Mathematics

Planning Guide

Grade 2 *Measurement*

Shape and Space (Measurement)

Specific Outcomes 2, 3, 4, 5

This Planning Guide can be accessed online at:
http://www.learnalberta.ca/content/mepg2/html/pg2_measurement/index.html

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This *Planning Guide* addresses the following outcomes from the Program of Studies:

Strand: Shape and Space (Measurement)

Specific Outcomes:

2. Relate the size of a unit of measure to the number of units (limited to nonstandard units) used to measure length and mass (weight).
3. Compare and order objects by length, height, distance around and mass (weight), using nonstandard units, and make a statement of comparison.
4. Measure length to the nearest nonstandard unit by:
   - using multiple copies of a unit
   - using a single copy of a unit (iteration process).
5. Demonstrate that changing the orientation of an object does not alter the measurements of its attributes.

Curriculum Focus

This sample targets the following changes to the curriculum:

- The general outcome now focuses on using direct or indirect measurement to solve problems; whereas the previous mathematics curriculum focused on estimating, measuring and comparing using standard units of measure for length and primarily nonstandard units for other measures.
- The specific outcomes have changed dramatically in number and scope. The 2007 curriculum has only five specific outcomes in Grade 2 Measurement, whereas in the previous mathematics curriculum there were seventeen. The first one not included in this section is about time and substantially differs from the outcomes nine through thirteen in the previous curriculum that dealt with time. The emphasis on time has shifted to Grade 3. Outcome fourteen was about reading a thermometer and is not in the new curriculum under Measurement, but would be a required skill in the science unit on Heat and Cold. Outcomes fifteen through seventeen relate to money and are no longer included in Shape and Space in the new curriculum. This leaves specific outcomes one through eight of the previous curriculum to be compared with outcomes two through five of the new curriculum.
- The previous mathematics curriculum included specific outcomes that focused on using the standard units of centimetre, decimetre and metre to measure a length. These no longer apply since units of measurement at Grade 2 are limited to nonstandard units in the current curriculum.

- Specific outcomes four and five of the previous curriculum were about measuring and comparing areas with nonstandard units and recognizing that various shapes could have the same area. Area and capacity are in the Grade 1 measurement section of Shape and Space in the recent curriculum and then does not show up again until Grade 4, although it is likely to be covered in Grade 3 as they develop a deeper understanding of multiplication.
- Specific outcome six of the previous curriculum focused on capacity, whereas there is no mention of capacity in the grades 2 and 3 curricula now. It is part of the Grade 1 measurement expectations, along with area.
- Specific outcome eight: recognition that the size and shape of an object does not determine its mass is no longer included as either a specific outcome or an indicator of achievement in the new curriculum; however, it may be one of the Big Ideas that students gain while comparing masses of identical containers that hold different contents.
- The new curriculum focus is on relationships and processes. Specific outcomes two, three and five are about relationships. The first relates the number of nonstandard units of measure required to the size of the nonstandard unit; the smaller the unit chosen, the more units required. Specific outcome three looks at the relationship of length, height, circumference and mass of one item to the length, height, circumference and mass of other items. The fifth specific outcome is the conservation of measurement attributes regardless of orientation. The fourth specific outcome focuses on the two processes that can be used during measurement with non-standard units, iteration, or repeated use of a single copy of the unit, and the alternative process, using multiple copies of the unit.

What Is a Planning Guide?

Planning Guides are a tool for teachers to use in designing instruction and assessment that focuses on developing and deepening students' understanding of mathematical concepts. This tool is based on the process outlined in *Understanding by Design* by Grant Wiggins and Jay McTighe.

Planning Steps

The following steps will help you through the Planning Guide:

Step 1: Identify Outcomes to Address

Guiding Questions

- What do I want my students to learn?
- What can my students currently understand and do?
- What do I want my students to understand and be able to do based on the Big Ideas and specific outcomes in the program of studies?

Big Ideas

- Measurement allows for the comparison and ordering of objects in either ascending or descending order.
- Students need time to understand the attribute to be measured before the focus shifts to following a specified process or learning the standard units of measure. Working with nonstandard units allows the students to direct their attention to the attributes being measured, such as linear measures of length, height, width and circumference or the attributes of mass (weight).
- There is an inverse relationship between the size of the unit of measure and the number of units used to measure a length or mass. For example, if you use a smaller unit of measure, the numeric measure will increase.
- Linear measurement may be done using one of two processes. The first is iteration, in which only one copy of the unit of measure is used repeatedly. The second process uses multiple copies of the unit of measure.
- Changing the orientation of an object does not alter its measurement attributes (conservation of measurement attributes).
- The distance around an object can be measured by using something flexible, such as a string, which is then laid out in a straight line to allow easier measurement.
- Understanding measurement and making estimates of measurements are based upon personal familiarity with the unit of measurement being used. The experiences that build the familiarity can also allow the development of benchmarks useful in making future estimates and monitoring for errors.
- Precision in measurement can vary and students will need to gain an understanding of measurement error.
- Development of reasoning with the transitive property if A= B and B= C then A= C. This allows students to deduce that A is longer than C, if they know that A is equal or longer than B and B is longer than C, for example.

Sequence of Outcomes from the Program of Studies

See http://education.alberta.ca/teachers/core/math/programs.aspx for the complete program of studies.

Grade 1

Specific Outcomes

1. Demonstrate an understanding of measurement as a process of comparing by:
   - identifying attributes that can be compared
   - ordering objects
   - making statements of comparison
   - filing, covering or matching.

Grade 2

Specific Outcomes

2. Relate the size of a unit of measure to the number of units (limited to nonstandard units) used to measure length and mass (weight).
3. Compare and order objects by length, height, distance around and mass (weight), using nonstandard units, and make a statement of comparison.
4. Measure length to the nearest nonstandard unit by:
   - using multiple copies of a unit
   - using a single copy of a unit (iteration process).
5. Demonstrate that changing the orientation of an object does not alter the measurements of its attributes.

Grade 3

Specific Outcomes

3. Demonstrate an understanding of measuring length (cm, m) by:
   - selecting and justifying referents for the units cm and m
   - modelling and describing the relationship between the units cm and m
   - estimating length, using referents
   - measuring and recording length, width and height.
4. Demonstrate an understanding of measuring mass (g, kg) by:
   - selecting and justifying referents for the units g and kg
   - modelling and describing the relationship between the units g and kg
   - estimating mass, using referents
   - measuring and recording mass.
5. Demonstrate an understanding of perimeter of regular and irregular shapes by:
   - estimating perimeter, using referents for cm or m
   - measuring and recording perimeter (cm, m)
   - constructing different shapes for a given perimeter (cm, m) to demonstrate that many shapes are possible for a perimeter.

Step 2: Determine Evidence of Student Learning

- What evidence will I look for to know that learning has occurred?
- What should students demonstrate to show their understanding of the mathematical concepts, skills and Big Ideas?

Using Achievement Indicators

As you begin planning lessons and learning activities, keep in mind ongoing ways to monitor and assess student learning. One starting point for this planning is to consider the achievement indicators listed in the *Mathematics Kindergarten to Grade 9 Program of Studies with Achievement Indicators*. You may also generate your own indicators and use them to guide your observation of the students.

