Mathematics

Planning Guide

Grade 3
Introducing Division

Number
Specific Outcome 12

This Planning Guide can be accessed online at:
http://www.learnalberta.ca/content/mepg3/html/pg3_introducingdivision/index.html

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Planning Guide: *Grade 3 Introducing Division*

Strand: Number
Specific Outcome: 12

This *Planning Guide* addresses the following outcomes from the program of studies:

Strand: Number	
Specific Outcomes:	12. Demonstrate an understanding of division (limited to division related to multiplication facts up to 5 x 5) by: • representing and explaining division using equal sharing and equal grouping • creating and solving problems in context that involve equal sharing and equal grouping • modelling equal sharing and equal grouping using concrete and visual representations, and recording the process symbolically • relating division to repeated subtraction • relating division to multiplication.

Curriculum Focus

This sample targets the following changes to the curriculum:

- The general outcome for number strand in the revised program of studies states simply that the goal of this entire strand is to "develop number sense." The number strand is no longer divided into "number concepts" and "number operations." Number sense includes, for example, the ability to partition numbers and the ability to reason multiplicatively. These abilities apply to both numbers and operations.
- In the previous program of studies, general outcomes at each grade level specified magnitudes of numbers and particular operations, implying that number sense develops in a linear fashion. A more current understanding of student's mathematical development is that number sense deepens over time. Students' understanding of even 1-digit numbers is not fully developed until they are able to think of numbers abstractly (as quantities that do not necessarily refer to specific objects or as progressions on a scale).
- Two specific outcomes from the previous program of studies referred to both multiplication and division. In one outcome, students were to demonstrate the operations of multiplication and division; in another outcome, students were to calculate products and quotients using estimation and mental mathematics strategies. In the revised program of studies there is an outcome requiring students to demonstrate their understanding of division exclusive of multiplication. Although students are required to be able to relate multiplication and division, the separation of multiplication and division into discreet outcomes supports students' understanding of these two separate processes.

- Whereas the previous program of studies specified products to 50 and their corresponding quotients, the revised program of studies specifies multiplication to 5 × 5 and related division. The priority at Grade 3 is for students to construct a solid understanding of the process of division, which will be the basis for problem solving as well as recall and mental mathematics strategies in later grades.
- The revised program of studies requires that students relate multiplication to division and division to multiplication, again ensuring that understanding is in place before students are expected to use inverse operations to verify solutions.
- The revised program of studies includes relating division to repeated subtraction, ensuring that students connect their understanding of division to their prior knowledge.
- Understanding of the process of division is supported by the requirement that students create, as well as solve, problems.
- "Sharing" division (where an amount is divided between a specified number of groups) and "grouping" division (where an amount is divided into groups of a specified size) are both included in the specific outcome for division to ensure that students experience models and problems of both types of division.

What Is a Planning Guide?

Planning Guides are a tool for teachers to use in designing instruction and assessment that focuses on developing and deepening students' understanding of mathematical concepts. This tool is based on the process outlined in *Understanding by Design* by Grant Wiggins and Jay McTighe.

Planning Steps

The following steps will help you through the Planning Guide:

Step 1: Identify Outcomes to Address

Guiding Questions

- What do I want my students to learn?
- What can my students currently understand and do?
- What do I want my students to understand and be able to do based on the Big Ideas and specific outcomes in the program of studies?

Big Ideas

- In order to divide numbers effectively, students need to understand division in terms of narrative stories, concrete examples and mathematical models.
- Students need to construct their own understanding of division by using personally meaningful strategies in a problem-solving context.
- Division can be thought of in different ways; for example as repeated subtraction, as the number of groups of a particular size that can be made from a given whole, as the size of a particular number of groups that can be made from a given whole, and as the inverse of multiplicative relationships in rate, ratio and proportion situations.
- The ability to understand division is related to multiplicative understanding, which develops over a number of years as a students' mathematical thinking becomes more abstract and relational.
- Relating the idea of partitioning a whole into equal groups to the idea of arriving at a whole by repeating equal groups helps students relate division to multiplication.
- Relating the idea of partitioning an array into rows and columns to the idea of creating an array by repeating rows or columns helps students relate division to multiplication.

