Planning Guide

Grade 3

2-Digit Mental Mathematics

Number

Specific Outcomes 6 and 7

This Planning Guide can be accessed online at:
http://www.learnalberta.ca/content/mepg3/html/pg3_2digitmentalmathematics/index.html

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Planning Guide: *Grade 3 2-Digit Mental Mathematics*

Strand: Number
Specific Outcomes: 6, 7

This *Planning Guide* addresses the following outcomes from the program of studies:

Strand: Number	
Specific Outcomes:	6. Describe and apply mental mathematics strategies for adding two 2-digit numerals, such as: • adding from left to right. • taking one addend to the nearest multiple of ten and then compensating • using doubles. 7. Describe and apply mental mathematics strategies for subtracting two 2-digit numerals, such as: • taking the subtrahend to the nearest multiple of ten and then compensating • thinking of addition • using doubles.

Curriculum Focus

This sample targets the following changes to the curriculum:

- The general outcome for number in the revised program of studies states simply that the goal of this entire strand is to "develop number sense." The number strand is no longer divided into "number concepts" and "number operations." Number sense includes, for example, the ability to partition numbers and the ability to reason multiplicatively. These abilities apply to both numbers and operations.
- In the previous program of studies, general outcomes at each grade level specified magnitudes of numbers and particular operations, implying that number sense develops in a linear fashion. A more current understanding of student's mathematical development is that number sense deepens over time. Students' understanding of even 1-digit numbers is not fully developed until they are able to think of numbers abstractly (as quantities that do not necessarily refer to specific objects or as progressions on a scale).
- There were no specific outcomes in the previous program of studies regarding learning and teaching mental mathematics strategies. Mental mathematics strategies were referred to as possible methods for solving addition or subtraction, and students were required to justify the choice of these strategies. However, the strategies themselves were not taught. Very few students invent the most efficient mental mathematics strategies (strategies involving compensation) without the help of structured teaching, and most students benefit from

discussion of strategies involving place value splitting or using doubles. Subtraction is significantly more difficult for many students compared to addition. Teaching students how to think of subtraction as addition not only helps them with subtraction, it builds understanding of part–whole relationships.

- The inclusion of mental mathematics strategies for 2-digit numbers in the Grade 3 curriculum recognizes the importance of the ability to add and subtract "in the moment," without supportive technology (either paper and pencil or calculator), in daily life and in the workplace. Efficient mental mathematics strategies greatly reduce the difficulty of these calculations.

What Is a Planning Guide?

Planning Guides are a tool for teachers to use in designing instruction and assessment that focuses on developing and deepening students' understanding of mathematical concepts. This tool is based on the process outlined in *Understanding by Design* by Grant Wiggins and Jay McTighe.

Planning Steps

The following steps will help you through the Planning Guide:

Step 1: Identify Outcomes to Address

Guiding Questions

- What do I want my students to learn?
- What can my students currently understand and do?
- What do I want my students to understand and be able to do based on the Big Ideas and specific outcomes in the program of studies?

Big Ideas

- Computation is most efficient when supported by understanding.
- There are strategies that make mental computation easier to perform.
- Partitioning numbers in different ways can make mental computations easier to perform.
- Two-digit numbers can be split into tens and ones and mental computations can be done on these parts before they are recombined to arrive at a solution.
- Known facts, for example doubles, can be used in a variety of ways to help perform mental computations.
- An understanding of the combinations that make 10 supports effective mental computations.
- The ability to count by combinations of tens and ones supports effective mental computations.
- Knowing doubles facts as they apply to 1-digit numbers supports an understanding of doubles of multiples of 10.
- Understanding, memory and estimation sense are all supported when component parts with greater magnitude (the left-most columns in place value splitting) are operated on before those with lesser magnitude.
- Thinking of the operations of addition and subtraction in terms of parts and wholes helps support mental computation and understanding.

Sequence of Outcomes from the Program of Studies

See http://www.education.alberta.ca/teachers/program/math/educator/progstudy.aspx for the complete program of studies.

