Mathematics

Planning Guide

Grade 2 *Increasing Patterns*

Patterns and Relations

Specific Outcome 2

This Planning Guide can be accessed online at:
http://www.learnalberta.ca/content/mepg2/html/pg2_increasingpatterns/index.html

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Planning Guide: *Grade 2 Increasing Patterns*

Strand: Patterns and Relations
Specific Outcome: 2

This *Planning Guide* addresses the following outcomes from the Program of Studies:

Strand: Patterns and Relations	
Specific Outcome:	2. Demonstrate an understanding of increasing patterns by: • describing • reproducing • extending • creating numerical (numbers to 100) and non-numerical patterns using manipulatives, diagrams, sounds and actions.

Curriculum Focus

This sample targets the following changes to the curriculum:

- The general outcome has changed in its view that algebraic thinking begins in the earliest math work done with pattern beginning in Kindergarten. The 1997 curriculum denoted algebra as grades 10–12 specific and focused heavily on using patterns to describe what was in the students' environment. The new curriculum states, "Learning to work with patterns in the early grades helps students develop algebraic thinking, which is foundational for working with more abstract mathematics in the higher grades" (Alberta Education, 2007, p. 8).
- The specific outcomes have changed considerably. Students were expected to compare patterns in Grade 1 in the previous curriculum, but are not formally expected to do so until Grade 3 in the new curriculum. There was no mention of increasing patterns in the 1997 curriculum, nor was there mention of repeated patterns and decreasing patterns until the latest curriculum. The new curriculum is more focused on students using patterns to make predictions and to justify their reasoning when solving routine and non-routine problems.

What Is a Planning Guide?

Planning Guides are a tool for teachers to use in designing instruction and assessment that focuses on developing and deepening students' understanding of mathematical concepts. This tool is based on the process outlined in *Understanding by Design* by Grant Wiggins and Jay McTighe.

Planning Steps

The following steps will help you through the Planning Guide:

Step 1: Identify Outcomes to Address

Guiding Questions

- What do I want my students to learn?
- What can my students currently understand and do?
- What do I want my students to understand and be able to do based on the Big Ideas and specific outcomes in the program of studies?

Big Ideas

Mathematics is often referred to as the science of patterns. Patterns permeate every aspect of mathematics. The brain is a pattern seeker, so the development of students' abilities to recognize, analyze and generally become proficient pattern seekers will not only pave the way for their success in mathematics, but in all learning.

- There are patterns that do not repeat, but increase, so the pattern does not lie in a repeated core, but in the manner the pattern grows, that is, the relationship between the steps.
- Patterns can be seen widely in the environment, actions, behaviours, direction or orientation, sound, art, nature, songs, music, math, stories and poems.
- Patterns can be expressed in a variety of ways. They can be translated from one mode to another.
- Pattern relationships can be described by a pattern rule.

Sequence of Outcomes from the Program of Studies

See http://education.alberta.ca/teachers/core/math/programs.aspx for the complete program of studies.

Grade 1

Specific Outcomes

1. Demonstrate an understanding of repeating patterns (two to four elements) by:
   - describing
   - reproducing
   - extending
   - creating

   patterns using manipulatives, diagrams, sounds and actions.

Grade 2

Specific Outcomes

2. Demonstrate an understanding of increasing patterns by:
   - describing
   - reproducing
   - extending
   - creating

   numerical (numbers to 100) and non-numerical patterns using manipulatives, diagrams, sounds and actions.

Grade 3

Specific Outcomes

1. Demonstrate an understanding of increasing patterns by:
   - describing
   - extending
   - comparing
   - creating

   numerical (numbers to 1000) and non-numerical patterns using manipulatives, diagrams, sounds and actions.

2. Demonstrate an understanding of decreasing patterns by:
   - describing
   - extending
   - comparing
   - creating numerical (numbers to 1000) and non-numerical patterns using manipulatives, diagrams, sounds and actions.

Step 2: Determine Evidence of Student Learning

Guiding Questions

- What evidence will I look for to know that learning has occurred?
- What should students demonstrate to show their understanding of the mathematical concepts, skills and Big Ideas?

Using Achievement Indicators

As you begin planning lessons and learning activities, keep in mind ongoing ways to monitor and assess student learning. One starting point for this planning is to consider the achievement indicators listed in the *Mathematics Kindergarten to Grade 9 Program of Studies with Achievement Indicators*. You may also generate your own indicators, and use them to guide your observation of the students.

The following indicators may be used to determine whether students have met this specific outcome. Can students:

- identify and describe increasing patterns in a variety of given contexts; e.g., hundred chart, number line, addition tables, calendar, tiling pattern or drawings?
- represent the relationship in a given increasing pattern concretely and pictorially?
- identify errors in a given increasing pattern?
- explain the rule used to create a given increasing pattern?
- create an increasing pattern and explain the pattern rule?
- represent a given increasing pattern using another mode; e.g., colour to shape?
- solve a given problem using increasing patterns?
- identify and describe increasing patterns in the environment; e.g., house/room numbers, book pages, calendar, pine cones, leap years?
- determine missing elements in a given concrete, pictorial or symbolic increasing pattern, and explain the reasoning?
- extend increasing patterns?

Sample behaviours to look for related to these indicators are suggested for some of the activities listed in **Step 3, Section C: Choosing Learning Activities** (p. 13).

Step 3: Plan for Instruction

Guiding Questions

- What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should I use?
- How will I meet the diverse learning needs of my students?

A. Assessing Prior Knowledge and Skills

Before introducing new material, consider ways to assess and build on students' knowledge and skills related to patterns. In Kindergarten and Grade 1, students have encountered repeating patterns. In Grade 1 the repeating patterns contained up to four elements in the core pattern and these were non-numerical: manipulatives, sounds, actions and diagrams only. In Grade 2, these patterns have been extended to contain as many as five elements in the core pattern and numerical patterns with numbers up to 100. Grade 2 is the first introduction to increasing patterns. In Grade 3 they will be reinforced and extended to include more complicated growing patterns, as well as comparing increasing patterns. In Grade 3, decreasing patterns are also introduced.

Some examples of ways to access students' readiness to be introduced to increasing patterns follow:

Given repeating patterns, can students continue the patterns correctly?

- Start with the very basic student, action or sound patterns.
- These could be two students standing and one sitting or two facing the door and one the windows.
- Other patterns to use are clap, snap, clap, patch (hands open, palms down on quads) or twirl, touch your toes, touch your head and hands on your waist.
- A pattern can be made with arm positions, such as a raised arm as in a right turn signal, out to the side like a left turn signal, dropped at one's side and then held out with hand raised as in a stop signal.
- Patterns can be shown with boot or running shoe alignment, such as two facing one direction and the third in the sequence at right angles to the first two.
- Class calendars often show repeating patterns and that is another opportunity to reinforce and extend the skill early in Grade 2.

For specific ideas, see 1a. Patterns can be considered in the context of the student day, such as the class schedule, opening exercises, recesses, movements in the gym, music, songs, and stories. Students can discuss repeating patterns in their home life and routines, such as lessons and practice routines, meals, getting ready for school, bedtimes and other such repeating sequences.

- Identify the core in a repeating pattern. Given repeating patterns with three to five elements, can students identify in some way the core pattern?

- Have the students translate repeating patterns into other modes. Ask the students how they could represent the patterns described above or others using colours, shapes, numbers, letters or other things. Unifix cubes can be used to show colour. Pattern blocks can be used to represent shapes.
- Have the students verbalize their descriptions of patterns and the thinking that allowed them to deduce patterns or predict extensions of the patterns.
- Have the students create repeating patterns and share them with the class, including numerical ones that are new to them in Grade 2.

Common misconceptions, do students:

- include the first item in the next repetition of the pattern core as the last member of the pattern core? This often happens, since seeing the repeated item starting the next loop of the pattern is the first signal the student has that the pattern is repeating. Having students loop the pattern core elements and compare each to the next will help students who fall into this trap.
- become confused when the same item appears in two places in the core pattern? Continue work with manipulatives complete core patterns. Translating to another mode such as letters may help these students see the core.

If a student appears to have difficulty with these tasks, consider further individual assessment, such as a structured interview, to determine the student's level of skill and understanding. See **Sample Structured Interview: Assessing Prior Knowledge and Skills** (p. 10).

Sample Structured Interview: Assessing Prior Knowledge and Skills

Directions	Date:	
	Not Quite There	**Ready to Apply**
Show the student three or four sets of manipulatives or pictorial representations, several of which are organized in a repeating pattern and others randomly arranged. **"Show me any of these that have a repeating pattern."**	• Does not identify all the repeating patterns or mistakes some randomly arranged groups for repeating patterns.	Correctly identifies all the repeating patterns.
Show the student a pictorial representation of a number of repeating patterns with cores of varying lengths. **"Circle one loop or core of the pattern in each of these patterns."**	• Does not understand what to do, since the idea of a core is unknown. • Loops some of the items in the patterns, but not all the core or more than the core. • Circles the core plus the first item of the next repetition.	• Accurately circles the core in each case.
Show the student several concrete or pictorial repeating patterns**.** **"Describe to me the pattern used in each of these."**	• Cannot describe the pattern in words. • Descriptions are insufficient. • Descriptions are incorrect for that pattern.	• Describes the patterns accurately and completely.
Present five or more repeating patterns of which two have the same pattern, but are shown in different modes. **"Show me which ones are the same pattern, just shown in different modes."**	• Does not see any of them as the same. • Mistakes some that are not the same pattern as alternate modes for the same pattern.	• Accurately matches all the same patterns shown in different modes and does not confuse any that have no twin.