Sequence of Outcomes from the Program of Studies

See http://www.education.alberta.ca/teachers/program/math/educator/progstudy.aspx for the complete program of studies.

Grade 2	Grade 3	Grade 4
Specific Outcomes	**Specific Outcomes**	**Specific Outcomes**
9. Demonstrate an understanding of addition (limited to 1- and 2-digit numerals) with answers to 100 and the corresponding subtraction by: • using personal strategies for adding and subtracting with and without the support of manipulatives • creating and solving problems that involve addition and subtraction • using the commutative property of addition (the order in which numbers are added does not affect the sum) • using the associative property of addition (grouping a set of numbers in different ways does not affect the sum) • explaining that the order in which numbers are subtracted may affect the difference.	12. Demonstrate an understanding of division (limited to division related to multiplication facts up to 5 x 5) by: • representing and explaining division using equal sharing and equal grouping • creating and solving problems in context that involve equal sharing and equal grouping • modelling equal sharing and equal grouping using concrete and visual representations, and recording the process symbolically • relating division to repeated subtraction • relating division to multiplication.	7. Demonstrate an understanding of division (1-digit divisor and up to 2-digit dividend) to solve problems by: • using personal strategies for dividing with and without concrete materials • estimating quotients • relating division to multiplication.

Step 2: Determine Evidence of Student Learning

Guiding Questions

- What evidence will I look for to know that learning has occurred?
- What should students demonstrate to show their understanding of the mathematical concepts, skills and Big Ideas?

Using Achievement Indicators

As you begin planning lessons and learning activities, keep in mind ongoing ways to monitor and assess student learning. One starting point for this planning is to consider the achievement indicators listed in *The Alberta K–9 Mathematics Program of Studies with Achievement Indicators*. You may also generate your own indicators and use them to guide your observation of the students.

The following indicators may be used to determine whether or not students have met this specific outcome. Can students:

- identify events from experience that can be described as equal sharing?
- identify events from experience that can be described as equal grouping?
- illustrate, with counters or a diagram, a given story problem, presented orally, that involves equal sharing, and solve the problem?
- illustrate, with counters or a diagram, a given story problem, presented orally, that involves equal grouping, and solve the problem?
- listen to a story problem; represent the numbers, using manipulatives or a sketch; and record the problem with a number sentence?
- create and illustrate, with counters, a story problem for a given number sentence; e.g., $\frac{6}{3} = 2$?
- represent a given division expression as repeated subtraction?
- represent a given repeated subtraction as a division expression?
- relate division to multiplication by using arrays and writing related number sentences?
- solve a given problem involving division?

Sample behaviours to look for related to these indicators are suggested for some of the activities listed in **Step 3, Section C: Choosing Learning Activities** (p. 10).

Step 3: Plan for Instruction

Guiding Questions

- What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should I use?
- How will I meet the diverse learning needs of my students?

A. Assessing Prior Knowledge and Skills

Before introducing new material, consider ways to assess and build on students' knowledge and skills related to division. For example:

- Use a narrative situation; for example, read the story, *The Doorbell Rang*, by Pat Hutchins (1989, Greenwillow Books), and have students model or draw division situations from the story and make reasonable predictions or estimations about outcomes.
- Create an activity where students have to share or divide a quantity that will yield a remainder. For example, use paper circles to represent cookies. Create groups of three or four students, and give 26 cookies to each group. How would they divide these cookies between them? Have them talk about how they decided to deal with the remainder. They may push those extra cookies away or give them to you, decide that some students can have one more, or cut the cookies into fractional parts. All are acceptable ways, as long as the students are able to accept the reality of this kind of situation and attempt to deal with it.
- Ask students if they can help to figure out real situations involving equal sharing, such as how many people would be on each team if the class were divided into four teams for a gym activity.
- Ask students if they can help to figure out real situations involving equal grouping, such as how many relay teams could be created from the number of students in the class if each team had four members.