Grade 2	Grade 3	Grade 4
Specific Outcomes	**Specific Outcomes**	**Specific Outcomes**
10. Apply mental mathematics strategies, such as: • using doubles • making 10 • one more, one less • two more, two less • building on a known double • thinking addition for subtraction for basic addition facts and related subtraction facts to 18.	6. Describe and apply mental mathematics strategies for adding two 2-digit numerals, such as: • adding from left to right • taking one addend to the nearest multiple of ten and then compensating • using doubles. 7. Describe and apply mental mathematics strategies for subtracting two 2-digit numerals, such as: • taking the subtrahend to the nearest multiple of ten and then compensating • thinking of addition • using doubles.	5. Describe and apply mental mathematics strategies, such as: • skip counting from a known fact • using doubling or halving • using doubling or halving and adding or subtracting one more group • using patterns in the 9s facts • using repeated doubling to determine basic multiplication facts to 9 × 9 and related division facts.

Step 2: Determine Evidence of Student Learning

Guiding Questions

- What evidence will I look for to know that learning has occurred?
- What should students demonstrate to show their understanding of the mathematical concepts, skills and Big Ideas?

Using Achievement Indicators

As you begin planning lessons and learning activities, keep in mind ongoing ways to monitor and assess student learning. One starting point for this planning is to consider the achievement indicators listed in *The Alberta K–9 Mathematics Program of Studies with Achievement Indicators*. You may also generate your own indicators and use them to guide your observation of the students.

The following indicators may be used to determine whether or not students have met these specific outcomes. Can students:

- add two given 2-digit numerals, using a mental mathematics strategy, and explain or illustrate the strategy?
- explain how to use the "adding from left to right" strategy; e.g., to determine the sum of 23 + 46, think 20 + 40 and 3 + 6?
- explain how to use the "taking one addend to the nearest multiple of 10 and then compensating" strategy; e.g., to determine the sum of 28 + 47, think 30 + 47 – 2 or 50 + 28 – 3?
- explain how to use the "using doubles" strategy; e.g., to determine the sum of 24 + 26, think 25 + 25; to determine the sum of 25 + 26, think 25 + 25 + 1 or doubles plus 1?
- apply a mental mathematics strategy for adding two given 2-digit numerals?
- subtract two given 2-digit numerals, using a mental mathematics strategy, and explain or model the strategy used?
- explain how to use the "taking the subtrahend to the nearest multiple of 10 and then compensating" strategy; e.g., to determine the difference of 48 – 19, think 48 – 20 + 1?
- explain how to use the "adding on" strategy; e.g., to determine the difference of 62 – 45, think 45 + 5, then 50 + 12 and then 5 + 12?
- explain how to use the "using doubles" strategy; e.g., to determine the difference of 24 – 12, think 12 + 12 = 24?
- apply a mental mathematics strategy for subtracting two given 2-digit numerals.

Sample behaviours to look for related to these indicators are suggested for some of the activities listed in **Step 3, Section C: Choosing Learning Activities** (p. 9).

Step 3: Plan for Instruction

Guiding Questions

- What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should I use?
- How will I meet the diverse learning needs of my students?

A. Assessing Prior Knowledge and Skills

Before introducing new material, consider ways to assess and build on students' knowledge and skills related to mental computation of 2-digit numbers. For example:

- Take 10 minutes at the beginning of mathematics lessons to do mental mathematics. Have students work on addition and subtraction questions involving 1- and 2- digit numbers. Ask them to figure out answers without using pencil and paper, and to keep their answers to themselves until most of the class has an answer. Write down all the answers students come up with. Ask them how they figured out the answer and write that down. Check to see if students understand the approaches used by their peers.
- To see if students are able to use doubles for addition and subtraction to 18, ask them to solve 12 – 6 or 6 + 8.
- To see if students are able to use a making tens strategy for addition and subtraction to 18, ask them to solve 9 + 7 or 7 + 4.
- To see if students are able to build on a known double for addition and subtraction to 18, ask them to solve 7 + 8.
- To see if students are able to use addition to subtract, ask them to solve 17 – 13.

