Mathematics

Planning Guide

Grade 6
Angles and Triangles

Shape and Space
(Measurement)
Specific Outcome 1

Shape and Space
(3-D Objects and 2-D Shapes)
Specific Outcomes 4 and 5

This Planning Guide can be accessed online at:
http://www.learnalberta.ca/content/mepg6/html/pg6_anglestriangles/index.html

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Strand: Shape and Space (3-D Objects and 2-D Shapes)
Specific Outcomes: 4 and 5

This *Planning Guide* addresses the following outcomes from the program of studies:

Strand: Shape and Space (Measurement)	
Specific Outcome:	1. Demonstrate an understanding of angles by: • identifying examples of angles in the environment • classifying angles according to their measure • estimating the measure of angles, using 45^o, 90^o and 180^o as reference angles • determining angle measures in degrees • drawing and labelling angles when the measure is specified.
Strand: Shape and Space (3-D Objects and 2-D Shapes)	
Specific Outcomes:	4. Construct and compare triangles, including: • scalene • isosceles • equilateral • right • obtuse • acute in different orientations. 5. Describe and compare the sides and angles of regular and irregular polygons.

Curriculum Focus

This sample targets the following changes to the curriculum:

- The general outcome focuses on describing the characteristics of 3-D objects and 2-D shapes, and analyzing the relationships among them, whereas the previous program of studies focused on describing, classifying, constructing and relating 3-D objects and 2-D shapes, using mathematical vocabulary.
- The specific outcomes focus on constructing and comparing different types of triangles as well as describing and comparing the sides and angles of regular and irregular polygons. The previous program of studies included constructing and classifying triangles according to the measure of their sides in Grade 5 and according to the measure of their angles in Grade 6. Regular polygons were sorted according to the number of lines of symmetry in Grade 6.

What Is a Planning Guide?

Planning Guides are a tool for teachers to use in designing instruction and assessment that focuses on developing and deepening students' understanding of mathematical concepts. This tool is based on the process outlined in *Understanding by Design* by Grant Wiggins and Jay McTighe.

Planning Steps

The following steps will help you through the Planning Guide:

Step 1: Identify Outcomes to Address

Guiding Questions

- What do I want my students to learn?
- What can my students currently understand and do?
- What do I want my students to understand and be able to do based on the Big Ideas and specific outcomes in the program of studies?

Big Ideas

Principles and Standards for School Mathematics states that instructional programs relating to the geometry standard should enable all students to "analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships" (NCTM 2000, p. 164). This document goes on to explain that students should use "drawings, concrete materials and geometry software to develop and test their ideas … about why geometric relationships are true" (NCTM 2000, p. 166).

Van de Walle and Lovin (2006) explain the role of geometric properties in describing the similarities and differences among various shapes:

> What makes shapes alike and different can be determined by an array of geometric properties. For example, shapes have sides that are parallel, perpendicular or neither; they have line symmetry, rotational symmetry or neither; they are similar, congruent or neither (p. 204).

In Van de Walle and Lovin (2006, pp. 207–208), the van Hiele levels of understanding in geometry describe the development in students' learning:

Level 0: Visualization
The objects of thought at Level 0 are shapes and what they look like. … It is the appearance of the shape that defines it for the student. … The products of thought at Level 0 are classes or groupings of shapes that seem to be alike.

Level 1: Analysis
The objects of thought at Level 1 are classes of shapes rather than individual shapes. … At this level, students begin to appreciate that a collection of shapes goes together because of properties; e.g., all cubes have six congruent faces and each of the faces is a square. … The products of thought at Level 1 are the properties of shapes.

Level 2: Informal Deduction
The objects of thought at Level 2 are the properties of shapes. … For example, four congruent sides and at least one right angle can be sufficient to define a square. Rectangles are parallelograms with a right angle. … The products of thought at Level 2 are relationships among properties of geometric objects.

Level 3: Deduction
The objects of thought at Level 3 are relationships among properties of geometric objects. …
The products of thought at Level 3 are deductive axiomatic systems for geometry.

Level 4: Rigour
The objects of thought at Level 4 are deductive axiomatic systems for geometry. … The products of thought at Level 4 are comparisons and contrasts among different axiomatic systems of geometry.

The focus in Grade 6 is on levels 1 and 2.