The following indicators may be used to determine whether the students have met this specific outcome. Can students:

- explain why one of two given nonstandard units may be a better choice for measuring the length of an object?
- explain why one of two given nonstandard units may be a better choice for measuring the mass (weight) of an object?
- select a nonstandard unit for measuring the length or mass (weight) of an object, and explain why it was chosen?
- estimate the number of nonstandard units needed for a given measurement task?
- explain why the number of units of measurement will vary depending upon the unit of measure used?
- estimate, measure and record the length, height, distance around or mass (weight) of a given object, using nonstandard units?
- compare and order the measure of two or more objects in ascending or descending order, and explain the method of ordering?
- explain why overlapping or leaving gaps does not result in accurate measures?
- count the number of nonstandard units required to measure the length of a given object, using a single copy or multiple copies of a unit?
- estimate and measure a given object, using multiple copies of a nonstandard unit and using a single copy of the same unit many times, and explain the results?
- estimate and measure, using nonstandard units, a given length that is not a straight line?
- measure a given object, change the orientation, re-measure and explain the results?
- recognize that if two nonstandard units are used to measure the same length or mass, more of the smaller unit will be required?

Sample behaviours to look for related to these indicators are suggested for some of the activities listed in **Step 3, Section C: Choosing Learning Activities** (p. 12).

Step 3: Plan for Instruction

Guiding Questions

- What learning opportunities and experiences should I provide to promote learning of the outcomes and permit the students to demonstrate their learning?
- What teaching strategies and resources should I use?
- How will I meet the diverse learning needs of my students?

A. Assessing Prior Knowledge and Skills

Before introducing new material, consider ways to assess and build on students' knowledge and skills related to measurement. For example:

1. Do the students recognize the various possible measuring attributes?
   Give students some pairs of objects to compare. Ask them to explain what they were comparing and how the two objects compare.

   You will need to have available a variety of materials for filling, covering and matching. Filling may require enough beans, rice, sand or foam chips to fill at least one of the boxes or containers. Matching lengths will require a number of nonstandard units such as linking cubes, plastic links, paperclips or other items of which you have an abundant supply and that are not too large a unit for measurement of the items to be measured. If you are measuring larger items, such as the width of a desk or door and the height of people, you will need larger units, such as decimetre rods from the base ten blocks, straws, Popsicle sticks or lengths of yarn and string. You will also need some items for covering areas such as tiles, decks of cards, photos or dominoes.

   For example, given several boxes, do students identify linear measurement possibilities such as the height of the boxes, the distance around the outside of the boxes (perimeter) and the width and length of the boxes? Do they only measure by matching length, width and height and verbalize in terms of which box is taller/shorter, longer/shorter or wider/narrower? The term "narrower" may not be in your students' everyday vocabulary. This is an opportunity to check that they know what the word means and encourage them to use it.

   Do students lift the boxes and use hand balancing to try to determine which is heavier and which is lighter? The boxes being compared need to have distinctly different weights.

   Do they also recognize that the boxes could be compared based upon the attribute of capacity, how much they will hold? Do they have suggestions as to what nonstandard units might be used to compare the capacities of the two boxes? The boxes should be different shapes, not ones that can be placed one inside the other and make comparison easy without any measurement.

Do students also consider the attribute of area when comparing the boxes? They may only look at the bottom or lid of the box when considering the area, which is how much space it will take up on the counter; however, if the question is which box would require the most wrapping paper, the area to be considered would include all six sides of the box.

If students' knowledge of these measurement attributes seems weak, you may want to provide more opportunities for the students to strengthen their knowledge of those attributes even as you are setting out the learning activities for students to meet the Grade 2 objectives for measurement. It is likely that the students' knowledge of area and capacity are less developed than their knowledge of linear measure. This does not necessarily reflect on their Grade 1 learning experiences, but the fact that in their home and community environment they will have had more opportunities to use linear measures. They could possibly have had more opportunities to experiment with capacity than area, but their exposure to the language related to capacity has likely been limited to "more" and "less."

2. If students need additional experiences with these measurement attributes, you can follow up with assessment based upon determining the ordering of three or more given objects, as in the Grade 1 curriculum.

   If a student appears to have difficulty with these tasks, consider further individual assessment, such as a structured interview, to determine the student's level of skill and understanding. See **Sample Structured Interview: Assessing Prior Knowledge and Skills** (p. 10).

Sample Structured Interview: Assessing Prior Knowledge and Skills

Name: **Directions**	Date: **Not Quite There**	**Ready to Apply**
Place two boxes of different shapes (not nesting boxes) in front of the student. It is best if these boxes are also of obviously different weights. Have in view and accessible to the student items that can be used to measure the capacity, such as rice or beans, and small containers that could be used to transfer these items to the boxes. Also, have some items that could be used as nonstandard units for measuring the areas, such as cards, photos or dominoes. Ask the student, **"How many ways can you measure and compare these boxes?"** If the student does not seem to know how to respond, probe with, **"Can you show me in as many ways as you can how these boxes are different?"** The necessary prerequisites for the Grade 2 measurement concepts are in linear measure and mass; however, the expectations for Grade 1 include knowledge of filling and covering for capacity and area, as well as mass and linear measure. So if your student has gaps in these aspects of measurement, it would be worthwhile providing opportunities to remedy those gaps before the end of Grade 2.	• Doesn't understand how to compare the boxes by measurements, only compares their colour. • Compares length, width and/or height only by matching. • Compares weight only by lifting them one in each hand or one after the other.	• Compares mass/weight and linear measures of at least two of width, height ("taller/shorter") and length. • May compare capacity by filling the smaller looking box with rice or beans; then transferring them into the larger looking box to check if there is still room in the box for more. • If the student also compares the surface area of the lid of the box or the bottom by covering it with units, you have a student who has very well developed measurement concepts/skills.

B. Choosing Instructional Strategies

Consider the following general strategies for teaching:

- Access prior knowledge on using measurement in the real world. What things do students know that are measured and why are they measured?
- Use linear measure and mass (weight) to measure things that are familiar and of importance to students, such as body measurements and masses of items that are common in the classroom or in their play, such as erasers, pencils, crayons, Plasticine, balls, staplers and scissors.
- Check for conservation of length and mass. Do students recognize that items rearranged, transformed or subdivided maintain the same measurement attributes?
- Have the students estimate prior to comparing or measuring, as it enhances motivation and involvement. Plus it gives you as the teacher more information on what students have gained from past experiences. If they are not applying benchmarks to improve their new estimates, it gives you the opportunity to ask questions and remind students about previous experiences so that they begin to see the value in taking note of results and recalling these referents when making future estimates.
- Encourage the students to measure the height, length, distance around or mass after each estimate so that they develop a better sense of these four measurement attributes and some benchmarks.
- Make both direct and indirect comparisons; e.g., standing two students back-to-back to compare their heights and indirectly comparing the height of two students in different classes by using paper strips to compare them or by considering the counts of their heights in the same nonstandard units. Even more indirect is the logic involved in the comparing measurements, such as student A is the same height as student B and student B is the same height as student C, so can they deduce that student A is also the same height as student C?
- Have the students compare their results and discuss reasons for any discrepancies.
- Accept a range of estimates—within 10% to 20% of the actual measure is reasonable (Van de Walle 2001, p. 295).
- Measure the length or mass with various nonstandard units, so students can develop an understanding of the inverse relationship between the size of the nonstandard unit and the numeric outcome, as well as improve their accuracy in making estimates.
- Provide situations that encourage students to begin to see the need for standard units to measure, for example using hand spans or the length of one's foot to measure the same items or various sized washers as weights to measure masses and discuss the variations in results.

C. Choosing Learning Activities

The following learning activities are examples that could be used to develop student understanding of the concepts identified in Step 1.

