Mathematics

Planning Guide

Grade 4 *Transformations*

Shape and Space (Transformations)

Specific Outcomes 5, 6

This Planning Guide can be accessed online at:
http://www.learnalberta.ca/content/mepg4/html/pg4_transformations/index.html

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Planning Guide: *Grade 4 Transformations*

Strand: Shape and Space (Transformations)
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This *Planning Guide* addresses the following outcomes from the Program of Studies:

Strand: Shape and Space (Transformations)

Specific Outcomes:

5. Demonstrate an understanding of congruency, concretely and pictorially.
6. Demonstrate an understanding of line symmetry by:
   5. identifying symmetrical 2-D shapes
   6. creating symmetrical 2-D shapes
   7. drawing one or more lines of symmetry in a 2-D shape.

Curriculum Focus

This sample targets the following changes in the curriculum:

- The General Outcome focuses describing and analyzing position and motion of objects and shapes, whereas the previous mathematics curriculum focused on using numbers and direction words to describe the relative positions of object in two dimensions, using everyday contexts.
- The Specific Outcomes focus on congruency and symmetry, whereas the previous mathematics curriculum focused on placing an object on a grid with oral and written directions as well as symmetry. Congruency was introduced in Grade 3 in the previous mathematics curriculum.

What Is a Planning Guide?

Planning Guides are a tool for teachers to use in designing instruction and assessment that focuses on developing and deepening students' understanding of mathematical concepts. This tool is based on the process outlined in *Understanding by Design* by Grant Wiggins and Jay McTighe.

Planning Steps

The following steps will help you through the Planning Guide:

Step 1: Identify Outcomes to Address

Guiding Questions

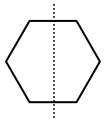
- What do I want my students to learn?
- What can my students currently understand and do?
- What do I want my students to understand and be able to do based on the Big Ideas and specific outcomes in the program of studies?

Big Ideas

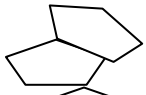
Congruency and symmetry are geometric properties. These properties can be used to determine what makes some shapes alike and different.

Congruent 2-D shapes are "geometric figures that have the same size and shape" (Alberta Education 1990, p. 198). Symmetrical 2-D shapes are geometric figures "that can be folded in half so that the two parts are congruent" (Alberta Education 1990, p. 205).

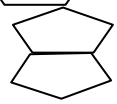
Symmetrical and congruent shapes are closely connected. Any symmetrical shape can be divided into two congruent parts along the line of symmetry; however, not every composite shape made up of congruent figures is symmetrical. For example,



This regular hexagon is symmetrical. The line of symmetry shown in the diagram divides the hexagon into two congruent shapes, each shape is a pentagon.



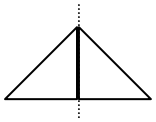
This composite shape is made up of two congruent pentagons. It is **not** symmetrical.



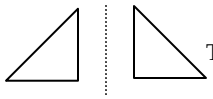
This composite shape is made up of two congruent pentagons. It **is** symmetrical.

It is the relation of congruent shapes to one other in the composite shape that determines whether or not this composite shape is symmetrical.

A shape remains the same size and shape when transformed using translations, reflections or rotations; i.e., the object and the image in these transformations are congruent. Symmetrical shapes form a subset of reflections. A reflection results in a symmetrical composite shape when the mirror line used to reflect a shape aligns with one side of the shape. For example:



This reflection results in a composite shape that is symmetrical.



The mirror line and the axis of symmetry coincide.

This reflection does not result in a composite shape but rather two separate shapes that are congruent. The mirror line is shown in the diagram but it is not the axis of symmetry.

Sequence of Outcomes from the Program of Studies

See http://education.alberta.ca/teachers/core/math/programs.aspx for the complete program of studies.

Grade 3
Specific Outcomes
There are no outcomes in Shape and Space (Transformations).

Outcomes from Shape and Space (3-D Objects and 2-D Shapes) include:

6. Describe 3-D objects according to the shape of the faces and the number of edges and vertices.
7. Sort regular and irregular polygons, including:
   - triangles
   - quadrilaterals
   - pentagons
   - hexagons
   - octagons

   according to the number of sides.

Grade 4
Specific Outcomes

5. Demonstrate an understanding of congruency, concretely and pictorially.
6. Demonstrate an understanding of line symmetry by:
   - identifying symmetrical 2-D shapes
   - creating symmetrical 2-D shapes
   - drawing one or more lines of symmetry in a 2-D shape.

Grade 5
Specific Outcomes

8. Identify and describe a single transformation, including a translation, rotation and reflection of 2-D shapes.

Step 2: Determine Evidence of Student Learning

Guiding Questions

- What evidence will I look for to know that learning has occurred?
- What should students demonstrate to show their understanding of the mathematical concepts, skills and Big Ideas?

Using Achievement Indicators

As you begin planning lessons and learning activities, keep in mind ongoing ways to monitor and assess student learning. One starting point for this planning is to consider the achievement indicators listed in the *Mathematics Kindergarten to Grade 9 Program of Studies with Achievement Indicators*. You may also generate your own indicators and use them to guide your observation of the students.

The following achievement indicators may be used to determine whether the students have met this specific outcome.

- Determine if two given 2-D shapes are congruent and explain the strategy used.
- Create a shape that is congruent to a given 2-D shape and explain why the two shapes are congruent.
- Identify congruent 2-D shapes from a given set of shapes shown in different orientations.
- Identify corresponding vertices and sides of two given congruent shapes.
- Identify the characteristics of given symmetrical and nonsymmetrical 2-D shapes.
- Sort a given set of 2-D shapes as symmetrical and nonsymmetrical and explain the process.
- Complete a symmetrical 2-D shape given half the shape and its line of symmetry and explain the process.
- Identify lines of symmetry of a given set of 2-D shapes and explain why each shape is symmetrical.
- Determine whether or not a given 2-D shape is symmetrical by using a Mira or by folding and superimposing.
- Create a symmetrical shape with and without manipulatives and explain the process.
- Provide examples of symmetrical shapes found in the environment and identify the line(s) of symmetry.
- Sort a given set of 2-D shapes as those that have no lines of symmetry, one line of symmetry or more than one line of symmetry.
- Explain the connections between congruence and symmetry using 2-D shapes.

Some sample behaviours to look for in relation to these indicators are suggested for many of the instructional activities in **Step 3, Section C, Choosing Learning Activities** (p. 11).

Step 3: Plan for Instruction

Guiding Questions

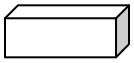
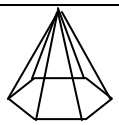
- What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should I use?
- How will I meet the diverse learning needs of my students?