Definitions (Alberta Education 1990, pp. 197–206)

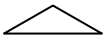
- Polygon: a closed figure with three or more sides.
- Regular Polygon: a polygon that has all sides congruent and all interior angles congruent.
- Triangle: a polygon with three sides.
- Scalene Triangle: a triangle with no sides of equal length.
- Isosceles Triangle: a triangle with only two sides of equal length.
- Equilateral Triangle: a triangle with all three sides of equal length.
- Right Triangle: a triangle with one right angle (an angle equal to 90^o).
- Obtuse Triangle: a triangle with one obtuse angle (an angle greater than 90^o and less than 180^o).
- Acute Triangle: a triangle in which all three angles are acute (less than 90^o).
- Congruent Figures: geometric figures that have the same size and shape.

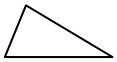
The types of triangles can be cross-classified. Examples (Small 2009, p. 96):

Acute equilateral triangle or equiangular triangle: 

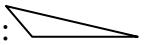
Acute isosceles triangle: 

Right isosceles triangle: 

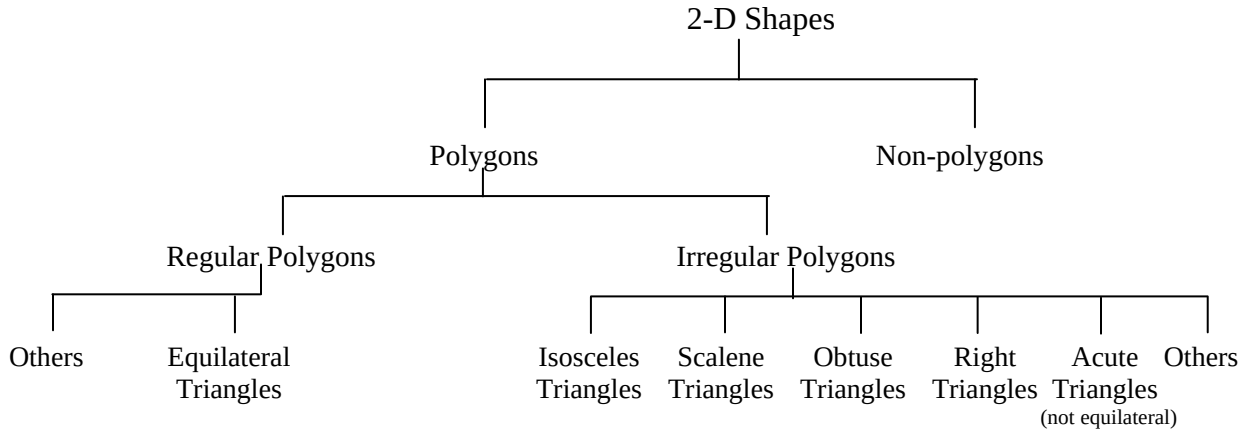
Obtuse isosceles triangle: 

Acute scalene triangle: 

Right scalene triangle: 

Obtuse scalene triangle: 

Example: Relationships among geometric shapes can be shown using a tree diagram.



The sum of the angles in any triangle is 180^o. Angles opposite congruent sides are equal; e.g., each angle in an equilateral triangle is 60^o, the angles in a right isosceles triangle measure 45^o, 45^o and 90^o.

Sequence of Outcomes from the Program of Studies

See http://www.education.alberta.ca/teachers/program/math/educator/progstudy.aspx for the complete program of studies.

Grade 5

Shape and Space (3-D Objects and 2-D Shapes)

Specific Outcomes

7. Identify and sort quadrilaterals, including:
   - rectangles
   - squares
   - trapezoids
   - parallelograms
   - rhombuses

   according to their attributes.

Grade 6

Shape and Space (Measurement)

Specific Outcomes

1. Demonstrate an understanding of angles by:
   - identifying examples of angles in the environment
   - classifying angles according to their measure
   - estimating the measure of angles, using 45^o, 90^o and 180^o as reference angles
   - determining angle measures in degrees
   - drawing and labelling angles when the measure is specified.

Shape and Space (3-D Objects and 2-D Shapes)

Specific Outcomes

4. Construct and compare triangles, including:
   - scalene
   - isosceles
   - equilateral
   - right
   - obtuse
   - acute

   in different orientations.

