Mathematics

Planning Guide

Grade 5 *Estimation Strategies*

Number

Specific Outcome 2

This Planning Guide can be accessed online at:
http://www.learnalberta.ca/content/mepg5/html/pg5_estimationstrategies/index.html

Table of Contents

Planning Guide: *Grade 5 Estimation Strategies*

Strand: Number
Specific Outcome: 2

This *Planning Guide* addresses the following outcomes from the Program of Studies:

Strand: Number	
Specific Outcome:	2. Use estimation strategies, including • front-end rounding • compensation • compatible numbers in problem-solving contexts.

Curriculum Focus

The changes to the curriculum targeted by this sample include:

- The general outcome focuses on number sense; whereas the previous math curriculum specified applying arithmetic operations to whole numbers and decimals, and to illustrate their use in creating and solving problems.
- The specific outcome includes using specific estimation strategies (front-end rounding, compensation and compatible numbers) in problem-solving contexts; whereas the previous math curriculum included estimating, mentally calculating, computing or verifying products (3-digit by 2-digit) and quotients (3-digit divided by 1-digit) of whole numbers.

What Is a Planning Guide?

Planning Guides are a tool for teachers to use in designing instruction and assessment that focuses on developing and deepening students' understanding of mathematical concepts. This tool is based on the process outlined in *Understanding by Design* by Grant Wiggins and Jay McTighe.

Planning Steps

The following steps will help you through the Planning Guide:

Step 1: Identify Outcomes to Address

Guiding Questions

- What do I want my students to learn?
- What can my students currently understand and do?
- What do I want my students to understand and be able to do based on the Big Ideas and specific outcomes in the program of studies?

Big Ideas

Number Sense
Number sense plays a major role the conceptual understanding of estimation strategies. Number sense is "an intuitive feeling for numbers and their various uses and interpretations; an appreciation for various levels of accuracy when figuring; the ability to detect arithmetical errors; and a common-sense approach to using numbers" (Reys 1992, p. 3).

Curriculum and Evaluation Standards identifies various aspects of number sense:

1. understanding the meanings of numbers,
2. having an awareness of multiple relationships among numbers,
3. recognizing the relative magnitude of numbers,
4. knowing the relative effect of operating on numbers and
5. possessing referents for measures of common objects and situations in the environment.

Estimation
Estimation is a mental "process of producing an answer that is sufficiently close to allow decisions to be made" (Reys 1986, p. 22). The types of estimation include quantity, computation and measurement. The focus for estimation described in the Grade 5 Number strand, specific outcome 2, is on computational estimation.

The first four aspects of number sense listed above provide the focus for computational estimation. *Principles and Standards for School Mathematics* states that estimating first and then calculating "provides a tool for judging the reasonableness of calculator, mental and paper-and-pencil computations" (NCTM 2000, p. 155). Expanding upon this further it states:

> "Students should be encouraged to explain their thinking frequently as they estimate. As with exact computation, sharing estimation strategies allows students access to others' thinking and provides many opportunities for rich class discussions" (NCTM 2000, p. 156).

As students estimate first and then calculate, they refine their estimation strategies. When estimating, students focus on "the meaning of the numbers and the operations" (Van de Walle and Lovin 2006, p. 125).

Front-end Strategy

The front-end strategy is a method of estimating computations by keeping the first digit in each of the numbers and changing all the other digits to zeros. This strategy can be used to estimate sums, differences, products and quotients. Note that the front-end strategy always gives an underestimate for sums, products and quotients (2- or 3-digit divided by 1-digit).

Example:
You buy a hamburger for $4.79 and a drink for $1.26. Will a $5 bill cover the cost?

Solution:
Total the front-end (dollar) amounts.

$4 + $1 = $5

Since the front-end strategy always gives an underestimate for a sum, $5 will not cover the cost.

Compensation Strategy

The compensation strategy is a method of adjusting a computational estimate to make it closer to the calculated answer. This strategy is used with the front-end and compatible numbers strategies to provide better estimates.

Example:
You buy a hamburger for $4.79 and a drink for $1.26. Will a $5 bill cover the cost?

Solution:
Total the front-end (dollar) amounts.

$4 + $1 = $5

Using the compensation strategy, group the cent amounts to form dollars:

$0.26 and $0.79 together make a little more than $1.

Final estimate including front-end and compensation:

$5 + $1 = $6

Answer to the problem: A $5 bill will not cover the cost because the cost is a little more than $6.

Compatible Numbers Strategy

The compatible numbers strategy is a method of estimating computations by using "friendly" or "nice" numbers that can be easily calculated mentally. Compatible numbers can be used in estimating sums, differences, products and quotients.

Example:
You are to divide $435 evenly among 7 people. About how much money should each person receive?

Solution:
Using compatible numbers, change 435 to a number closest to 435 and evenly divisible by 7; i.e., 420.

$420 \div 7 = 60$

Each person will receive about $60.

Using compensation, think "420 is less than 435 so the estimated answer is a little less than the calculated quotient."

Final estimate using compatible numbers and compensation:
Each person will receive a little more than $60, about $62.

Note: When using compatible numbers to estimate quotients with a single digit divisor, the 3-digit dividend is changed to the nearest multiple of the divisor using multiples of 10, keeping the divisor unchanged.

Sequence of Outcomes from Program of Studies

See http://education.alberta.ca/teachers/core/math/programs.aspx for the complete program of studies.

Grade 4	Grade 5	Grade 6
Specific Outcomes	**Specific Outcomes**	**Specific Outcomes**
3. Demonstrate an understanding of addition of numbers with answers to 10 000 and their corresponding subtractions (limited to 3- and 4-digit numerals) by: • using personal strategies for adding and subtracting • estimating sums and differences • solving problems involving addition and subtraction.	2. Use estimation strategies, including: • front-end rounding • compensation • compatible numbers in problem-solving contexts.	8. Demonstrate an understanding of multiplication and division of decimals (1-digit whole number multipliers and 1-digit natural number divisors).
6. Demonstrate an understanding of multiplication (2- or 3-digit by 1-digit) to solve problems by: • using personal strategies for multiplication with and without concrete materials • using arrays to represent multiplication • connecting concrete representations to symbolic representations	5. Demonstrate, with and without concrete materials, an understanding of multiplication (2-digit by 2-digit) to solve problems. 6. Demonstrate, with and without concrete materials, an understanding of division (3-digit by 1-digit), and interpret remainders to solve problems. 11. Demonstrate an understanding of addition and subtraction of decimals (limited to thousandths).	

