Mathematics

Planning Guide

Grade 3

Introducing Multiplication

Number

Specific Outcome 11

This Planning Guide can be accessed online at:
http://www.learnalberta.ca/content/mepg3/html/pg3_introducingmultiplication/index.html

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Planning Guide: *Grade 3 Introducing Multiplication*

Strand: Number
Specific Outcomes: 11

This *Planning Guide* addresses the following outcomes from the program of studies:

Strand: Number	
Specific Outcomes:	11. Demonstrate an understanding of multiplication to 5 × 5 by: • representing and explaining multiplication using equal grouping and arrays • creating and solving problems in context that involve multiplication • modelling multiplication using concrete and visual representations, and recording the process symbolically • relating multiplication to repeated addition • relating multiplication to division.

Curriculum Focus

This sample targets the following changes to the curriculum:

- The general outcome for the number strand in the revised program of studies states simply that the goal of this entire strand is to "develop number sense." The number strand is no longer divided into "number concepts" and "number operations." Number sense includes, for example, the ability to partition numbers and the ability to reason multiplicatively. These abilities apply to both numbers and operations.
- In the previous program of studies, general outcomes at each grade level specified magnitudes of numbers and particular operations, implying that number sense develops in a linear fashion. A more current understanding of student's mathematical development is that number sense deepens over time. Students' understanding of even 1-digit numbers is not fully developed until they are able to think of numbers abstractly (as quantities that do not necessarily refer to specific objects or as progressions on a scale).
- Two specific outcomes from the previous program of studies referred to both multiplication and division. In one outcome, students were to demonstrate the operations of multiplication and division; in another outcome, students were to calculate products and quotients using estimation and mental mathematics strategies. A different outcome dealing with the recall of addition and subtraction facts also required the recall of multiplication facts to 49. In the revised program of studies there is an outcome requiring students to demonstrate their understanding of multiplication in a variety of specified ways, and a similar outcome for

division. Although students are required to be able to relate multiplication and division, the separation of multiplication and division into discreet outcomes supports students' understanding of these two separate processes.

- Whereas the previous program of studies specified products to 50, the revised program of studies specifies multiplication to 5 x 5. The priority at Grade 3 is for students to construct a solid understanding of the process of multiplication, which will be the basis for problem solving as well as recall and mental mathematics strategies in later grades.
- The revised program of studies requires that students relate multiplication to division and division to multiplication, again ensuring that understanding is in place before students are expected to use inverse operations to verify solutions.
- The revised program of studies includes relating multiplication to repeated addition, ensuring that students connect multiplication to their prior knowledge, and recognizing that most students in Grade 3 think additively rather than multiplicatively. This outcome prepares students for the transition from additive reasoning to multiplicative thinking that will begin for some of them as early as Grade 3.
- Similarly, students are exposed to multiplication both as equal groupings, which can be interpreted additively, and arrays, which support the transition to multiplicative thinking.
- Understanding of the process of multiplication is supported by the requirement that students create, as well as solve, problems.

What Is a Planning Guide?

Planning Guides are a tool for teachers to use in designing instruction and assessment that focuses on developing and deepening students' understanding of mathematical concepts. This tool is based on the process outlined in *Understanding by Design* by Grant Wiggins and Jay McTighe.

Planning Steps

The following steps will help you through the Planning Guide:

Step 1: Identify Outcomes to Address

Guiding Questions

- What do I want my students to learn?
- What can my students currently understand and do?
- What do I want my students to understand and be able to do based on the Big Ideas and specific outcomes in the program of studies?

Big Ideas

- In order to multiply numbers effectively, students need to understand multiplication in terms of concrete examples and mathematical models.
- Students need to construct their own understanding of multiplication by using personally meaningful strategies in a problem-solving context.
- Multiplication can be thought of in different ways; for example, as repeated addition, as numbers of equal groups, as arrays and as proportional relationships.
- Students develop a robust understanding of multiplication over a number of years.
- Although multiplication can be understood as repeated addition, a complete understanding of multiplication is more complex, and involves the ability to think of numbers as ways to represent relationships as well as concrete quantities, the ability to think in terms of many-to-one relationships and the ability to reason proportionally.
- Being able to relate the idea of arriving at a whole by repeating equal groups to the idea of partitioning a whole into equal groups is the foundation for relating multiplication to division.