Present to the student different repeating patterns, such as hop, skip, jump or clap, clap, snap, snap, patch. **"Please show me another way that we could make this same pattern."** **"Using a different mode this time, show me another way we could make this same pattern."**	• The student does not translate the pattern correctly, for example states, abbc, for aabbc. • The student only translates to one mode.	• The student accurately translates the pattern presented in each case to a different mode.

B. Choosing Instructional Strategies

Consider the following general strategies for teaching increasing patterns.

- Start with physical materials that allow students to make changes to experimental extensions without fear of error.
- Get students talking about how patterns can grow, so they learn from each other while accumulating the vocabulary necessary to describe increasing patterns.
- Students often enjoy extending patterns and when doing so with concrete materials may extend these much further than in written formats.
- Build the transfer from math concepts to other subjects. For example, while establishing the concept of increasing patterns, look for these cumulative songs and stories and share these in and out of math time. Some possibilities are the following books and songs, *The House that Jack Built*, *Chicken Licke*", *The Old Woman and Her Pig, The Bag I'm Taking to Grandma's* and *The Great Big Enormous Turnip.* There are many cumulative songs, such as "Old Macdonald Had a Farm," "The Old Lady who Swallowed a Fly," "I Had a Cat and the Cat Pleased Me," "The Green Grass Grows All Around," "Alouette," "The Twelve Days of Christmas," "Going on a Bear Hunt" and "An Old Austrian Went Yodelling." Librarians and music teachers will be valuable resources in collecting some of these to share with your class.
- Consider the amount of materials required for some patterns that grow quite quickly. Their rapid growth and the limitations of space and materials may limit students to making only one step of the pattern at a time and then transferring it onto grid paper before building the next step. Groups of students could work together to reduce the quantity of materials required. Students, either individually or in small groups, can work with different manipulatives and/or patterns if supplies are limited and/or the increasing pattern grows quickly.
- The brain is a pattern seeker and makes sense of the world through patterns. Helping students see pattern as a fundamental of mathematics and all learning will help them become better students.
- Since pattern is critical to mathematical thinking, pattern work should begin early in the year and extend throughout the year's study of mathematics. This will influence students' beliefs about mathematics, which influences their learning.
- Students who recognize that mathematics is based upon patterns will see relationships between problems and between mathematical concepts, finding it easier to learn and recall concepts and skills. They will likely persevere in problem solving because they expect the world of mathematics to make sense and possible solutions to exist.

C. Choosing Learning Activities

The following learning activities are examples that could be used to develop student understanding of the concepts identified in Step 1.

Sample Activities:

1. **Identify and Describe Increasing Patterns in a Variety of Given Contexts; and Represent the Relationship in a Given Increasing Pattern, Concretely and Pictorially** (p. 14)
2: **Identify Errors in Given Increasing Pattern** (p. 26)
3: **Explain the Rule Used to Create a Given Increasing Pattern** (p. 29)
4: **Create an Increasing Pattern, and Explain the Pattern Rule** (p. 31)
5: **Represent a Given Increasing Pattern, Using Another Mode; e.g., colour to shape** (p. 32)
6: **Solve a Given Problem Using Increasing Patterns** (p. 33)
7: **Identify and Describe Increasing Patterns in the Environment; e.g., house/room numbers, book pages, calendar, pine cones, leap years** (p. 36)
8: **Determine Missing Elements in a Given Concrete, Pictorial or Symbolic Increasing Pattern, and Explain the Reasoning** (p. 37)
9: **Extending Increasing Patterns** (p. 38)

Sample Activity 1: Identify and Describe Increasing Patterns in a Variety of Given Contexts; and Represent the Relationship in a Given Increasing Pattern, Concretely and Pictorially

(Specific Outcome 2, Achievement Indicators a, b)

1. Calendar

Frequently teachers use the monthly calendar to develop patterns that students can predict, identify, extend, verify, describe and translate into other modes. In Kindergarten, Grade 1 and early Grade 2, patterns used will be repeating patterns that vary in number of elements, form, colour, position and/or size. Now students in Grade 2 are being introduced to a new type of pattern, one based upon the relationship between progressive steps in the pattern. The calendar may be a good way to gradually unveil a growing or increasing pattern while introducing these patterns with other materials. The following is a sample of a plan that includes moving from repeating patterns to increasing patterns on the monthly calendar done daily with students using numerals for each date written upon varied forms.

September: a repeating pattern, such as oak leaf, maple leaf and poplar leaf.
October: repeating patterns, such as turkey, pumpkin, ghost, jack-o-lantern.
November: a repeating pattern, such as poppy, poppy, cross, cross, cross.
December: a repeating pattern, such as evergreen tree, wreath, star, wreath, evergreen tree. With the onset of the new year, introduce the concept of increasing patterns.
January: an increasing pattern, such as snowflake, sled, snowflake, snowflake, sled, sled, snowflake, snowflake, snowflake, sled, sled, sled.
February: an increasing pattern, such as red heart, pink heart, red heart, pink heart, pink heart, red heart, pink heart, pink heart, pink heart. You may have introduced growing patterns in the fall to work with skip counting by 2s, 5s and 10s.

Look For …

Do students:

- ☐ identify more than one possibility for growing patterns as the first few elements are unveiled? For example, an unfolding abaabbaaabbb pattern could reasonably be predicted as an abaabaaabaaaab pattern when only the first four elements are known.

Note that the increasing patterns in skip counting are limited to a constant increase, unlike the increasing patterns suggested in the January and February calendar ideas above.

2. Do You Know My Pattern?

As January begins, or whenever you wish to introduce increasing patterns in which the increase varies in a pattern, start playing a game with students on the overhead, if available, as it makes it easy to vary manipulatives and allows all students to see. Tiles are a good beginning manipulative. After placing one tile on the overhead, ask the students if that gives them enough information to know what your pattern is going to be. Students may predict, but if they do, simply put the next tile in your pattern on the overhead. If they predicted and were wrong, simply tell them the pattern they were thinking of is an interesting pattern, but not the one you are building this time. Gradually students will come to the realization that you can

never predict the pattern with confidence on just one tile, and even two is risky. At least three tiles or steps are generally needed before one can identify the pattern and possibly more, depending upon the length of its core in a repeating pattern or the length of at least three steps in an increasing pattern.

The following are examples of growing patterns you might begin with on the overhead and the questions that you might ask.

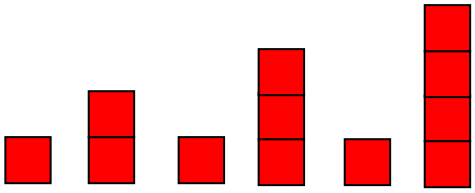
Pattern one:

Do you know my pattern?

Now do you think you know my pattern?

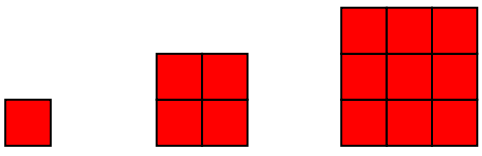
Students are likely to predict at this point that the next step is three tiles set out in a horizontal line. You may ask a student to come up and show the class step three. If the student is correct and lays down three tiles in a line horizontally, move on to ask what is different about this pattern compared to the patterns we have learned about in Kindergarten, Grade 1 and so far in Grade 2. If the student predicted that the next step was to return to a single tile or some other variation, respond with, "That would be an interesting pattern, but is not my pattern this time." Then show the class step three (three tiles drawn one after the other horizontally). Given three steps, can they predict what the next step will be? Ask the students how this pattern is different from the ones they usually work with in math. They will likely come up with some way of expressing that the patterns are growing or getting bigger or that they do not repeat and stay the same over and over. Students may note that these patterns are not connected like repeating patterns, as sometimes each step is shown as being discreet, but not always. The tiles could be lined up in a pattern as follows.

Pattern two:



Ask the students if they can tell you what tiles make up step one, step two and so on. Can they predict the next step?

Pattern three:



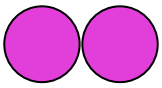
Do students understand the need to keep shapes proportional by analyzing its growth in more than one direction and keeping the growth constant in both planes? Can they see that if the first step is a square, the second grows by one up and one across—still a square shape—then the third step will be a three by three square and not a two by three rectangle?

3. Patterns with Circular Manipulatives

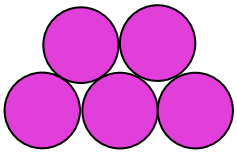
Another day play the game "Do You Know My Pattern," as above, but use bingo chips or pennies as manipulatives to make increasing patterns. The lesson described will first ask students to work together to describe the increasing patterns that are studied. It serves as preparation for using this differently shaped manipulative and further develops the language for describing growing patterns. It also models how to translate the pattern into numerals for study to make generalization possible. The activities in this lesson will help students develop the necessary language for successfully "Building Behind the Shield" in the next lesson idea. Your students may not be ready for some of the numerical work involved in the study of a sophisticated increasing pattern, but will likely be able to identify, extend, describe and reproduce such a pattern. The mathematics behind these patterns may challenge some of your students and will let others know that patterns are not just art and fun, but mathematical. For these students delving into the mathematics or functions will have to wait until Grade 3 or later; however, it is helpful for you as a teacher to see this and know this is where it is leading your students. It points to the mathematical value in studying increasing patterns and how the sophistication of the mathematics in increasing patterns will take your students forward right into high school. Therefore, this curriculum objective is more important than perhaps it is first perceived.

Pattern one: Using the overhead projector, show steps of the pattern and ask questions.

Step one:  Ask, "Do you know my pattern?"

Step two:  Ask "Do you know my pattern now?"