- estimating products
- applying the distributive property.

7. Demonstrate an understanding of division (1-digit divisor and up to 2-digit dividend) to solve problems by:
   - using personal strategies for dividing with and without concrete materials
   - estimating quotients
   - relating division to multiplication.

Step 2: Determine Evidence of Student Learning

Guiding Questions

- What evidence will I look for to know that learning has occurred?
- What should students demonstrate to show their understanding of the mathematical concepts, skills and Big Ideas?

Using Achievement Indicators

As you begin planning lessons and learning activities, keep in mind ongoing ways to monitor and assess student learning. One starting point for this planning is to consider the achievement indicators listed in the *Mathematics Kindergarten to Grade 9 Program of Studies with Achievement Indicators.* You may also generate your own indicators and use them to guide your observation of the students.

The following indicators may be used to determine whether or not students have met this specific outcome. Can students:

- provide a context for when estimation is used to:
  - make predictions
  - check the reasonableness of an answer
  - determine approximate answers?
- describe contexts in which overestimating is important?
- determine the approximate solution to a given problem not requiring an exact answer?
- estimate a sum or product using compatible numbers?
- estimate a quotient using compatible numbers?
- estimate the solution to a given problem using compensation, and explain the reason for compensation?
- select and use an estimation strategy for a given problem?
- apply the front-end strategy to estimate:
  - sums; e.g., 253 + 615 is more than 200 + 600 = 800
  - differences; e.g., 974 – 250 is close to 900 – 200 = 700
  - products; e.g., the product of 23 × 24 is greater than 20 × 20 = 400 and less than 25 × 25 = 625
  - quotients; e.g., the quotient of 831 ÷ 4 is greater than 800 ÷ 4 = 200?

Sample behaviours to look for related to these indicators are suggested for some of the activities listed in **Step 3, Section C: Choosing Learning Activities** (p. 14).

Step 3: Plan for Instruction

Guiding Questions

- What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should I use?
- How will I meet the diverse learning needs of my students?

A. Assessing Prior Knowledge and Skills

Before introducing new material, consider ways to assess and build on students' knowledge and skills related to estimating sums, differences, products and quotients. For example:

- On a trip you travel 4250 km the first week, 3755 km the second week and 2115 km the third week.
  i. Estimate how many kilometres you travel during the three weeks. Explain your thinking.
  ii. Do you think your estimate is more or less than the calculated answer? Explain your reasoning.
- You have 4 pieces of chocolate that each weigh 253 g. Estimate whether the total weight of these 4 pieces of chocolate is more or less than 1000 g or 1 kg. Explain your thinking.
- During one summer, Marcie travels 7185 km while Jimmy travels 4205 km.
  i. Estimate how much farther Marcie travelled than Jimmy during the summer. Explain your thinking.
  ii. Do you think your estimate is more or less than the calculated answer? Explain your reasoning.
- You have 62 stickers to share equally among 3 people.
  i. Estimate how many stickers each person receives. Explain your thinking.
  ii. Do you think your estimate is more or less than the calculated answer? Explain your reasoning.
- Judy used the following estimation strategy to estimate the sum of 365 and 437.
  Judy's thinking: I used the front-end strategy. $365 \rightarrow 300$
  $437 \rightarrow 400$
  $300 + 400 = 700$
  My estimate for the sum of 365 and 437 is about 700.

  How could you adjust Judy's estimate to make it closer to the calculated sum? Explain your thinking without doing the actual calculation.

  Subtraction questions: $685 - 217 = ?$ $685 - 274 = ?$

  Estimate (front-end strategy): $600 - 200 = 400$ $600 - 200 = 400$

  i. Which estimate is closer to the actual difference? Explain your thinking without doing the actual calculation.

ii. For each estimated difference, is 400 an overestimate or an underestimate? Explain your thinking without doing the actual calculation.

If a student appears to have difficulty with these tasks, consider further individual assessment, such as a structured interview, to determine the student's level of skill and understanding. See **Sample Structured Interview: Assessing Prior Knowledge and Skills** (p. 10).

Sample Structured Interview: Assessing Prior Knowledge and Skills

Directions	Date:	
	Not Quite There	**Ready to Apply**
On a trip you travel 4250 km the first week, 3755 km the second week and 2115 km the third week. **a. Estimate how many kilometres you travel during the three weeks. Explain your thinking.** **b. Do you think your estimate is more or less than the calculated answer? Explain your reasoning.**	Calculates first and then estimates rather than the reverse. Provides an estimate but is unable to explain his or her thinking. Does not know whether the estimate is more or less than the calculated answer. Correctly states that the estimate is more or less than the calculated answer but is unable to explain his or her reasoning.	Uses and explains appropriate estimation strategies, such as the front-end strategy, together with compensation to estimate the sum: a. Example: 4250 → 4000 3755 → 3000 2115 → 2000 9000 Compensation: 250 + 755 is about 1000. Also, 2115 is 2000 + 115, so I have 115 more to add on. Final estimate: 9000 + 1000 + 100 = 10 100. I travel about 10 100 km. b. 9000 is less than the calculated answer but when I compensated, my final answer of 10 100 is very close to the calculated answer.
You have 4 pieces of chocolate that each weigh 253 g. Estimate whether the total weight of these 4 pieces of chocolate is more or less than 1000 g or 1 kg. Explain your thinking.	Adds 4 and 253 rather than estimating the product of 4 and 253. Uses an estimation strategy that does not provide an accurate estimate. Provides an accurate estimate but is unable to explain his or her thinking.	Uses a personal estimation strategy to obtain an accurate estimate and explains his or her thinking. Example: 4×253 is a little more than $4 \times 250 = 1000$.