5. Describe and compare the sides and angles of regular and irregular polygons.

Grade 7

Shape and Space (3-D Objects and 2-D Shapes)

Specific Outcomes

3. Perform geometric constructions, including:
   - perpendicular line segments
   - parallel line segments
   - perpendicular bisectors
   - angle bisectors.

Step 2: Determine Evidence of Student Learning

Guiding Questions

- What evidence will I look for to know that learning has occurred?
- What should students demonstrate to show their understanding of the mathematical concepts, skills and Big Ideas?

Using Achievement Indicators

As you begin planning lessons and learning activities, keep in mind ongoing ways to monitor and assess student learning. One starting point for this planning is to consider the achievement indicators listed in *The Alberta K–9 Mathematics Program of Studies with Achievement Indicators*. You may also generate your own indicators and use these to guide your observation of students.

The following indicators may be used to determine whether or not students have met this specific outcome. Can students:

- identify the characteristics of a given set of triangles according to their sides and/or their interior angles?
- sort a given set of triangles and explain the sorting rule?
- identify a specified triangle from a given set of triangles; e.g., isosceles?
- draw a specified triangle; e.g., scalene?
- replicate a given triangle in different orientation and show that the two are congruent?
- sort a given set of 2-D shapes into polygons and non-polygons and explain the sorting rule?
- demonstrate congruence (sides-to-sides and angles-to-angles) in a regular polygon by superimposing?
- demonstrate congruence (sides-to-sides and angles-to-angles) in a regular polygon by measuring?
- demonstrate that the sides of a given regular polygon are the same length and that the angles of a regular polygon are the same measure?
- sort a given set of polygons as regular or irregular and justify the sorting?
- identify and describe regular and irregular polygons in the environment?
- classify a given set of angles according to their measure; e.g., acute, right, obtuse, straight, reflex?
- estimate the measure of an angle, using 45^o, 90^o and 180^o as reference angles?
- measure, using a protractor, given angles in various orientations?
- draw and label a specified angle in various orientations, using a protractor?

Sample behaviours to look for related to these indicators are suggested for some of the activities listed in **Step 3, Section C: Choosing Learning Activities** (p. 12).

Step 3: Plan for Instruction

Guiding Questions

- What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should I use?
- How will I meet the diverse learning needs of my students?

A. Assessing Prior Knowledge and Skills

Before introducing new material, consider ways to assess and build on students' knowledge and skills related to counting. For example:

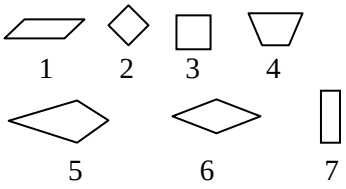
Sort the following quadrilaterals in two different ways and explain the sorting rule for each way.

- Explain why all rhombuses are parallelograms.

- Describe two similarities and two differences between trapezoids and parallelograms.

If a student appears to have difficulty with these tasks, consider further individual assessment, such as a structured interview, to determine the students' level of skill and understanding. See **Sample Structured Interview: Assessing Prior Knowledge and Skills** (p. 10).

Sample Structured Interview: Assessing Prior Knowledge and Skills

Directions	Date:	
	Not Quite There	**Ready to Apply**
Place the following problem before the student. **Sort the following quadrilaterals in two different ways and explain the sorting rule for each way.**  1 2 3 4 5 6 7	Sorts the quadrilaterals in only one way and may have difficulty explaining the sorting rule. OR Sorts the quadrilaterals in two different ways but is unable to explain at least one of the sorting rules.	Sorts the quadrilaterals in two different ways and explains the sorting rules using precise mathematical language. Sample Answers: • Sort 1 Quadrilaterals that have all sides the same length are numbers 2, 3 and 6. The rest of the quadrilaterals do not have sides the same length. • Sort 2 Quadrilaterals with two pairs of parallel sides are numbers 1, 2, 3, 6 and 7. The remaining two quadrilaterals do not have two pairs of parallel sides.
Say, **"Explain why all rhombuses are parallelograms."**	Attempts to explain but the explanation is incorrect or confusing. OR Knows what a parallelogram is but does not know what a rhombus is.	Explains in detail why all rhombuses are parallelograms. Sample Answer: A rhombus is a quadrilateral with two pairs of parallel sides and four congruent sides. A parallelogram is a quadrilateral with two pairs of parallel sides. Since all rhombuses are quadrilaterals with two pairs of parallel sides, all rhombuses are parallelograms.