If a student demonstrates step three correctly, ask the students how they could describe this pattern to someone so that they could build it even if they could not see it. If the students do not identify step three correctly, show step three:

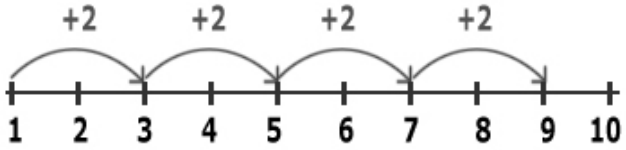


Then ask the students if they can build step four. Ask how the students would explain what happens to build the next row. Can they generalize that the next step takes the bottom row as its top row number and adds one to the row beneath it with two rows being constant? Ask the students to translate the pattern into numbers. Done directly it would look like:

0		1		2		3	
1	then	2	then	3	then	4	but as sums it is 1, 3, 5, 7 …

This can be shown on a number line and students can easily see the pattern is growing by twos, beginning with one, thus odd numbers. The change of plus two is constant from step to

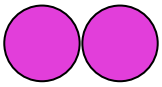
step. This constant rate of growth can be found by the student, but using the information the students have at their disposal so far would only allow them to solve for the number of circles used to build step 9 by building all the preceding steps. They have only found the recursive relationship. In years to come they will learn to find the functional relationship, which will allow them to find the number of circles for any step without constructing the intervening models. For example, in this case, if *n* is the step number, the number of circles required will be $2n-1$. At that stage of pattern study, the power of algebraic thinking becomes apparent.



Ask the students how we could explain the pattern of change in numbers instead of words. As students share that it is adding two to the last sum, it can be pointed out that the language of mathematics helps us describe patterns in a very compact way; it doesn't take a lot of space, time or words. In other words, it is short and sweet.

Pattern two:

Step one:  Ask, "Do you know my pattern?"

Step two:  Ask "Do you know my pattern now?"

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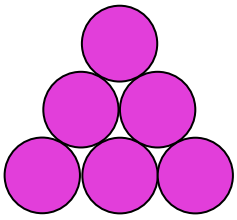
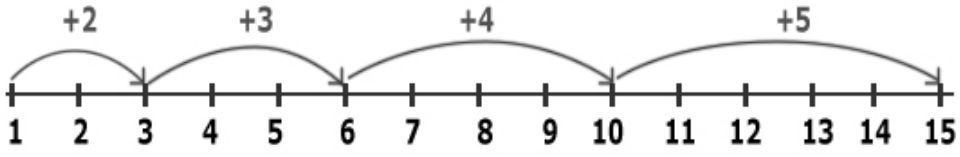


Then ask the students if they can build step four. Ask how the students would explain what happens to build the next row. Can they generalize that the next step always adds another row at the bottom, being one greater than the last bottom row? Ask the students to translate the pattern into numbers. Done directly it would look like:

1 then 1 2 then 1 2 3, but as sums it is 1, 3, 6, 10, 15, 21 **…**

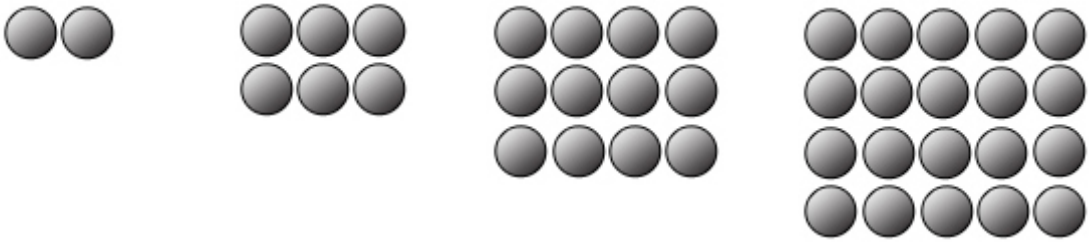
This is a good time to show the students how to read this pattern on a number line as the change from one step to another as below.



Ask the students how we could explain this pattern in numbers instead of words (as in the sums of circles used in the series above). Ask the students if the change in this growing pattern is constant or changing. Help them in this way to notice that the growth may be a fixed amount or an amount that increases in a pattern or predictable way. As students share that it is adding the next counting number to the previous sum of circles, again it can be pointed out that mathematics helps us describe patterns in a very succinct way. Students may also have noticed from these last two patterns that growing patterns may begin the same way, but can be very different. Comparison of increasing patterns begins in Grade 3, so it is not necessary for students to go farther than being aware of this possible divergence of patterns in terms of making comparisons.

Learning to Chart the Change from Step to Step

Students need to see modeling of methods to analyze the change between steps of the pattern. The example below uses a chart instead of a number line.



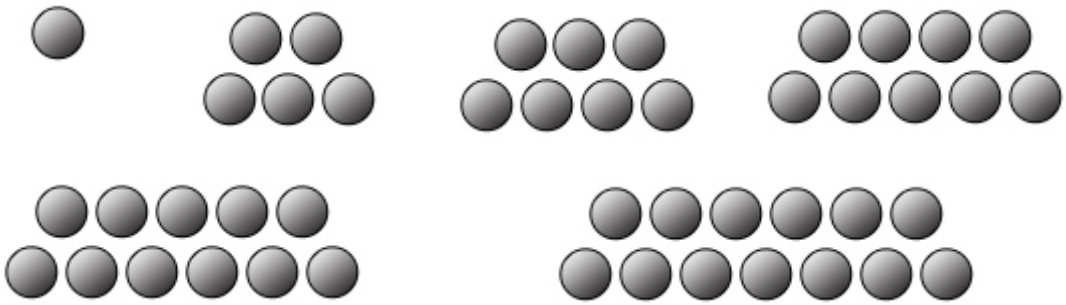
Marking the circles from the previous step in each succeeding step helps students easily see the change (Van de Walle and Lovin 2006, p. 282).

Students need to be guided to learn how to chart the changes they observe and study these changes for patterns that will allow them to predict and test the next steps and eventually use rules or generalizations to calculate steps that are further along in the sequence. Building models for these steps would take a great deal of time and material, so math becomes an efficient way to solve the problem. This learning is helpful before students are faced with using sophisticated increasing patterns to solve problems.

In the example above, students can be guided to list the steps in sequence, then underneath these, record the number of circles at that step. Below this, students can record the change in the number of dots, as below. At Grade 2 it is only necessary for the students to represent the relationship concretely and pictorially, so this level of recording and discussion would only be done with teacher guidance and recording. The students do need to be able to explain rules that generate patterns, however, so an introduction to this type of charting and analysis will prepare students for future learning and challenge your most able students.

Steps	1	2	3	4	5	6	7	8
Number of circles	2	6	12	20	30	?	?	?
		+4	+6	+8	+10	+?		

Another pattern for consideration:



Ask the students how we could describe this pattern in words. With guiding questions it may be described as "the top row represents the number of the step and that is also the number of rows in step, with each row being one more than the one above it." This is another example of a pattern that grows in two directions.

Building Behind the Shield

Now that students have some idea of how to build increasing patterns and describe them in words, ask them to create a growing pattern of their own using the pennies behind a shield made of two sheets of heavy paper or light cardboard taped together to block their work from their partner's view. Their objective is to describe the making of their growing pattern to their partner carefully enough that the partner is able to build the same increasing pattern on the other side of the shield without viewing the original. The student giving directions may, however, observe the partner's developing pattern and give additional directions to the partner to try and correct any misunderstandings. When pairs are done or time is called, the shields are lifted and students talk about what made it difficult to explain or understand and by what rule(s) the pattern is growing. This activity also helps accomplish achievement indicator e, which is to create an increasing pattern and explain the pattern rule.

4. Increasing Patterns on the Hundred Chart

If you have been finding and describing patterns on the hundred chart, you have been finding and describing increasing patterns. Using the hundred chart to show skip counting by 2s, 5s or 10s is using it to look at increasing patterns that grow by a constant amount. This is the type of increasing patterns you will probably include in your plans for the fall of the year.

Name Patterns in September

Give the students a blank ten-by-ten matrix. Ask them to print their first name, one letter per box, over and over until the matrix is full with no spaces between the copies of their name. Students should start at the upper left and print across each row until the row is full and then drop down to the left of the subsequent row, placing the next letter of their name in the first box of that row. This means that unless they have one, five or ten letters in their first name, they will not be starting the second row with the first letter of their name. Ask the students to choose a favourite colour and colour in the boxes where the last letter of their name appears. If students have that letter other places in their name, they should only colour it when it is in the last position. Doing a sample of this on an overhead transparency as you explain the directions will clarify the instructions. When the students are done, they are to hang them on a bulletin board or the front chalkboard. After they are completed, together the class should study them and see if they can organize them into groups that show the same pattern. When asked why they think these students within these groups have the same pattern on their charts, some of the answers may surprise you. Students often suggest ideas such as because they are all boys or girls. It is not as obvious to them as to you that the patterns are based upon the number of letters in their names. This is a good activity for getting to know one another's names in September, and prepares students for the following lesson.

Building the Skip Counting Patterns with Manipulatives

Introducing the skip counting patterns on the hundred chart can be integrated with manipulative work and extended to calculator work. The manipulatives and the calculators help to motivate and involve all students, as well as lay ground work for their skip counting in Grade 2 and multiples in Grade 3. The easiest manipulative to make these skip counting patterns, such as abababab, counting by 2s, or aaaabaaaab, counting by 5s, is two colours of Unifix cubes, used to build trains as close to the length or width of the classroom as time and materials will allow. Different groups of students need to use different pairs of colours so that a colour shortage of Unifix does not immediately develop. Multilink cubes can also be used, but take longer and more strength to connect into the trains. They do, however, have the advantage of not breaking apart unexpectedly. When students have created these long trains with a repeating pattern, have them break them into sets of ten starting from the left end and place each ten-cube length of train one under another as they break them off. Laying them in a box lid helps hold them in position so the students can study them for the visual patterns they create. Preserving and displaying one sample of each of the patterns created in this way will allow the students to compare these patterns to ones they later generate on calculators and record on a hundred chart. It is useful at this point to make the connection between these manipulative patterns and the same patterns seen on the hundred chart when skip counting or to Name Patterns from the previous lesson. For example, if you cover the multiples of five on the hundred chart, students will see the pattern of two vertical lines of the hundred chart that

match up with the vertical lines of the second colour in the manipulative version of the aaaab pattern. Likewise, students who have names five letters in length had Name Patterns the same. The checkerboard pattern is clear on the hundred chart when skip counting by 2s and matches the manipulative box lid with the abababab pattern.