Sequence of Outcomes from the Program of Studies

See http://www.education.alberta.ca/teachers/program/math/educator/progstudy.aspx for the complete program of studies.

Grade 2	Grade 3	Grade 4
Specific Outcomes	**Specific Outcomes**	**Specific Outcomes**
9. Demonstrate an understanding of addition (limited to 1- and 2-digit numerals) with answers to 100 and the corresponding subtraction by: • using personal strategies for adding and subtracting with and without the support of manipulatives • creating and solving problems that involve addition and subtraction • using the commutative property of addition (the order in which numbers are added does not affect the sum) • using the associative property of addition (grouping a set of numbers in different ways does not affect the sum) • explaining that the order in which numbers are subtracted may affect the difference.	11. Demonstrate an understanding of multiplication to 5 × 5 by: • representing and explaining multiplication using equal grouping and arrays • creating and solving problems in context that involve multiplication • modelling multiplication using concrete and visual representations, and recording the process symbolically • relating multiplication to repeated addition • relating multiplication to division.	6. Demonstrate an understanding of multiplication (2- or 3-digit by 1-digit) to solve problems by: • using personal strategies for multiplication with and without concrete materials • using arrays to represent multiplication • connecting concrete representations to symbolic representations • estimating products • applying the distributive property.

Step 2: Determine Evidence of Student Learning

Guiding Questions

- What evidence will I look for to know that learning has occurred?
- What should students demonstrate to show their understanding of the mathematical concepts, skills and Big Ideas?

Using Achievement Indicators

As you begin planning lessons and learning activities, keep in mind ongoing ways to monitor and assess student learning. One starting point for this planning is to consider the achievement indicators listed in *The Alberta K–9 Mathematics Program of Studies with Achievement Indicators*. You may also generate your own indicators and use them to guide your observation of the students.

The following indicators may be used to determine whether or not students have met this specific outcome. Can students:

- identify events from experience that can be described as multiplication?
- represent a given story problem, using manipulatives or diagrams, and record the problem in a number sentence?
- represent a given multiplication expression as repeated addition?
- represent a given repeated addition as multiplication?
- create and illustrate a story problem for a given number sentence; e.g., 2 × 3 = 6?
- represent, concretely or pictorially, equal groups for a given number sentence?
- represent a given multiplication expression, using an array?
- create an array to model the commutative property of multiplication?
- relate multiplication to division by using arrays and writing related number sentences?
- solve a given multiplication problem?

Sample behaviours to look for related to these indicators are suggested for some of the activities listed in **Step 3, Section C: Choosing Learning Activities** (p. 10).

Step 3: Plan for Instruction

Guiding Questions

- What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should I use?
- How will I meet the diverse learning needs of my students?

A. Assessing Prior Knowledge and Skills

Before introducing new material, consider ways to assess and build on students' knowledge and skills related to multiplication. For example:

- Have students use skip counting in a concrete situation, such as counting money or counting out a snack to arrive at a total amount.
- Have students use counters or pictures to figure out a multiplication situation, such as how many treats would they need if they were having a party for five students and each student was going to get three treats. Are they able to conceive of this kind of a situation and arrive at an answer using concrete modelling and counting?
- Have students write a mathematics journal entry describing what they know about multiplication.

If a student appears to have difficulty with these tasks, consider further individual assessment, such as a structured interview, to determine the student's level of skill and understanding. See **Sample Structured Interview: Assessing Prior Knowledge and Skills** (p. 8).

Sample Structured Interview: Assessing Prior Knowledge and Skills

Directions	Date:	
	Not Quite There	**Ready to Apply**
Set out 5 dishes. Ask the student to put 2 counters in each dish. Say, **"Can you tell me how many that is in all?"** If the student counts each object individually, ask, **"Can you do that a faster way?"**	The student is unable to put an equal number of objects in each set. The student is unable to give the correct count for the total number of objects. The student needs to count each object one by one.	The student uses skip counting, doubling or a known fact to arrive at an answer in a multiplication situation.
Tell a story such as the following: You are planning a party. There will be a total of 6 friends at the party. You want to cook 2 hot dogs for each friend. How many hot dogs will you need to buy? Say, **"Please use counters or tallies to show me how you would figure this out."**	The student is not able to model the story. The student does not show 6 equal groups of 2 objects or tallies.	The student is able to relate a story with a multiplicative context to a number operation modelled using counters or tallies.