During one summer, Marcie travels 7185 km while Jimmy travels 4205 km. **a. Estimate how much farther Marcie travelled than Jimmy during the summer. Explain your thinking.** **b. Do you think your estimate is more or less than the calculated answer? Explain your reasoning.**	Calculates first and then estimates rather than the reverse. Provides an estimate but is unable to explain his or her thinking. Does not know whether the estimate is more or less than the calculated answer. Correctly states that the estimate is more or less than the calculated answer but is unable to explain his or her reasoning.	Uses and explains an appropriate estimation strategy and explains his or her thinking. Example: a. 7185 7000 4205 – 4000 Estimate: 3000 The amount dropped off each number is about the same: 185 is about the same as 205. Therefore, the estimate is close to the actual answer. Marcie travels about 3000 km farther than Jimmy during the summer. b. Since 205 is slightly more than 185, then the estimate is slightly more than the calculated answer.
You have 62 stickers to share equally among 3 people. **a. Estimate how many stickers each person receives. Explain your thinking.** **b. Do you think your estimate is more or less than the calculated answer? Explain your reasoning.**	Does not know which operation to use in the estimated answer. Calculates first and then estimates rather than the reverse. Provides an estimate but is unable to explain his or her thinking. Does not know whether the estimate is more or less than the calculated answer. Correctly states that the estimate is more or less than the calculated answer but is unable to explain his or her reasoning.	Uses and explains an appropriate estimation strategy and explains his or her thinking. Example: a. $62 \div 3$ is close to $60 \div 3 = 20$. Each person receives about 20 stickers. b. The estimate is less than the calculated answer because 60 is less than 62.

Judy used the following estimation strategy to estimate the sum of 365 and 437. **Judy's thinking: "I used the front-end strategy:** **365 → 300** **437 → 400** **300 + 400 = 700** **My estimate for the sum of 365 and 437 is about 700."** **How could you adjust Judy's estimate to make it closer to the calculated sum? Explain your thinking without doing the actual calculation.**	Does not know how to adjust Judy's estimate to make it closer. Provides a closer estimate; e.g., 800, but does not explain his or her reasoning.	Uses compensation to adjust Judy's estimate and explains the process. Example: 65 was dropped off 365 to make 300. 37 was dropped off 437 to make 400. 65 + 37 is about 100. A better estimate is 700 + 100 = 800.
Present the following two subtraction questions and their estimated differences to the student: Subtraction questions: 1. 685 – 217 = ? 2. 685 – 274 = ? Estimate (front-end strategy): 1. 685 – 217 = ? 600 – 200 = 400 2. 685 – 274 = ? 600 – 200 = 400 Ask the students: **a. Which estimate is closer to the actual difference? Explain your thinking without doing the actual calculation.** **b. For each estimated difference, is 400 an overestimate or an underestimate? Explain your thinking without doing the actual calculation.**	Does not know which estimate is closer to the actual difference. States which estimate is closer to the actual difference but does not explain his or her thinking. Does not know whether the 400 is an overestimate or an underestimate. States that 400 is an overestimate in at least one of the examples but does not explain why.	a. States that the estimated answer of 400 is a closer estimate for question 1 than for question 2. For question 2, about the same amount is dropped off each number in the front-end strategy; therefore, the estimate is close. For question 1, 85 is dropped off the first number and only 17 is dropped of the second number so the estimate will not be as close. b. In each case, 400 is an underestimate because you must consider the difference of 85 – 17 and also 85 – 74. The difference in each case must be added on to the 400 to make a closer estimate.

B. Choosing Instructional Strategies

Consider the following general strategies for teaching computational estimation.

- Encourage the students to take risks as they explore various computational estimation strategies. They must develop a comfort level in finding approximate answers to computation.
- "Create a classroom environment that encourages student exploration, questioning, verification and sense making" (Reys 1992, p.5).
- Have the students communicate their thinking as they estimate and then "share their reasoning with the class" (Reys 1992, p.5).
- Capitalize on class sharing by highlighting the estimation strategies that result in close estimates; e.g., combining compensation with other strategies such as front-end or compatible numbers.
- Provide opportunities for the students to explore the multiple relationships among numbers and among operations.
- Provide regular reinforcement so that students *always* estimate before they calculate to determine the reasonableness of their calculated answers. Van de Walle and Lovin (2006) state, "A good place to begin … computation is with estimation. Not only is it a highly practical skill, but it also helps [students] look at answers in ballpark terms and can form a check on calculator computation" (p.125).
- Provide a variety of problem-solving contexts in which students decide that an estimated answer is adequate and efficient.
- Provide a variety of problem-solving contexts in which students have the opportunity to explore various types of computational estimation strategies and then choose the strategy that works best for them in a given situation.

C. Choosing Learning Activities

The following learning activities are examples of activities that could be used to develop student understanding of the concepts identified in Step 1.

Sample Activities:

1. **Sorting Problems** (p.15)
2. **Estimating Using Front-end and Compensation Strategies** (p.16)
3. **Estimating Using Compatible Numbers and Compensation** (p.21)

Sample Activity 1: Sorting Problems

Look For …

Do students:

- ☐ correctly classify problems and explain the rules for sorting?
- ☐ exhibit flexibility in classifying the problems in more than one way?
- ☐ correctly identify problems where prediction is needed, an approximate answer is adequate or an exact answer is required?
- ☐ correctly identify problems where an overestimate is required?

As students solve a variety of problems, have them always estimate the answer first and then calculate if a calculation is necessary. Through discussion, have the students conclude that not all problems require an exact answer; an approximate answer might be adequate, sometimes an overestimate is important or perhaps a prediction is required that involves estimation. Classify the problems into these three groups. Reinforce the idea that estimating before calculating is *always* necessary to determine the reasonableness of calculated answers obtained using paper-and-pencil or a calculator.

Then provide the students with a variety of problem-solving contexts and have them sort the problems into groups using an open sort in which the students decide the name for each group and explain their reasoning. They might sort the problems according to which operation they would use to solve the problem. If necessary, use a closed sort to focus attention on estimating. The categories for the closed sort could include the following: prediction is needed, an approximate answer is adequate or an exact answer is required. Students could mark the problems that require an overestimate.

Sample Problems

a. Jenny and her 11 friends go out to eat at a restaurant. Each meal costs $21, including GST and the tip. Will $200 cover the cost of the meals?

b. There are 6 boxes of juice in a carton. About how many cartons of juice are needed for 38 students if each student drinks 3 boxes of juice?

c. Mark has $30 and wants to buy a CD for $12.89 and a book for $23.59. Does he have enough money to buy the CD and the book?

d. Jason wants to buy a bicycle for $250. If he saves $15 a month, Jason thinks that he will be able to buy this bicycle in 2 years. Is his thinking reasonable?

e. One hundred ten students are going to camp in vans. If each van can take 8 students, how many vans are needed?

f. Danny read 9 minutes a day. In about how many days will he read a total of 300 minutes?

g. Susie has 304 stickers to share equally among 4 friends. How many stickers does each friend receive?