Using the Calculator to Produce Skip Counting Patterns and Hundred Charts to Record

It is best if the students work in pairs with one doing the calculator work, while the other is marking on the hundred chart. Most students love using highlighters to quickly mark on the hundred chart the numbers called out by their partners with the calculators. The partners can reverse roles for the next pattern they produce. The completion of highlighting the whole square can be done after the person on the calculator has called the pattern to one hundred. The student with the calculator should enter 2, +, =, =, = … for counting by 2s and 5, +, =, = , = … for counting by 5s. Students should label these patterns with the calculator buttons pressed to generate them. They can also generate the patterns for other counting groups. When the highlighting is completed, students can then match up the patterns with the manipulative patterns of the trains stored in the box lids and with Name Patterns posted. If storing the manipulative patterns was a problem, students could colour in blank ten by ten grid charts to represent the patterns they found and title them according to the pattern they represent, such as aaaab or abababab. Students with these experiences will readily see the skip counting patterns on the hundred chart as numerical increasing patterns.

If you have removable numerals in a pocket hundred chart, just turn over the numbers you do not want to be visible. If you are using a transparency of a hundred chart, use opaque chips or pennies to obscure the numbers you do not wish to be shown. If you have neither, a hundred chart with numbers blacked out could be used. Other numerical patterns, beyond skip counting or multiples, that could be identified by students and described might include some of the following:

Pattern one: 1, 6, 7, 12, 13, 18, 19, 24, 25 …	(+5, +1, +5, +1, +5, +1)
Pattern two: 1, 2, 4, 8, 16, 32, 64 …	(doubling beginning with 1)
Pattern three: 1, 2, 4, 7, 11, 16, 22 …	(+ the next counting number beginning with 1)
Pattern four: 1, 3, 7, 13, 21, 31, 43 …	(+ the next even number beginning with 1)

Again studying these increasing patterns will require a discussion of what the change is from one number to the next and how the change is repeated or grows in a pattern. Using the number line to study the change in conjunction with the hundred chart will help students see the pattern in the change and ensure they understand the usefulness of a number line in decoding the pattern in these increasing patterns, which are really "sequences" (Van de Walle and Lovin 2006).

5. Patterns on the Addition Table

Students may not be familiar with using the addition table to see patterns. Before the students begin searching for patterns on the addition table, be sure they know what it is and how it works. Placing an addition table transparency on the overhead allows all students to see clearly. Using various coloured overhead non-permanent markers will enable the students to easily see different lines and patterns. You may also wipe the transparency clean and re-use it.

Ask the students what patterns they see on the addition table.

+	0	1	2	3	4	5	6	7	8	9	10
0	0	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10	11
2	2	3	4	5	6	7	8	9	10	11	12
3	3	4	5	6	7	8	9	10	11	12	13
4	4	5	6	7	8	9	10	11	12	13	14
5	5	6	7	8	9	10	11	12	13	14	15
6	6	7	8	9	10	11	12	13	14	15	16
7	7	8	9	10	11	12	13	14	15	16	17
8	8	9	10	11	12	13	14	15	16	17	18
9	9	10	11	12	13	14	15	16	17	18	19
10	10	11	12	13	14	15	16	17	18	19	20

6. Tile, Block or Square Patterns

Study patterns made with tiles, coloured squares, blocks, Unifix cubes or Multilink cubes and transfer them to paper.

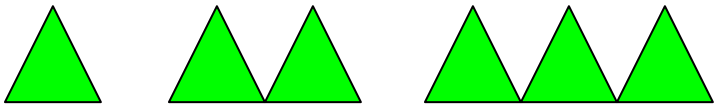
Studying patterns together before asking students to create their own will help ensure that all students have had opportunities to take away essential learnings. Beginning with a symmetrical base pattern is easiest. A pattern of a cross, three across and three down, is a good option. Using a different colour for each progressive round of manipulatives placed makes it easier to see and track the growth. The colours can be used to make a vertical bar graph to record how many of the manipulatives in each colour were required for each step.

When the students are ready to make their own increasing patterns to meet achievement indicator e, they could be assigned to create their own increasing patterns with any manipulatives. Should they elect to use Unifix cubes, students can keep a pattern aligned by hanging Unifix cubes on geoboard pegs/nails. They can also place Unifix cubes on a grid that has squares with matching dimensions to these cubes. Then, as the pattern is built out, the cubes can be lifted and a dot of the correct colour placed in the appropriate square on the grid. The complete colouring in of the squares can be done as a final step. To begin, students can use black cubes, squares or dark coloured blocks to lay down the central pattern. Next, they determine how the pattern will grow or what the rules are. Now the individual or group grows the pattern according to the rules. Switching the colour of manipulative for each step will make it easier for the students to keep track of the pattern growth. Finally, they transfer

it onto grid paper by colouring the squares according to the manipulative colours, which are then lifted away. This activity accomplishes e, creating an increasing pattern.

7. Pattern Block Patterns

Students can be given lessons using overhead pattern blocks. This way they can learn to name the shapes, if they are not already known, and accumulate some experiences making increasing patterns with the shapes before setting out to construct increasing patterns independently or in small groups. Some possible patterns to explore together follow. If you have enough pattern blocks available, it would be ideal if students could build their own models as you work on the overhead.

Pattern one: 

Ask the students what they predict will come next.

Pattern two:  Ask students if they know your pattern yet.

Next, show four green triangles made into one larger triangle as seen below.

"Can you show me what the third step in this increasing pattern will be?" It is very possible that a student will discover the next largest triangle with three rows, the bottom having five triangles, two inverted, the middle row having three triangles with the centre one inverted and the top row, a single triangle as below.

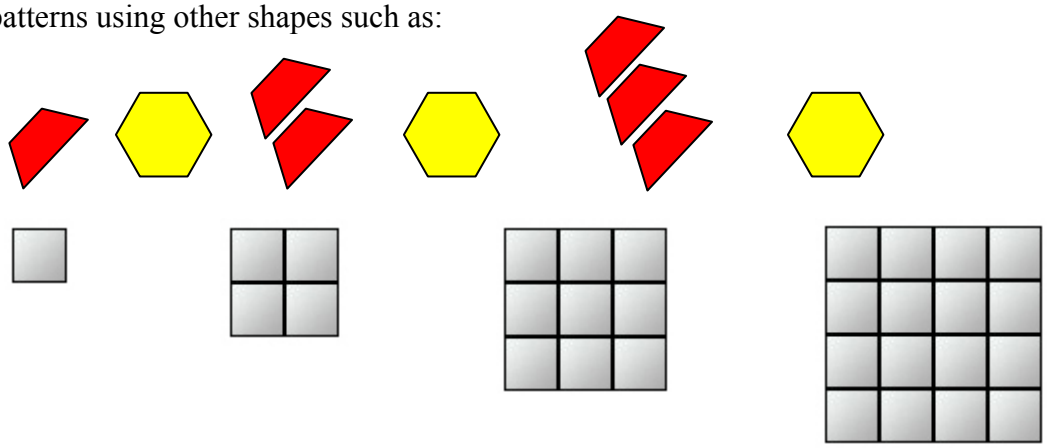
When this step has been discovered and shared, help the students find the data available. "How many rows does this triangle have? How many triangles are in each row? How many triangles does this step use in total? Is there a pattern we can figure out that will tell us how it changes from step to step?" If this is not enough data to let students uncover the addition of a successive odd number of triangles being added to create the next larger bottom row, then go ahead and build the fourth step together so it becomes clearer. If they predict it, ask how they can verify their prediction. Then build it to check.

Since patterns such as this require a large number of a particular pattern block, you may need ways around a lack of concrete materials. Students may be given pattern block stickers, but again this requires a large number of shaped stickers and is an expense. Running them off on paper and having kids or volunteers cut them ready for students to glue into their patterns is a possibility, but is also labour intensive. The easiest way is to invest the time in making templates or to purchase commercial ones. To make templates, you need a utility knife, a marker that will write on plastic lids and a collection of plastic lids. Trace the shapes from the pattern blocks onto the lids and, using the utility knife, cut the shapes out. Now students have templates to make the pattern block shapes. As each progressive step is made, students can transfer it to a pictorial image on a paper and re-use those pattern blocks to make the next step in the pattern.

As students run out of materials to construct their patterns, ask them to share how they solved for the next steps in the patterns. Some will have manipulated materials or 2-D shapes representing the materials, others will have drawn diagrams, others mentally manipulated figures and some converted the pattern to numbers to make it easier to see the relationships or growth pattern.

Other Pattern Block Patterns

Make patterns using other shapes such as:



If students have made the growing triangles pattern in pattern two above, ask them if they think the pattern for making increasingly bigger squares will be different or the same before building these. Then construct the growing squares to verify or defy their predictions.

Ongoing assessment:

As you share various representations of increasing patterns with the class and ask them if they know the next step in your pattern, you can assess their recognition of increasing patterns in a variety of forms. You can also evaluate their ability to describe the patterns, which should be increasing in sophistication of language and organization as the work with patterns progresses throughout the year.