Sample Activities:

Teaching Linear Measurement

1. **The Right Sized Unit** (p. 14)
2. **Estimating Linear Measures** (p. 15)
3 **Two Ways to Measure Directly, How to Measure the Distance Around a Shape and Measurement Error** (p. 16)
4 **Changing Orientation Does Not Change Measurement Attributes** (p. 18)
5 **Practising Measurement with Single and Multiple Units** (p. 19)
6 **Measurement and All About Me** (p. 20)
7 **Relating the Size of Measurement Units to the Number of Units** (p. 21)
8 **Ordering (Seriating)** (p. 22)
9 **Crooked Paths** (p. 23)
10. **Making the Transition to Standard Linear Measurement Tools** (p. 24)

Teaching Measurement of Mass

1. **Becoming Acquainted with Mass Measurement Instruments** (p. 26)
2. **Selecting the Right Nonstandard Units** (p. 28)
3. **Estimating the Number of Nonstandard Units** (p. 29)
4. **Conservation of Mass** (p. 30)
5. **Ordering According to Mass/Weight, Ascending and Descending as Required** (p. 31)
6. **Relating the Number of Units to the Mass/Weight of the Unit Used** (p. 32)

Teaching Linear Measurement

Sample Activity 1: The Right Sized Unit

Students need to be able to select a nonstandard unit of an appropriate length to make linear measures and explain why it is the right sized unit for the task. You need to have a variety of possible nonstandard measurement manipulatives available. These could include: base ten rods, multilink cubes, centicubes, coffee stir sticks, straws, Popsicle sticks, string, yarn, paperclips, nails, screws, washers, pennies, marbles, paper strips of various lengths or footprints cut out of heavy paper and plastic links.

1. As a beginning step, ask the students which of two notably different sized nonstandard units they would choose to measure a specific object and why. Some examples follow.
   - If you want to measure the width of the door, which would be a better choice: straws or centicubes? Why?
   - If you want to measure the width of your math book, which would be a better choice: centicubes or stir sticks? Why?
   - If you want to measure the length of the soccer field, which would be a better choice: skipping ropes or straws? Why?

   Students will soon begin to select nonstandard units that are large enough to do the job efficiently, but with reasonable accuracy. For example, they may tell you that there are not enough straws to measure the field and that it would take too long to measure it with straws.

2. The second step is to ask the students what nonstandard units they would select to measure a given object if they could freely choose and why. Some possible items you could present to students include: the height of their bicycle tire, a chalkboard eraser, their glue sticks, the garbage can, each other, the teacher's desk, their desk, the door, a window, a bookcase, the chalkboard ledge, a pencil and a roll of masking tape. They can then focus on the lengths of items such as their stride, their feet, their fingers, a paperclip, a desk top, a library or textbook, playing cards, a sheet of paper, a bulletin board, the class calendar, a piece of art paper and a poster in the classroom. During this activity, a discussion of length and width may arise. You may use the convention that the longer side is the length and the shorter is the width. It is also a good time to discuss "depth." For example, students can measure the depth of a book shelf, aquarium or box. Distances will likely be of interest to the students, but keep in mind that length as a linear distance between two objects or points is more abstract than measurement of length as a property of an object. Measurement units may need to vary with greater distances. For example, a tracing of a foot might be quite reasonable for measuring the distance from a particular student's desk to the teacher's desk, but might seem too small for measuring the length of the gym or the distance from the classroom to the gym or office.

Sample Activity 2: Estimating Linear Measures

As students begin exploring measurement, ask them to estimate each measure they are going to make. If you ask them to estimate how many of a particular nonstandard unit would be required to measure an object or person before they have any experiences measuring objects with nonstandard units, students are likely to be hesitant to make estimates, as they have no referents or benchmarks. When they do make estimates, they are apt to vary widely. Encourage the students to make estimates even as they are beginning to explore measurement, as not only does it develop the habit, but it will allow them to see how much better they have become at estimating through experience.

Look For …

Do students:

- ☐ hesitate or seem reluctant to make an estimate or guess?
- ☐ become more comfortable with making estimates as they accumulate benchmarks or referents through experience?
- ☐ refine their estimates based on their experiences?
- ☐ erase estimates after the actual measures and make them both match?

Some students will naturally recall and draw on their past measuring experiences to refine their predictions. Other students may need you to draw their attention to how relating past measurement outcomes with the same nonstandard unit improved their accuracy in measurement predictions that followed. Remind the students of the outcome of a previous measurement. Then ask how the new item to be measured compares to the previous item measured. Is it taller/longer or shorter? Is it nearly as long or about half or less? They can use these comparisons to refine their estimates. Make sure the students understand that no estimate is wrong, but they can make better and better estimates with practice.

You may have to model for students estimates and actual measurements that do not match, as they may still feel that their estimate is wrong unless it matches the actual measure precisely. When they see that your estimates and measures vary, they may be more willing to record their estimates and not adjust them after the actual measurement is made.

Sample Activity 3: Two Ways to Measure Directly, How to Measure the Distance Around a Shape and Measurement Error

Ask the students to measure a variety of objects using nonstandard units while working in pairs. Have each pair agree upon and use the same nonstandard units. The advantage of working in pairs is that students need to address any differences in the measurements they find. Give them a list of items for both students to measure and have them record their estimates and actual measurements, as well as the units used. Your list might include: the width of the seat of their chairs, the length of their math books, the height of their glue sticks, the height of the door handle, the depth of the bookshelf, the width of their pencil cases, the length of their erasers and pencils and the distance around a can that is in the room, such as a coffee can for lost and found pencils or that holds spare rulers.

Observe the students as they work on their tasks. Notice whether they are using multiple copies of the same unit to measure or whether they are using a single unit and placing it down repeatedly (iteration). Also note how the students approach the problem of measuring a can's curved surface.

Look For …

Do students:

- ☐ select suitable sized nonstandard units?
- ☐ only measure with one item, using it repeatedly?
- ☐ take care not to overlap or leave gaps when measuring?
- ☐ understand why discrepancies occur in their measurements even when made with the same nonstandard units?
- ☐ solve the problem of measuring curved surfaces by using flexible chains of nonstandard units such as paperclip and plastic chain links or string which is then oriented in a straight line for measurement?

After the students have completed their measurements, ask them how they made their measurements. During the sharing, if iteration is described, have the students explain how it can be done. A discussion might then follow about the advantages and disadvantages of iteration, which should lead to discussion of the difficulties of leaving gaps or overlapping, thus creating measurement error. Some discussion of measurement error may have arisen during the discussion of which sized nonstandard unit was best.

If the students use too big a unit, it is hard to make an accurate measurement at the end. A measurement of the width of the door might end up being three straws and about a half. The soccer field might have been 20 skipping ropes and a bit long. On the other hand, too small a unit leaves the potential for measurement error due to gaps or overlapping. Students can discuss how they might prevent gaps and overlapping. If the units are stuck together, such as linking cubes, or if they are stretched taut like a chain of paperclips, gaps and overlap can be eliminated. Measuring by the iteration method has the potential for a large measurement error due to gaps and overlap, but it has the advantage of not requiring multiples of the nonstandard unit.

Ask the students to share their methods for measuring the distance around (circumference of) a can. Some students may have found flexible items like a paperclip chain. Others may have taken a string or yarn and stretched it around the circumference and then laid it out straight to make a measurement in a straight line. Students need to use these alternative methods when measuring circumferences of other items during future measurement tasks; however, some students may not trust these measurements because they do not yet understand the concept of conservation of measurement attributes through changes in orientation. The following activities should provide most students an opportunity to gain this understanding.