Sample Activity 2: Estimating Using Front-end and Compensation Strategies

i. Multiplying 2-digit by 2-digit Whole Numbers

Draw on prior knowledge by reviewing estimation strategies used by students in solving multiplication problems (2- or 3-digit by 1-digit).

In preparation for using the front-end strategy to estimate the product of two 2-digit numbers, discuss the pattern in multiplying by 10: $10 \times 10 = 100$, $10 \times 20 = 200$, $10 \times 30 = 300$, $10 \times 40 = 400$ and so on. To differentiate instruction, have the students use skip counting as necessary to establish the pattern. Also, use 10×10 as the basis for the other products; e.g., $10 \times 20 = 10 \times 2 \times 10 = 2 \times 10 \times 10$.

Continue with other patterns using multiples of 10:

- $20 \times 20 = 400$, which can be written as "$2 \times 2 \times 10 \times 10 = 400$"
- $20 \times 30 = 600$, which can be written as "$2 \times 3 \times 10 \times 10 = 600$"
- $20 \times 40 = 800$, which can be written as "$2 \times 4 \times 10 \times 10 = 800$."

Have the students generalize a rule for multiplying two 2-digit numbers using multiples of ten.

Look For …

Do students:

- ☐ effectively multiply using multiples of 10?
- ☐ apply their knowledge of multiplication number facts?
- ☐ generalize a rule for multiplying two 2-digit numbers using multiples of ten?
- ☐ generalize that the front-end strategy always produces an underestimate, so compensation is needed to refine the estimate?
- ☐ communicate clearly how to use the front-end strategy?
- ☐ communicate clearly how to use compensation to make an estimate more accurate?

Provide the students with problem-solving contexts requiring the multiplication of two 2-digit whole numbers. Model estimating the product using the front-end strategy. Then encourage the students to refine the estimate by using compensation.
An example is provided below.

Problem:
To raise money at school, 24 students each sold 36 chocolate bars. Estimate how many chocolate bars the students sold.

Solution:
Through discussion, have the students decide the operation used in this problem: multiplication.
Write "front-end strategy" on the board and state that this strategy will be used to estimate the product.
Explain that the front-end strategy uses only the first digit in each number and replaces the other digits with zeros; therefore, 24 becomes 20 and 36 becomes 30. Review multiplying by 10s and also rewriting each number as a product of 10.
Write "24×36" on the board.

Then rewrite it using the front-end strategy: 20 × 30.
If necessary, rewrite "20 × 30" as "2 × 3 × 10 × 10 = ?"
Finally, write "20 × 30 = 600."

This completes the estimation using the front-end strategy, but encourage the students to refine the estimate by using compensation. Explain that the compensation strategy is used to adjust the estimate to make it closer to the actual product. Ask the students whether 600 is more or less than the actual product and why they think so. A sample explanation might be "Since the digits in the ones place were replaced by zeros, then 24 x 36 is greater than 600." Through discussion, have the students generalize that the front-end strategy for finding the product of two numbers is always an underestimate; therefore, compensation is needed to refine the estimate.

Write "24 × 36 is greater than 600" on the board. Underline the 4 in 24 and the 6 in 36 to focus attention on the digits that were dropped using the front-end strategy.

Have the students decide what number must be added to 600 to make the estimate more accurate. Sample response: "4 was dropped from 24, and 4 groups of 40 is 160. 6 was dropped from 36, and 6 groups of 20 is 120. 160 + 120 = 280, so using compensation, the adjusted estimate is 600 + 280 = 880."
Answer to the problem: The students sold about 880 chocolate bars.

Encourage the students to calculate the answer to the problem using a personal strategy and then compare their calculated answer to the estimated answer.

ii. Dividing 3-digit Numbers by 1-digit Numbers

Draw on prior knowledge by reviewing estimation strategies used by students in solving divisions problems (1-digit divisor and up to 2-digit dividend).

In preparation for using the front-end strategy to estimate the quotient of a 3-digit number by a 1-digit number, discuss the pattern in dividing multiples of 100 by a single digit number: $800 \div 2 = 400$ because $2 \times 400 = 800$, $800 \div 4 = 200$ because $4 \times 200 = 800$, $800 \div 8 = 100$ because $8 \times 100 = 800$.

Provide the students with problem-solving contexts requiring the division of a 3-digit number by a 1-digit number. Model estimating the quotient using the front-end strategy. Then encourage the students to refine the estimate by using compensation. For example:

Problem:
A string that is 685 cm long is to be cut into 3 pieces of equal length. Estimate how long each piece will be.

Solution:
Through discussion, have the students decide that the operation used in this problem is division.
Write "front-end strategy" on the board and state that this strategy will be used to estimate the quotient in the problem.
Explain that the front-end strategy uses only the first digit in each number and replaces the other digits with zeros.
Therefore, 685 is rewritten as "600."

Write "$685 \div 3$" on the board.
Then rewrite it using the front-end strategy: $600 \div 3$.
If necessary, rewrite the division as a multiplication: $3 \times ? = 600$.
Finally, write "$600 \div 3 = 200$."

Look For…

Do students:

- ☐ effectively divide using multiples of 100 in the dividend?
- ☐ apply their knowledge of division number facts?
- ☐ explain the connection between division and multiplication?
- ☐ generalize that the front-end strategy always produces an underestimate when dividing a 3-digit number by a 1-digit number so compensation is needed to refine the estimate?
- ☐ communicate clearly how to use the front-end strategy?
- ☐ communicate clearly how to use compensation to make an estimate more accurate?

This completes the estimation using the front-end strategy but encourage the students to refine the estimate by using compensation. Explain that the compensation strategy is used to adjust the estimate to make it closer to the actual quotient. Ask the students whether or not 200 is more or less than the actual quotient and why they think so. A sample explanation might be: "Since the digits in the ones and tens place were replaced by zeros, then $685 \div 3$ is greater than 200."