If a student appears to have difficulty with these tasks, consider further individual assessment, such as a structured interview, to determine the student's level of skill and understanding. See **Sample Structured Interview: Assessing Prior Knowledge and Skills** (p. 8).

Sample Structured Interview: Assessing Prior Knowledge and Skills

Directions	Date:	
	Not Quite There	**Ready to Apply**
Set out 5 dishes. Give the student 15 counters. Say, **"Can you share out these counters evenly and fairly? How many are in each dish?"** If the student divvies out each object individually, ask, **"Can you do that a faster way?"**	The student does not put an equal number of objects in each dish. The student is unable to say the number of counters in each dish by counting the number in one dish. The student needs to divvy out the objects one by one.	The student uses estimating and grouping to create equal shares, and accurately counts one set to answer the question, knowing that each dish will have the same amount.
Tell a story such as the following: You are having some friends over to play. There will be 4 children at your house. You have 16 cookies. If you divide up the cookies fairly, how many cookies will each child get? Say, **"Please use counters or tallies to show me how you would figure this out."**	The student is not able to model the story. The student does divide 16 objects 4 ways by making fair and equal shares.	The student is able to relate a story with a division context to a number operation modelled using counters or tallies.

B. Choosing Instructional Strategies

Consider the following guidelines for teaching about division:

- Use an "Assessment for Learning" approach to ensure that students understand the learning intentions for all activities, understand what distinguishes quality work, receive descriptive feedback about their progress and have opportunities for self and peer assessment. For example, have students write about their understanding of division situations and notation both before and after learning about division, and have them compare their answers to see what they have learned. (This activity is adapted from *About Teaching Mathematics: A K–9 Resource* (p. 203) by Marilyn Burns, Copyright 2000 by Math Solutions Publications.)
- Invite students to estimate answers before solving problems and to reflect on their predictions after a task has been completed.
- Teach division through a problem-solving approach. Use contexts that are real or relevant to your own students. Make sure each student understands division in a story context or as an investigation of a real-life problem, and relates numbers and symbols to concrete objects and relationships, before moving on to abstract representations of division using just numbers and symbols outside of a context.
- Use different contexts for division, including both equal grouping and equal sharing problems.
- Relate division to different multiplicative contexts, such as comparison problems (something is a number of times as small as something else) and area or array problems (where the total is known but one of the sides is not).
- Encourage students to think about, discuss, act out or explain the situation in a problem before beginning to solve it.
- Encourage learners to model problems with objects or draw pictures, diagrams or charts to represent the elements and relationships in problems in order to help them build connections between visual and kinesthetic understanding and abstract mathematical understanding.
- Encourage students to write equations that first describe the way they have already represented the action in a problem and solved the problem, and later translate a problem into an equation in order to solve it.
- Plan a significant amount of time for students to compare strategies and approaches with the rest of their class at the end of a problem-solving lesson. During this time, ask questions about the efficiency and mathematical thinking of particular responses in order to encourage greater abstraction and mathematical elegance.
- Differentiate equal sharing division problems by allowing students to use different solution strategies, starting with using counters or tallies to share one by one or by trial and error for students who are just beginning to develop an understanding of equal sharing division, to estimating amounts and sharing out in groups, to building on known facts or relating division to multiplication as students develop more understanding of division.
- Differentiate equal grouping division problems by allowing students to use different solution strategies, starting with counting and trial and error for students who are just beginning to develop an understanding of equal grouping division, through repeated addition and subtraction and on to building on known facts and relating division to multiplication as students develop more understanding of division.
- If necessary, differentiate tasks by substituting smaller or larger numbers.

C. Choosing Learning Activities

The following learning activities are examples of activities that could be used to develop student understanding of the concepts identified in Step 1.

Sample Activities:

1. **Equal Grouping Division Problems** (p. 11)
2. **Equal Sharing Division Problems** (p. 12)
3. **Relating Division to Multiplication** (p. 14)

Sample Activity 1: Equal Grouping Division Problems

Build a Problem
(An example of an equal grouping problem occurs when you need to figure out how many relay teams you would have if you divided your class into relay teams of four members each.)