A. Assessing Prior Knowledge and Skills

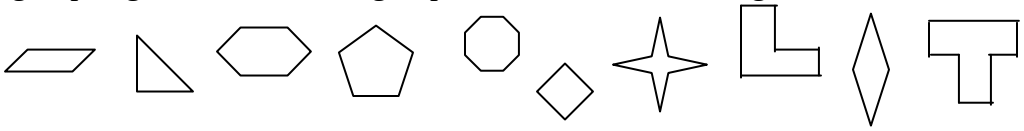
Before introducing new material, consider ways to assess and build on the students' knowledge and skills related to 3-D objects and 2-D shapes. Have 3-D objects available for the students to use as needed. Examples of assessments include the following:

- Label the following diagram using the words: face, edge and vertex.

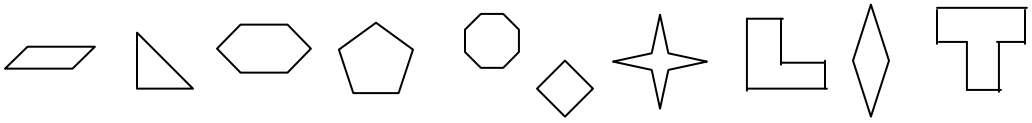
- Count the number of faces, edges and vertices of the 3-D objects shown in the chart below. Record your answers in the chart provided.

3-D Object	Number of Faces	Number of Edges	Number of Vertices
			
			
			

- Cut out the 2-D shapes and sort them according to the number of sides. Write the name of each group; e.g., the name of the group with three sides is triangles.



- Classify the following 2-D shapes into two groups: regular polygons and irregular polygons. Write R on regular polygons and I on irregular polygons. Explain your thinking.

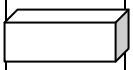
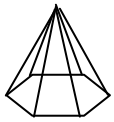


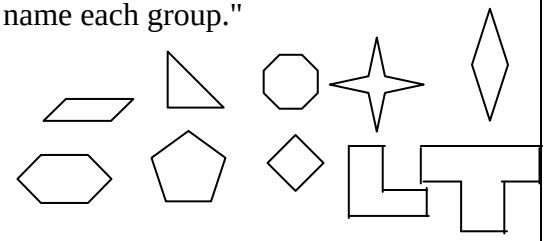
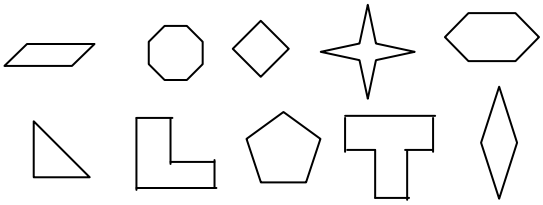
If a student appears to have difficulty with these tasks, consider further individual assessment, such as a structured interview, to determine the student's level of skill and understanding. See **Sample Structured Interview: Assessing Prior Knowledge and Skills** (p. 9).

Sample Structured Interview: Assessing Prior Knowledge and Skills

<table>
<tr><td>Directions</td><td colspan="2">Date:</td></tr>
<tr><td></td><td>Not Quite There</td><td>Ready to Apply</td></tr>
<tr><td>Provide the student with 3-D objects to use as needed. Take the cube and say, "Point to a face on this object." Then say, "Point to a vertex on this object." Then say, "Point to an edge on this object."

Place the following chart before the student.
Then say, "Count the number of faces, edges and vertices of the 3-D objects shown in the chart. Record your answer in the chart provided."

<table>
<tr><th>3-D Object</th><th>Number of Faces</th><th>Number of Edges</th><th>Number of Vertices</th></tr>
<tr><td>  </td><td></td><td></td><td></td></tr>
<tr><td>  </td><td></td><td></td><td></td></tr>
<tr><td>  </td><td></td><td></td><td></td></tr>
</table>
</td><td>• Confuses the terms: face, vertex and edge.
• Uses the terms appropriately but makes errors in counting.
• Thinks that a cylinder has 3 faces instead of 2.
• Thinks the cylinder has no edges instead of 2.</td><td>• Knows the terms (face, vertex and edge) and counts the correct number for each of the given 3-D objects.</td></tr>
</table>

Cut the following 2-D shapes out and place them before the student. Say, "Sort the following 2-D shapes according to the number of sides and name each group." 	• Does not recognize that the L-shaped figure is a hexagon or the T-shaped figure is an octagon. • Sorts the 2-D shapes but has difficulty naming at least one of the groups; e.g., may say that the group has four sides rather than saying they are all quadrilaterals.	• Sorts the 2-D shapes correctly and names the groups, using the mathematical terms.
Place the following 2-D shapes in front of the student. Say, "Classify the following 2-D shapes into two groups: regular polygons and irregular polygons. Explain your thinking." 	• Does not sort the 2-D shapes into the correct groups. • Sorts the 2-shapes into the groups correctly except for the star and the elongated rhombus, thinking that these shapes are regular when they are not. • Cannot explain why a given shape is regular.	• Sorts the 2-D shapes correctly into the two groups and explains that all the regular polygons have equal sides and equal corners whereas the other polygons do not.

B. Choosing Instructional Strategies

Consider the following general strategies for teaching congruence and symmetry with 2-D shapes:

- Access prior knowledge on 2-D shapes so that the terminology is understood and applied as needed.
- Use everyday contexts to introduce congruence and symmetry, drawing the students' prior experiences in the real world.
- Include many hands-on activities to establish the concept of congruence prior to symmetry. The students need to know the characteristics of two congruent shapes before they can understand that the symmetrical parts of a 2-D shape are congruent.
- Use a variety of manipulatives to construct the meaning of congruence and symmetry such as pattern blocks, tiles, Miras, paper folding, geoboards and cardboard shapes; e.g., stick men—one as the original, the other as a congruent partner.
- Through exploration, provide opportunity for the students to generalize the Big Ideas about congruence and symmetry and record these ideas in various ways, such as using Frayer models.
- Have the students share their ideas about sorting various sets of 2-D shapes and provide follow-up activities to address any misconceptions that may arise.
- Have the students justify their thinking by explaining why they use a particular strategy to create congruent and/or symmetrical 2-D shapes.
- Encourage flexible thinking by having the students sort sets in more than one way or create symmetrical 2-D shapes in more than one way.

C. Choosing Learning Activities

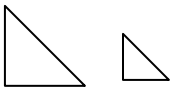
The following learning activities are examples of activities that could be used to develop student understanding of the concepts identified in Step 1.

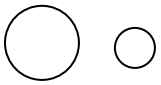
Sample Activities:

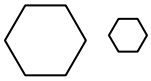
1. **Teaching Congruency** (p. 12)
2. **Teaching the Identification of Symmetrical 2-D Shapes** (p. 16)
3. **Teaching the Creation of Symmetrical 2-D Shapes** (p. 18)
4. **Teaching Students to Draw One or More Lines of Symmetry** (p. 21)

Sample Activity 1: Teaching Congruency

1. Congruency: Same Size and Shape
   a. Provide the students with cutouts of pairs of 2-D shapes and ask them what is the same about each pair. Examples of pairs:
   b.