Sample Activity 2: Identify Errors in Given Increasing Pattern
(Specific Outcome 2, Achievement Indicator c)

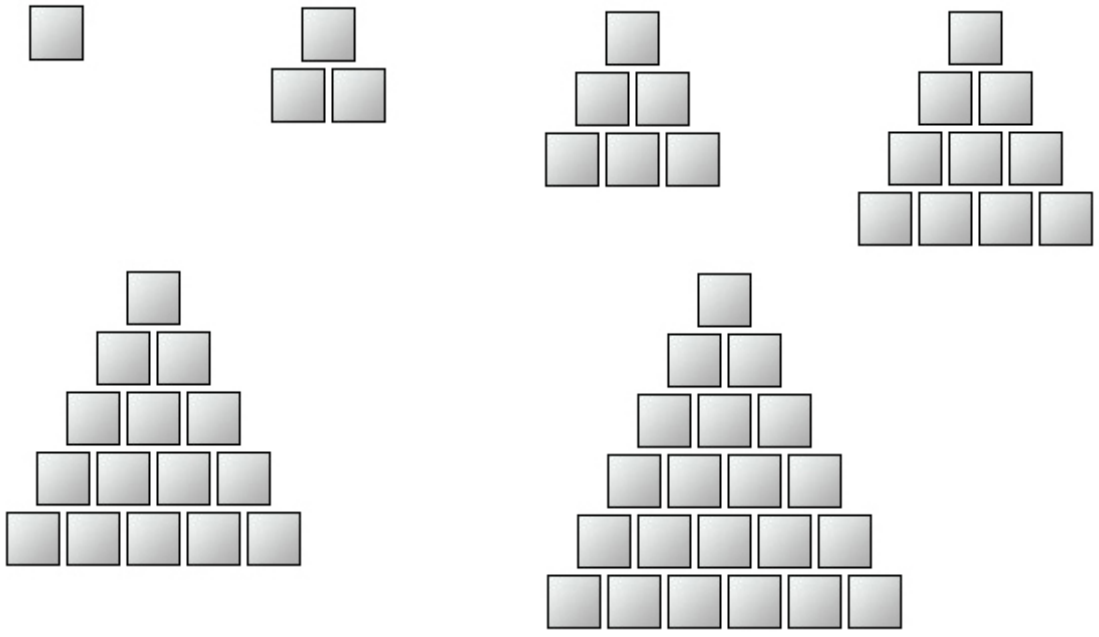
Error Hunt

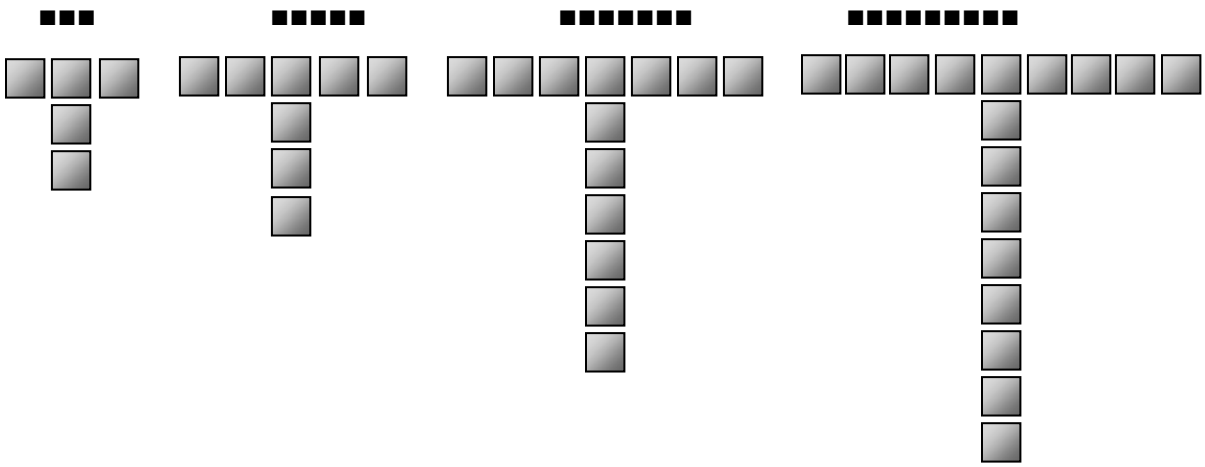
Construct some increasing patterns with concrete materials that have an error. Ask the students to stand when they think they have identified an error as you uncover progressive steps in the pattern. Have the students describe where the error lies and how they knew it was an error. Then have them tell and show how to correct it. Vary the manipulative with which the patterns are constructed. This process is very teacher-centred at this stage. As soon as students have experienced a few of these patterns, ask the students to make one or more growing patterns from a manipulative in the classroom. Ask them to include an error, but to make sure there is enough information included in the steps so that others will be able to see what the pattern should have been and how to correct it. Ask the students to make a correct copy of the pattern as well. These can be photographed or scanned and put on the computer so that they can be accessed by the entire class. Alternatively, this project can be at an activity centre so only a few students each day make their increasing patterns and a version with an error. The other students visit it during the day and at the end of the day discuss the error and how it could be corrected.

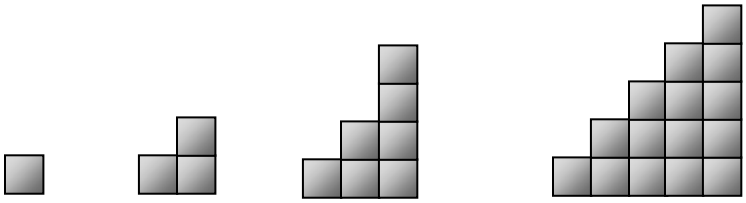
Here are some possible patterns with errors that you could show students. Ask them to find the errors in the following patterns and be ready to correct them. Also, ask them to explain how they knew there was an error and how they knew what the correct steps were.

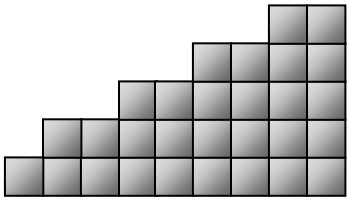
1. ■ ■■ ■■■ ■■■■■ ■■■■■■

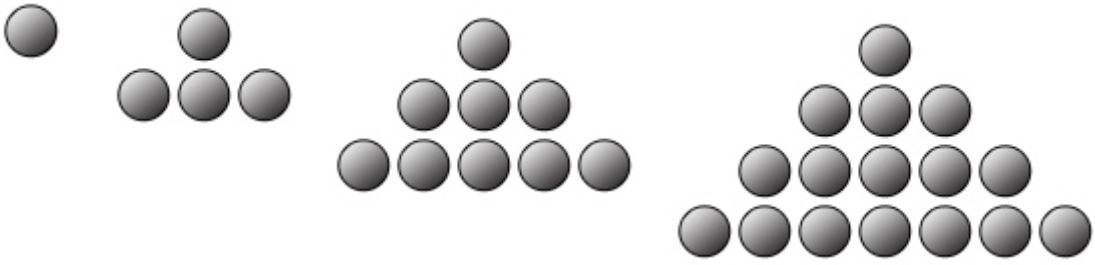
2.



3. 

4. 

5. 

6. 

7. 2, 4, 6, 8, 10, 14, 16, 18, 20

8. 1, 2, 4, 8, 32, 64

9. 1, 2, 4, 7, 11, 16, 29

10. 5, 10, 15, 20, 30, 35, 40

11. 10, 20, 30, 50, 60, 70

Ongoing assessment:

Be sure all students are asked to share their solutions when the above questions done are in class. When the students create their own, observe whether they accurately create increasing patterns and which kinds they attempt, as well as whether they understand these patterns well enough to make their error with sufficient information for detection and correction. You will want to check these before using them as a test for students. Otherwise, you will have to rely on students to tell the creators that their pattern cannot be detected and corrected because it does not contain enough information. Sometimes feedback from the other students is quite valuable, but you also do not want this activity to confuse them.

Sample Activity 3: Explain the Rule Used to Create a Given Increasing Pattern (Specific Outcome 2, Achievement Indicator d)

What's My Rule with Number Machines

Students may have been describing the rules for particular patterns in some of the other pattern activities in which patterns were made with manipulatives and numbers. This activity is geared to focus completely on finding the rules or function for a given increasing pattern. The sequences involved will all show arithmetic growth; that is each series grows by a constant amount. This game can be done with the teacher preparing sets of cards in which the numbers on the backs of each set of cards all increase by the same amount in relation to the number on the front of the card. Students are shown the numbers and asked what the rule is. This activity is more fun if you build a number or function machine. The machine flips over the cards as they pass through, so when you insert the cards with the smaller number showing, it exits the machine flipped over so the larger number is showing. Continue to insert cards and observe the numbers they emerge as until the students can tell you the rule. Then insert the remaining cards to check that they also follow the students' rule. Ask the students to prepare their own sets of cards for the number machine, which can be used with the class machine over a number of days. If all students have created milk carton "computer tutors" to help them practise their facts, these tutors can be used as number machines.

Constructing a Number Machine

Materials needed:

- Clean two-litre milk carton, knife, stapler, tagboard and parcel tape (it adheres well to the wax carton).

Construction

- Create cards that are 8 cm by 5 cm.
- Take a clean two-litre milk carton and staple the top shut (or slit the top open and fold down the flaps if you prefer a flat-topped machine).
- Cut a slit in the front of the carton. The bottom of the slit is 5 cm from the base of the carton. The slit should be 7 cm wide and 2 cm high.
- Cut a second slit so the top of the slit is about 15 cm above the base of the carton. It should also be 7 cm wide and 2 cm high.
- Cut a piece of tagboard about 30 cm long and 7 cm wide.
- Thread the tagboard through the top opening and out through the bottom opening.
- Tape the tagboard to the upper edge of the top of the opening. The tagboard forms a curved slide inside the milk carton.
- Experiment with the length of the slide so that when a card is inserted in the top it will flip over and easily exit at the bottom.