Through discussion, have the students generalize that the front-end strategy for finding the quotient of 3-digit number divided by a 1-digit number is always an underestimate; therefore, compensation is needed to refine the estimate.
Write "$685 \div 3$ is greater than 200" on the board.

Have the students make suggestions for a value of the refined estimate. Sample response: "Since 85 was dropped in the front-end estimate, then add 85 ÷ 3 to the estimate. 85 is about 90. 90 ÷ 3 = 30. 200 + 30 = 230. Each piece of string is about 230 cm long."

Encourage the students to calculate the answer to the problem using a personal strategy and then compare their calculated answer to the estimated answer.

Emphasize that the front-end strategy is only used if the hundreds digit is divisible by the divisor. Otherwise, the compatible numbers strategy is more effective to use in estimating the quotient.

iii. Adding and Subtracting Decimals

Draw on prior knowledge by reviewing students' estimation strategies in solving problems by adding and subtracting whole numbers (limited to 3- and 4-digit numerals).

In preparation for using the front-end strategy to estimate the sum or difference of decimals, review place value and the meaning of decimals.

Explain that the front-end strategy requires that the decimal part of the number be replaced by zeros, leaving whole numbers to add or subtract.

Review the front-end strategy for adding and subtracting whole numbers and have the students apply this strategy to appropriate problems. Encourage them to use compensation as needed to refine the estimated answers.

Provide the students with problem-solving contexts requiring the addition or subtraction of decimals. Model estimating the product using the front-end strategy. Then encourage the students to refine the estimate by using compensation. For example:

Look For …

Do students:

- ☐ explain that the digits after the decimal are not used in the front-end estimation of sums and differences of decimals?
- ☐ apply their knowledge of addition and subtraction number facts?
- ☐ explain the connection between addition and subtraction?
- ☐ generalize that the front-end strategy always produces an underestimate, so compensation is needed to refine the estimate?
- ☐ communicate clearly how to use the front-end strategy?
- ☐ communicate clearly how to use compensation to make an estimate more accurate?

Problem:
You have 136.5 cm of red ribbon and 26.2 cm of green ribbon.

a. Estimate how much ribbon you have in all.
b. Estimate how much more red ribbon you have than green ribbon.

Solution:

a. Through discussion, have the students establish the fact that the operation used in this problem is addition. Write "front-end strategy" on the board and state that this strategy will be used to estimate the sum in the problem. Explain that the front-end strategy uses only the first digit in each number and replaces the other digits with a zero. Therefore, 136.5 is rewritten as "100." Also, 26.2 is rewritten as "20."

   Write "136.5 + 26.2" on the board.
   Then rewrite it using the front-end strategy: 100 + 20.

   Finally, write "100 + 20 = 120."

   This completes the estimation using the front-end strategy, but encourage the students to refine the estimate by using compensation. Explain that the compensation strategy is used to adjust the estimate to make it closer to the actual sum. Ask the students whether 120 is more or less than the actual sum and why they think so. A sample explanation might be: "Since many of the digits were replaced by zeros, then 136.5 + 26.2 is greater than 120." Write "136.5 + 26.2 is greater than 120" on the board.

   Have the students make suggestions for a value of the refined estimate. Sample response: "Since 36 and 6 were dropped in the front-end estimate and their sum is about 40, add 40 the estimate. 120 + 40 = 160. The total length of the two ribbons is about 160 cm long."

   Encourage the students to calculate the answer to the problem using a personal strategy and then compare the calculated answer to the estimated answer.

b. Use a similar process in finding the difference between the two decimals. The front-end strategy will result in 120 – 40 = 80. When compensating to obtain a closer estimate, the students should discuss the effect of dropping the 36 and the 6; i.e., 36 – 6 = 30. Through discussion, have the students summarize that the difference of 30 must be added to the front-end estimate to make a closer estimate.
   Final estimate: 80 + 30 = 110.
   Answer to the problem: The red ribbon is about 110 cm longer than the green ribbon.

   Encourage the students to calculate the answer to the problem using a personal strategy and then compare their calculated answer to the estimated answer.

Sample Activity 3: Estimating Using Compatible Numbers and Compensation

i. Multiplying 2-digit by 2-digit Whole Numbers

The compatible numbers strategy is a method of estimating computations by using "friendly" or "nice" numbers that can be easily calculated mentally.

Provide the students with problem-solving contexts requiring the multiplication of two 2-digit whole numbers. Model estimating the product using the compatible numbers strategy. Then encourage the students to refine the estimate by using compensation, if necessary. For example:

Problem:
To raise money at school, 24 students each sold 36 chocolate bars. Estimate how many chocolate bars the students sold.

Solution:
Through discussion, have the students establish the fact that the operation used in this problem is multiplication. Write "compatible numbers strategy" on the board and state that this strategy will be used to estimate the product in the problem. Review the meaning of compatible; i.e., things that are compatible work well together. Explain that compatible numbers are numbers that you can multiply easily in your head.

Look For …

Do students:

- ☐ effectively multiply using multiples of 10?
- ☐ apply their knowledge of multiplication number facts?
- ☐ generalize a rule for multiplying two 2-digit numbers using multiples of ten?
- ☐ explain the reason for using compensation along with the compatible numbers strategy to refine the estimated product?
- ☐ communicate clearly how to use the compatible numbers strategy?
- ☐ communicate clearly how to use compensation to make an estimate more accurate?

Explain that the compatible numbers strategy often uses numbers that are multiples of 10 because these numbers are easy to multiply in your head. Therefore, 24 becomes 20 and 36 becomes 40. Review multiplying by 10s and rewriting each number as a product of 10. Write "24 × 36" on the board.

Then rewrite it using the compatible numbers strategy: 20 × 40.
If necessary, rewrite "20 × 40" as "2 × 4 × 10 × 10 = ?"
Finally, write "20 × 40 = 800."

This completes the estimation using the compatible numbers strategy. Have the students discuss whether or not they think 800 is a close estimate to the actual product and why or why not. A sample explanation might be: "Since 24 was decreased to 20 and 36 was increased to 40, then the product should be very close because the increase and decrease even things out." Explain that sometimes compensation is not needed because the estimated answer is already very close to the actual product.

Final estimate: 800
Answer to the problem: The students sold about 800 chocolate bars.

Encourage the students to calculate the answer to the problem using a personal strategy and then compare the calculated answer to the estimated answer.