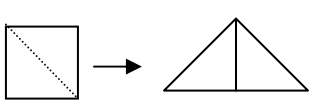
i) 

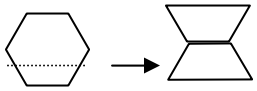
ii) 

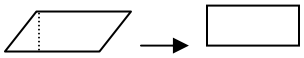
iii) 

Through discussion, have the students verbalize that the 2-D shapes in each pair are the same shape with different areas or sizes. Have the students draw or cut out another set of 2-D shapes that are the same shape but different size or area.

c. Provide the students with the first shape in each of the following examples. Have the students cut the shape and rearrange it to make a different 2-D shape. A sample of a new arrangement is shown. Ask the students what is the same about the original shape and the rearranged shape.

i) 

ii) 

iii) 

Through discussion, have the students verbalize that the 2-D shapes in each pair have the same area or size but different shapes. Have the students draw or cut out another set of 2-D shapes that have the same area but different shapes.

Look For …

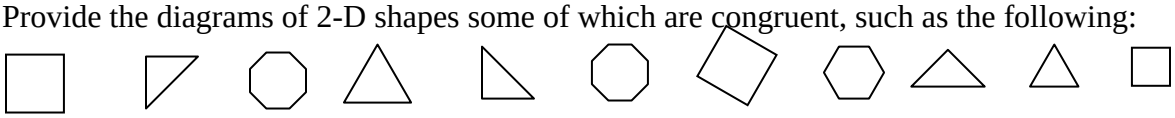
Do students:

- ☐ apply their understanding of area to congruency?
- ☐ justify whether or not two shapes are congruent?
- ☐ apply congruency to everyday context?

d. Present the students with the following problem:
Make a hexagon that is the same size and shape as the yellow pattern block hexagon. Explain how you do it.
Have the students draw another pair of congruent 2-D shapes and explain their process.
Introduce the name, congruent, for shapes that have the same size and shape.

e. Have the students find congruent shapes in the classroom and at home. They should be prepared to explain why these shapes are congruent.

2. Sorting Congruent Shapes
Provide the diagrams of 2-D shapes some of which are congruent, such as the following:



Ask the students to:

- put a check mark on shapes that are congruent to 
- put an X on shapes that are congruent to 
- shade in the shapes that are congruent to 

Have the students explain the strategy they used to determine if the shapes were congruent. Suggest that they trace and cut out the three shapes and then superimpose them on the given shapes to prove congruency.

Look For …
Do students:
- ☐ use an appropriate strategy to determine congruence?
- ☐ recognize that congruent shapes can have different orientations?

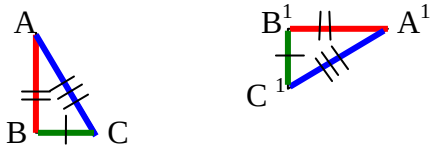
3. Create a Copy to Make Congruent Shapes
Provide the students with pattern blocks. Have the students work in pairs. One student makes a design with the pattern blocks and the other student copies the design so that the two designs are congruent. Have the students draw the two congruent designs on isometric dot papers. They could cut out one design and superimpose it on the other design to check for congruency.
Adaptations:

- Have the students create congruent designs on geoboards and draw the designs on square dot paper or geopaper. They could cut out one design from the dot paper and superimpose it on the other design to check for congruency.

- Have the students use other ways to create congruent designs, such as drawing a design on folded paper and cutting out the two designs.

Look For …
Do students:
- ☐ create congruent shapes using a variety of strategies?
- ☐ connect the concrete and pictorial modes by drawing shapes made with concrete materials?
- ☐ explain why the created copy is congruent to the original shape?

4. Corresponding Parts of Congruent Shapes
Have the students label corresponding vertices and colour-code corresponding sides of congruent pairs of 2-D shapes that they created or are presented to them. Instead of colour-coding the corresponding sides, the students may wish to use markings on the sides as shown below. Include examples that have the congruent shapes in different orientations as shown in the diagram.



Have the students justify that they have identified the corresponding sides and vertices correctly by tracing one shape complete with the markings and superimposing it on the other congruent shape. The labelled vertices and colour-coded or marked sides should match.

5. Frayer Model for Congruency
   Have the students complete a **Frayer Model** to consolidate their understanding of congruency. This model can be completed together as a class or in groups, or independently depending on the needs of the students.

   An example of a Frayer Model for congruency is shown below:

Look For …

Do students:

- ☐ write the definition in their own words?
- ☐ describe the essential characteristics of congruence as well as the nonessential characteristics?
- ☐ provide a variety of examples and nonexamples of congruence and explain why they sorted them as they did?

Frayer Model

Definition

Two shapes are congruent if they have the same size and shape.

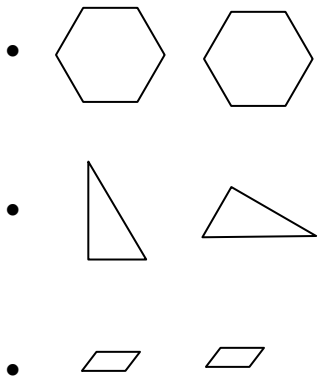
Characteristics

Essential Characteristics
Congruent shapes must have:
- corresponding sides congruent
- corresponding vertices congruent
- the same area
- the same shape.

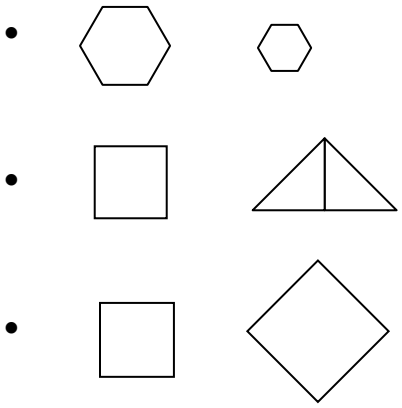
Nonessential Characteristics
Congruent shapes may:
- include any number of shapes
- be large or small
- include regular or irregular shapes
- have different positions (orientation).

