 5: Follow-up on Assessment

Guiding Questions

- What conclusions can be drawn from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction?

A. Addressing Gaps in Learning

If students are having difficulty in solving the basic facts using strategies, check for the following:

- Do students know their addition facts using strategies? If not, go back to teaching strategies for addition before continuing any work on subtraction.
- If students need addition work, can they solve for the facts with manipulatives? For example, can they show making tens if they have a ten frame? Start with small numbers for the addends. Make sure each doubles-related strategy is in place. Are students reverting to counting on for most facts? Have the students practise with flash cards that cue them to think of the strategy and how to use it. Play more games that reinforce the strategy. Be sure that students have mastery of the combinations for ten before revisiting the making tens strategies. Also, check how students visualize the teen numbers. For example, ask how they see a number like 16. Do they report visualizing 10 and 6 more or do they report 1 and 6? If the latter is the case, they need to work on place value before moving forward.
- If students have addition strategies but are struggling with the subtraction strategies, more work with the number families and triangular flash cards is in order. Solving problems structured to automatically trigger a student to think of addition for subtraction would also be helpful. Once "think addition" is used reliably with sums to ten, the other options for solving subtraction facts with addition sums greater than ten can be revisited. Spend more time with manipulatives such as two ten frames when reteaching building up or back down through ten.

B. Reinforcing and Extending Learning

Students who have achieved or exceeded the outcomes will benefit from ongoing opportunities to apply and extend their learning. These activities should support students in developing a deeper understanding of the concept and should not progress to the outcomes in subsequent grades. If students are proficient using these strategies, challenge them to apply them to two-digit addition and subtraction. Can a student take a problem such as 23 + 48 and mentally alter it to 20 + 1 + 2 + 40 + 8 and by combining the 2 and 8, make a 10 to add to 20 + 40 for a total of 70 and the 1 left over from 3. When a student is asked 42 – 28, can they convert it to 42 – 30 = 12 and then add 2 more to compensate for giving an extra 2 to the 28 to make it 30? You may want to focus student thinking on relationships and patterns they already use with questions such as, "Is there any way that knowing the double 6 + 6 can help you solve mentally 16 + 6? What about 26 + 6? What about 36 + 6? Could you mentally solve 26 + 36?"

Bibliography

Alberta Education. *The Alberta K–9 Mathematics Program of Studies with Achievement Indicators*. Edmonton, AB: Alberta Education, 2007.

_____. *Diagnostic Mathematics Program, Division I, Operations*. Edmonton, AB: Alberta Education, 1990.

Baratta-Lorton, Mary. *Mathematics Their Way: An Activity-Centered Mathematics Program for Early Childhood Education.* Reading, MA: Addison-Wesley Publishing Company, Inc., 1976.

Baratta-Lorton, Robert. *Mathematics … A Way of Thinking*. Reading, MA: Addison-Wesley Publishing Company, Inc., 1977.

Calkins, Trevor. *The Power of Ten: Brain Compatible Strategies for Learning to Add, Subtract and Regroup*. Victoria, BC: Power of Ten Educational Consulting, 2003.

Garland, Cynthia. *Mathematics Their Way Summary Newsletter*. Saratoga, CA: The Center for Innovation in Education, 1990.

National Council of Teachers of Mathematics. *Principles and Standards for School Mathematics.* Reston, VA: NCTM, 2000.

Richardson, Kathy. *Assessing Math Concepts: Changing Numbers.* Bellingham, WA: Mathematical Perspectives, 2002.

_____. *Assessing Math Concepts: Hiding Assessment.* Bellingham, WA: Mathematical Perspectives, 2002.

_____. *Assessing Math Concepts: More/Less Trains.* Bellingham, WA: Mathematical Perspectives, 2002.

_____. *Assessing Math Concepts: Ten Frames.* Bellingham, WA: Mathematical Perspectives, 2002.

Van de Walle, John A. *Elementary and Middle School Mathematics: Teaching Developmentally.* 4th ed. Boston, MA: Addison Wesley Longman, Inc., 2001.

Van de Walle, John A. and LouAnn H. Lovin. *Teaching Student-Centered Mathematics: Grades K–3.* Boston, MA: Pearson Education, Inc., 2006.