Sample Activity 4: Changing Orientation Does Not Change Measurement Attributes

Before the students set out to practise these measurement processes and explain any discrepancies, it may be important for them to understand that changing the orientation of an object does not change its measurement attributes. Otherwise, students may try to explain the differences in results between iteration and measuring with multiple copies as due to movement of the objects being measured. Students at a pre-stage to conservation of length often think that a linear measure of an object is changed when it is moved, subdivided or rearranged. A classic example is a necklace that has been stretched out to its full length, then rearranged into a circle. When a student who does not understand conservation of length is asked if the necklace in the circular formation is the same size, longer or shorter than it was in the straight line formation, he or she will often declare the necklace in the circular formation shorter, despite having observed you moving the same necklace from one arrangement to the other.

Look For …

Do students:

- ☐ predict a difference in linear measures when the object's orientation is changed?
- ☐ do students perform the measurements of the object in both orientations?
- ☐ do students simply do a quick check that their first measurement was accurate by doing a second measurement when the item is repositioned?
- ☐ do students cease making the second measurement when the object's orientation has changed?

Initially students can be asked to measure the length of a table leg from the table to the floor. Then they can predict and measure the height of the table from the floor or the leg from the floor to the bottom of the table. They can then tackle items for which the orientation changed, such as a textbook length as it lies on the desk or floor. Then they can predict what its height will be when it is measured standing up. Then measure its height by leaning the book against a wall or placing it on the chalkboard ledge. Discuss the results. Try this with several other items that have been reoriented between measurements. A can standing and then on its side would be another good challenge. Finally, students can measure their partners' heights and foot lengths in two orientations. They may find it easier to measure by tracing around each other on paper. These tracings will even allow the students to measure the perimeter of themselves later as they develop skills measuring around curves. As they measure their various body parts, the results can be written on their 2-D tracings. They can colour or decorate these tracings and hang them around the room or sit them in their chairs for parent–teacher–student conferences. Consider using a booklet called "All About Me" or "How I Measure Up" in which they record their measurements, rather than recording them directly on the tracing. Not only is it neater and easier to read, but it provides places for the estimates to be written in advance of each measurement. Once enough measurements have been done so that the class has concluded that orientation does not change measurement attributes, it is safe to move on to further measurement practice.

Sample Activity 5: Practising Measurement with Single and Multiple Units

Provide some opportunities for the students to use the processes of measurement. Give the students some items to measure both by using multiple copies and then a single copy of the same unit and have them compare their results. Student partners could be asked to measure the items using first just one of their selected nonstandard units (iteration) and then multiple copies. Then they should compare their results to see if they differ and if so try and explain why. This will focus them on the advantages and disadvantages of using each process. Further discussion of measurement error can follow.

Sample Activity 6: Measurement and All About Me

Now the students are ready to make many measurements. They may especially find it interesting to measure themselves. To add to the mathematical interest of these measurements, comparisons can be made between some of the body parts, such as the following:

1. “Look at the circumference of various people's wrists and then their ankles.” Using string or paperclip chains allows these measurements to be displayed one along side the other for visual comparison of a number of students' results. Take a string that is labelled with a piece of masking tape for one person’s ankle and hang it beside one representing the person's wrist. How do they compare if you measure them with paperclips? Is an ankle about two more paperclips in circumference than a wrist?
2. How do the lengths of your arms compare to the circumference of your head?
3. How does your neck size compare to the distance from your collar bone to your navel?

A list of the various measurements you want students to make can be recorded in their booklets or can be written up on a chart. Consider also visiting the various comparisons you wish made as well.

You may wish to have the students apply their subtraction skills by not only noting which feature is longer, but by how much. As students work, observe whether they are calculating the differences through a third measurement or by subtracting. For example, if asked who is taller, you or your partner, students may solve this by direct comparison, either standing back-to-back or by laying their tracings beside one another. If they have the actual numbers of units written down for their heights, they may compare numbers, looking for the larger number, assuming both measurements were made with the same nonstandard units.

If the units used were not the same, then it is a good time to talk about how this was a problem when people who measured with one set of units offered to sell or buy something from someone in another part of the world who measured with a different set of units or units with the same name but a different size. Cloth is sold in rolls called bolts. If one country made bolts in a small size and the other country was making theirs in a larger size, when they discussed selling and buying bolts, one of the merchants was going to feel cheated. They had to find a way to make sure they both were talking about the same sized bolts, and so they needed to have standard measurements or measurements that everyone would know meant the same thing. That is why we use common units of measure, such as centimetres and grams.

When students are asked to state how much taller one student is than the other, they may use a third measurement to assess the difference in heights. They may measure the amount that extends beyond their matching heights. This is most likely to occur if students have made paper strips that represent their height to solve the challenge of preparing a class list that will line up the class from shortest to tallest for the school photographer. Using a cash register roll makes it easy to create strips of paper of various heights or lengths.

Sample Activity 7: Relating the Size of Measurement Units to the Number of Units

Assign the students to measure with at least two different nonstandard units a variety of classroom objects. Remind them to make estimates before the measurement and record both the estimate and the actual measurement along with the units used. As students work, observe their estimates. Do the estimates of the second measurement reflect that students are considering the relationship of the units and adjusting their estimates accordingly? For example, if students first measured with nails that are about half as long as the rods they are using for the second measurement, is their second estimate smaller than the actual measure with the nails? Is it about half as big? Direct the students not to make the second estimate until after the first actual measurement. Once students have made their estimates and measurements, discuss the results. Ask questions such as: Was your second nonstandard unit longer or shorter than the first unit used? Did you use more or fewer of the second units? As you record some of the students' findings, you can ask them if they see a pattern. Once they determine that if you use a smaller unit, the number of units required is more and, conversely, if you use a larger unit, the number of units required are fewer, you can ask them to watch out for any measurements they make for which this is not the case or challenge them to find an example for which this is not true.

Look For …

Do students:

- ☐ make both estimates before doing the first measurement?
- ☐ make second estimates that show an intuitive knowledge of the inverse relationship between the size of the unit and the numeric outcome?
- ☐ make second estimates that reflect an approximation of the size comparison of the two nonstandard units used (e.g., half the size, then twice the number) or are estimates only appropriately smaller or larger, not proportionally so?
- ☐ think they might find and so are searching for an example for which this inverse relationship is not true or do they appreciate that this is a universal truth.

Sample Activity 8: Ordering (Seriating)

Students can work in small groups or individually to measure and compare various linear measures of items to be compared, including some curved surfaces. This could be done with all students measuring simultaneously or in a rotation to a centre. Be sure the activity sheet requires them to order some of the items in ascending order and others in descending order so that students become accustomed to checking which they were asked to do.

Look For …

Do students:

- ☐ measure all the items with the same nonstandard unit?
- ☐ pay attention to whether the task asks for ascending or descending order?
- ☐ measure items relatively accurately without careless gaps or overlap?
- ☐ select appropriate nonstandard units?
- ☐ have techniques for measuring curved surfaces?

Sample Activity 9: Crooked Paths

It is important to compare lengths that are not straight lines. Students have done so by measuring the distance around various items, or their circumference. They can also do so by measuring paths that do not follow straight lines. You can make crooked or curvy paths on the floor with masking tape, as well as on paper. Have the students predict which of two paths is longer. Then, using string or rope longer than either path, students measure the paths. Students will sometimes feel that one path "looks longer" based on it being more spread out than another more compact path. Ensure that students experience a comparison in which the longer path is the more compact path. Push the students to produce a more convincing argument than a path "looks longer." Students who can use the rope or string to measure the crooked path must have a transitive understanding of measurement. That is, they need to be able to understand that “if A is equal in length to B (the rope) and B is equal in length to (or shorter/longer than) C, then A must be equal in length to (or shorter/longer than) C” (Van de Walle and Lovin 2006, p. 229). It is possible that a Grade 2 student may not yet have a transitive understanding of measurement.