Note: Compensation is usually not needed when one factor is increased the other factor is decreased to make compatible numbers. However, compensation is needed when finding the product of two 2-digit numbers using compatible numbers in the following examples:

- $43 \times 84 = ?$ Compatible numbers: 40×80. Estimated product is 3200.
  The estimate is too low because both factors in the product were decreased. Using compensation, the final estimated product is about 3600.
- $79 \times 58 = ?$ Compatible numbers: 80×60. Estimated product is 4800.

The estimate is too high because both factors in the product were increased. Using compensation, the final estimated product is about 4600.

ii. Dividing 3-digit Numbers by 1-digit Numbers

In preparation for using the compatible numbers strategy to estimate the quotient of a 3-digit number by a 1-digit number, review division facts and explain that the numbers used in division facts are "nice" or "friendly" numbers because you can divide them in your head.

Also, review the pattern in dividing multiples of 100 by a single digit number: $800 \div 2 = 400$ because $2 \times 400 = 800$, $800 \div 4 = 200$ because $4 \times 200 = 800$, $800 \div 8 = 100$ because $8 \times 100 = 800$.

Provide the students with problem-solving contexts requiring the division of a 3-digit number by a 1-digit number. Model estimating the quotient using the compatible numbers strategy. Then encourage the students to refine the estimate by using compensation, if necessary. For example:

Look For…

Do students:

- ☐ effectively divide using multiples of 100 in the dividend?
- ☐ apply their knowledge of division number facts?
- ☐ explain the connection between division and multiplication?
- ☐ explain that the compatible numbers strategy will produce an underestimate or an overestimate so compensation is needed to refine the estimate?
- ☐ communicate clearly how to use the compatible numbers strategy?
- ☐ communicate clearly how to use compensation to make an estimate more accurate?

Problem:
A string that is 315 cm long is to be cut into 4 pieces of equal length. Estimate how long each piece will be.

Solution:
Through discussion, have the students establish the fact that the operation used in this problem is division.
Write "compatible numbers strategy" on the board and state that this strategy will be used to estimate the quotient in the problem. Explain that the compatible numbers strategy requires that the numbers in the problem be rewritten using common number facts. Through discussion, have the students realize that numbers close to 315 that can be easily divided by 4 include 280 and 320.

Write "315 ÷ 4" on the board.
Then rewrite it using the compatible numbers strategy: 280 ÷ 4 and also 320 ÷ 4.
If necessary, rewrite the divisions as a multiplication: 4 × ? = 280 and 4 × ? = 320.
Finally, write 280 ÷ 4 = 70 and 320 ÷ 4 = 80.

Explain that the quotient of 315 ÷ 4 is greater than 280 ÷ 4 = 70 and less than 320 ÷ 4 = 80.
Have the students decide whether the estimate is closer to 70 or 80. Possible explanation: "Since 315 is closer to 320 than it is to 280, then the estimate is closer to 80, say 78.
Answer to the problem: Each piece of string is about 78 cm long.

Encourage the students to calculate the answer to the problem using a personal strategy and then compare the calculated answer to the estimated answer.

iii. Adding and Subtracting Decimals

In preparation for using the compatible numbers strategy to estimate the sum or difference of decimals, review place value and the meaning of decimals. Explain that often an approximate value for a decimal that is used in estimation is the closest whole number that the decimal represents. For example, 3.255 can be approximated as about 3 or the value of 4.885 is about 5.

When using the compatible numbers strategy, the decimal part of the number may or may not be used to approximate the value of the decimal, depending on the choice of compatible numbers. When the decimal has two or more digits before the decimal, these digits are rewritten as a multiple of 10 or 100 for ease of mental computation.

After the students approximate decimals by using whole numbers, then have them use the compatible numbers and compensation strategies that they learned previously for adding and subtracting whole numbers.

Provide the students with problem-solving contexts requiring the addition or subtraction of decimals. Model estimating the product using the compatible numbers strategy. Then encourage the students to refine the estimate by using compensation. For example:

Look For …

Do students:

- ☐ explain how the digits after the decimal may be used to approximate the value of a decimal?
- ☐ explain when the digits after the decimal may be used in estimating the sum or difference?
- ☐ apply their knowledge of addition and subtraction number facts?
- ☐ explain the connection between addition and subtraction?
- ☐ explain the reason for using compensation along with the compatible numbers strategy to refine the estimated sum or difference?
- ☐ communicate clearly how to use the compatible numbers strategy?
- ☐ communicate clearly how to use compensation to make an estimate more accurate?

Problem:
You have 136.5 cm of red ribbon and 26.2 cm of green ribbon.

a. Estimate how much ribbon you have in all.
b. Estimate how much more red ribbon you have than green ribbon.

Solution:

a. Through discussion, have the students establish the fact that the operation used in this problem is addition. Write "compatible numbers strategy" on the board and state that this strategy will be used to estimate the sum in the problem.
   Explain that the compatible numbers strategy uses numbers that are easy to add in your head, such as multiples of 10 or 100. Therefore, 136.5 can be rewritten as 140. Also, 26.2 can be rewritten as 30.

   Write "136.5 + 26.2" on the board.
   Then rewrite it using the compatible numbers strategy: 140 + 30.

Finally, write "140 + 30 = 170."

This completes the estimation using the compatible numbers strategy, but encourage the students to refine the estimate by using compensation. Explain that the compensation strategy is used to adjust the estimate to make it closer to the actual sum. Ask the students whether 170 is more or less than the actual sum and why they think so. A sample explanation might be: "Since 136.5 is less than 140 and 26.2 is less than 30, then 136.5 + 26.2 is less than 170." Write "136.5 + 26.2 is less than 170" on the board.

Have the students make suggestions for a value of the refined estimate. Sample response: Since 4 was added to 36 to make it 40 and 4 was also added to 16 to make it 30, then decrease the estimate by 8.
Final estimate: 170 – 8 = 162.
Answer to the problem: The total length of the two ribbons is about 162 cm long.

Encourage the students to calculate the answer to the problem using a personal strategy and then compare the calculated answer to the estimated answer.

b. Use a similar process for finding the difference between the two decimals. The compatible numbers strategy will result in rewriting 136.5 – 26.2 as 140 – 30 = 110. Explain that this is a good estimate and compensation is not needed because 4 was added to each of the numbers, thereby keeping the difference constant. Use other simpler examples to reinforce the concept of constant difference; e.g., 8 – 5 = 10 – 7 because 2 is added to 8 and also to 5.
Answer to the problem: The red ribbon is about 110 cm longer than the green ribbon.