If a student appears to have difficulty with these tasks, consider further individual assessment, such as a structured interview, to determine the student's level of skill and understanding. See **Sample Structured Interview: Assessing Prior Knowledge and Skills** (p. 8).

Sample Structured Interview: Assessing Prior Knowledge and Skills

Directions	Date:	
	Not Quite There	**Ready to Apply**
Give the student the problem 15 + 3. Ask the student to solve the problem mentally, but have paper or counters available if the student is unable to provide a correct answer using mental mathematics. If the student appears to be counting from one, ask, **"Can you do it without counting everything?"** Then say, **"Please explain what you did to get that answer."**	Is not able to use counting on as a mental mathematics strategy.	Is able to use counting on or fact knowledge, without using counters, writing or representing on paper.
Give the student the problem 16 – 2. Ask the student to solve the problem without using paper or counters, but have these available to use if needed. Say, **"Please explain what you did to get that answer."**	Is not able to use counting back as a mental mathematics strategy.	Is able to use counting back or fact knowledge, without using counters, writing or representing on paper.
Give the student the problem 4 + 6 to answer. Say, **"Please explain what you did to get that answer."**	Does not know the different ways to make 10 from single digit addends without using a counting strategy.	Is able to make 10 from single digit addends by using knowledge of partitions of 10 rather than by counting.
Give the student the problems 7 + 7, 8 + 8 and 9 + 9 to answer. Say, **"Please explain what you did to get that answer."**	Does not know doubles facts for addition to 18.	Shows knowledge of doubles facts for addition to 18 by answering at least two of the three questions correctly, without having to count.
Give the student the problem 10 – 8 to answer. Say, **"Please explain what you did to get that answer."**	Is not able to think of subtraction as addition for an equation using numbers less than 10.	Is able to relate subtraction to addition, reverse numbers and count on or use fact knowledge to get a correct answer.

B. Choosing Instructional Strategies

Consider the following guidelines for teaching about mental computation of 2-digit numbers:

- Use an "Assessment for Learning" approach to ensure that students understand the learning intentions for all activities, understand what distinguishes appropriate strategies, receive descriptive feedback about their progress and have opportunities for self and peer assessment. For example, use the "Traffic Lights" for 2-digit mental computation master found at the end of this document. Have students use this tool for self-assessment of learning intentions both before and after learning about mental computation of 2-digit numbers.
- Teach mental mathematics in short 10-minute minilessons to help students develop particular mental mathematics strategies that might be useful in the problem-solving lessons and investigations that follow. Suggest to students that they might find mental mathematics strategies helpful when problem solving. Help students develop particular strategies by asking planned questions that encourage the use of the strategy and increase in difficulty and sophistication.
- In problem-solving situations, accept strategies that show less sophistication, such as counting on and counting back. If students are ready, encourage them to try more abstract strategies. Mental mathematics strategies are naturally differentiated so that students can successfully arrive at an accurate answer with a strategy that makes sense to them. This activity helps struggling students move toward more fluency in partitioning and manipulating numbers and parts of numbers. This fluency develops over time and through experience, leading to greater mathematical understanding, efficiency and abstraction.
- Ensure that there is adequate time for students to share and compare mental mathematics strategies with the rest of the class. During this time, ask if other students understand and can explain the strategy or use it to answer a similar question. Discuss the relative efficiency of particular responses in particular situations. Some strategies are equally efficient in some situations. Efficiency also depends on the level of understanding of the person using the strategy.
- Consider teaching mental computation throughout the year, and highlighting strategies in problem-solving contexts where they are useful, so that students, who may not have thought of using them to solve a problem, can appreciate their effectiveness.
- Use games that reinforce mental mathematics strategies.