Examples of equal grouping problems have, in the past, been less common in school mathematics than equal sharing problems. It is important to give students experience of both types of problems. The advantage of equal grouping division problems is that they promote the development of repeated subtraction strategies, where equal sharing problems do not. Solving equal grouping problems will help your students develop efficient personal strategies for dealing with division, and will help them think about division in more abstract ways.

Use problems that are relevant to your students according to their own context. For example, if your students have experience organizing an open house, party or other event where participants will sit at tables, you can give them a problem in which they need to figure out how many tables will be needed for 25 participants, if each table sits four people.

Solve the Problem
Have students get down to work to solve this problem using personal invented strategies and whatever manipulatives, diagrams or symbols they choose. Circulate throughout the classroom and make notes as to the solution strategies students use and how they explain what they are doing in response to teacher questioning. Fast finishers can go on to another question; for example, how many small jugs of juice will be needed if each jug is good for three people?

Share Solutions
When most of the class has arrived at a solution, choose three or four students, who used different strategies, to draw their solutions on the board, on flip chart paper or on transparencies to use with an overhead. Call the class together and have each of the students you have chosen explain their strategy to the class. Have other students reflect on what they understood about the presentation and how it compares to their approach.

Follow-up Activities
Pose similar problems in subsequent mathematics classes, always leaving time for discussion. Modify problems to address learning needs that arise during working sessions or discussions. If students do not move past drawing and counting strategies, try modelling that you can draw a shape to represent a table, juice jug, or whatever the problem is about, and label it with a numeral. This might help to push students to think in groups, rather than counting one by one. Eventually, students will start to think just with numbers and not with drawing; however, using drawing and counting strategies is acceptable at the Grade 3 level. Students will also need to learn to use numbers to repeatedly subtract the group amount and count the number of subtractions, in order to show understanding of using repeated subtraction to solve a division problem.

Extensions
When you are satisfied that students have an understanding of division and a repertoire of solution strategies, have students make up problems for each other or the class.

Students who demonstrate that they understand division can work with problems with larger numbers.

The following activity is adapted from *Young Mathematicians at Work: Constructing Multiplication and Division* (pp. 57–63) by Catherine Twomey Fosnot and Maarten Dolk, Copyright 2001 by Heinemann.

Sample Activity 2: Equal Sharing Division Problems

(An example of an equal sharing division problem occurs when you need to figure out how many students would be on each team if you divided your class into four equal teams.)

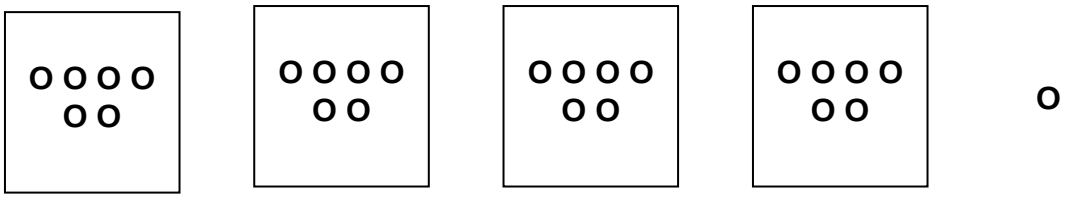
Create equal sharing problems that are appealing to your students. Problems can be fanciful, imaginative or humorous as well as realistic. For example, students often enjoy problems involving animals (e.g., a farmer had 23 cows to distribute as equally as possible between 6 pens) or imaginative situations where they are the hero (e.g., Charlie won 17 trucks in a lottery and decided to divide them between himself and 3 friends). Students can suggest characters and objects for problems that the class writes together, with the teacher supplying appropriate numbers.

It is not necessary to teach students about the difference between equal sharing and equal grouping problems. The different types of division problems do not need to be taught discreetly, and students do not need to learn to identify the different types. Because of the narrative action in these two types of problems, however, students will use different approaches to solve them until their thinking has developed to the point where they just see them as division problems. It is important that teachers understand which type of problem is being solved and discussed, because the development of abstract understanding and the use of increasingly sophisticated solution strategies is different for the two types of division.