Encourage the students to calculate the answer to the problem using a personal strategy and then compare the calculated answer to the estimated answer.

Step 4: Assess Student Learning

Guiding Questions

- Look back at what you determined as acceptable evidence in Step 2.
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

In addition to ongoing assessment throughout the lessons, consider the following sample activities to evaluate students' learning at key milestones. Suggestions are given for assessing all students as a class or in groups, individual students in need of further evaluation, and individuals or groups of students in a variety of contexts.

A. Whole Class/Group Assessment

Estimation: Assessment Task
In this assessment task, students will demonstrate their understanding of estimating products and quotients of whole numbers. They will estimate the number of lines of print (products) in four of Hans Christian Andersen's stories. They will then justify their choice of estimation strategy and explain how to improve accuracy of estimates. Students will then estimate the number of pages read each day (quotients) for four students who each read a different novel. They will communicate key ideas related to estimating quotients.

Task-specific Criteria:

- to assist in developing the scoring guide
- should be written in a way that answers "how well" the student achieved the outcome(s)

Each student will:

- select and use estimation strategies and communicate effectively how and why they were used
- use compensation as needed to make a closer estimate and justify its use
- adjust the dividend in the division problems to the nearest multiple of the divisor (multiples of 10)
- communicate effectively key points to remember when estimating division problems.

Early finishers can estimate the number of words in each of the novels when each line contains about 9 words.

Title: Estimation—Student Assessment Task

Maren and her friends each read a different story by Hans Christian Andersen. Maren said she had read the most because her story had the most pages. Nicholas pointed out that his story had more lines on a page so he actually read the most. Finally, Brooklyn and Jaxon decided they should all estimate the number of lines in their stories to settle the argument.

Name	Story	Number of Pages	Lines per page
Maren	The Ugly Duckling	29	18
Nicholas	The Steadfast Tin Soldier	16	25
Brooklyn	The Princess and the Pea	24	17
Jaxon	The Fir Tree	18	19

1. Select and use strategies to estimate the number of lines in each story to find out who read the most. Show all your thinking below.

The Ugly Duckling	The Steadfast Tin Soldier
The Princess and the Pea	The Fir Tree

Explain which person read the most.

2. Explain why you used the estimation strategy or strategies that you chose.

Maren and her friends also read novels as shown in the chart below.

Name	Novel	Number of pages	Days to read novel
Maren	*A Series of Unfortunate Events: The Hostile Hospital*	255	6
Nicholas	*The Bellmaker*	336	5
Brooklyn	*Charlotte's Web*	184	3
Jaxon	*Sunwing*	243	8

3. Use estimation strategies to find about how many pages per day each student read.

A Series of Unfortunate Events: The Hostile Hospital	*The Bellmaker*
Charlotte's Web	*Sunwing*

4. What is important to remember when estimating division problems?

Student _____

SCORING GUIDE
Estimation: Student Assessment Task

Level / Criteria	4 Excellent	3 Proficient	2 Adequate	1 Limited *	Insufficient / Blank *
Estimates the product (2-digit by 2-digit) **Question 1**	Shows a thorough understanding of estimation by selecting appropriate strategies, including compensation, to make a close estimate.	Shows a clear understanding of estimation by selecting appropriate strategies and uses compensation to make a reasonable estimate.	Shows an understanding of one type of estimation strategy but does not use compensation to make an accurate estimate.	Shows some understanding of estimating but fails to apply a complete the estimation strategy or doesn't estimate—multiplies original numbers and then may or may not adjust the product to represent an estimate.	No score is awarded because there is insufficient evidence of student performance based on the requirements of the assessment task.
Justifies choice of estimation strategies **Question 2**	Justifies selection of estimation strategies with sound mathematical support, including a clear explanation for using compensation to improve accuracy.	Justifies selection of estimation strategies with clear support, including some reference to using compensation.	Justifies selection of estimation strategies with minimal support, but makes no reference to using compensation.	Identifies the estimation strategy with no explanation.	No score is awarded because there is insufficient evidence of student performance based on the requirements of the assessment task.
Estimates the quotient (3-digit divided by 1-digit) **Question 3**	Shows a thorough understanding of estimation by adjusting the dividend to the nearest multiple of the divisor (multiples of 10) and effectively using compensation to improve accuracy.	Shows a clear understanding of estimation by adjusting the dividend to a multiple of the divisor (multiples of 10) and using compensation to improve accuracy.	Shows an understanding of estimation by adjusting both the dividend and the divisor to workable numbers or by adjusting only the dividend (not a close multiple of 10) with no use of compensation to improve accuracy.	Shows some understanding of estimation but fails to apply a complete estimation strategy or doesn't estimate—divides original numbers and then may or may not adjust the quotient to represent an estimate.	No score is awarded because there is insufficient evidence of student performance based on the requirements of the assessment task.
Identifies the essential elements of estimating quotients **Question 4**	States and illustrates with examples the importance of adjusting the dividend to make it divisible by the divisor and then clearly explains how to use compensation to improve accuracy.	States the importance of adjusting only the dividend to make it divisible by the divisor and explains that compensation is necessary to improve accuracy.	States the importance of adjusting the dividend in general terms with no mention of compensation to improve accuracy.	States the importance of adjusting the dividend in vague terms.	No score is awarded because there is insufficient evidence of student performance based on the requirements of the assessment task.

***** When work is judged to be limited or insufficient, the teacher makes decisions about appropriate intervention to help the student improve.

B. One-on-one Assessment

1. Ask the student to explain to you the difference between estimating an answer to a problem and calculating the answer using paper-and-pencil. If necessary, coach the student by providing an example of a problem and having the student estimate the answer prior to calculating it. Have the student explain his or her thinking when making an estimate. Provide guidance as necessary to use the front-end or compatible numbers strategy. Encourage the student to refine the estimate by using compensation.

2. Present the following problem to the student and have him or her read it orally.
   You have a piece of string and cut off 46.8 cm, leaving 138.6 cm. Estimate the length of string you had at the beginning.