C. Choosing Learning Activities

The following learning activities are examples of activities that could be used to develop student understanding of the concepts identified in Step 1.

Sample Activities:

1. **Adding Left to Right** (p. 10)
2. **Going to the Nearest Ten** (p. 11)
3. **Thinking of Addition** (p. 12)
4. **Using Doubles** (p. 14)

Sample Activity 1: Adding Left to Right

Setting the Task
Use the following or a similar sequence of questions to help students learn to add using the greater powers of 10, found at the left of numbers in standard place value notation, before the lesser powers of 10. Adding left to right is natural for students who have number sense for the magnitude of the numbers they are adding. Students do not add right to left unless they have been taught to do so. The mental activity of adding right to left works in opposition to number sense; students are unlikely to think about the magnitude of numbers and use it to estimate and evaluate numbers unless they are adding left to right.

Ask students to "solve it in your head, keep it to yourself" rather than calling out numbers as they perform their mental mathematics. Once a majority of the students in the class indicate they have a solution, begin to collect all the different solutions, without judging or ranking the possibilities. Then ask different students how they arrived at their solution. The main goal of this activity is to explore possible solution strategies, and consider their potential for accuracy and efficiency. Nonetheless, there can only be one right answer to these types of questions. As students share their solution strategies, encourage reflection and discussion about how different solutions came about and encourage the students themselves to articulate correct answers, as well as reasoning about incorrect answers.

Try This
This sequence is designed to encourage students to add left to right and to develop this as a strategy to use in mental mathematics. Any of the following questions can be skipped and the sequence condensed, if it is evident that most students already understand the strategy to the level of that particular question in the sequence. Alternatively, more questions can be added at any point to build a more resilient understanding of the strategy. For each question presented to students, it is important for them to explain how they arrived at their answer.

Ask, "What is 10 + 5?"
Ask, "What is 10 + 9?"
Ask, "What is 10 + 10?"
Ask, "What is 10 + 15?"
Ask, "What is 20 + 10?"
Ask, "What is 20 + 15?"
Ask, "What is 21 + 10?"

Ask, "What is 21 + 15?"
Ask, "What is 20 + 20?"
Ask, "What is 24 + 25?"
Ask, "What is 40 + 20?"
Ask, "What is 40 + 30?"
Ask, "What is 42 + 37?"
Ask, "What is 36 + 25?"

Follow-up Activity
Reinforce mental mathematics strategies for 2-digit numbers by having students play games with cards or dice. For example, have students play a game like "Pig." Two to six players can play this game at a time. The first player throws a pair of dice, adds the amounts together and remembers the sum. A player can roll as many times as he or she wishes in succession in a single turn, keeping a running total of the score, but if the player rolls a 1 his or her turn is over and the player receives no points for that turn. Players keep their points if they stop before they roll a 1. Points are recorded at the end of each turn. Other possible rules include losing all points and starting at zero if they roll a double 1. The first person to collect a total of 100 points wins.

Sample Activity 2: Going to the Nearest Ten

Setting the Task
Use the following or similar sequences of questions to help students learn to add and subtract by taking one addend or a subtrahend to a multiple of 10 and then compensating. In order to use this strategy, students will need to be able to partition 10 efficiently, and will therefore need to be fluent users of the "making 10" strategy from single-digit mental mathematics.

Ask students to "solve it in your head, keep it to yourself" rather than calling out numbers as they perform their mental mathematics. Once a majority of the class has a solution, begin to collect all the different solutions, without judging or ranking the possibilities. Then ask different students how they arrived at their solution. The main goal of this activity is to explore possible solution strategies, and consider their potential for accuracy and efficiency. Nonetheless, there can only be one right answer in these cases. As students share their solution strategies, encourage reflection and discussion about how different solutions came about and encourage students to self-correct and articulate the reasons for different answers.