Most students relate to equal sharing problems because they have been in situations where it was important to create fair shares. However, it is quite difficult for students to invent more abstract strategies for these problems on their own. The narrative action in equal sharing division problems does not lend itself to repeated subtraction strategies. Students only begin to invent more abstract and sophisticated solution strategies for equal sharing problems when they are able to relate division to multiplication and think about division abstractly, solving it using similar strategies regardless of whether it is equal grouping or equal sharing.

To begin with, students will get the idea of distributing equal shares one by one and then in small groups using trial and error, creating increasingly reasonably sized groups by estimating the likely size of a group based on the number of objects that are left to distribute. Students will begin by drawing or modelling equal sharing narratives using pictures, tallies or counters. They will distribute counters until they are all used up. For example, they might create a finished piece of work that looks similar to the graphic below:

25 rescued Dalmatians need to be shared fairly between 4 friends.



As well, students should be encouraged to describe the outcome of the problem and write it down on their paper, or dictate it orally for someone to scribe; e.g., each friend got 6 Dalmatians and there was an extra one.

Encourage students to distribute counters or make tallies more than one at a time. Ask if they think there are enough objects to take them two at a time and distribute them that way? What about in larger groups? As they begin to distribute in groups, their work will look very similar to the work shown above, but they will be using a more difficult strategy, one involving partitioning and estimating. It is important that the teacher observes these strategies and makes notes, or asks students to explain their strategies in an individual interview or in a group discussion.

Even so, students will either count out objects and then count the size of the groups (and copy their work onto paper) or they will make tallies or marks and need to count and recount as they go, making sure they have distributed all the items that are represented by the dividend. This is very inefficient, because it involves a lot of recounting. Encourage them to keep track of amounts, using numbers to make note of the total distributed so far somewhere on their page (or make a mental note and keep a running tally), so when they distribute the next batch they can add everything together, instead of recounting. This will be too difficult for some Grade 3 students.

Continue to use questioning to encourage those students who are able to use larger groups and friendly numbers to do so, and to record amounts to keep a running total. Next, if students are comfortable with the strategies used so far, encourage them to record the amounts using written numerals in a chart, rather than using marks or tallies on a drawing. For example, if a student knew how to count by fives, they might use the following strategy, which is more efficient than divvying up by ones or twos:

25 rescued Dalmatians need to be shared fairly between 4 friends.

5 1	5 1	5 1	5 1	20 used 4 more used 1 left over

A chart is quick to draw, helps students keep track of their work and allows them to add up numbers easily. Students need to have had the opportunity to make sense of equal sharing problems using drawings. They can use a chart if it seems quicker and easier to them and it makes sense. Numbers can be written vertically or horizontally, with or without an addition sign. At this stage students should be able to write a sentence with their solution in it, for example, "Each friend got 6 Dalmatians with one left over." Encourage students to translate this into mathematical notation, writing a complete division equation. Many students will need to see this modelled a number of times before they are able to write it themselves.

As students also work with equal grouping problems and multiplication as it relates to division, they will begin to recognize equal sharing problems as division problems. If they have made this abstraction, they might use repeat subtraction and known facts to solve equal sharing problems.

Sample Activity 3: Relating Division to Multiplication

Multiplicative thinking involves the ability to think of a multiplicative situation in terms of part–whole relationships. When students are able to think this way, they are also able to use this understanding in division situations. With a little help and guidance, they are able to connect multiplication with division in a meaningful way. An important model for part–whole relationships in multiplication and division is the array model of multiplication. This model is also a key to helping students connect multiplication and division. Connecting equal grouping division to equal sharing division is an important step in understanding multiplication and division. It will help students learn to use part–whole reasoning to connect multiplication and division.