Directions	Date:	
	Not Quite There	**Ready to Apply**
Say, **"Describe two similarities and two differences between trapezoids and parallelograms."**	Provides a limited description of one or two similarities and one or two differences between trapezoids and parallelograms. OR Knows what a parallelogram is but does not know what a trapezoid is.	Describes two similarities and two differences between trapezoids and parallelograms. Sample Answer: Similarities • Both are quadrilaterals; i.e., they both have four sides. • Both have at least one pair of parallel sides. Differences • Parallelograms have two pairs of parallel sides, whereas trapezoids have only one pair of parallel sides. • Parallelograms have opposite angles congruent, whereas trapezoids do not have opposite angles congruent.

B. Choosing Instructional Strategies

Consider the following instructional strategies for teaching constructing and comparing triangles, and describing and comparing regular and irregular polygons.

- Connect new geometric concepts to previous concepts learned.
- Connect the geometric concepts to the real world.
- Connect classification of triangles according to angles with classification of angles according to their measure.
- Integrate the specific outcomes; e.g., focus on triangles as examples of polygons and the equilateral triangle as an example of a regular polygon where all angles have the same measure.
- Provide hands-on experiences by having students construct various 2-D shapes and then focus on their properties.
- Use concept attainment with examples and non-examples of a given geometric concept to promote discussion and critical thinking so that students construct their own meaning and write their own definition.
- Have the students sort 2-D shapes into groups and explain the sorting rule.
- Have the students discuss the similarities and differences between two geometric concepts.
- Consolidate learning of the various geometric concepts through different strategies; e.g., Frayer Model, tree diagram and semantic feature chart.

C. Choosing Learning Activities

The following learning activities are examples of activities that could be used to develop student understanding of the concepts identified in Step 1.

Sample Activities:

1. **Concept Attainment for Geometric Concepts** (p. 13)
2. **Open and Closed Sorts** (p. 15)
3. **Connections among Geometric Concepts** (p. 22)
4. **Tree Diagrams** (p. 24)
5. **Property Lists for Regular Polygons and Different Kinds of Triangles** (p. 25)
6. **Minimal Defining Lists** (p. 26)
7. **What Is My Shape?** (p. 27)
8. **Frayer Model** (p. 28)
9. **True or False** (p. 29)
10. **Semantic Feature Chart** (p. 30)
11. **Congruency** (p. 31)
12. **Using Technology to Draw 2-D Shapes and Determine Congruency** (p. 32)
13. **Constructing 2-D Shapes without Technology** (p. 33)

Sample Activity 1: Concept Attainment for Geometric Concepts

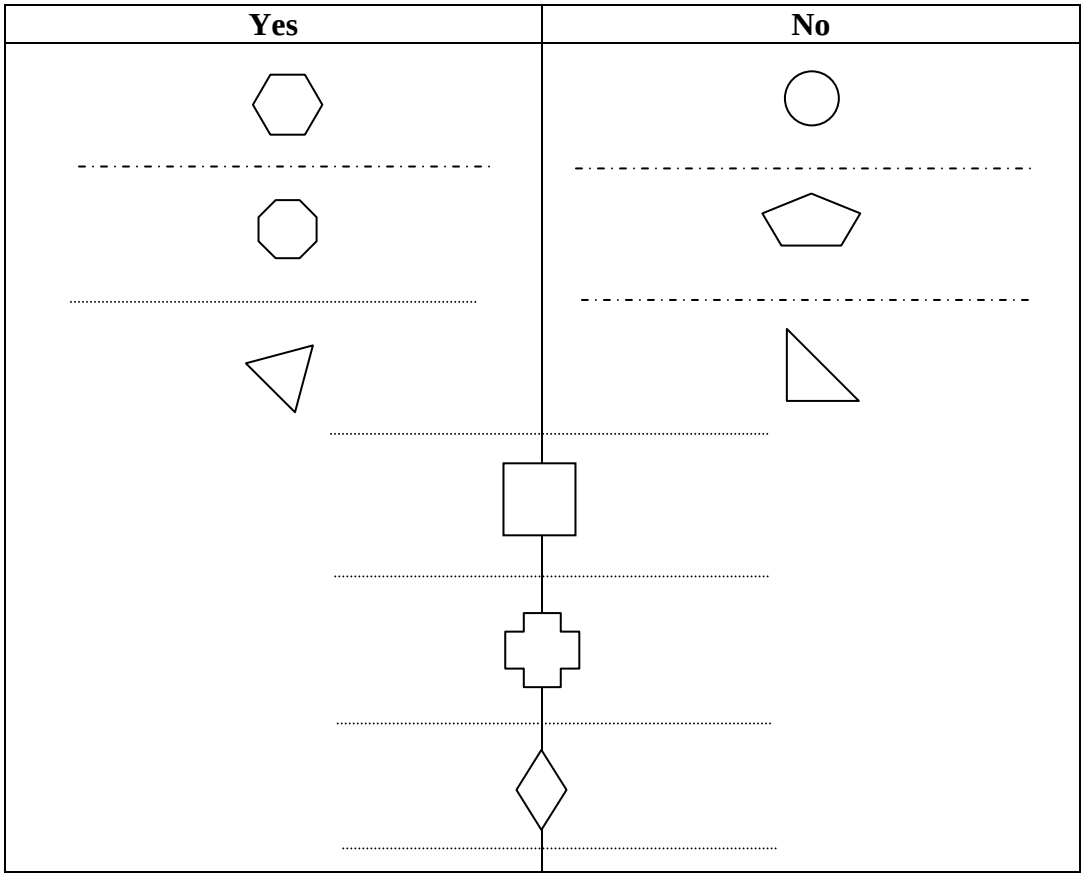
Use concept attainment to stimulate student thinking about what visual representations show the various concepts by focusing on examples and non-examples.