Try This
This sequence is designed to encourage students to learn to move to a multiple of ten and compensate as a strategy to use in mental mathematics. Any of the following questions can be skipped and the sequence condensed, if it is evident that most students already understand the strategy to the level of that particular question in the sequence. Alternatively, more questions can be added at any point to build a more resilient understanding of the strategy. For each question presented to students, it is important for them to explain how they arrived at their answer.

Ask, "What is 20 + 10?"
Ask, "What is 20 + 15?"
Ask, "What is 18 + 6?"
Ask, "What is 19 + 15?"
Ask, "What is 29 + 17?"
Ask, "What is 38 + 14?"
Ask, "What is 47 + 38?"
Ask, "What is 22 + 59?"
Ask, "What is 50 – 10?"
Ask, "What is 50 – 9?"
Ask, "What is 52 – 9?"
Ask, "What is 63 – 20?"
Ask, "What is 63 – 17?"
Ask, "What is 87 – 38?"

Sample Activity 3: Thinking of Addition

Setting the Task
Use the following or a similar sequence of questions to help students learn to subtract by thinking of addition. This approach works best when the subtrahend is relatively close to the minuend. It makes less sense for equations like 54 – 6 = 48. In this case, students are more likely to work backward from 54. In a problem like 54 – 47, however, it makes sense to start with the subtrahend and determine the difference by adding on. The sequence below helps students understand this.

Follow a similar approach to that described in sample activities 1 and 2 to work with students on this strategy – asking questions and discussing strategies with the class as a whole in 10-minute minilessons.

Try This
The following sequence is designed to encourage students to think of addition in appropriate subtraction situations and to develop this as a strategy to use in mental mathematics. Any of the following questions can be skipped and the sequence condensed, if it is evident that most students already understand the strategy to the level of that particular question in the sequence. Alternatively, more questions can be added at any point to build a more resilient understanding of the strategy. For each question presented to students, it is important for them to explain how they arrived at their answer.

Ask, "What is 54 – 6?"
Ask, "What is 53 – 49?"
Ask, "What is 42 – 36?"
Ask, "What is 93 – 63?"
Ask, "What is 93 – 72?"
Ask, "What is 85 – 66?"

Follow-up Activity
Reinforce adding and subtracting strategies by playing a game like "Hundred Chart Leap Around." The goal of this game is to rack up the fewest number of leaps. For each group of two to four of players, you will need a hundred chart as a game board, a different coloured counter for each player and a grab bag of cards or chips with the numbers 1 through 100 on them. For the first move, each player draws a card with a number on it. Starting from zero, they leap to this target number by ones, fives or tens using as few leaps as possible. They can leap both forward and backward in a single turn. For example, if the target number is 68 and the player chooses the most efficient approach, the player will leap to 70 in leaps of ten, then backward two, for a total of 9 leaps. The player can also make the play in 10 leaps, by leaping 6 tens to 60, 1 five to 65 and 3 ones to 68.

For each subsequent turn, a player will draw a new number and leap from his or her present position to a new position in the least possible number of leaps. If the player on 68 draws 44, the player might leap back by 2 tens to 48, by 1 five to 43 and then forward by 1 one to 44.

The number of leaps is agreed upon and recorded on a score sheet at the end of each player's turn. Each player takes six turns. At the end of six turns, the player with the least number of leaps wins.

The simplest version of this game is played on a hundred chart with numbers on it. If the game is played on a blank hundred chart, however, students will need to use more of their number sense and mental calculation abilities. In this case, if the number cards are cut from another hundred chart of the same size (or the game is played using a pocket chart), they can be placed on the game board as they are drawn. This will help the players, as well as the teacher, keep track of the play. Drawing a vertical red line down the middle of the blank hundred chart, or colouring the left half of the chart in, can also help students visualize numbers, as they will be able to subitize the lines of five.

A more difficult version of this game is played with no game board. Players who are able to use mental mathematics strategies for addition and subtraction will be able to perform these calculations in their head. They may have some difficulty keeping track of their leaps with no game board. In this scenario, you might want to suggest they keep track using fingers or tally marks.