In order to help students connect equal sharing to equal grouping, devise two division problems that use the same numbers and can be modelled using an array. One problem, should be an equal sharing problem and one problem should be an equal grouping problem. For example, pose the following two problems to students and ask them to draw a picture as part of their solution for each problem:

1. Fifteen children went to the movies. They sat near each other with 3 children in the first row and 3 children sitting behind them in each row after that until they all had a seat. How many rows of children were there?
2. Fifteen children went to the movies. They sat in 3 rows, with the same number in each row. How many children were in each row?

Gather as a class and discuss solutions to the problem and any relationship between the two problems. What can you do to the array in the first problem to make it match the array in the second problem (rotate 90 degrees). This activity encourages students to move beyond narrative thinking when solving problems, to think in terms of numbers and relationships, and to employ models that support abstract mathematical thinking to solve problems. In this case, using arrays allows them to think in terms of parts and wholes, and relate equal sharing division to equal grouping division.

To connect multiplication to division, pose another pair of problems:

1. If there are 24 students in the class and the teacher wants groups of four to do an activity, how many groups will there be?
2. If the teacher wants to give out pencils to students who are in groups, and she puts 4 pencils in each of 6 cans, how many pencils is that in all?

After students solve both problems, convene the class for a discussion. What is the relationship between the two problems? How were they solved? Were they multiplication or division problems? In many cases, students will use multiplication or repeated addition to solve the first problem, and the class might conclude that they are both multiplication problems. It isn't helpful to correct this—in fact, it is an important insight to be able to see the multiplicative relationships in a division problem.

To reinforce the relationship between multiplication and division, ask the students to solve a problem like:

The teacher had 24 pencils to share between 6 tables. How many pencils were on each table?

Ask the students whether this is a division or multiplication problem. What makes it a division problem? What if she were sharing out the pencils 6 to each table and wondering how many tables she could get from 24 pencils? Are these the same situations? What is the relationship between the numbers in these two situations? Can we use an array to draw this to help us see why the numbers are the same, even though the situations are different? Can we make multiplication problems using the same array?

What are the relationships between all these different problems? This is a far more powerful illustration of the relationship between multiplication and division than teaching "fact families," which students can learn to write without thinking about quantity or relationship. Make sure students all get the chance to articulate their understanding of all the different relationships between multiplication and both types of division. They can do this by teaching another student, writing about it in a mathematics journal entry, or speaking up in the group about their understandings.

Step 4: Assess Student Learning

Guiding Questions

- Look back at what you determined as acceptable evidence in Step 2.
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

In addition to ongoing assessment throughout the lessons, consider the following sample activities to evaluate students' learning at key milestones. Suggestions are given for assessing all students as a class or in groups, individual students in need of further evaluation, and individual or groups of students in a variety of contexts.

A. Whole Class/Group Assessment

Introducing Division

The following test, Understanding Division, can be given to the whole class, to small groups or to individual students, either as a written test or orally. It can be broken up by giving one question or a few questions at a time. The test can be easily modified using different numbers to give students many chances to succeed.

Give students the opportunity to read questions and discuss the meaning of the question orally with a partner before beginning to solve the problem on their own on paper.

Circulate while students are completing each question and discuss the meaning of the question with them. Ask students to explain their thinking, and make notes when their thinking is not clearly illustrated in the solution they have recorded on paper. Make additional notes of which students need additional help interpreting questions, and which students need additional time to talk about the meaning of questions before they can begin to work on a solution, using an appropriate strategy that makes sense of the meaning of the question.

Understanding Division

Name _____

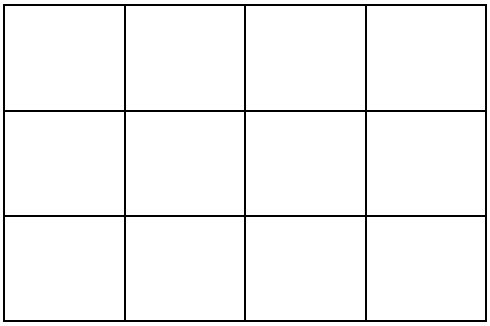
1. How many cars would you need to take students on a field trip if each car could take 3 students and there were 20 students? Show how you would figure this out using numbers, drawing or counters. Write an equation to show your answer.