B. Choosing Instructional Strategies

Consider the following guidelines for teaching about multiplication:

- Use an "Assessment for Learning" approach to ensure that students understand the learning intentions for all activities, understand what distinguishes quality work, receive descriptive feedback about their progress and have opportunities for self and peer assessment. For example, have students write about their understanding of a multiplication situation and notation both before and after learning about multiplication, and have them compare their answers to see what they have learned. (This activity is adapted from *About Teaching Mathematics: A K–9 Resource* (p. 203) by Marilyn Burns, Copyright 2000 by Math Solutions Publications.)
- Invite students to estimate answers before solving problems and to reflect on their predictions after a task has been completed.
- Teach multiplication through a problem-solving approach. Use contexts that are real or relevant to your own students. Make sure each student understands multiplication in a story context or as an investigation of a real-life problem, and relates numbers and symbols to concrete objects and relationships, before moving on to abstract representations of multiplication using just numbers and symbols outside of a context.
- Use different contexts for multiplication, including equal grouping problems, multiplicative comparison problems (something is a number of times as large as something else), combinations and area or array problems.
- Encourage students to think about, discuss, act out or explain the situation in a problem before beginning to solve it.
- Encourage learners to model problems with objects or draw pictures, diagrams or charts to represent the elements and relationships in problems in order to help them build connections between visual and kinesthetic understanding and abstract mathematical understanding.
- Encourage students to write equations that first describe the way they have already represented the action in a problem and solved the problem, and later translate a problem into an equation in order to solve it.
- Plan a significant amount of time for students to compare strategies and approaches with the rest of their class at the end of a problem-solving lesson. During this time, ask questions about the efficiency and mathematical thinking of particular responses in order to encourage greater abstraction and mathematical elegance.
- Differentiate multiplication problems by allowing students to use different solution strategies, starting with counting for students who are just beginning to develop an understanding of what multiplication is, to skip counting, repeated addition, repeated doubling and building on known facts as students develop multiplicative understanding.
- If necessary, differentiate tasks by substituting smaller or larger numbers.
- Use student's literature to teach about equal groups; for example, use *Each Orange Had 8 Slices: A Counting Book,* by Paul Giganti (1992, HarperCollins) or *What Comes in 2s, 3s & 4s*, by Suzanne Aker (1990, Aladdin Paperbacks). Lessons can be planned around these or similar books that involve students solving problems suggested by the books, students making individual pages to be compiled into a similar class book (What comes in 5s, 6s, 7s and 8s?), or students devising problems based around some of the examples in the books for their peers to solve.

C. Choosing Learning Activities

The following learning activities are examples of activities that could be used to develop student understanding of the concepts identified in Step 1.

Sample Activities:

1. **Solving and Writing Multiplication Problems** (p. 11)
2. **Using Repeated Addition** (p. 12)
3. **Exploring Arrays** (p. 13)
4. **Many Ways to Multiply** (p. 14)

Sample Activity 1: Solving and Writing Multiplication Problems

Build a Problem
Create a set of problems that are relevant to your students according to their own context. For example, at a school with outdoor ping-pong tables, the following problems are relevant:

- Ping-Pong balls are sold in packages of 4. The principal started the year by buying 4 packages. How many Ping-Pong balls is that?
- There are approximately 5 weeks during the fall when students can play Ping-Pong outdoors. If they go through 3 balls a week, how many will they need?
- Did the principal buy enough?
- There are 4 weeks during the month of May when students can play Ping-Pong. How many balls will they need?
- If the Ping-Pong balls cost 2 dollars a package, how much did the principal spend in the fall?
- How many packages will the principal need to buy for the month of May?
- What will that cost?
- Can you predict how many packages the principal will need to buy for the month of June? What will that cost?

Note that these problems are specific to a particular context where students understand the need for purchasing a certain number of a specific piece of equipment at regular intervals, because they need and use that equipment. In the Ping-Pong ball example, numbers that refer to time, packaging and cost are realistic. Begin by asking just one question from the problem set. Do not model a specific way to solve this problem; however, have students discuss, rephrase or dramatize the problem in order to understand the narrative action in the problem.