Sample Activity 4: Using Doubles

Setting the Task
Use the following or a similar sequence of questions to help students gain experience with adding and subtracting 2-digit numbers using their knowledge of the sums of doubles. Students will need to know their doubles facts from 1-digit mental mathematics, as well as near doubles in order to perform similar calculations with 2-digit numbers. This fact knowledge is often gained through life experience, but it is possible that students will not know and remember sums of doubles to 9 + 9. If this is the case, they will need experience with visual representations (as well as numerical representations) of these facts until they have automatic recall.

Follow a similar approach to that described in sample activities 1 and 2 to work with students on this strategy – asking questions and discussing strategies with the class as a whole in 10-minute minilessons.

Try This
The following sequence is designed to encourage students to use doubling and near doubling strategies to perform mental mathematics calculations. Any of the following questions can be skipped and the sequence condensed, if it is evident that most students already understand the strategy to the level of that particular question in the sequence. Alternatively, more questions can be added at any point to build a more resilient understanding of the strategy. For each question presented to students, it is important for them to explain how they arrived at their answer.

Ask, "What is 4 + 4?"
Ask, "What is 14 + 14?"
Ask, "What is 14 + 15?"
Ask, "What is 13 + 15?"
Ask, "What is 14 + 17?"
Ask, "What is 40 + 40?"
Ask, "What is 39 + 41?"
Ask, "What is 38 + 40?"
Ask, "What is 39 + 39?"
Ask, "What is 60 – 30?"
Ask, "What is 60 – 31?"
Ask, "What is 60 – 29?"
Ask, "What is 50 – 25?"
Ask, "What is 52 – 25?"

Follow-up Activity
Have students play games involving doubling to reinforce mental mathematics involving doubles. For example, have them draw dominoes from a grab bag, add the numbers together and then double the total in order to get their score for that turn. If a player draws a double, they can double it twice. That is, if a player draws double 3, which is 6, they double it to make 12 and then double that to make 24. Each player takes five turns. The one with the highest score at the end of the game wins.

Step 4: Assess Student Learning

Guiding Questions

- Look back at what you determined as acceptable evidence in Step 2.
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

In addition to ongoing assessment throughout the lessons, consider the following sample activities to evaluate students' learning at key milestones. Suggestions are given for assessing all students as a class or in groups, individual students in need of further evaluation, and individual or groups of students in a variety of contexts.

A. Whole Class/Group Assessment

2-Digit Mental Mathematics

The following test can be given to the whole class, to small groups or to individual students, either as a written test or orally. It can be broken up by giving one question or a few questions at a time. The test can be easily modified using different numbers to give students many chances to succeed.

Instruct students to do each problem in their head and to write down what they did in order to get the answer.

Circulate while students are completing each question to ensure that students are doing mental mathematics rather than using personal or learned pencil and paper strategies. Students should be spending the majority of their time writing about their mental process rather than solving the question itself. Note the time students are taking to solve the equation.

Allow students, who have difficulty expressing themselves in writing, to dictate their answers and have someone scribe for them, without discussing their strategy or prompting them in any way.

Alternatively, keep notes during classroom activities (as to which students understand and use which strategies), and assess for gaps in your records rather than giving this whole assessment to all your students.

Note that the following questions do not necessarily elicit the strategy being tested, as students' experience, understanding, preferences and style vary widely. The strategy that a student chooses to use may be equally or more efficient than the target strategy. If this is a problem, give a similar question, agree on an answer and ask students to write how a person would use the target strategy to solve the problem. For example, ask, "How would a person use the doubling strategy to solve the mental mathematics question of 40 – 18?"

Understanding Mental Mathematics

Name _____

Solve the following using mental mathematics. Explain what you did to get the solution.