Use a "Yes/No" chart displayed on the overhead projector or drawn on the board. Tell students that you are thinking of a geometric concept and will place examples of this concept in the "Yes" column and non-examples in the "No" column.

Have students explain the differences between the figures in the two columns by describing the properties, not naming the shape. Use student responses to decide on the next example and non-example to use.

Example Illustrating Concept Attainment

The following example of concept attainment focuses on regular polygons.



Getting Started

Place a regular hexagon in the "Yes" column and a circle in the "No" column. Students may say that the figure in the "Yes" column has straight sides and the figure in the "No" column does not have straight sides. Through discussion, establish that both figures are 2-D but the one on the left has straight sides, which is an important property of the concept.

Congruency
You also want to focus on the congruency of the sides; therefore, show the next example with a regular octagon in the "Yes" column and an irregular pentagon in the "No" column. Have students explain how the two examples in the "Yes" column are the same and how they are different from the two examples in the "No" column; e.g., the two figures in the "Yes" column each have congruent sides, whereas the two figures in the "No" column do not have congruent sides.

Look For …
Do students:
- ☐ explain how the examples in the "Yes" column are the same by focusing on the characteristics they have in common but are different from the examples in the "No" column?
- ☐ decide into which column to place a given set and justify their choice?
- ☐ transfer the learning about properties to write definitions in their own words?

Equal Angles
Similarly, to focus on equal angles, place an equilateral triangle in the "Yes" column and a right triangle in the "No" column. Again, have students describe the properties of all the examples in the "Yes" column that do not apply to the non-examples in the "No" column.

Classifying Polygons
Place the next example, a square, on the line between the "Yes" and the "No" columns and have students decide where they think the square should go and justify their response. Since you are focusing on the properties of regular polygons, place the square in the "Yes" column because all sides are congruent and all angles are congruent; therefore, it is a regular polygon. Similarly, continue with other figures placed on the line between the "Yes" and "No" columns. After discussion, place each example in the correct column to continue attaining the concept of regular polygons.

Final Steps
Proceed with examples and non-examples until the class discussion summarizes that all the examples in the "Yes" column have all the following properties that do not apply to the non-examples in the "No" column:

- a 2-D figure with straight sides
- all sides are congruent
- all interior angles are congruent.

At the very end, invite students to suggest a name that applies to all the examples in the "Yes" column but does not apply to any of the non-examples in the "No" column; i.e., regular polygons.

Then have students write their own definition of a regular polygon by describing the necessary and sufficient conditions and drawing a diagram.

Use a similar process for developing understanding of the different types of triangles and have students write a definition for each type of triangle along with a diagram.

Sample Activity 2: Open and Closed Sorts

Provide students with a variety of 2-D shapes, including shapes that are polygons (regular and irregular as well as triangles of all kinds) and those that are not polygons. *Use BLM 1–6 on pages 16–21*.