Solve the Problem
Have students get down to work to solve that problem using personal strategies and whatever manipulatives, diagrams or symbols they are comfortable using. Circulate throughout the classroom and make notes as to what solution strategies students use and how they explain what they are doing in response to teacher questioning.

Share Solutions
When most of the class has arrived at a solution, choose three or four students who used different strategies to draw their solutions on the board, on flip chart paper or on transparencies to use with an overhead. Call the class together and have each of the students you have chosen explain their strategy to the class. Have other students reflect on what they understood about the presentation and how it compares to their approach.

Follow-up Activities
Continue posing problems from this or another set, at a rate of one or two problems a day, always leaving time for discussion and modifying the problem set in order to address learning needs that arise during working sessions or discussions. When you are satisfied that students have an understanding of multiplication and a repertoire of solution strategies, have students make up problems for each other or the class according to a set of stipulations, such as the numbers in the problems need to equal 5 or less, the context needs to be realistic and the problem needs to relate to real experience.

Extensions
Students who demonstrate that they understand multiplication can work with problems with products greater than 25.

Sample Activity 2: Using Repeated Addition

Act Out a Problem

As a warm-up activity, pose a problem in a narrative context. For example, how many cookies would you need for 5 students, if each student got 4 cookies? Select students to act out the story or illustrate it using drawing or manipulatives in front of the class. Ask students to figure out the answer on their calculators without using the multiplication key. Have them write down or explain how they got their answer to the rest of the class. Have students compare answers obtained using repeated addition with the answer they get by using the multiplication key. If they don't already know how to press the equals key repeatedly in order to add the same number a number of times, teach them this shortcut once they have shown they can find the answer using repeated addition. Pose alternate problems on different days to practise repeated addition, both with and without a narrative context, and then introduce the conventional way to write an multiplication expression using symbols and numbers.

The above activity is adapted from *Elementary and Middle School Mathematics: Teaching Developmentally* (p. 130) by John A. Van de Walle and Sandra Folk, Copyright 2005 by Pearson Education Canada Inc.

Circles and Stars

Have students work with a partner for the following activity. Each pair should have a die and scraps of paper (or they can make 8-page booklets). Show students how to play the game "Circles and Stars." The first partner rolls a die and draws that many large circles. The second partner does the same. Each partner then rolls the die again, in turn, and draws that many stars inside each of their circles. Together, participants figure out each of the totals. The total for each drawing is written on the drawing, under the circles and stars.

Have students play seven rounds with their partner. The winner is the one with the most stars overall, which requires students to add up the stars in all of their drawings.

Next, ask students to work with their partner to figure out appropriate equations to write under each drawing. They can write equations showing repeated addition; they should also be taught how to write multiplication equations. They should then translate the meaning of the equation into words, such as "2 groups of 3 equals 6" for the equation 2 × 3 = 6.

Have pairs join up with other groups to compare the numbers and products that resulted from playing the game. Explore their observations and discoveries in a whole class discussion.

The above activity is adapted from *About Teaching Mathematics: A K–9 Resource* (p. 196) by Marilyn Burns, Copyright 2000 by Math Solutions Publications.

Sample Activity 3: Exploring Arrays

Using Arrays to Develop an Understanding of Multiplication
Depending on the environment in which students live and work, different examples of arrays from real-life contexts can be brought into classroom discussions and problem solving. Working with arrays helps students construct an understanding of multiplication that is more abstract than skip counting or successive addition, one in which they are able to conceive of numbers of groups and numbers in each group at the same time, without having to account for each item.

The development of students' understanding of arrays begins with a completely linear understanding of items in an array. Next they learn to think about either successive rows or successive columns as groups of equal size, skip counting or using repeated addition to determine a product. Students then learn to think about both rows and columns at the same time. Finally, students learn to recognize that the units along the edge of an array represent a multiplicative situation, one in which a corner unit can be counted in both a row and a column. This level of abstraction indicates true multiplicative reasoning.

Arrays in Daily Life
Arrays might be found in the daily life of students in the packaging of food or drink items in rectangular boxes, in architecture (e.g., windows), in building construction (e.g., floor tiles), in repeated patterns on fabrics, such as those used in rectangular window coverings, and in baking trays. Once you have found some examples from contexts to which your students can relate, there are several ways to manipulate these contexts to help students move through the developmental stages of understanding arrays and develop multiplicative reasoning.