Congruency

Examples



Nonexamples



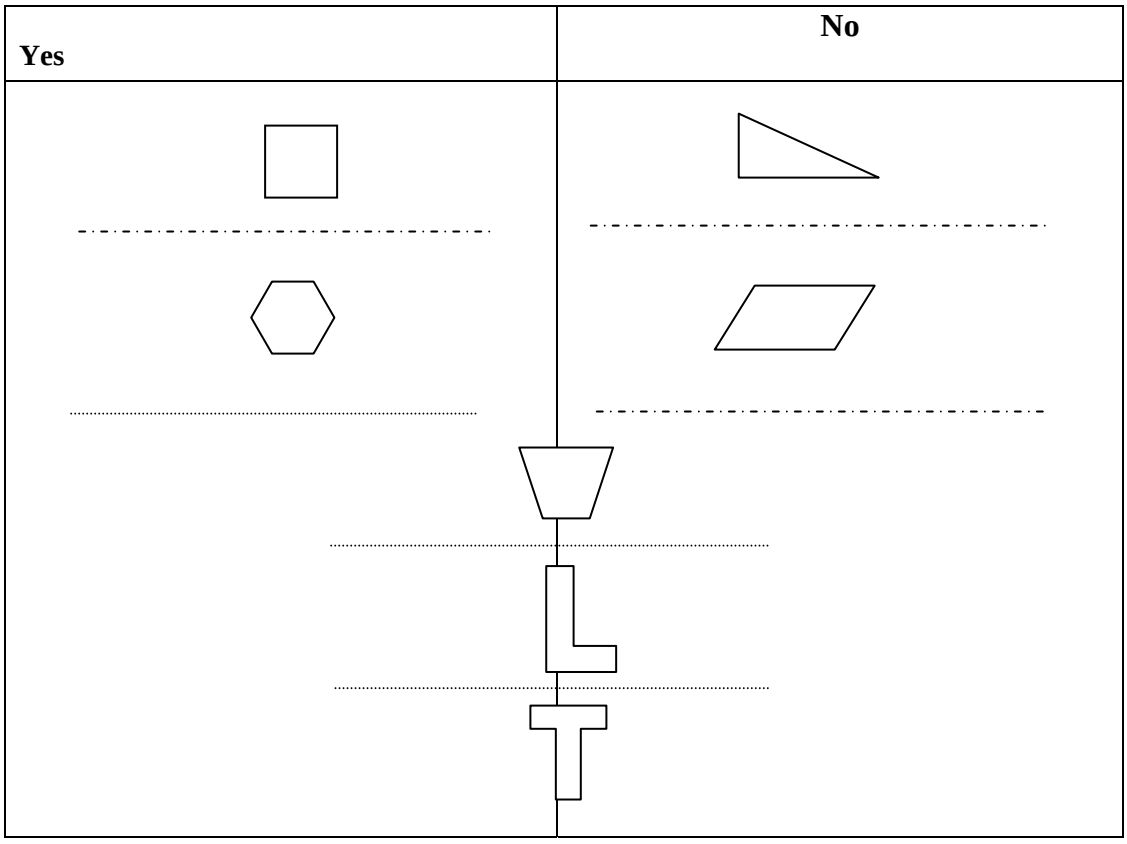
Format adapted from D. A. Frayer, W. C. Frederick and H. J. Klausmeier, *A Schema for Testing the Level of Concept Mastery* (Working Paper No. 16/Technical Report No. 16) (Madison, WI: Research and Development Center for Cognitive Learning, University of Wisconsin, 1969). Adapted with permission from the Wisconsin Center for Education Research, University of Wisconsin-Madison.

Sample Activity 2: Teaching the Identification of Symmetrical 2-D Shapes

1. Concept Attainment for Symmetrical 2-D Shapes
   Use concept attainment to stimulate the students' thinking about what visual representations show symmetrical 2-D shapes.

   a. Use a Yes/No chart as shown below on the overhead projector or drawn on the board.

   Place an example of a symmetrical 2-D shape in the "Yes" column and a nonexample in the "No" column. Example:



b. Have the students explain the difference between the two. Use the students' responses to decide the next example and nonexample to use. For example, if you place a square in the "Yes" column and a scalene triangle (all sides of different lengths) in the "No" column, the students might say that one has 4 sides (quadrilateral) and the other has 3 sides. Since you want to focus on symmetry, show the next example with a symmetrical hexagon and a nonsymmetrical quadrilateral (parallelogram). Have the students explain how the two examples in the "Yes" column are the same and how they are different from the two examples in the "No" column.

c. Place the next example on the line between the "Yes" and the "No" columns and have the students decide where they think the example should go and justify their response. After discussion, place the example in the correct column to continue attaining the concept of symmetry.

   Continue with examples and nonexamples until the class discussion summarizes that:

Look For …

Do students:

- ☐ explain how the examples in the "Yes" column are the same by focusing on the characteristics they have in common but are different from the examples in the "No" column?
- ☐ decide into which column to place a given set and justify their choice?
- ☐ transfer the learning about symmetry to other everyday contexts?

- the examples in the "Yes" column all show shapes that can be cut in half
- introduce the word symmetrical and draw the axis of symmetry to show the halves for each symmetrical shape
- the axis of symmetry that cuts each symmetrical shape in half produces two congruent shapes (trace and fold the shapes to prove congruency)
- symmetrical 2-D shapes can be regular or irregular.

Extension:

- After the students begin to explain how the examples are the same and also different from the nonexamples, encourage them to create shapes and place them on the Yes/No chart, justifying their placement. Encourage the students to use folding or Miras to show that designs are symmetrical. A Mira is made out of red plexiglass and can be used to view the reflection of a 2-D shape through the glass rather than on it, as with a mirror. If the object reflects perfectly on the image when the Mira is placed on the 2-D shape along the axis of symmetry, then the shape is symmetrical.
- Use shapes from everyday contexts and decide if the shapes are symmetrical or not, and why.
- Use concept attainment to develop or review understanding of congruent 2-D shapes.

Adapted from W. George Cathcart, Yvonne M. Pothier and James H. Vance, *Learning Mathematics in Elementary and Middle Schools* (2nd ed.) (Scarborough, ON: Prentice-Hall Canada, 1997), p. 186. Adapted with permission from Pearson Education Canada.

Sample Activity 3: Teaching the Creation of Symmetrical 2-D Shapes

1. Paper Folding
   Build on the students' knowledge in identifying symmetrical 2-D shapes by tracing the shapes, cutting them out and folding them to show that the halves are congruent. Ask the students to fold a piece of paper and create a design along the fold line. Review that the unfolded shape is symmetrical because the two halves are congruent. Have the students share their symmetrical 2-D shapes. Using paint makes the designs eye appealing for a bulletin board display.

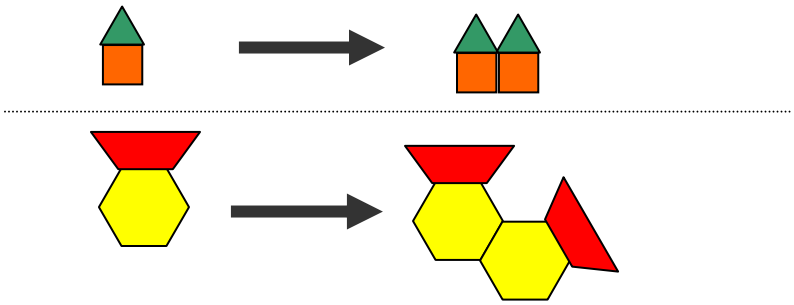
Look For …

Do students:

- ☐ use a variety of strategies to create symmetrical shapes?
- ☐ explain the process used to create symmetrical shapes?
- ☐ connect the concrete to the pictorial mode by drawing the symmetrical shapes made by using concrete materials?