1. 23 + 42 =

2. 39 + 27 =

3. 48 – 19 =

4. 62 – 45 =

5. 26 + 24 =

6. 24 – 12 =

SCORING GUIDE
2-Digit Mental Mathematics

Level / Criteria	4 Excellent	3 Proficient	2 Adequate	1 Limited	Insufficient / Blank
Calculating Left to Right	I was able to perform mental mathematics quickly, using an efficient strategy. I clearly explained my strategy so others could understand. ☐	I was able to use and explain this strategy. My answer was correct or close to correct. I was able to self-correct with minimal prompting. ☐	I was able to explain how this strategy worked, but I was not able to solve this equation correctly. ☐	I struggled to use or explain this strategy with understanding. ☐	I was not able to use or explain this strategy. ☐
Going to the Nearest Ten	I was able to perform mental mathematics quickly for both addition and subtraction, using an efficient strategy. I clearly explained my strategy so others could understand. ☐	I was able to use and explain this strategy. My answer was correct or close to correct. I was able to self-correct with minimal prompting. ☐	I was able to explain how this strategy worked, but I was not able to solve this equation correctly. ☐	I struggled to use or explain this strategy with understanding. ☐	I was not able to use or explain this strategy. ☐
Thinking of Addition	I was able to perform mental mathematics quickly, using an efficient strategy. I clearly explained my strategy so others could understand. ☐	I was able to use and explain this strategy. My answer was correct or close to correct. I was able to self-correct with minimal prompting. ☐	I was able to explain how this strategy worked, but I was not able to solve this equation correctly. ☐	I struggled to use or explain this strategy with understanding. ☐	I was not able to use or explain this strategy. ☐
Doubling	I was able to perform mental mathematics quickly for both addition and subtraction, using an efficient strategy. I clearly explained my strategy so others could understand. ☐	I was able to use and explain this strategy. My answer was correct or close to correct. I was able to self-correct with minimal prompting. ☐	I was able to explain how this strategy worked, but I was not able to solve this equation correctly. ☐	I struggled to use or explain this strategy with understanding. ☐	I was not able to use or explain this strategy. ☐

Name _____

B. One-on-one Assessment

1. Ask the student to solve 24 + 32 using mental mathematics and tell you how he or she figured out the answer.

   Use the following accommodations, if necessary:

   - Prompt the student to use and explain the left-to-right strategy.
   - Have the student use ten bars and unit cubes to show his or her thinking.

2. Ask the student to solve 19 + 28 using mental mathematics and tell you how he or she figured out the answer.

   Use the following accommodations, if necessary:

   - Prompt the student to use and explain the strategy of going to the nearest 10 in one of the addends and compensating.
   - Have the student use ten bars and unit cubes or written numbers to show his or her thinking.

3. Ask the student to solve 43 – 29 using mental mathematics and tell you how he or she figured out the answer.

   Use the following accommodations, if necessary:

   - Prompt the student to use and explain the strategy of going to the nearest 10 (in the subtrahend) and then compensating.
   - Have the student use drawings or written numbers to show his or her thinking.

4. Ask the student to solve 43 – 37 using mental mathematics and tell you how he or she figured out the answer.

   Use the following accommodations, if necessary:

   - Prompt the student to use and explain the strategy of thinking of addition in order to subtract.
   - Have the student use drawings or written numbers to show his or her thinking.

5. Ask the student to solve 19 + 20 using mental mathematics and tell you how he or she figured out the answer.

   Use the following accommodations, if necessary:

   - Prompt the student to use and explain how doubling a number could help solve this problem.
   - Have the student use ten bars and unit cubes, diagrams or numbers to show his or her thinking.

6. Ask the student to solve 22 – 11 using mental mathematics and tell you how he or she figured out the answer.

   Use the following accommodations, if necessary:

   - Prompt the student to use and explain how thinking of doubling could help him or her solve this problem.
   - Have the student use ten bars and unit cubes to show their thinking.