- Once the successful length of the slide has been determined, cut the slide leaving about 1 cm to fold over the lower edge of the bottom opening and tape the slide into place.

If you wish to make the number machine fancy, it can be decorated or even made to look like an animal's head. It can be made to be more attractive and sturdy by covering it with plastic adhesive paper. If you choose to decorate the carton, it is suggested that you do so before you tape down the slide so that the cards do not get caught on the covering as they exit the bottom of the slide.

Making the Cards

- Cards should be 5 cm by 8 cm.
- Cut a small triangle off the upper right-hand corner of all cards to ensure they are inserted into the machine with the correct side up. This will make sure that the intended pattern is created.
- Producing each pattern set on a different colour of cardstock helps them from becoming mixed up. Alternatively, each set could be uniquely stamped to allow you and the students to keep the sets organized.

Sample Patterns for cards:

Set one:

Front:	3	7	9	2	8	12	15	4	0
Back:	5	9	11	4	10	14	17	6	2

Rule: Plus two

Set two:

Front:	2	4	5	6	3	9	15	19	12
Back:	3	5	6	7	4	10	16	20	13

Rule: Plus one

Set three:

Front:	2	4	5	6	3	9	15	9	12
Back:	7	9	10	11	8	14	20	14	17

Rule: Plus five

Set four:

Front:	3	7	9	2	8	12	15	4	0
Back:	13	17	19	12	18	22	25	14	10

Rule: Plus ten

Set five:

Front:	3	7	9	2	8	1	5	4	6
Back:	6	14	18	4	16	2	10	8	12

Rule: Double the number

Set six:

Front:	3	7	9	2	8	1	5	4	6
Back:	7	15	19	5	17	3	11	9	13

Rule: Double the number plus one

Play "What's My Rule"

Deposit a card in the upper slot of the machine or just show students the front of the card if you have no machine. Ask the students if they know your rule yet. Then drop a second card in the opening or show it and ask the students if they know your rule yet. It is handy to record these numbers as you proceed. It may be done by drawing a T chart and on the left-hand side listing the numbers as cards go in or on the front of cards and then on the right-hand side the corresponding numbers that came out or were on the back side of the cards. Some teachers label the T chart "In" and "Out" and some draw the shape of the number machine upper opening on the left and the exit opening shape on the right.

Continue until the students have hypothesized a rule and then continue through the remaining cards to verify that the rule works for all the cards.

Sample Activity 4: **Create an Increasing Pattern, and Explain the Pattern Rule** (Specific Outcome 2, Achievement Indicator e)

1. Have the students create increasing patterns with flat toothpicks glued onto coloured construction paper or tagboard. Have them describe their pattern rule on the back or under their pattern covered with a flap of paper. Share them with the class and see if the class can figure out the rules.

2. Ask the students to create a booklet of increasing patterns that they have made with manipulatives or numerals and record the rules on the backside of the page on which each pattern is illustrated.

   Ongoing assessment:

   By examining the students' growing pattern booklets, you can identify those students who are having difficulty with the concept of increasing patterns. You should also be able to note the level of sophistication of their growing patterns and the language used to describe them. This assessment alone should tell you which of your students understand the concept and can create and describe increasing patterns. The rubric for creating and describing increasing patterns found under assessment in this planning guide can be used as you evaluate their booklets.

3. Building Behind the Shield described in 1c could accomplish this outcome as long as the students are asked to explain their pattern rules.

4. Making sets of cards for the number machine in number three is another way students have created increasing patterns. If they have to present them with an explanation of their rule, they will have met this criterion.

Sample Activity 5: **Represent a Given Increasing Pattern, Using Another Mode; e.g., colour to shape** (Specific Outcome 2, Achievement Indicator f)

Ask the students to show another way to represent the same pattern with different manipulatives or symbols. As examples, ask them how a snap, clap, snap, clap pattern could be represented in colours. After they suggest variations such as red, blue, red, blue or yellow, orange, yellow, orange, ask how it could be represented in letters. After they suggest alternatives such as abababab, ask if there are any other ways to represent this pattern. If they don't offer suggestions, ask if it could be done with numbers or other materials in the classroom. What about with sounds, actions or shapes? Ask the students to represent given increasing patterns in other modes. You can show a series on an overhead or have a series made up at a centre. Allow time for sharing and encourage variety. Students may give you more variety if you show only several patterns and ask them to represent one or all in as many different ways or modes as they possibly can. You may want to discuss whether all number examples count as one mode, so that an abababab pattern is not represented numerically over and over again with varying numbers or repeatedly with varying manipulatives. To integrate this with sorting, the students could sort their representations onto a bulletin board that was labeled for the various categories or modes identified, such as sounds, actions, letters, shape, colour, position, numerals and manipulatives. Keep the patterns simple in this activity, as the focus is on varying the mode.

Ongoing assessment:

If the students have individual white boards or chalkboards, it is easy to assess their translation to other modes of increasing patterns in classes in short activities that students will enjoy and will serve as a review of your teaching. Through actions, sounds and manipulatives, on the board and/or overhead show the students increasing patterns. Ask them to show each pattern in another mode on their boards. Each time, some of these patterns may be shared for variety and reinforcement with the class. On a given signal such as your raised hand, students hold up their boards toward you so you can see what they produced. If you do not have individual chalkboards or white boards, students may use the back of recycled paper to print their translations. Alternatively, half or a third of the class may go to the board at a time to take turns showing you their transfer to another mode. Just reviewing the student's creations from the above activity will show you which students have the ability and their level of flexibility to use a wide range of modes.

Sample Activity 6: Solve a Given Problem Using Increasing Patterns
(Specific Outcome 2, Achievement Indicator f)

Most of the problems with increasing patterns that students will be solving at Grade 2 are ones that they will eventually do with multiplication. A few may use more complicated growing patterns. Below is a sampling of types of problems from which you can create other similar problems.

1. Star had a quarter that says "25 CENTS" on it. She bought a glass of lemonade at her friend Stan's lemonade stand down the street for 5¢. Her little brother and four of her friends came along. She would like to buy them all a glass of lemonade. Does she have enough money? How many more glasses can she buy? What do you think she should do?

   Students may have to work out the necessary calculations for how much it costs for varying numbers of glasses of lemonade in some approximation of the chart below.

glasses	1	2	3	4	5	6
cost	5¢	10¢	15¢	20¢	25¢	30¢

   Did the students note that if she counts herself, her brother and four more friends, there would be six glasses needed? Since she only has 25 cents, they might conclude that she should share her glass with her little brother and buy each of her friends one, assuming that she hasn't finished her lemonade yet. She might tell her friends that she does not have enough money for all of them and see if any of the friends have money of their own to purchase drinks for themselves. She might decide to go home and get more money or ask her mother for another five cents.

   As the problems vary with items to be purchased and costs, students may need to rely more heavily on the charted calculations. They might be good at skip counting by fives and recognize how to apply this skill with money, but find it difficult to count items other than money or add amounts such as three, seven or nine. A second question of this type could ask the students to calculate the number of crayons that are in the centre box if the teacher dumped in seven small boxes that contained eight crayons each.

2. Another variation on this question is as follows. Paul's father said that he would give him a hockey card for every day that he practised the piano. Paul asked his dad if he would give him one for the day he had his lesson, too and his dad agreed. Paul wonders if he practises every day for the next four weeks, how many more hockey cards he could add to his collection of 25 cards. He wonders if he will have over 50 cards if he does not miss a day. Can you help him answer these questions?

3. As much as possible, it is desirable to have problems that students could actually experience in their lives. This motivates them to solve the problem and helps them see the value of mathematical learning, as well as making it easier for them to understand and thus solve the problems. The following is a common practical problem. Jane has a new small photo album that takes four pictures per page, two on each side. She has seven pages back and front filled. How many photos does she have in her book so far? Bringing in a photo album that takes two

photos on each side of the page and allowing students to place photos in that many pages will allow them to see this concretely. Then connect it to the pictorial stage by showing how this could be drawn in a picture or diagram, especially for a class that does not have an album like this to help them. Finally, connect it to the symbolic stage by sharing methods for recording what they see in the book with numbers and mathematical symbols. This problem can be done repeatedly for different sized albums. There are albums that take three and four photos per side of page. Card collector albums usually take nine cards per page as both the front and back of each card needs to be viewed.

4. This question is built on a little more complicated increasing pattern because the change is not constant but grows in a predictable way by a pattern. Your teacher asked if you would like to make a deal on your recesses this week. The teacher said that instead of having a ten minute recess each morning and afternoon, you could instead have a special deal just this week. The deal is that you would be allowed 1 minute of recess Monday morning and 2 minutes Monday afternoon. Tuesday morning you would get 4 minutes recess and Tuesday afternoon 8 minutes. Recesses would continue to be given in this pattern. The teacher reminded students that Wednesday was early dismissal and there would be no afternoon recess and that there was no school on Friday for students this week. Should the students take the teacher's special offer or not? Explain your answer.

   The students will set out to calculate the number of minutes of recess and find their sum to compare with the sum of the regular recess minutes as below.