2. Create a Copy to Show Symmetrical Designs Using Manipulatives
   a. Have the students work in pairs. Provide them with pattern blocks and isometric dot paper. Instruct one student in each pair to create a design using two pattern blocks. The other student in the pair is then to copy the design (reflect the design) to make a composite symmetrical 2-D design. The students may create the symmetrical design using a vertical, horizontal or oblique axis of symmetry. (Note: The vertical line of symmetry is easiest for students to use in creating symmetrical designs.)

Examples: Original Design Composite Symmetric Shape



   b. Have the students copy the symmetric 2-D designs made with pattern blocks onto isometric dot paper and explain how they know the designs are symmetrical. The students may wish to cut out the designs and fold them along the axis of symmetry to determine if the designs are symmetrical. Alternately, the students may wish to trace the design using tracing paper, then fold the design along the axis of symmetry to determine if the two halves fit perfectly on each other.
      Adaptations:
      a. Have the students use geoboards instead of pattern blocks to create the copies. Have them transfer the symmetrical designs onto square dot paper and justify that they are symmetrical.
      b. Have the students use Miras to create the copies. A Mira is made out of red plexiglass and can be used to view the reflection of a 2-D shape through the glass rather than on it, as with a mirror. Instruct the students to reach behind the Mira and draw the image that they see in the Mira.

Adapted from W. George Cathcart, Yvonne M. Pothier and James H. Vance, *Learning Mathematics in Elementary and Middle Schools* (2nd ed.) (Scarborough, ON: Prentice-Hall Canada, 1997), p. 186. Adapted with permission from Pearson Education Canada.

3. Create a Copy to Show Symmetrical Designs without Using Manipulatives

Look For …

Do students:

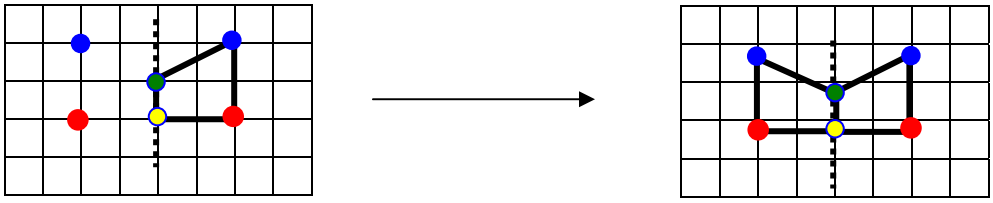
- ☐ transfer their understanding of corresponding vertices and sides between congruent shapes when drawing symmetrical shapes?
- ☐ verify that their shapes are symmetrical by folding them along the axis of symmetry or by using a miras?
- ☐ progress from using vertical axes symmetry to using horizontal and finally oblique lines axes of symmetry when drawing symmetrical designs on grids?

a. Provide the students with square and/or isometric dot paper with half of one symmetrical design drawn on it along with the axis of symmetry. Centimetre grid paper can also be used. Use a transparency of the dot paper with the half design to model the process.

b. Start with a vertical axis of symmetry since it is the easiest axis for the students to reflect the design. Have the students focus on the position of the vertices, reminding them that congruent shapes have corresponding vertices. Review corresponding vertices of congruent shapes if necessary—see the activities for congruency.

c. On the opposite of the axis of symmetry, place a point to show a vertex that is corresponding to one of the vertices on the half design shown. If the original vertex is two spaces away from the axis of symmetry, then the reflected vertex is also two spaces away from the axis of symmetry on the other side. Continue marking the corresponding vertices for each vertex. Then join the vertices to complete the symmetrical design. Justify that the completed design is symmetrical by cutting out the design and folding it or by using a Mira placed along the axis of symmetry.

Example:



d. Provide other similar examples for reinforcement. Then provide examples in which the axis of symmetry is horizontal and then oblique.

e. Have the students work in pairs. One student makes half a design and decides on which line segment of the design the axis of symmetry should be placed. The other student draws the completed symmetrical 2-D design. They must justify why the design is symmetrical. The students may wish to trace the design using tracing paper, then fold the design along the axis of symmetry to determine if the corresponding sides and angles fit

perfectly over one another. Provide the students with Miras as another way to check for symmetry.

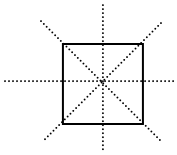
Sample Activity 4: Teaching Students to Draw One or More Lines of Symmetry

1. How Many Lines of Symmetry?

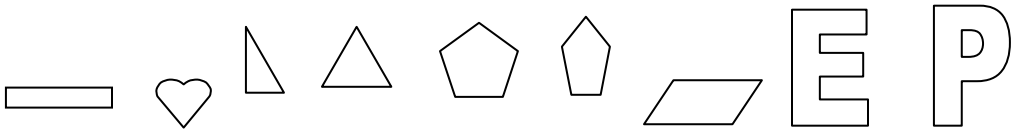
   a. Ask the students how they might draw lines of symmetry on a symmetrical 2-D shape. Some may suggest folding the shape and then drawing the line of symmetry along the fold. Others may suggest using a Mira and positioning it so that half the design reflects perfectly on the other half and then drawing the line of symmetry along the Mira, using it as a ruler. Have the students show different ways and discuss each strategy.

   b. Provide examples of 2-D shapes with one line of symmetry, two lines of symmetry and no lines of symmetry. Have the students draw the lines of symmetry.

      Example:

   c. Provide the students with a collection of 2-D shapes drawn on paper, such as the following :

      Instruct the students to decide which shapes are symmetrical and draw lines of symmetry on those shapes. Remind the students that some of the symmetrical shapes may have one line of symmetry or more than one line of symmetry. The students should be prepared to explain the process they used to draw the lines of symmetry.

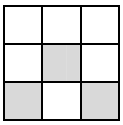
      Extensions:

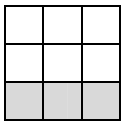
      c. Have the students create shapes with no lines of symmetry, one line of symmetry or more than one line of symmetry.
      d. Have the students sort a set of 2-D shapes into groups. A closed sort would include the possible categories, such as shapes having no lines of symmetry, one line of symmetry or more than one line of symmetry. An open sort would allow the students to create their own groups, name the groups and justify why each shape fits within a given group.

2. Symmetrical Patterns on Grid Paper

Provide examples of 3 by 3 squares on grid paper. Shade 3 small squares so that the figure has one line of symmetry.

Examples:





Challenge the students to:

e. make as many different patterns with one line of symmetry by shading in three small squares
f. make patterns with two lines of symmetry by shading in three small squares
g. shade in four small squares and make figures with one, two or no lines of symmetry
h. make figures with more than two lines of symmetry.

Look For …

Do students:

- ☐ create different 3 by 3 squares with 3 shaded squares so that the resulting design has no lines of symmetry, one line of symmetry or more than one line of symmetry?
- ☐ transfer their learning to 4 by 4 squares?
- ☐ explain the process used in creating the designs?