   Use the following prompts to guide the student's thinking, if necessary:

   - State the problem in your own words.
   - What do each of the numbers in the problem represent – a part or a whole?
   - What is the unknown in the problem – a part or a whole?
   - What number sentence could you write to show the meaning of the problem?
   - What operation will you use to solve the problem? Explain.
   - Use an estimation strategy that makes sense to you to find the answer to the problem. Explain your thinking as you write the numbers. (Hint: provide guidance in using the front-end or compatible numbers strategies, if necessary.)
   - Explain how you know your estimate is quite close to the calculated answer. (Hint: have the student use compensation, if appropriate, to refine the estimate.)
   - Calculate the answer to the problem using paper and pencil to record your personal strategy.
   - Compare your calculated answer with your estimated answer.

3. Use the same procedure as outlined in Question 2 with the following problem:
   There are 52 candies in each of 23 bags. Estimate how many candies there are in all the bags.

4. Have the students complete the following:
   Create a problem that requires only an estimated answer to solve it. Solve the problem you created by estimating the answer and explaining your thinking.

C. Applied Learning

Provide opportunities for the students to use computational estimation strategies in a practical situation and notice whether or not the strategies transfer. For example, ask a student if a $10 bill will cover the cost of buying a milkshake for $3.98 and a sub for $6.59. Have him or her explain the thinking done to solve the problem. Does the student:

- use estimation in solving this problem?
- explain clearly why a $10 bill would not cover the cost of the two items?
- apply the estimation skills in solving other problems? For example: About how many photo pages are needed for 220 photos if each page holds 8 photos?

Step 5: Follow-up on Assessment

Guiding Questions

- What conclusions can be made from the assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction?

A. Addressing Gaps in Learning

Students who have difficulty solving problems using estimation will enjoy more success if one-on-one time is provided in which there is open communication to diagnose where the learning difficulties lie. Assessment by observing a student solving problems will provide valuable data to guide further instruction. Success in problem solving depends on a positive climate in which the students are confident in taking risks. By building on the understanding that each student already has and accommodating the individual learning styles, success will follow.

If the difficulty lies in understanding the problem, use the following strategies:

- Provide problems that relate to the student's interest; use the student's name in the problem.
- Use smaller numbers in the problem initially.
- Have the student paraphrase the problem.
- Provide base-ten materials for the students to represent the problem as needed.
- Guide the student in determining what the numbers represent – a part or a whole.
- Guide the student in determining what the unknown in the problem represents – a part or a whole.
- Have the student decide which operation should be used and why.
- Ask guiding questions to show the connections between addition and subtraction and also between multiplication and division.
- Provide a graphic organizer such as the K–N–W–S chart.

If the difficulty lies in computational estimating, use the following strategies:

- Use the base-ten materials to focus on the place values of the numbers and the relationship among the place values.
- Use smaller numbers initially and connect them to larger numbers; e.g., connect 30 to 300 to 3000.
- Convince the student of the need for estimating by citing many real-world examples of where estimating is needed.
- Review number facts and place value.
- Emphasize flexibility in estimating, capitalizing on the student's methods and fine tuning them for correctness and efficiency.
- Take small steps, using the front-end or compatible numbers strategy without compensation first and then adding the compensation when the student sees the need for it in providing a better estimate.

Other strategies for estimating products and quotients are available on pages 250–253 of the *Diagnostic Mathematics Program, Division II, Operations*.

B. Reinforcing and Extending Learning

Students who have achieved or exceeded the outcomes will benefit from ongoing opportunities to apply and extend their learning. These activities should support students in developing a deeper understanding of the concept and should not progress to the outcomes in subsequent grades.

Consider strategies such as the following:

- Provide tips for parents on practising computational estimation at home or in the community. For example:
  - Take the students shopping and have them estimate the total grocery bill prior to going through the check out.
  - Collect cash register receipts, cover the totals and have the students estimate the total; or, tear off the totals and have the students match the receipts with the correct totals using estimation.
  - Talk to your students about data in the newspaper and magazines and encourage them to add and subtract mentally and explain how they are doing it.
  - Integrate estimation activities into the daily activities of the students; e.g.,
    - After reading a novel for a specific number of days, estimate how many pages were read per day.
    - If you brush your teeth twice a day for two minutes each time, about how many minutes do you brush your teeth in a year?
- Have the students create computational problems that require an estimated answer. These problems can be displayed in chart on the bulletin board.
- Have the students estimate sums, differences, products and quotients using a variety of estimation strategies and then decide which strategy provides the most accurate estimate and why.
- Continually challenge the students to refine their estimates using compensation and explain the reason for compensation.
- Have the students critique other students' estimation strategies and explain why they work or not. Which would be the most efficient and why?
- Have the students write an explanation for a computational estimation strategy so that everyone in the class can understand it.

Estimation grids

- Encourage the students to play games involving estimation; e.g.,
  The goal of this game is to capture four cells in a row (vertically, horizontally, or diagonally) on the grid below. Divide the class into two teams. Display the grid and the factor board shown below on an overhead projector. Each team takes turns choosing two factors from the factor board, estimates the product to match a number on the grid. If the product is on the grid, the team captures that cell. The teacher or a chosen student has a calculator to check if the product of the two factors chosen is on the grid. The first team to capture four cells in a row on the grid is the winner.

Grid

187	1189	1769	943	697
1403	319	1219	1037	437
901	1159	323	551	2501
1007	253	1537	671	391
583	779	3233	667	451

Factor Board

11	17	19	23
29	41	53	61

- Encourage the students to think about how large the answer will be (a 2-digit number? a 3-digit number?); e.g., provide the students with the following computations. Have the students estimate how many digits the answer to each computation will have and explain their thinking.

1. $132 + 579$
2. $132 + 979$
3. $1567 – 438$
4. $5086 – 145$
5. 45×89
6. 98×99
7. $348 \div 4$
8. $735 \div 6$

K What facts do I **KNOW** from the information in the problem?	**N** Which information do I **NOT** need?	**W** **WHAT** does the problem ask me to find?	**S** What **STRATEGY** will I use to solve the problem?

Adapted with permission from Mary Lee Barton and Clare Heidema, *Teaching Reading in Mathematics: A Supplement to Teaching Reading in the Content Areas Teacher's Manual* (2nd ed.) (Aurora, CO: McREL (Mid-continent Research for Education and Learning), 2002), p. 113.

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