Open Sort
Instruct students to sort the 2-D shapes into groups and describe the sorting rule. Then, have students sort the shapes in a different way and describe the sorting rule.

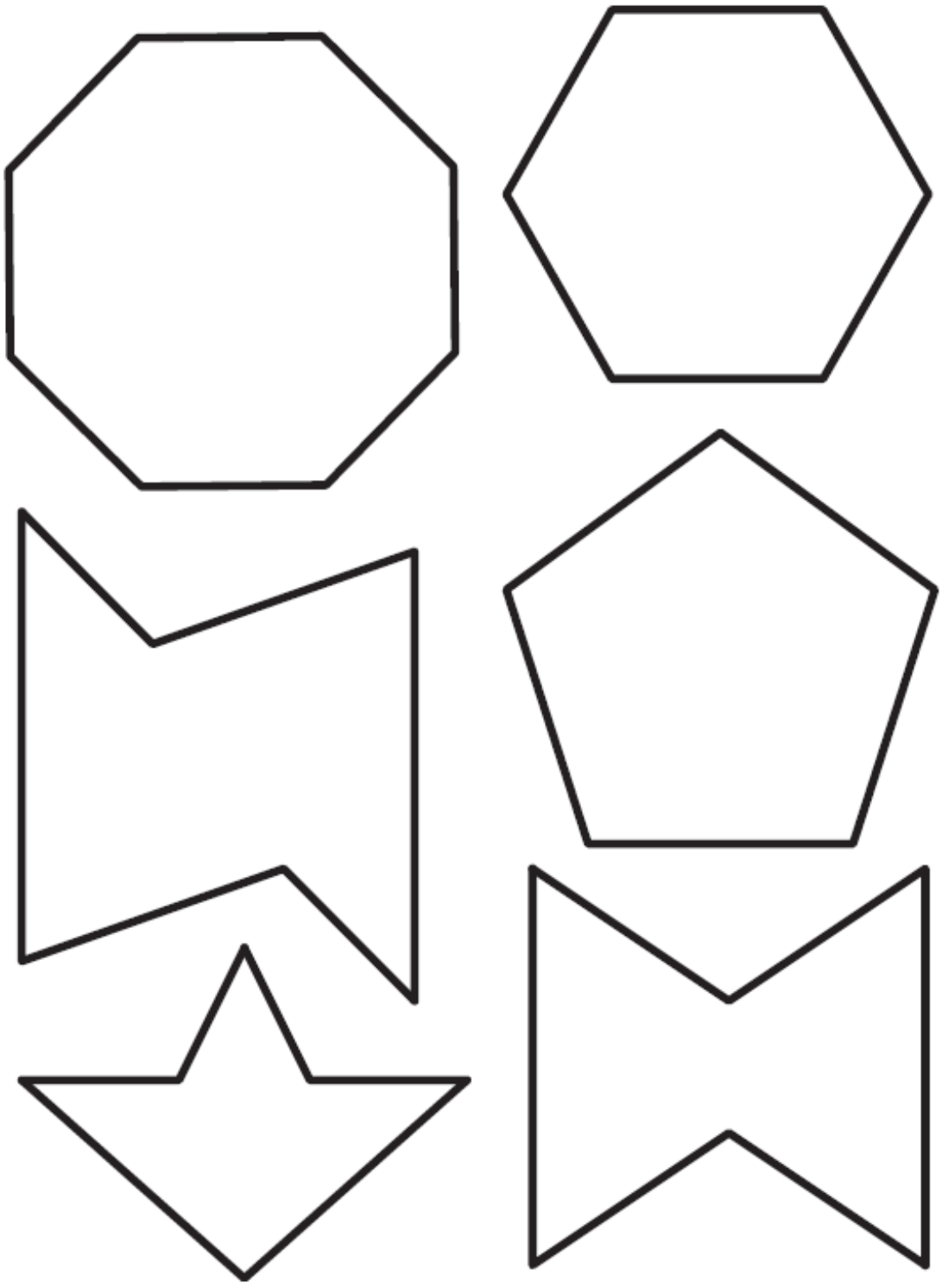
Closed Sort
Provide students with the categories into which they are to sort the 2-D shapes; e.g., polygons and not polygons, regular polygons and irregular polygons. Have students sort the 2-D shapes into the given categories and explain their reasoning.