Sample Activity 10: Making the Transition to Standard Linear Measurement Tools

Students in the third grade are expected to use standard units and measurement devices for metres and centimetres. To gain full value from the work of measuring with nonstandard linear units, students need to make the transition to measuring devices. In doing so, they compare the nonstandard units and the standard measurement lengths. Students should learn that it is the spaces, not the marks on a ruler that are being counted. You may want to discuss with the Grade 3 teachers in your school which grade will cover the transition to standard measurement devices. One way it can be done is to have the students place paperclips along a strip of paper and mark off each clip. Another way is to have the students make a simple measuring instrument, such as a narrow oaktag strip about 5 cm wide that has two coloured rectangles of paper (5 cm long each) glued alternately so that the students see that the critical part of the ruler is the coloured spaces, not the markings. For this activity to be of value to the students, they will need to transfer this understanding to how this works on a ruler. Some metre sticks are available commercially with alternating coloured decimetres. If students are not ready for this, then it can be done in Grade 3 (Van de Walle and Lovin 2006, pp. 232, 233).

Teaching Measurement of Mass

Introduction

Mass is often considered a synonym for weight. Mass is the amount of matter in an object and weight refers to the pull of gravity on that object. So if a person were on the moon, their mass would stay the same as on earth, but their weight would be different (less) due to the reduction of gravitational pull on the moon. Since on earth mass and weight are very close, we tend to use these terms interchangeably, particularly in elementary school.

Sample Activity 1: Becoming Acquainted with Mass Measurement Instruments

1. The Need for Scales

Ask the students to hold out their hands, take one item in each hand and tell you which one weighs more. Begin with familiar classroom items, such as a stapler and a glue stick or a pair of scissors and a marker. Progress to manually weighing items with less obvious differences in weight, such as a box of pins and a glue stick or two notebooks, one thin with larger pages and one thick with smaller pages. As the students express their differences of opinions or that they are uncertain, point out the need for equipment to make accurate measures.

Students may be surprised if they are using size to predict the relative weight of some items. For example, give them a small can of something dense like salmon, tuna or beans and ask them to compare the weight with a can of Chinese noodles, which may be in a much larger can, but weighs noticeably less. Likewise, they can later compare things like the same sized bottles filled with a variety of items such as sand and macaroni or spices, which vary widely in weight but come in the same sized containers. Sugar and sweeteners in the same sized containers each weigh notably different amounts. Students will discover that weight is influenced by density, not simply by container size.

Weight is a relative measure. You may also ask the students if an item is heavy or not, such as a bag of sugar and a bag of chips. Some students may state that the sugar is not that heavy for them. If they all agree it is heavy, ask them if they think their parents would say the sugar was heavy. If any students claim an item is not heavy for themselves or their parents, ask them to hold the item for a period of time. As time passes, ask if it seems to become heavier and why. Heavy and light are relative terms. Whatever you pick up tends to be compared to what you picked up just prior or to what you pick up thereafter. So a beach ball feels much lighter than a basketball. However, it may not be so easy to determine how the weight of a volleyball compares to that of a basketball because they are very close in weight. Weight is also relative to your strength. What is a light weight for one person may be a heavy weight for another. Weight is relative in more ways than one.

2. Balance Scales

A balance pan scale is actually used to measure the mass of objects. Students need to have experience with these scales to know what the varying positions tell them about the mass of the objects being compared. Students with experience know that when the pans are balanced, the items have equal mass and when they are unbalanced, the higher item is lighter than the lower item. The difference in the relative levels of the pans is related to the difference between the objects' masses or weights. The larger the difference in pan levels, the greater the difference between their masses/weights.

Find out whether your students are familiar with a balance pan scale. Ask them if they know what it is for. Do they respond that it tells whether something is heavier or lighter than what is in the other pan or whether they are the same weight? If so, move on to check whether they know that the heavier item will tip its pan lower and that the pans will balance if the items weigh the same amount. Students with experience on see-saws will likely understand this. If students do not know what a balance pan scale does, then place a variety of objects opposite one another in the pans and observe what happens. Can the students figure out what is happening and why? Over the course of the year, you want to have one balance pan scale or several in the room and give all students opportunities to work with them directly.

Homemade equal-arm balance scales can be made by mounting a stake or dowel to a thick wooden block as the base. Drill a hole in the centre of a cross piece about 30–50 cm long and mount it over a nail hammered into the wooden upright on the base. Alternatively, you can use a crossbar of peg board two rows of peg holes deep. In this case, the dowel or stake would have a hook inserted upon which the crossbar would pivot. At each end of this swinging crossbar attach two pans. These pans can be made from the plastic lids of several containers, plastic containers, the bottoms of milk cartons or from small aluminum foil baking pans. Thread strings through three or four holes equally spaced on the sides of these pans and tie them to nails or hooks inserted in the ends of the crossbar. Adjust as needed until the pans hang in balance on each end of the crossbar. For several other alternatives, see *It's the Thought that Counts* (Lowery 1986, p. 172).

Sample Activity 2: Selecting the Right Nonstandard Units

You will need a variety of nonstandard units for students to use when finding the mass of objects. Once objects have a mass greater than a kilogram, standard weights will likely be needed, so select items to weigh that are less than a kilogram. Some possible nonstandard units could be nails, screws, washers, tiles, paperclips, pennies, sets of all the triangles, squares, trapezoids or hexagons from your pattern blocks, marbles, golf balls, Unifix cubes, multilink cubes, centicubes and other such manipulatives that you have in multiples and are the same size and weight. Items to weigh can include a plastic bug, a small toy car, a large hair barrette, a comb, a bar of soap, a package of gum, a shoe, a mitten, various balls, along with the many other items that are found in a Grade 2 classroom. When students suggest a nonstandard unit and find that one unit is heavier than the item to be weighed, they will want to revise their selection for a lighter nonstandard unit. As students select nonstandard units for weighing items, ask them to explain why they chose that unit or why they revised their choice of unit.

Sample Activity 3: Estimating the Number of Nonstandard Units

From the earliest measurements of mass done in the class, have the students estimate what the measure will be. As with linear measurement, until the students have some benchmarks or referents, they will likely be hesitant and estimates will vary widely. As they accumulate knowledge of the mass of some items, they will become more confident and more accurate. Again reassure them that no estimate is incorrect, but that the more measurements they do, the closer their estimates are likely to be to the actual measures.

Set up a station or, if you have a sufficient number of scales, organize an activity on which all the students can work in small groups. Give the students a list of the items to be weighed with a place to record their estimates and the actual weight/mass for each item. Follow up with discussion of nonstandard units chosen, the difficulties with making estimates and improvements in accuracy.

Sample Activity 4: Conservation of Mass

Some students may still be at a pre-conservation of mass stage. You can check by showing them two equal balls of Plasticine, one in each pan of the scales. Once they have noted that the masses are equal, as the pans are balanced, take out one ball of Plasticine.

- In plain view of the student, flatten the ball into a pancake.
- Before setting it back on the scale, ask the student whether it will be heavier, lighter or the same mass/weight as the other ball of Plasticine still on the scale.
- Students who expect it to be heavier because the mass has been redistributed have not yet attained conservation of mass.
- Then place the pancake of Plasticine in the pan and let the student observe that it does in fact still weigh the same.
- Allow the student opportunities to explore how mass does not vary with the arrangement of the object, then retest him or her.
- Place two identical decks of cards in elastic bands on the scale.
- Take one deck off the scale after the student has observed that they have equal mass.
- Remove the elastic band and spread the cards out.
- Before placing them back on the scale, ask the student which group of cards will weigh more or will they both weigh the same.
- Then place the spread out deck in the pan and let the student see that they both still weigh the same.