C. Applied Learning

Everyday experiences provide frequent opportunities to practise and perform mental mathematics calculations. Adding up a bill for purchases at a store or calculating distances in a car trip are just two examples where mental mathematics is a good choice for figuring out an answer. Games can provide further opportunities to practise calculations. Teachers can encourage mental mathematics by:

- creating experiences that simulate shopping
- having students participate in fundraisers, such as book or bake sales where shoppers need to add together purchases and make change, especially where items are selling for small amounts (under 50 cents)
- providing opportunities to play games that involve mental mathematics
- challenging students to make up their own games that involve mental mathematics
- introducing students to the online game "Ghostblasters" that students can play against each other to practise 2-digit mental subtraction: http://resources.oswego.org/games/Ghostblasters3/ghostsub3.html.

Step 5: Follow-up on Assessment

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction?

A. Addressing Gaps in Learning

Students who have difficulty with mental mathematics strategies may have gaps in their understanding of number. They may need more experiences with counting. Visual materials like ten frames may help them to develop the number sense they need to perform mental calculations. Activities involving dice or cards can also help build meaningful associations between representations of numbers and actual quantities. Having students practise mental mathematics skills using smaller numbers, e.g., numbers to 20, while developing understanding of larger 2-digit numbers, can help develop fluency.

For example:

- Ask students who are struggling with questions, like 42 + 37, to solve similar problems with smaller numbers, like 12 + 17.
- Photocopy sets of tiny ten frame cards with several tens included in each set. Students can use these to illustrate solutions to mental mathematics questions or to make up 2-digit equations for each other to solve.
- Allow students to use a calculator to perform or check calculations some of the time. Have them predict outcomes before entering anything into the calculator. Encourage them to break numbers up in different ways and add or subtract the pieces.
- Allow students to use ten bars and unit cubes to illustrate mental mathematics calculations.
- If available, use an arithmetic rack with 100 beads to illustrate student's thinking during discussions. Individual student arithmetic racks (also called rekenreks) with 100 beads also exist for students to use as visual models of mental mathematics.

B. Reinforcing and Extending Learning

Students who have achieved or exceeded the outcomes will benefit from ongoing opportunities to apply and extend their learning. These activities should support students in developing a deeper understanding of the concept and should not progress to the outcomes in subsequent grades.

Consider strategies, such as the following.

- Have students organize a mental mathematics competition or computation "bee." They will need to write 2-digit addition and subtraction questions on cards, and calculate the correct answers. The teams or individuals with the most correct answers will win. There can be a time limit for each question, but speed should not be the goal.

- Teach alternate subtraction strategies, such as cancelling out common amounts to make the question simpler (e.g., take 50 away from both numbers in the equation 77 – 58 = ? to make 27 – 8). Another strategy is the principle of constant difference. For constant difference, students add or subtract the same amount from each number in a subtraction equation, knowing the answer won't change. Instead of solving 97 – 59 =? they can add three to each number to create the question 100 – 62, which is much easier to solve.
- Encourage students to play more complex, challenging games. For example, in the game "Closest to 100," two people play with a regular card deck that has had the tens and face cards removed. Each player gets six cards in their hand. From these cards they need to select four cards to make two 2-digit numbers that add up to a number that is as close as possible, or equal, to 100. The difference between their number and 100 is their score for that turn. The cards that have been used are discarded and each player gets four more cards. Play continues until the deck is used up. Players add up their total score. The player with the lowest total wins.

Traffic Lights for Mental Addition

Colour the light
- green for **yes**
- yellow for **maybe**
- red for **no**

Student Name _____

Before		After
O	I can add numbers in my head by adding tens first, then ones, and adding those numbers together.	O
O	I can add numbers in my head by changing them to make them friendlier, then changing back.	O
O	I can add numbers in my head by using what I know about doubles.	O

Traffic Lights for Mental Subtraction

Colour the light
- green for **yes**
- yellow for **maybe**
- red for **no**

Student Name _____

Before		After
O	I can subtract numbers in my head by changing numbers to make them friendlier, then changing back.	O
O	I can subtract numbers in my head by thinking about addition.	O
O	I can subtract numbers in my head by using what I know about doubles.	O

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