Adapted from W. George Cathcart, Yvonne M. Pothier and James H. Vance, *Learning Mathematics in Elementary and Middle Schools* (2nd ed.) (Scarborough, ON: Prentice-Hall Canada, 1997), p. 187. Adapted with permission from Pearson Education Canada.

Have the students share their work and discuss their strategies.

3. Frayer Model for Symmetry

Have the students complete a Frayer Model to consolidate their understanding of symmetry. This model can be completed together as a class or in groups or independently depending on the needs of the students.

An example of a Frayer Model for symmetry is shown below:

Look For …

Do students:

- ☐ write the definition in their own words?
- ☐ describe the essential characteristics of symmetry as well as the nonessential characteristics?
- ☐ provide a variety of examples and nonexamples of symmetrical shapes and explain why they sorted them as they did?

Frayer Model

Definition

A shape is symmetrical if it can be folded in half so that the two parts are congruent. (Alberta Education 1990, p. 205)

Characteristics

Essential Characteristics
Symmetrical shapes must have:

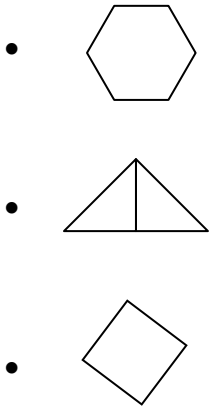
- two congruent parts separated by a line of symmetry
- corresponding vertices and sides matching when the shape is folded along the axis of symmetry

Nonessential Characteristics
Symmetrical shapes may:

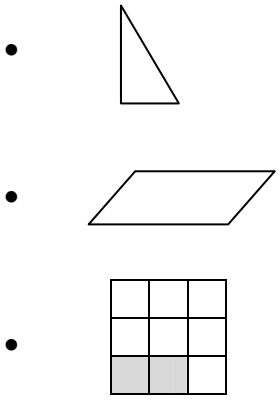
- be large or small
- include regular or irregular shapes
- have different positions (orientation)
- have more than one line of symmetry.

Symmetry

Examples



Nonexamples



Format adapted from D. A. Frayer, W. C. Frederick and H. J. Klausmeier, *A Schema for Testing the Level of Concept Mastery* (Working Paper No. 16/Technical Report No. 16) (Madison, WI: Research and Development Center for Cognitive Learning, University of Wisconsin, 1969). Adapted with permission from the Wisconsin Center for Education Research, University of Wisconsin-Madison.

Step 4: Assess Student Learning

Guiding Questions

- Look back at what you determined as acceptable evidence in Step 2.
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Sample Assessment Tasks

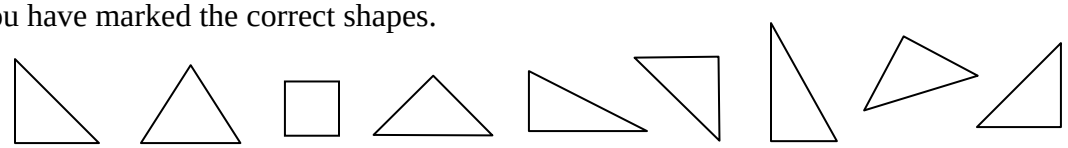
In addition to ongoing assessment throughout the lessons, consider the following sample activities to evaluate students' learning at key milestones. Suggestions are given for assessing all students as a class or in groups, individual students in need of further evaluation, and individual or groups of students in a variety of contexts.

A. Whole Class/Group Assessment

Note: Performance-based assessment tasks are under development.

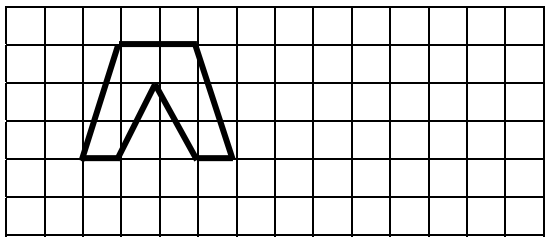
Provide the students with rulers, scissors, pattern blocks, isometric dot paper, square dot paper, scrap paper, tracing paper and Miras.

1. Put an X on all the shapes that are congruent to the first shape. Explain how you know that you have marked the correct shapes.

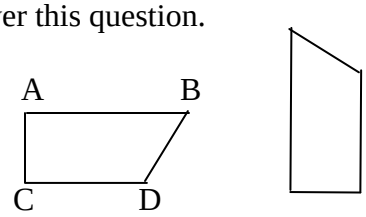


2. a. Is the shape drawn on the grid paper symmetrical? Explain how you know.

   b. Draw a shape on the grid paper that is congruent to the given shape. Explain how you know the shape you drew is congruent to the other shape.



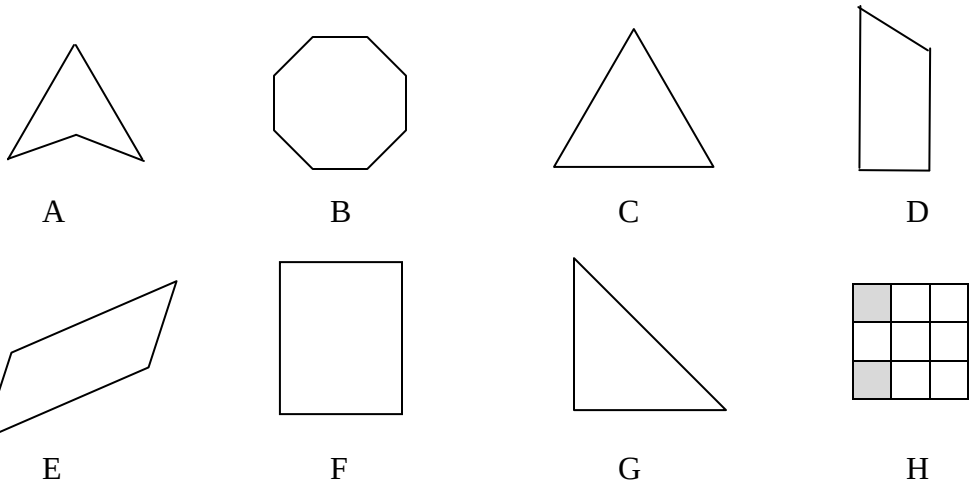
3. Use the following shapes to answer this question.



   a. Mark or colour the diagrams to show the corresponding sides on the congruent shapes.

   b. Label the corresponding vertices of the second congruent shape by using A^1 , B^1, C^1, and D^1.

4. Use the following shapes to answer parts (a), (b) and (c) of this question.



   a. Circle all the symmetrical shapes.