If you are looking for more activities to give to a student who needs them to understand conservation, see the Lawrence Lowery book listed in the bibliography. Research has shown that conservation of number must precede conservation of length, which will precede conservation of mass. The ages at which these are achieved vary somewhat. It is more likely that some Grade 2 students will not attain conservation of mass than conservation of length. Given sufficient opportunities to work with the scales and various materials, however, students will face evidence that challenges their current beliefs if they are still pre-stage or transition stage for conservation of mass.

Sample Activity 5: Ordering According to Mass/Weight, Ascending and Descending as Required

Now students need time to practise weighing various items and ordering them according to their mass from heaviest to lightest and from lightest to heaviest.

- Things to weigh will need to be carefully chosen so that they do not weigh more than a kilogram.
- Items heavier than that tend to require standard weights for efficient weighing with nonstandard units.
- The items need to be sufficiently different in weight so that students can discriminate between them with the nonstandard units that are available.
- It is best to make a collection of things that you have available for weighing and then weigh them to check the differences between their weights and what measures would result with some of the nonstandard units you have ready for use.
- Items could include: a plastic spoon, a metal spoon, a mug, an earring, an eraser, a pencil, a glue stick, a pen, various shoes, a cap, a toque, a mitten, spice jars the same size but full of spices that vary dramatically in weight, cans or packages of food, a tempera paint block, a paint brush, small toys and a deck of cards.
- It is important that all students have the opportunity to directly use the balance pan scales.
- If you do not have enough scales for all the students to work in pairs weighing items, set up a centre and a schedule for rotation through that centre until everyone has had one or more turns.
- Be sure the activity sheet that you create lists places for an estimate as well as the actual mass and the units.
- Students also need to be able to explain why they chose the units they did. You can ask the students to tell you why on the recording sheet or ask them to be ready to explain why to you as you visit them as they work.
- Students need to be able to explain the method they used to order the items. This explanation is likely to focus them back on their place value concepts. It also is a way of drawing students' attention to whether or not they have ordered the items in ascending order when the directions called for descending order.
- Be aware that students may use the transitive property and logic, rather than numbers, to order three items. If the scissors weighed more than the tape and the ball weighed more than the scissors, the student could deduce that the correct ascending order is ball, scissors, tape.
- As always, you want to encourage students to be aware of multiple ways to solve problems.

Sample Activity 6: Relating the Number of Units to the Mass/Weight of the Unit Used

Set up an activity requiring students to weigh items each with two or more different nonstandard units. Require an estimate first for each nonstandard weight to be used. You may dictate which units you want them to use or they may be given the freedom to use any two of the nonstandard units available to them. Your objective is to have the students study the pattern that emerges between the change in mass of the nonstandard units and the number of units required. You may want to ask for the first estimate and leave a place for them to complete the actual mass and state the nonstandard units used. Then, before they make their estimates of the number of second nonstandard units that will be required, ask if they predict it will take more or less of the next nonstandard unit. Following the notation of whether it will take more or less, they can make their estimates of how many it will take. If you have dictated the nonstandard units to be used in each case, it is easier to discuss as a group their measures and the inverse relationship that exists between the size of the unit and the number required.

Step 4: Assess Student Learning

Guiding Questions

- Look back at what you determined as acceptable evidence in Step 2.
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

In addition to ongoing assessment throughout the lessons, consider the following sample activities to evaluate students' learning at key milestones. Suggestions are given for assessing all students as a class or in groups, individual students in need of further evaluation, and individual or groups of students in a variety of contexts.

A. Whole Class/Group Assessment

Linear Measure

Name: _____ Date: _____

You and a group of friends were arguing about the length of some of your sports equipment. You were planning ways to check who was right. What ideas would you give the group about how to measure each of the things you were comparing?

1. One item you wanted to check was the length of your ice hockey sticks. Which of the following nonstandard units would you use to measure them and why?
   a. Unifix cubes b. base ten rods c. straws

   Why? _____

   _____

2. You also wanted to measure the length of your floor hockey sticks. Which one of the nonstandard units would you chose and why?
   a. centicubes b. Unifix c. stir sticks

   Why? _____

   _____

3. You wanted to measure the distances around a basketball and around a soccer ball so you could find out how much bigger the basketball is than the soccer ball. How would you measure the balls?

   _____

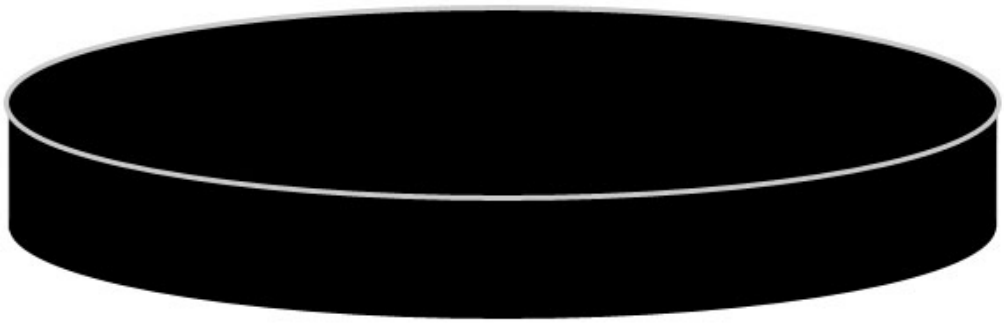
   _____

4. When you measured the distance across a Frisbee, it was 11 Unifix cubes wide. If you remeasured the width using centicubes, how many centicubes wide do you predict it would be and how did you make your prediction?

   _____

   _____

5. a. You only have one paperclip in your pocket and you want to measure the width of this puck. How would you do it?



_____

_____

b. How can you measure carefully so you will have the least possible measurement error?

_____

_____

c. How would you measure the width of this puck if you had a box of paperclips?

_____

_____

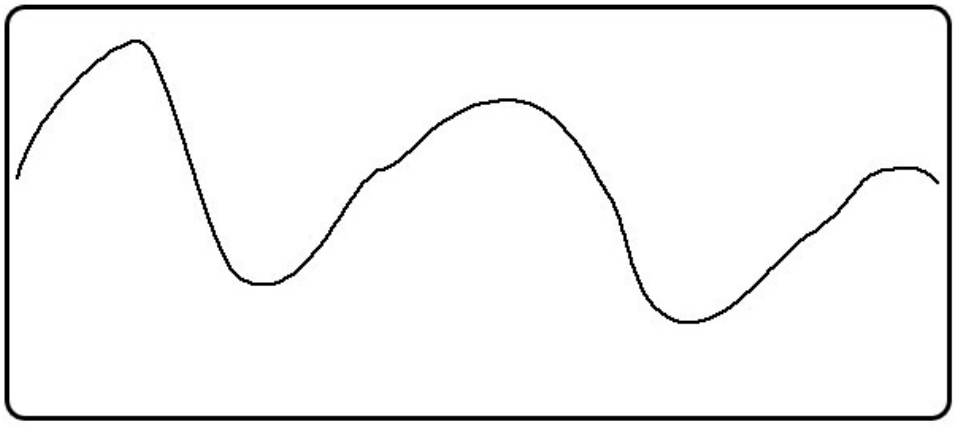
6. Your coach asked you all to bring to the game Thanksgiving weekend a can of food to give to the food bank. You wanted to find out whose can was tallest and whose can was biggest around. Before you measure your can, make your estimates of the distance around and height. Use the paperclips the teacher put out to make your measurements. Print the number that is on the can here:
Can number: _____