2. What would you type into the calculator to figure out your answer to question 1?

3. What would you type into a calculator to figure out an answer to question 1 if both the division and multiplication key on your calculator were broken?

4. There are 19 students on a field trip. Their teacher needs to divide them into 4 groups. How many students will be in each group? Show how you would figure this out using numbers, drawing or counters. Write an equation to show your answer.

5. Write as many equations as you can think of to describe this array. Make sure to include multiplication and division equations.



SCORING GUIDE
Introducing Division

Level / Criteria	4 Excellent	3 Proficient	2 Adequate	1 Limited	Insufficient / Blank
Equal Grouping Division	I solved an equal grouping problem using an efficient strategy. My written work was organized and clearly communicated my thinking. I wrote an appropriate division equation to help solve my problem. ☐	I solved an equal grouping problem and showed my thinking on paper. I was able to write an equation to illustrate my thinking about the problem. ☐	I used an appropriate strategy to attempt to solve an equal grouping problem. I was unable to write a complete and accurate equation to illustrate my solution. ☐	I was unable to solve an equal grouping division problem. My work shows limited understanding of division. ☐	I was not able to show any understanding of how to solve a division problem. ☐
Division Notation and Division as Repeated Subtraction	I can translate a division problem into notation in order to solve it using a calculator. I showed that I was also able to think of the problem in terms of repeated addition or subtraction. ☐	I showed the division problem on a calculator. I also showed an alternate strategy or related expression on the calculator. ☐	I was able to use the calculator to show an expression that related to the division problem. ☐	My response does not show understanding of division or a solution strategy that would work for this problem. ☐	I was not able to answer this question in any way. ☐
Equal Sharing Division	I solved an equal sharing problem using an efficient strategy. My written work was organized and clearly communicated my thinking. I wrote an appropriate division equation to help solve my problem. ☐	I solved an equal sharing problem and showed my thinking on paper. I was able to write an equation to illustrate my thinking about the problem. ☐	I used an appropriate strategy to attempt to solve an equal sharing problem. I was unable to write a complete and accurate equation to illustrate my solution. ☐	I was unable to solve an equal sharing division problem. My work shows limited understanding of division. ☐	I was not able to show any understanding of how to solve a division problem. ☐
Relating Division to Multiplication	I was able to show at least four correctly written equations to illustrate an array, including at least one multiplication equation and one division equation. My work was well organized and clearly laid out. ☐	I showed at least one division and one multiplication equation to describe an array. My equations were complete and correct. ☐	I showed either a division equation or a multiplication equation to describe an array. ☐	I used numbers to label or describe an array. ☐	I was not able to use numbers to label or describe an array. ☐

Name _____

B. One-on-one Assessment

1. Ask the student to make up a story problem showing division into groups of equal size. Then ask the student how they would figure out the solution.

   Use the following accommodations, if necessary:

   - Suggest contexts, such as creating groupings of snack items for sale, creating sets of a favourite type of toy and creating groups of students to participate together in an activity.
   - Prompt the student to find a quicker way to count groups if he or she appears to be counting and recounting individual items.
   - Provide a calculator for the student to show his or her understanding of the operation of division or successive subtraction/addition (students are more likely to use addition to build successive groups as they work toward the total, rather than subtracting groups from the total).

2. Ask the student to make up a story problem showing division into equal groups. Then ask the student how he or she would figure out the solution.

   Use the following accommodations, if necessary:

   - Suggest contexts, such as dividing up toys, supplies or snack items between friends.
   - Prompt the student to find a quicker way to distribute and count groups if he or she is distributing and counting individual items.
   - Provide a calculator for the student to show his or her understanding of the operation of division.

3. Show the student a 5 × 3 array. Ask him or her to make up a few different division and multiplication story problems based on the array, and ask the student to explain how he or she would solve these problems.