Special Offer	Monday	Tuesday	Wednesday	Thursday
A.M.	1	4	16	32
P.M.	2	8	X	64
Regular Recesses				
A.M.	10	10	10	10
P.M.	10	10	X	10

   Students who complete this information might never do the addition calculation as it is unnecessary. By counting regular recess times by tens or twenties, they may notice that the total minutes is 70 minutes, or just over an hour. The students who focus on Thursday's large numbers of minutes under the special offer will be able to see that Thursday's recesses alone will total 96 minutes, more than an hour in just the afternoon recess. Those students will not think it necessary to add all the times in the special offer to know that it is a better deal than the regular recesses. You might still have some student opt for the regular recesses because that student will be away from school on Thursday. Another student might state that the students would not have enough time to go to the washroom and get drinks on Monday and they would not be able to concentrate and do their work with such short breaks. It is important that students learn that their reasons are as important as their calculations and it may not be just the bottom line that influences the final decision. We want students to be thinkers, not just number pluggers.

5. If you receive $2 allowance four times a month and your grandma offers to match whatever you save per month up to $5, and triple whatever you save each month beyond the five dollars, what is the most you could save in a month? In 4 months? In a year? Explain your thinking. Why do you think your grandma would make you this kind of offer? (Grandma is probably hoping to motivate her grandchild to make saving rather than spending a habit.) Ask the students to share how they solved the problem. You may have students work in groups.

6. You live on a farm and raise lambs. Your mother sheep, called a ewe, has twin lambs every year. If each of her lambs grows up and has twins the next year, what is the most sheep your flock could have in the fourth year if all the twins are female and if you did not buy or sell any?

	Year 1	Year 2	Year 3	Year 4
Mother sheep	1	3	**9**	**27**
Lambs born	2	6	**18**	**54**
Total sheep	3	9	**27**	**81**

   Ask the students why it is unlikely that you would have 81 sheep at the end of four years. Discuss reasons such as half of the lambs are likely to be males (rams) and would not bear lambs, some sheep might not have twins every year, some lambs might not survive.

7. Create calendar problems such as the following. If you get paid your allowance when your dad gets paid every two weeks and were last paid on the 11th, when will you be paid next? Ask the students how they solved this problem. Some will have looked at a calendar and dropped down two boxes and others will have thought that two weeks is 7 and 7 or 14 and added that to 11 to get 25. Other students may have started at 11 and counted on 10 and then 4 more by ones. If they share their strategies, they can start to evaluate them for efficiency and learn from each other.

Ongoing assessment:

Begin with observations of student contributions as you study these types of problems. Repeating variations of these problems, you hope to see more students showing their understanding of how to approach the problem and to successfully find and use the increasing patterns to solve them. Simple problems may be given to the students to do showing that they know how to use diagrams or recording to solve, such as in the cost of seven candies at the store that were six cents each or how many photographs can be placed in six pages if the album pages take four pictures each. These simple story problems can be evaluated based on a point system. Do the students show some attempt to chart or draw a diagram to lay out the pattern? If they used a diagram, did they convert what they showed in it to numerals? Are most of the calculations of the increasing amounts correct? If there is an error early on in the addition, only deduct from the possible points allowed in the question once or a fraction of the marks allowed, rather than discounting all their calculations. Did they find the correct answer? Was the answer to the question written in a sentence or with units to show that the student knows what they were asked and have found out?

Sample Activity 7: Identify and Describe Increasing Patterns in the Environment; e.g., house/room numbers, book pages, calendar, pine cones, leap years
(Specific Outcome 2, Achievement Indicator h)

1. Dwelling/Room/Road Number Patterns

With the students study the pattern of the room numbers in the school and together create a description of the pattern. Ask the students where else there are increasing patterns in their neighbourhoods or places they have been. If they are rural students, they may note the numbering of the range roads or the rural routes. In some cities like Edmonton and Calgary, the streets follow a numbering system. As dwellers in or visitors to apartments, town houses, condos or single family homes, they will be able to share some interesting patterns observed in their numbering. They may have visited hospitals or hotels and noticed the numbering system of rooms there. A note to parents will encourage them to help their children take note of these patterns in their environment and help prepare children to describe the patterns for the class. The students will note that house numbers usually increase by two on each side of the street, one side being odd numbers and the other even. In apartments, condos and hospitals, they may notice the pattern is that the digit on the far left tells the floor of the room or unit and the numbers increase by one. Odds and evens may be on opposite sides of the hall.

2. Library Patterns

Look for patterns in the library. The Dewy decimal system presents some number patterns. As well, there may be patterns regarding the locations of the reference, paperbacks, hardcover and picture books.

3. Calendar Patterns

Ask the students if they drop down from one box on a calendar what the pattern is. Students may notice that it is a week or seven days later. Can they tell you what the next Tuesday will be, if you tell them this Tuesday is the first? How do they know? If Halloween was Wednesday, October 31 this year, what was the date two weeks before? What day of the week was it?

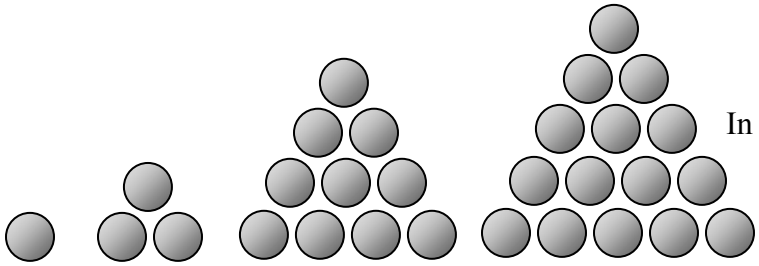
Explain to students that leap year means that we add an extra day to the calendar that year. Since the earth actually takes 365 ¼ days to complete its orbit, every fourth year the extra day is added to February to keep us on track with the seasons. Explain that we have had Leap Year in 2000, 2004 and again in 2008. When will the next one be after 2008? Will we have Leap year in 2015? How do you know?

Sample Activity 8: Determine Missing Elements in a Given Concrete, Pictorial or Symbolic Increasing Pattern, and Explain the Reasoning

(Specific Outcome 2, Achievement Indicator i)

1. **What's Missing?**

   Using any or all the types of increasing patterns sampled in the previous lessons, create sequences and remove a step in the pattern. Have the students determine what the missing element is and explain their reasoning. Begin with concrete examples and move to pictorial before symbolic. At the pictorial stage they can be done by recording each step of the pattern on a separate card. If these cards are displayed in a pocket chart, students should be able to tell you what step is missing and what it looks like. Then the missing step can be inserted in the pattern. Concrete examples can be photographed digitally and viewed on a computer to make it easier for everyone to see. Photographs of the corrected version can follow the corresponding pattern with the omission. This is a very similar indicator to 2c, identifying errors in a given pattern, and you may wish to address these indicators together. An example of a pictorial sequence with a missing element and a symbolic sequence with an omitted step follow.



In this case, step three is omitted.

2. **1, 3, 6, 10, 21, 28, 36** In this case, 15 is omitted.

   Ongoing assessment:

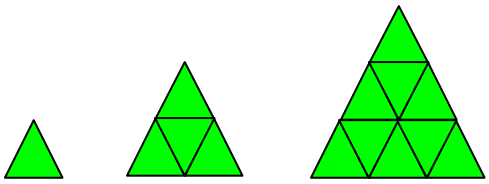
   Evaluate the whole class with a paper and pencil set of patterns with missing elements including some pictorial and some numeric. Meet with those students who are unable to complete it with few errors, show them some manipulative examples and see if the students can find the errors. What you hope to discover is which students do not yet have the ability to discern the pattern and then the error. Some of the students may just not have had the concentration and fine motor or mental energy or skills to complete the paper and pencil task, but actually can see the patterns and the errors.

Sample Activity 9: Extending Increasing Patterns

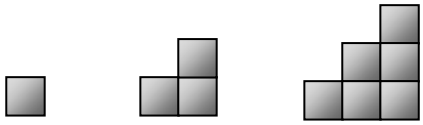
Extending increasing patterns should come early in their work on increasing patterns, not at the end. It should precede students creating their own increasing patterns. As always, begin with concrete examples, moving to pictorial and finally symbolic representations. Here are just a few examples.

Add the next two steps of each of the following patterns.

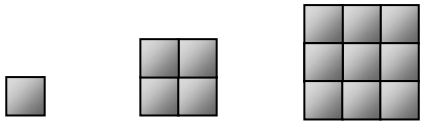
1.



2.



3.



4. 1, 4, 9, 16, _____, _____

5. 5, 10, 20, 35, _____, _____

6. 1, 2, 4, 7, 11, 16, _____, _____

Ongoing assessment:

Evaluate the whole class with a paper and pencil set of increasing patterns beginning with some pictorial then moving to some numeric. Be sure that the majority of the numeric patterns grow by a constant, so that the activity is not too difficult for most Grade 2 students. Meet with those who are unable to complete it with few errors, show some manipulative examples and see if the students can extend them. If they can extend patterns with manipulatives, ask them to extend patterns on a hundred chart. If successful with that, you will know the student knows how numeric increasing patterns should continue, but for some reason did not do this on paper. Moving one more step to the abstract, check to see whether the student can extend a simple increasing pattern when given the first few numbers orally, such as 2, 4, 6, 8, 10. What happens if you orally give the student more numeric increasing patterns to extend? Some of the students may just not have the fine motor skill or mental energy to complete the paper and pencil task, but actually can see the patterns and know how to extend them.

Step 4: Assess Student Learning

Guiding Questions

- Look back at what you determined as acceptable evidence in Step 2.
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

In addition to ongoing assessment throughout the lessons, consider the following sample activities to evaluate students' learning at key milestones. Suggestions are given for assessing all students as a class or in groups, individual students in need of further evaluation, and individual or groups of students in a variety of contexts.