Height: estimate: _____ paperclips actual measure: _____ paperclips

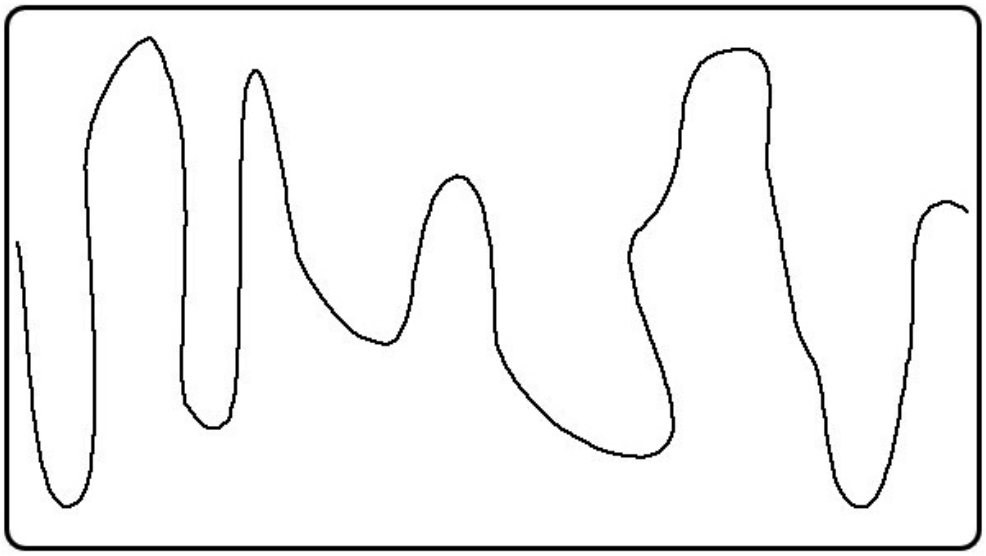
Distance Around: estimate: _____ paperclips actual measure: _____ paperclips

7. When you were skating down the ice, your path looked like the line below. Your friend's path running down the soccer field is below that. First print whose path you predict is longer. Measure each path. Whose path was longer and by how much? You may use any of the materials the teacher has provided.

   Path down the ice in hockey: This path is _____ _____ long.



   Path down the field in soccer: This path is _____ _____ long.



   The path that is longer is the _____ path by _____ _____ .

8. Measure the diagonal of this paper from the upper left corner to the bottom right corner. Put a green dot on the upper left corner and a red dot on the lower left corner.
   It is _____ _____ long.
   how many *units*
   Now turn the page sideways and estimate the length of the diagonal starting at the green dot and going to the red dot.
   I estimate it is _____ _____ long.

   I think I am right because _____

   _____

   _____

9. Measure and order the following objects at one of the measurement stations in the classroom. You may use any of the nonstandard units found there, but print which ones you used.

   a. The height of the box of macaroni is _____ _____ tall.
   how many *units*

   The height of the box of crackers is _____ _____ tall.
   how many *units*

   The height of the box of cereal is _____ _____ tall.
   how many *units*

   Order these objects from tallest to shortest:

   _____ , _____, _____

   b. The length of the small can of tomato paste is _____ _____ long.
   how many *units*

   The length of the can of beans is _____ _____ long.
   how many *units*

   The length of the can of juice is _____ _____ long.
   how many *units*

   Order these objects from smallest to largest:

   _____ , _____, _____

c. The distance around the small can of tomato paste is _____ _____ around.
how many *units*

The distance around the can of beans is _____ _____ around.
how many *units*

The distance around the can of juice is _____ _____ around.
how many *units*

Order these objects from largest to smallest:

_____ , _____, _____

Measure of Mass/Weight and Linear

Name: _____ Date: _____

Your twin cousins asked you to help them organize their room. They have some shelves on which to put their toys. They want to put the lightest toys on the top shelf and organize the rest by weight so that the heaviest things are on the bottom shelf. Your cousins have made an equal arm balance scale. They found some things in the garage, their room and their parents' computer room to use for nonstandard units: paperclips, washers, nails, marbles, pennies, blocks and golf balls.

Please help them by answering the questions below.

1. The twins wondered which one of these nonstandard units to use to weigh their

   a. little metal cars

      paperclips or small nails? They should use the
      _____ .

      Why?
      _____ .

   b. hockey cards

      golf balls or pennies? They should use the
      _____

      Why?
      _____

   c. rock collection

      marbles or golf balls? They should use the
      _____

      Why?
      _____

   d. plastic whistle

      paperclips or pennies? They should use the
      _____

      Why?
      _____

2. How many marbles do you estimate would have the same weight as a baseball glove?

   I estimate the baseball glove would have the same weight as _____ marbles.

   You couldn't find the marbles and so decided to use pennies instead. Do you estimate it would take more, fewer or the same number of pennies as marbles to equal the baseball glove?

   I estimate it will take _____ pennies than/as marbles to equal the weight of the baseball glove.

   How many pennies? _____

   Explain why you think so. _____

   _____

3. When it is your turn and no other student is there, go to a measurement station. Estimate the weight of the action figure there in the nonstandard units of your choice. Record your estimate and print the units used. Circle the letter of the station where you worked. Then weigh the action figure and record the actual weight and the units.

   I was at station A, B, C or D.

   My estimate: _____ _____.
   how many *units*

   My action figure actually weighed _____ _____ .
   how many *units*

4. a. While at the measurement station, weigh the following three items found there and put them in order from the heaviest to the lightest:

	plastic boat	rubber ball	metal car
Estimate with units			
Actual with units			

My order is
First: _____,
Second: _____,
Third: _____.

b. Weigh the second collection and order them from the lightest to the heaviest.

	eraser	scissors	roll of masking tape
Estimate with units			
Actual with units			

My order is
First: _____,
Second: _____,
Third: _____.

5. As the students placed things on the shelves, they needed to know how much room to leave for things, so they measured how long items were and marked the amount of room with a piece of tape on the shelf. They didn't have a ruler, so they used the things they had gathered from home or things they borrowed from their classroom. Here is a picture of the toy car they were placing on the shelf.

What nonstandard unit would you use to measure it and why?

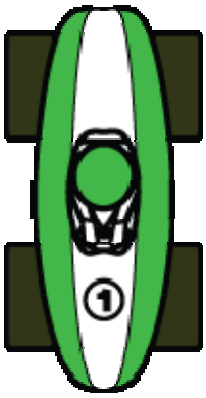
_____

_____

My estimate is _____ _____ .

Actual measure: _____ _____ .

Then the students decided it would be better if the car faced the other way, as shown below.



What length of shelf space do you predict it will need facing this way? _____

_____

How did you figure this out? _____

_____

B. One-on-one Assessment

Measurement

Directions	Date:	
	Not Quite There	**Ready to Apply**
Present the student with a single paperclip and say, **"Show me how you would measure this straw if you only had this one paperclip. Before you do, how many paperclips long do you estimate the straw to be?"**	• States that the straw cannot be measured with only one paperclip. • Does not give an estimate or gives one that is far from reasonable. • Measures the paperclip with fingers and uses three or four fingers to represent the size of the paperclip and measures to estimate. • Does not show the process of iteration. • Leaves unreasonable gaps or overlaps when measuring by iteration.	• Estimates reasonably using visualization of about how many paperclips would measure the length. • Uses the paperclip by flipping it or holding the end point with a finger to measure the number of paperclips it would take to measure the length of the straw.