Try This
Use small arrays, with five or fewer items in each row or column. Students can subitize these numbers, which means they are less likely to count everything by ones, and more likely to think in groups. This encourages students to use skip counting, successive addition and doubling strategies to calculate products. To further discourage counting by ones, arrays can have hidden or covered items; for example, a tiled floor that is somewhat obscured by furniture, or patterns on a window covering where one of two blinds or curtains is pulled up or back. The questions are always, "How many?" and "How did you figure that out?" whether problems are posed to individuals, small groups or the whole class, on the board, orally or on a printed worksheet. Most of the time, students should record their thinking in words, either written or scribed, and translate their thinking into conventional expressions or equations using numbers and symbols.

Extension
An extension activity is to use an array repeatedly. As students calculate, for example, the number of drinks in four 6-packs, they are encouraged to use doubling to arrive at a product. A more difficult example involves using a combination of filled and partially filled arrays, such as 2 filled and 2 partially filled egg cartons or muffin tins (examples of either 3 × 2 arrays or larger examples with products of 12, 16 or 20). When arrays are partially filled, always show either complete rows or complete columns. Using combinations of filled and partially filled arrays encourages students to actually see relationships between different numbers and groupings, and builds multiplicative reasoning visually (the way ten frames build number sense and pave the way for mental mathematics).

Sample Activity 4: Many Ways to Multiply

Prior Knowledge
Draw on prior knowledge by discussing with all students in the class whether they have noticed any numbers appearing repeatedly in the multiplication activities they have done so far. What observations can students make when comparing these numbers to each other, for example, in relation to repeated doubling or successive addition?

Setting the Task
For this activity, students will work in small groups. You will need a large number of coloured tiles (at least 24 for each group), as well as 2 cm grid paper. If you have access to different shaped chocolate boxes (especially ones with a variety of shapes, including one with a single row of candies), bring them in to show to students.

To set a context, ask the students to imagine that their job is to design flat, rectangular boxes to sell chocolates or candies. For example, if you wanted to make a box that would hold four candies, what are all the possible designs? Have the students suggest and then construct these designs with their tiles.

Pose the following problem: what are all the design possibilities for boxes that will hold 6, 9, 12, 18 and 24 candies? Have students find the possibilities using their tiles, then make and cut out the design using the grid paper. Students can use a large sheet of paper to create a poster of their results and present their findings in a class discussion once this investigation is complete.

Variations and Extensions
This type of investigation lends itself to a variety of variations and extensions. For example, have the students themselves generate the numbers they wish to investigate, based on the initial discussion described in this sample activity. Compare results for the different numbers and draw conclusions based on doubling or repeated groupings. Make predictions about the possible shapes and numbers of solutions for larger boxes such as would hold 30, 36 or 48 candies. Discuss possibilities for boxes with two or three layers. Explore the possible shapes of boxes for square numbers, such as 1, 4, 9, 16 and 25. Make and test conjectures about the number of solutions for squares of odd numbers versus squares of even numbers. Make all possible arrays for all the numbers from 1 to 12 or higher.

Follow-up Discussion
The basic idea of this study is to investigate all possible ways to arrive at a given product. Have students write equations and also record their thoughts on paper throughout these investigations. Have frequent whole class discussions to provide opportunities for students to share and explore ideas about multiplication.

These discussions will also provide teachers with some excellent opportunities to introduce ideas related to geometry, such as the definition of a square as a special type of rectangle, or the understanding that area is measured using square units – therefore, all the rectangular arrays made for a given product have the same area.

Step 4: Assess Student Learning

Guiding Questions

- Look back at what you determined as acceptable evidence in Step 2.
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

In addition to ongoing assessment throughout the lessons, consider the following sample activities to evaluate students' learning at key milestones. Suggestions are given for assessing all students as a class or in groups, individual students in need of further evaluation, and individual or groups of students in a variety of contexts.

A. Whole Class/Group Assessment

Introducing Multiplication

The following test, Understanding Multiplication, can be given to all students in the class, to small groups or to individual students, either as a written test or orally. It can be broken up by giving one question or a few questions at a time. The test can be easily modified using different numbers to give students many chances to succeed.