   Use the following accommodations, if necessary:

   a. Prompt the student to talk about the array in terms of the total numbers of objects, as well as the number in each row or in each column.
   b. Suggest contexts, such as packing fruit (or other items) into boxes, sitting in a block of seats at a theatre, or creating floor covering using tiles.
   c. Prompt the student to use the same array to create different problems using both multiplication and division.
   d. Provide the student with a calculator to show how these different problems could be solved.

C. Applied Learning

Provide opportunities for students to experience and describe division situations and contexts in everyday life:

- Give students responsibility for distributing snack items, school supplies or craft supplies into equal portions for equal sharing division or sets of a particular size for equal grouping division. Have students figure out the size or number of sets and deal with remainders as they occur in real-world situations. Practise writing equations that correspond to these shared experiences of division.
- Have students use tiles to create arrays or make grid patterns to cut apart as part of an art project. Have them reassemble arrays or grids to create rectangles of different sizes. Talk about the numbers of rows and columns created using different totals, as well as about leftovers that result when different arrays are made; for example, when a 3×6 array is made from an original 4×5 array, leaving two tiles leftover.
- Have students figure out ways to cut a rectangular cake into a certain number of pieces. For example, if there are 23 students in the class, what is the best way to cut the cake?
- Have students figure out division problems like, if we have 25 minutes left in the day, and we want groups to be able to rotate around each of 4 stations, how many minutes will each group get at each station?

Step 5: Follow-up on Assessment

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction?

A. Addressing Gaps in Learning

Students who have difficulty understanding division will benefit from ongoing experiences with mathematical narratives and contexts that involve division situations. These repeat experiences allow students to develop understanding of the action of division, relate different types of division to each other, and relate multiplication to division.

For example:

- Have students make up their own problems to solve or make problems with or for a partner that involve personal details, such as their own names, birthdays, favourite foods, colours, toys or animals. Alternatively, have them imagine scenarios where they have objects to distribute or organize, such as animals on a farm, riches in a castle or cars in a mansion.
- Encourage students, who are having difficulty, to dramatize division narratives so they can imagine division in relationship to their lived experience.
- Create story problems that use small numbers, simple contexts and no extraneous information. Introduce difficulty gradually, and reduce difficulty again if it becomes a block to understanding or diminishes confidence to the point where a student cannot proceed.
- Provide students with the opportunity to solve problems using a choice or any combination of models, objects, counters, drawings, oral language, written language and symbols. Do not restrict solution strategies at first. Observe students' solution strategies, and encourage solution strategies that are just a little more difficult; for example, moving from distributing counters one by one to distributing in groups using tallies, to distributing by using numbers that you keep track of by skip counting.
- Allow students to figure out answers to problems in their own way, usually without writing an equation first. Encourage students to narrate answers to division problems using oral language that they or someone else can then write down. When they are fluent at this step, they can begin to write equations that describe what they have done. In the beginning, especially at the Grade 3 level, these equations are written after the problem has been solved, not before.
- Have students work with the book, *The Doorbell Rang*, by Pat Hutchins (1989, Greenwillow Books). They can use stuffed toys and make cookies out of paper in order to act out the division situations in this story to first predict and then check what will happen before they turn each page. Each time they figure out the solution to a situation, they can write the division equation, or choose from a set of pre-made equations on strips of paper, in order to describe the situation using mathematical notation.

B. Reinforcing and Extending Learning

Students who have achieved or exceeded the outcomes will benefit from ongoing opportunities to apply and extend their learning. These activities should support students in developing a deeper understanding of the concept and should not progress to the outcomes in subsequent grades.

Consider strategies, such as the following.

- Have students use their understanding of division to figure out more difficult applied learning problems, such as how to measure and divide a piece of paper so that strips of equal size can be cut for a craft activity; e.g., paper weaving.
- Have students write problems involving different kinds of division for their peers or family members to solve.
- Have students write a story for other students modelled on the book, *The Doorbell Rang*, by Pat Hutchins (1989, Greenwillow Books). They should use different numbers, and decide if their book is going to deal with remainders, or just present division situations with no remainders.

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