Set a box of paperclips on the table. Ask the student, **"How could you measure the length of this straw using paperclips as your unit?"**	• The student measures again using a single unit. • The student measures with multiple copies of the same unit, but with notable gaps or overlap.	• The student demonstrates measurement with multiple copies of the same unit, careful not to leave gaps or to overlap and maintaining a relatively straight line.
Present the student with a box of paperclips and a box of popsicle sticks. Ask the student, **"Would you measure the window with the paperclips or the Popsicle sticks? Why?"** After the student responds to these questions, ask, **"When would you use the other unit to measure something?"** You can ask about other measurements to see whether this was one careless mistake. For example, **"Which would you use to measure the width of your desk? The width of a milk carton?"**	• The student chooses the paperclips to measure the window and doesn't have a good measurement reason. • The student does not explain that the paperclips would be used for something smaller than a Popsicle stick or not as large as two or more Popsicle sticks, while the sticks are used for items that are longer than two or more sticks.	• Student selects nonstandard units appropriate to the item to be measured. • Student explains clearly how the unit is chosen to allow a reasonably accurate measurement of the length/width/height of or distance around the object to be measured.

Make sure there are string and paperclips or links that a student could use to measure a curved surface accessible and in view of the student. Show the student a cylinder or can and ask, **"How would you measure the distance around this cylinder/can?"**	• The student states it cannot be done or does nothing. • The student puts string or paperclips/links around the cylinder, but does not make the loop tight around the shape. • The student does not stretch out the string and measure it with some nonstandard units like the paperclips or does not count the paperclips or links used to measure the can/cylinder. • The student does not keep track of where the count of paperclips or links started and so double counts some of them.	• The student uses the string to circle the curved surface, accurately marking where it meets itself. He or she lays it out straight and carefully uses nonstandard units to calculate the length of the string. • Alternatively, the student used links or paperclips to make a chain until it was large enough to fit snugly around the can or cylinder. If part of a link is required to make the circumference, the student allows for this by stating a little smaller than the number used or a little larger than one less than the number used.
Provide both Popsicle sticks and paperclips and a ribbon about 30 centimetres long. Ask the student, **"How many Popsicle sticks long do you estimate this ribbon to be?"** **"Now measure it."** **"Would it take more or less or the same amount of paperclips to measure the length of the ribbon? Tell me how you know."**	• Does not estimate the length of the ribbon in Popsicle sticks as about three. • Does not know/state whether it would take more or less or the same number of paperclips as Popsicle sticks or answers incorrectly. • Does not have an explanation or the explanation is based upon irrelevant information or misconceptions.	• Estimates the length of the ribbon in Popsicle sticks as three. • Recognizes that it will take more paperclips than Popsicle sticks for the measurement and explains that if the unit is smaller, more of them will be needed.

Provide a number of nonstandard linear measuring units. Present the student with a number of items to measure one at a time with nonstandard units (so they cannot just be physically compared and ordered). Provide a recording sheet on which the student can note his or her estimates and the actual measures and then to sequence the items. **"I am going to give you some items to measure one at a time from this bag. Estimate how long each one is and then measure it. Record your estimate and the actual measure on this sheet. When you have measured all the items from this bag, put them in order from longest to shortest on the sheet."** Repeat with different objects but have the student order them from shortest to longest.	• The student does not measure the three items with the same nonstandard units. • The student does not make an estimate or makes wild estimates. • The student leaves gaps or overlaps units. • The student does not measure in a straight line or on a curve as appropriate. • The student estimates and measures, but does not order the items either from longest to shortest or shortest to longest.	• The student measures all three items with the same nonstandard units after making reasonable estimates. • The student orders them accurately according to the specified sequence. • If the student sequences in reverse, when asked if the items are in order now from longest to shortest, he or she quickly recognizes the error and corrects it. In this case, the only work with the student needs to be about checking that the ordering is being done as asked.
Show a student a balance pan scale with an item in one pan such as a shoe being balanced by a number of marbles. Ask the student, **"If we took out the marbles and replaced them with pennies, would it take more pennies, less pennies or the same amount of pennies as the marbles?"** **"Explain how you know."** Have any student who errors in response count the marbles as removed and replace them with pennies. Then have the student count the pennies and compare the number of pennies and marbles it took. Then repeat the question, **"Did it take more, less or the same amount of pennies as marbles? Why?"**	• Student answers incorrectly either that it takes the same number or more pennies. • Student does not answer or states, "I don't know." • Student explains based on the misconception that a penny has a mass equal to a marble. • Student explains based on the misconception that the smaller the unit, the smaller the number required.	• Student states that more pennies will be needed than marbles because they weigh less and so you need more of them.

Provide a balance pan scale, a variety of nonstandard units and three or more items to be weighed that vary in appearance – not three identical jars of products with varying densities. Ask the student, **"Weigh each of these after making an estimate of the weight and then order them from heaviest to lightest."**	• Student uses different units to weigh each item. • Student does not make estimates. • Student does not balance the pans before "reading" the weights. • Student does not sequence the items from heaviest to lightest or the inverse.	• Student estimates the weight of each item before weighing. • Student uses the same nonstandard unit to weigh each. • Student balances the pans before counting the units. • The student orders the items from heaviest to lightest. Or, when asked if they are now in order from heaviest to lightest, the student recognizes his or her directions and self-corrects.

C. Applied Learning

Provide opportunities for the students to use their understanding of measurement in a practical situation and notice whether or not their understanding transfers. For example, the challenges to measure and order the students or the teachers by height for the school photographer may be a chance for students to apply their learning. Other challenges, such as "Who has the lightest running shoe and what kind is it?" may be given to encourage students to use their measurement skills for scientific investigations. Include discussion of why a person would want to have the lightest running shoe possible. Also encourage the students to indicate measurements that they are interested in making.

Step 5: Follow-up on Assessment

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction?

A. Addressing Gaps in Learning

The best way for students to fill in gaps in their learning is to be involved in measurement activities. Discussion with peers and questioning by adults will allow the students to confront some of their misconceptions, learn new techniques, increase their knowledge and use of referents to make better estimates, measure with more accuracy, transition to understanding conservation of length and mass, and recognize the inverse relationship between the size of the units used and the number required. Beginning measurement work early in the year and revisiting it regularly throughout the year will allow students the time and opportunities required for maximum success. Leaving the equipment and manipulatives available for free time activities will give students additional opportunities to experiment with measurement. Be aware that if the students who are finished their work early are the only ones able to practise measurement, the students who most need it may be at a disadvantage.

B. Reinforcing and Extending Learning

Students who have achieved or exceeded the outcomes will benefit from ongoing opportunities to apply and extend their learning. These activities should support students in developing a deeper understanding of the concept and should not progress to the outcomes in subsequent grades.

Consider strategies such as the following:

- Provide tips for parents on how to make an equal arm balance scale at home and things the student might use for nonstandard units such as pennies, paperclips, golf balls and nails. Encourage the parents to:
  - ask the child to estimate first before weighing
  - ask the child to order items from heaviest to lightest and at times from lightest to heaviest
  - ask the child why a particular unit was chosen or exchanged for a different nonstandard unit
  - ask the child if the nonstandard unit were exchanged for a lighter/heavier nonstandard unit, would it take more or less or the same number

- Provide tips for parents on linear measure for reinforcement and extensions at home. For example:
  - Reinforce the need to estimate first and encourage them to use nonstandard units such as paperclips, spoons, straws, coffee stir sticks, pennies or other things they have in multiple units that the children can lay end to end.
  - Advise them about the various methods, such as measuring with a single unit or multiple units and measuring around curved surfaces.
  - Also mention the need for attention to measurement error through gaps, overlap or not keeping a straight line.
  - Let them know that one of the goals is to order items in both ascending and descending order by measurement, not just directly physically comparing lengths, heights, widths or distance around an object.
  - Let them know that students are measuring themselves at school, so they are prepared for their children to ask to make wrist, ankle and other body measurements and comparisons.

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