Circulate while students are completing each question and discuss the meaning of the question with them. Allow students, who have difficulty expressing themselves in writing, to dictate their answer to a question and have someone scribe for them. Make notes on how students approach tasks. Do students need help to interpret questions? Do they easily understand the meaning of the equations, or do they need to talk about equations and translate them into narrative language in order to understand them? If students are working on unusual or unexpected solutions, ask them to explain what they are thinking.

Understanding Multiplication

Name _____

1. Make up a story problem for the equation 6 x 3 = ?

2. Imagine the multiplication key on your calculator is broken. Write down what you would type into your calculator to solve the equation 4 x 3 = ? if you didn't know what the answer was.

3. Show 5 x 4 using an array. Solve for the total and write an equation showing your answer. Draw another array with the same product (total), but that illustrates a different equation. Write this equation, too.

4. Use the back of this paper to show all the possible ways to use multiplication to get a product (answer) of 24. Draw pictures and write an equation for each of the ways.

SCORING GUIDE
Introducing Multiplication

Level / Criteria	4 Excellent	3 Proficient	2 Adequate	1 Limited	Insufficient / Blank
Writing a Multiplication Problem	I wrote a believable story problem that involved 6 groups of 3 things. My problem illustrated an example of multiplication in a real-world context. ☐	I wrote a believable story problem that involved 6 groups of 3 things. ☐	I wrote a story problem that showed multiplication. ☐	My story problem did not show multiplication. ☐	I was not able to make up a story problem. ☐
Understanding Multiplication	I was able to show a way to solve a multiplication problem on a calculator without using the multiplication key. I wrote down all the steps clearly and in order. ☐	I showed that I could solve this problem on a calculator without using the multiplication key. ☐	I showed an alternate way to solve this problem, or I solved it correctly without showing how I would use a calculator with a broken multiplication key. ☐	My response does not show understanding of multiplication or any appropriate process to find a solution to this question. ☐	I was not able to answer this question in any way. ☐
Understanding Arrays	I made an array to show 5 x 4, wrote and solved this as an equation, made a second array with the same product and wrote a correct second equation. My work was well organized, detailed and showed understanding. ☐	I was able to make 2 arrays with the same product and write at least one correct corresponding equation. ☐	I was able to show an array and the correct product for 5 x 4. ☐	I could not show an array as a model for multiplication. ☐	I could not show an array or solve the problem 5 x 4 in any way. ☐
Different Equations for the Same Product	I systematically worked through possible solutions to this problem until I was sure I had them all. I organized my answer in such a way as to show relationships between solutions. ☐	I showed five or more possible solutions to this problem. I presented my solutions in the form of equations. ☐	I showed three or more ways to use multiplication to achieve a product of 24. ☐	I showed one or two possible ways to use multiplication to achieve a product of 24. ☐	I was not able to write a multiplication equation or show how to use multiplication to achieve a product of 24. ☐

Name _____

B. One-on-one Assessment

1. Ask the student to make up a story problem showing repeated groups of equal size. Then ask the student how he or she would figure out the solution or total in a situation like this.

   Use the following accommodations, if necessary:

   - Use manipulatives that increase the likelihood of a multiplicative narrative; e.g., small models of people or animals, dishes, food models.
   - Prompt the student to find a quicker way to count if he or she appears to be counting a total by ones.
   - Provide a calculator for the student to show his or her understanding of the operation of multiplication or successive addition.

2. Show the student an array with some of the elements concealed; for example, put a piece of paper over a corner of an array of 3 rows by 4 columns of identical objects. Show the array very quickly before partially covering it so that the student knows it is an array. Ensure that the student can see how many rows and columns in the array, even when it is partially concealed. Ask the student to tell you how he or she would figure out the total number of elements in the array.

   Use the following accommodations, if necessary:

   - Ask the student to talk about the array in terms of the numbers of objects in each row and number of objects in each column.
   - Prompt the student to find a quicker way if he or she appears to be visualizing the objects under the paper and counting by ones.
   - Provide a calculator for the student to show his or her understanding of the operation of multiplication or successive addition.

3. Group 16 objects in four groups of four. Confirm the total number of objects with the student. Ask the student to create a different multiplication situation from the same number of objects. Ask the student to describe both situations using equations.

   Use the following accommodations, if necessary:

   - Ask the student to combine or split the existing groups to make larger or smaller groups of equal size.
   - Ask the student to use oral language instead of equations to describe the multiplicative situation; i.e., "8 groups of 2 objects each makes 16."