A. Whole Class/Group Assessment

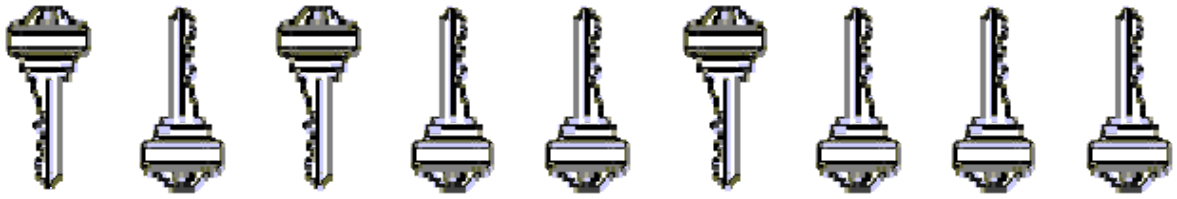
1. Extending Increasing Patterns
   Give the students the task on one day of extending several increasing patterns, include one pictorial and one numeric, which grow by a constant such as a tower of Unifix two high, then one four high, then one six high and ask the students to show the next two steps. The numeric pattern might be 4, 8, 12, 16, _____ , _____. Also include two samples of increasing patterns in which the growth increases by a pattern, such as the progressive steps or stairs pattern and a numeric version such as 1, 3, 6, 10, 15, _____, _____ .

   Evaluate students based on the following rubric:

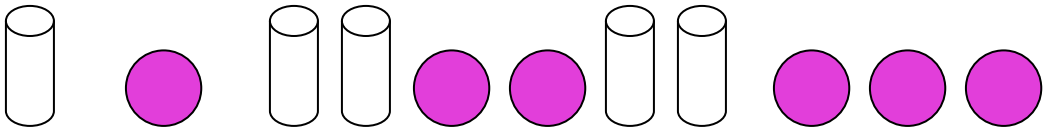
Rubric for Extending Increasing Patterns			
Not Yet	**Needs More Instruction**	**Achieved**	**WOW!**
• Does not extend the pattern given. • Can only do so with concrete or pictorial patterns. • May show a geometrically increasing pattern growing in only one direction, when it needed to grow in two. • Makes errors in how arithmetic sequences increase. Not just an error in addition when extending the pattern.	• Creates a close approximation of the next step of a geometrically increasing pattern. For example, one row may be short a counter in a pictorial representation where there are multiple rows and varying numbers of counters in each. • Usually is able to extend increasing patterns that grow by a constant reliably.	• Reliably extends increasing patterns that grow by a constant and can extend simple ones that grow geometrically in any format: concrete, pictorial or numeric.	• Extends all patterns accurately in any format.

2. Reproducing Patterns
   Give the students three or four pictorial increasing patterns and ask them to translate each one to a different mode. In each case, you want them to use a different mode to translate the pattern.

   a. clap, snap, clap, snap, snap, clap, snap, snap, snap

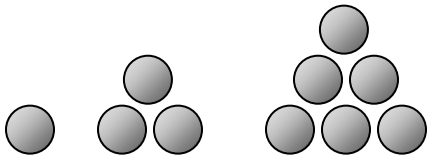


   b. hop, skip, hop, hop, skip, skip, hop, hop, skip, skip, skip, hop



   c. ■● ■■●● ■■■●●●

   d.



   In this case, you will have to determine whether students' translations are sufficient if they give you a sum for the total circles or do they need to show you the numbers in each row in a column? It seems most likely that they would translate this pattern to numerals. If it is too confusing based upon your students' experiences, leave this last pattern off.

Rubric for Reproducing Increasing Patterns			
Not Yet	**Still Requires Practice**	**Achieved**	**WOW!**
• Does not recognize two modes demonstrating the same pattern as identical patterns. • Does not translate a given pattern into another mode.	• Can translate some increasing patterns into other modes successfully, but occasionally makes errors or cannot translate into more than one or two other modes.	• Can translate increasing patterns encountered successfully into three or more other modes, such as letters, shape, colour, position or numerals.	• Shows great flexibility in translating increasing patterns into a wide variety of other modes.

3. Creating and Describing Increasing Patterns
   Assign the students the task of creating and describing as many different increasing patterns as they can in the time allowed. Allow about twenty minutes or long enough for students to complete at least two patterns.

   Assess their competence based upon the following rubrics.

Rubric for Creating Increasing Patterns			
Not Yet	**Needs More Instruction**	**Achieved**	**WOW!**
• Makes repeating patterns only or no patterns at all.	• Patterns created include both repeating and increasing patterns.	• Patterns created are only increasing patterns.	• Increasing patterns vary, including some that are arithmetic (grow by a constant) and some that are geometric (grow by a pattern).

Rubric for Describing Increasing Patterns			
Not Yet	**Needs More Practice**	**Achieved**	**WOW!**
• Description lacks sufficient language or organization to allow the teacher to be sure the student can describe the pattern. • Another student could definitely not construct the pattern from this description.	• The description allows the teacher to recognize the pattern, but steps may be missing or poorly or inadequately described. Other students might not build the same pattern from this description.	• The teacher and other students could recognize the pattern and construct it from the description without seeing an illustration. • The language is typical of a Grade 2 student in structure and vocabulary.	• Teacher and other students could construct this pattern from the description and the language is sophisticated for a Grade 2 student**.**

B. One-on-one Assessment

Give the student some samples of pictorial increasing and repeating patterns that are on strips of paper and have been shuffled. Ask the student to sort these into patterns that are repeating or increasing. Can the student sort these accurately?

If not, move back to the concept of pattern and ask the student to identify patterns in various settings from a Unifix train that is made up of cubes in random order with ones that have two colours in a repeating pattern to numbers in random order to numbers in a patterned sequence such as 2, 4, 6, 8. If the student recognizes pattern, move on to repeating patterns and check to see if the student can identify the core of repeating patterns in which the core may vary up to five elements.

If the student sorted the repeating and increasing patterns accurately, take the cards away and ask the student to build with any of the manipulatives you have on hand a repeating pattern (tiles, pennies or bingo chips, and pattern blocks). If the student constructs a repeating pattern, consider how basic it is, such as abababab. Ask the student to build one more with a different pattern. Can the student build a different pattern? Can the student build one with more than two elements, such as abcabcabc? If not, begin work on repeating patterns to increase the complexity of the repeating patterns in the student's repertoire.

If the student seems to have a reasonable grasp of repeating patterns, you want to find out what grasp of increasing patterns the student has. Ask the student to build with any of the manipulatives an increasing pattern. Observe whether the student builds a pattern that is arithmetic or geometric. With manipulatives, the student often builds a pattern that is geometric or grows by a pattern, but when moving to numerals, often builds patterns based upon the addition of a constant (arithmetic). If the student cannot build one increasing pattern, you need to start working to move the student from recognition that a pattern is not repeating but increasing in a predictable pattern, to being able to figure out how it is growing and extend such a pattern. If the student built a geometric increasing pattern, ask for a second and/or third to be sure that he or she does not just have one or two memorized. If they were all geometric, ask the student to show you a pattern on the hundred chart or with numerals. This one is more likely to be an arithmetic increasing pattern, such as counting by fives. If the student shows you one, ask him or her how it is growing. Then ask for several other increasing or growing patterns with numbers. Ask for an explanation of how each is increasing. If the student cannot show one on the hundred chart or with numerals, then you know that the difficulty lies within the transfer of increasing patterns to numerals or confusion exists because there are two types of increasing patterns. In the case of the latter, start building on the work to show students that both of these patterns are increasing, one by a constant and one by increasing amounts that unfold in a pattern. In the case of the former, start with manipulative and pictorial representations of patterns and connect them to their numeric representation.

C. Applied Learning

- Have the students identify patterns in the environment and discuss whether they are repeating or increasing patterns. Ask for parents' assistance in doing so in the students' everyday experiences and encourage them to prepare their children to share these examples with the class.
- Have the students do problems that entail using increasing patterns and point out to the students that they solved these problems by finding the increasing pattern and extending it. For example, involve the class in figuring out how much time it will take for everyone to share their work, if they are each given 2 minutes to share and it takes 1 minute between student sharing for the student to sit down and the next one to come up to the front. Encourage parents to involve their children in problems involving patterns. Warn parents that the students generally are not working with numbers larger than one hundred. For example, if mom and dad agree to spend $20 on each person's Christmas stocking and there are five people in the family, how much money will they spend on the stockings?
- Have the students organize with patterns in their environment. Ask the students how we can organize our class library so it will be easier to find books on topics that you like or easier to know if all the books have been returned?
- Have the students organize their data from science experiments in class to examine for patterns.
- Have the students do some experiments and look at data for growing patterns in situations where the actual experiment would not be a good use of time, is not wise or is not possible. For example, if a toy car accelerates down a ramp of a particular length and angle in a certain number of seconds, and the same car travels down two other ramps that are 15 centimetres longer but raised the same amount as the first and the acceleration times are recorded. Is there a pattern in the time taken that will allow you to predict the time it would take for the car to travel down a ramp longer than you have? If you wanted to know how high a basketball would bounce if it were dropped off the roof, could you figure it out without climbing on the roof? If it is dropped from 1 metre, how high does it bounce? If it is dropped from 2 metres, how high did it bounce? Does someone know how high the roof is?

Step 5: Follow-up on Assessment

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction?

A. Addressing Gaps in Learning

Suggestions are given in the previous section on one-on-one assessment.

B. Reinforcing and Extending Learning

Students who have achieved or exceeded the outcomes will benefit from ongoing opportunities to apply and extend their learning. These activities should support students in developing a deeper understanding of the concept and should not progress to the outcomes in subsequent grades. Many of these ideas have been described in the activities throughout the section on learning activities. Some of the activities were open-ended, like making a booklet of increasing patterns that the students could keep and add to over a long period of time.

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