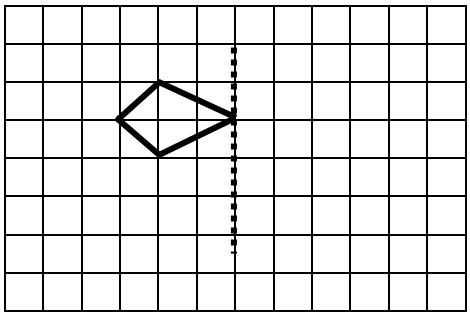
   b. Draw <u>all</u> the lines of symmetry on the symmetrical shapes.

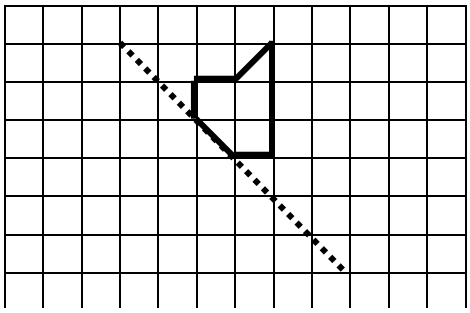
   c. Sort the shapes by placing the letter of each shape in the chart below:

Lines of Symmetry	**Letter Names for the Shapes**
No lines of symmetry	
One line of symmetry	
More than 1 line of symmetry	

5. Create a symmetrical 2-D shape that is different from any shape in this exercise. Draw it in the space below or draw it on the dot paper provided. Explain how you know your shape is symmetrical.

6. Complete each of the following diagrams to make a symmetrical shape using the line of symmetry shown (dotted line):

a. 

b. 

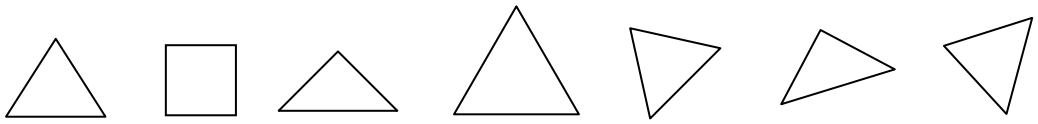
7. Give three examples of symmetrical shapes in your everyday world.

8. Explain how congruent shapes are part of symmetrical shapes.

B. One-on-one Assessment

Provide the student with a ruler, scissors, pattern blocks, isometric dot paper, scrap paper, tracing paper and Miras.

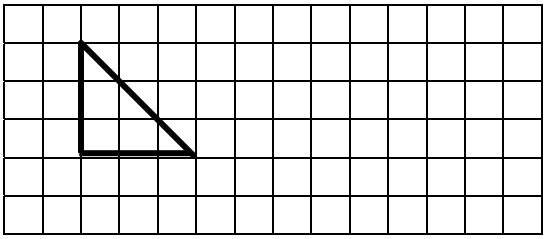
1. Provide the student with the following 2-D shapes. Say, "Put an X on all the shapes that are congruent to the first shape. Explain how you know which shapes are congruent to the first shape."



If the student has difficulty selecting the shapes congruent to the first shape, circle the first shape to focus on it, then suggest that the student trace the shape, cut it out and use it to find other congruent shapes. For a student who has difficulty tracing accurately, provide the cutout shapes for the student.
Remind the student that congruent shapes have the same size and shape—all the sides must match perfectly and all the vertices must match perfectly.

2. Provide the student with the grid containing the triangle below. Say,
"Is the shape drawn on the grid paper symmetrical? Explain how you know."

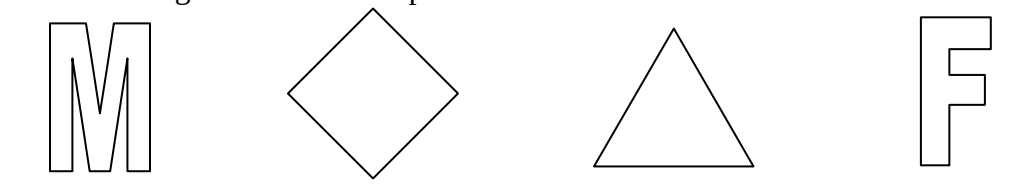
Then ask the student to draw a shape on the grid paper that is congruent to the given triangle. Have the student explain how he or she knows the shape that he or she draws is congruent to the given triangle.



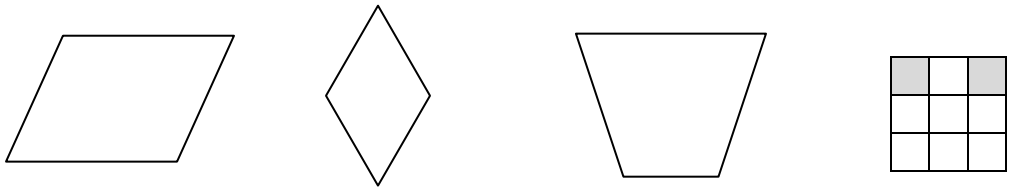
If the student has difficulty deciding whether or not the shape is symmetrical, have the student trace the shape on tracing paper, cut it out and fold it to see if the two halves match perfectly. If the student has difficulty tracing and cutting accurately, provide a cutout copy of the shape for the student to manipulate.

If the student has difficulty drawing a shape congruent to the given shape, suggest that the student use the cutout shape and trace it onto the grid. Alternately, suggest that the student draw one vertex and then count the spaces on the grid to decide where to place the next vertex. Then he or she should join the points for the vertices to complete the congruent shape.

3. Place the following labelled 2-D shapes before the student.



A B C D



E F G H

Have the student circle all the symmetrical shapes.

Then instruct the student to draw all the lines of symmetry on the symmetrical shapes. Finally, have the student sort the shapes by placing the letter of each shape in the chart provided.

Lines of Symmetry	Letter Names for the Shapes
No lines of symmetry	
One line of symmetry	
More than 1 line of symmetry	

If the student has difficulty choosing the symmetrical shapes, suggest that he or she trace the shapes on tracing paper, cut them out and fold them to determine if the two parts match perfectly. For a student who has difficulty tracing accurately, provide the cutout shapes for the student. Alternately, suggest that the student use a Mira to check for symmetry. Place the Mira on one of the shapes to model its use if necessary.

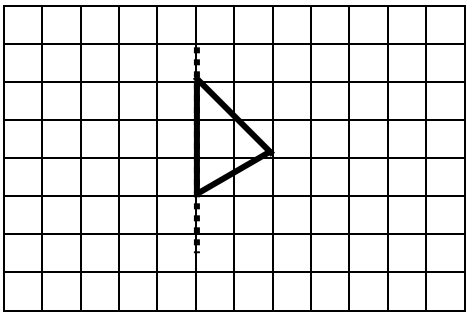
4. Provide the student with square dot paper, isometric dot paper, scrap paper, scissors and pattern blocks. Ask the student to create a symmetrical 2-D shape that is different from any of the shapes used so far in this exercise and draw it on plain paper or on the dot paper provided. Have the student explain how he or she knows that the shape is symmetrical.