C. Applied Learning

Provide opportunities for students to experience and describe multiplicative situations and contexts in everyday life.

- Plan a snack, party, event or craft that involves multiples. Have students figure out totals that will be needed by their team or their small group for everyone to have fair shares or to be able to participate in an activity.
- Go to http://illuminations.nctm.org/ActivityDetail.aspx?ID=3 to play the game "Bobbie Bear." Can students organize their answers to help predict the number of outfits? How does this game relate to students' understanding of arrays? Rich discussions are possible as students discuss their solution strategies after they play this game.
- Play a multiplication game that encourages students to skip count while keeping track of the number of groups they have counted, as well as to notice which numbers are more likely to occur as multiples of small numbers. Use a 6 x 6 chart, numbered consecutively from 1 to 36. Students can roll two dice and multiply the numbers by skip counting on the chart or keeping track with their fingers, leaving a coloured counter on the end product. Once a square is occupied another counter cannot cover it. After 10 turns or a set number of minutes, the student with the longest consecutive row, column or diagonal of counters in his or her colour wins the game.

Step 5: Follow-up on Assessment

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction?

A. Addressing Gaps in Learning

Students, who have difficulty understanding multiplication, will benefit from ongoing experiences with mathematical narratives and contexts that involve multiplicative situations. These repeat experiences allow students to develop the understanding, moving on from counting and modelling strategies for solving problems, through skip counting, successive addition and doubling strategies, to true multiplicative thinking.

For example:

- Have students make up their own problems to solve or make problems with or for a partner that involve personal details, such as their own names, birthdays, favourite foods, colours, toys, animals, characters, objects and ideas.
- Encourage students, who are having difficulty, to dramatize and model multiplicative narratives so they can imagine multiplication in relationship to their lived experience.
- Use small numbers, problems that can be solved using known skip counting sequences, simple contexts and no extraneous information. Introduce difficulty gradually, and reduce difficulty again if it becomes a block to understanding or diminishes confidence to the point where a student cannot proceed.
- Allow students to solve problems without using equations and to answer using language instead of symbols until they are able to fully understand mathematical notation. Provide direct translation of their narrative answers into mathematical symbols, and do this on a repeated basis. If necessary, provide a written translation tool or chart for students to use.
- Provide students with the opportunity to solve problems using a choice or any combination of models, objects, counters, drawings, oral language, written language and symbols. Do not restrict solution strategies at first. Observe students' solution strategies, and encourage solution strategies that are just a little more difficult; for example, moving from direct narrative modelling to using representative counters or tallies to represent objects, or from a counting all strategy to skip counting using known skip counting sequences.

B. Reinforcing and Extending Learning

Students, who have achieved or exceeded the outcomes, will benefit from ongoing opportunities to apply and extend their learning. These activities should support students in developing a deeper understanding of the concept and should not progress to the outcomes in subsequent grades.

Consider strategies, such as the following.

- Provide students with contexts for using their multiplication skills, such as designing construction projects, planning a garden, designing visual patterns in arrays (e.g., using pattern blocks and calculating numbers of elements they need to make the array larger), planning events, taking a recipe, pattern or picture and making an enlargement using multiplication.
- Have students use multiplication to solve area and volume problems that they model using tiles and cubes, but also describe using equations.
- Introduce students to simple, everyday multiplication problems using rate, multiplicative comparison and combinations.

Bibliography

Alberta Education. *The Alberta K–9 Mathematics Program of Studies with Achievement Indicators*. Edmonton, AB: Alberta Education, 2007.

Burns, Marilyn. *About Teaching Mathematics: A K–9 Resource.* 2nd ed. Sausalito, CA: Math Solutions Publications, 2000.

Fosnot, Catherine Twomey and Maarten Dolk. *Young Mathematicians at Work: Constructing Multiplication and Division*. Portsmouth, NH: Heinemann, 2001.

State of Victoria Department of Education, Employment and Training. *Early Numeracy Interview Booklet.* Melbourne, AU: State of Victoria, 2001.

Van de Walle, John A. and Sandra Folk. *Elementary and Middle School Mathematics: Teaching Developmentally*. Canadian ed. Toronto, ON: Pearson Education Canada Inc., 2005.