   If the student has difficulty creating a symmetrical shape, provide one or more of the following prompts as needed:

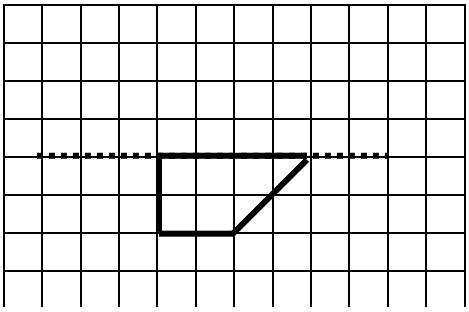
   - Fold a piece of scrap paper and draw half the design on one side. Cut it out and trace the unfolded design.
   - Use the pattern blocks to make a symmetrical 2-D shape and then copy it onto isometric dot paper provided.
   - Draw half a symmetrical design and the axis of symmetry along one of its sides. Then complete the symmetrical design by locating the corresponding vertices and sides on the opposite side of the axis of symmetry.

5. Ask the student to complete the following diagram to make a symmetrical shape using the line of symmetry shown (dotted line).

a.



b.



If the student has difficulty completing the symmetrical design, use the following prompts as needed:

- Look at one vertex. Count how many spaces it is from the axis of symmetry. Draw the partner for this vertex the same number of spaces in the opposite direction of the axis of symmetry. Continue with all the vertices except the ones lying on the axis of symmetry. Draw line segments to connect the vertices.
- Trace the design on tracing paper. Cut it out. Flip the design over and trace it on the other side of the axis of symmetry.

C. Applied Learning

Provide opportunities for the students to use their understanding of congruence and symmetry in a practical situation and notice whether or not this understanding transfers. For example, have the students collect leaves for science or an art project and then sort them as to whether they are symmetrical or not.

Does the student:

- sort the leaves correctly?
- explain why a given leaf is symmetrical or not?
- use congruency in describing two symmetrical parts?
- draw a different leaf showing symmetry if requested to do?
- complete the other half of a leaf to show symmetry when provided with the first half of the leaf?
- explain why two leaves are congruent or not congruent?

Step 5: Follow-up on Assessment

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction?

A. Addressing Gaps in Learning

Congruence
If the student is having difficulty with congruence, provide many opportunities for him or her to use concrete materials that are congruent, such as congruent pattern blocks or tiles. Place the congruent shape in a variety of orientations to reinforce the concept that shapes remain congruent when the orientation changes. Connect the work done with concrete materials to diagrams. Have the student draw the two congruent shapes using square or isometric dot paper.

To create congruent shapes, have the student draw a design on folded paper, cut it out and verify that the two shapes have the same shape and size. When matching congruent shapes to select congruent pairs or to identify corresponding sides and vertices, have the students cut out the shapes and superimpose on over the other. If the student is unable to cut out the shape because it is in a book, provide tracing paper so that he or she can trace the 2-D shape and superimpose it on other shapes to determine congruency and the corresponding sides and vertices.

Symmetry
If the student is having difficulty with symmetry, reinforce the connection to congruence by using folded paper to create symmetrical designs. Explain that the fold forms the axis of symmetry. Have the student identify symmetrical shapes by folding them. Provide tracing paper for the student to trace shapes out of books and then fold the cut out shapes to determine if they are symmetrical.
Use a variety of manipulatives to address the different learning styles. Provide experiences with folded paper, Miras, pattern blocks and geoboards. Ask probing questions to encourage the students to explain their thinking. Connect the manipulation of objects to diagrams by having the students draw the designs created.

When creating a symmetrical shape without manipulatives, have the students create half a design on grid paper and draw a vertical axis of symmetry along one of the sides of the design. Remind the students that the other half of the design must be congruent to the half design drawn. Review that congruent shapes have corresponding vertices. Have the students complete the other half by visually counting the spaces of corresponding vertices from the axis of symmetry. Continue work with vertical axes of symmetry until the students understand the process. Then include horizontal lines of symmetry as the students create and complete symmetric designs. Oblique lines of symmetry may be too difficult for some students so professional discretion must be used.

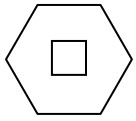
B. Reinforcing and Extending Learning

The students who have achieved or exceeded the outcomes will benefit from ongoing opportunities to apply and extend their learning. These activities should support the students in developing a deeper understanding of the concept and should not progress to the outcomes in subsequent grades.

Consider strategies, such as the following.

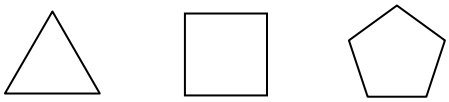
- Provide tips for parents on providing their children opportunities to apply their knowledge about congruence and symmetry.
  - When the child is observing a flower or an insect, take the opportunity to talk about symmetry that exists in nature.
  - Use paper folding to create various shapes in doing crafts with the children, such as making valentines, stars and happy faces.
  - Use paper folding and paint to make butterflies and other symmetrical designs.
  - In board games, such as checkers, have the child discover patterns that show symmetry on the board game.
  - Make snow angels and talk about why these shapes are symmetrical. Extend the talk to include suggestions as to how our body is symmetrical.
- Have the students sort everyday objects such as buttons, pencils, pens, paper clips and rulers as symmetrical or not and justify the process used.
- Have the students draw composite shapes using pattern blocks, indicating which shapes are symmetrical and why.
- Have the students explain which letters of the alphabet are symmetrical and why.
- Have the students explore palindromes; i.e., numbers that read the same frontward and backward, such as 88 and 121. Have them sort the palindromes into two sets, symmetrical and nonsymmetrical palindromes. They may wish to use a Mira to check their work. Typing the numbers so that there is consistency in spacing and writing the digits is recommended.
- Challenge the students to complete a symmetrical 2-D shape when given half the shape and its oblique line of symmetry. Oblique and horizontal lines of symmetry are more challenging for the students to use when completing a symmetrical 2-D shape.

- Challenge the students to find all the lines of symmetry in complex 2-D shapes without folding and explain how they know they are correct. For example:



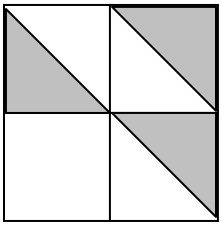
- Challenge the students to study the pattern below and answer the following:
  - Draw all the lines of symmetry for each 2-D shape and record the number below each shape.
  - Draw the fourth shape in the pattern and determine how many lines of symmetry are possible.
  - Explain how all the 2-D shapes are the same. (Answer: they are regular polygons.)

– Predict how many lines of symmetry the tenth diagram would have. Check your prediction by drawing the diagram.
– Write a generalization about symmetrical lines in the pattern shown.

Shape 

Number of
lines of symmetry: _____ _____ _____

- Challenge the students to create designs such as the following and ask a partner to make the design symmetrical by exchanging positions between two shapes within the design. For example, in the following design, exchange the positions between two triangles to make the design symmetrical. Can it be done in more than one way?



(Alberta Education 1990, p. 160)

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