Look For …

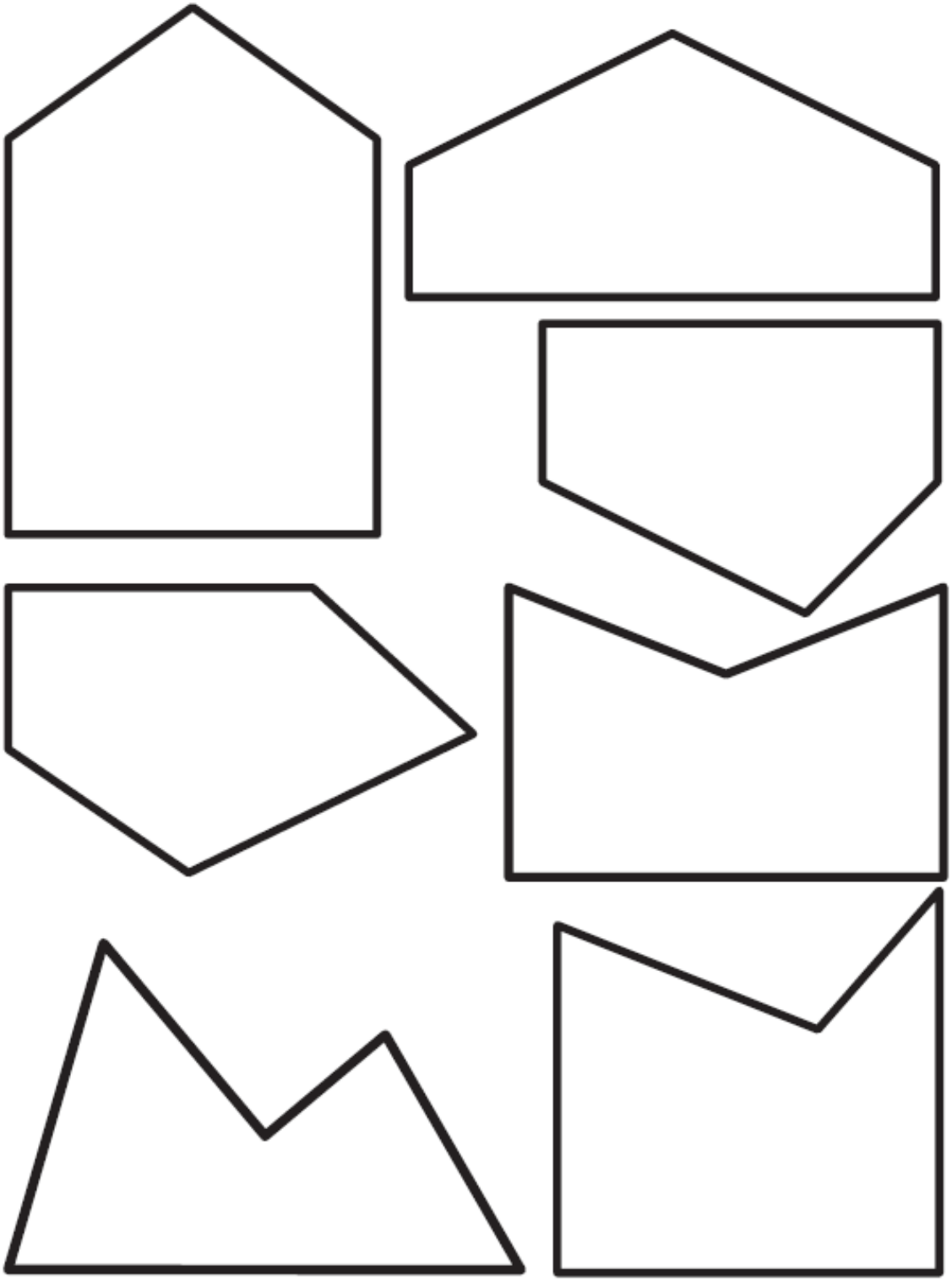
Do students:

- ☐ sort 2-D shapes into two or more groups and describe the sorting rule?
- ☐ sort 2-D shapes into two or more groups in a different way and describe the sorting rule?
- ☐ critique sorting rules created by other students?
- ☐ sort 2-D shapes into given categories and explain their thinking?

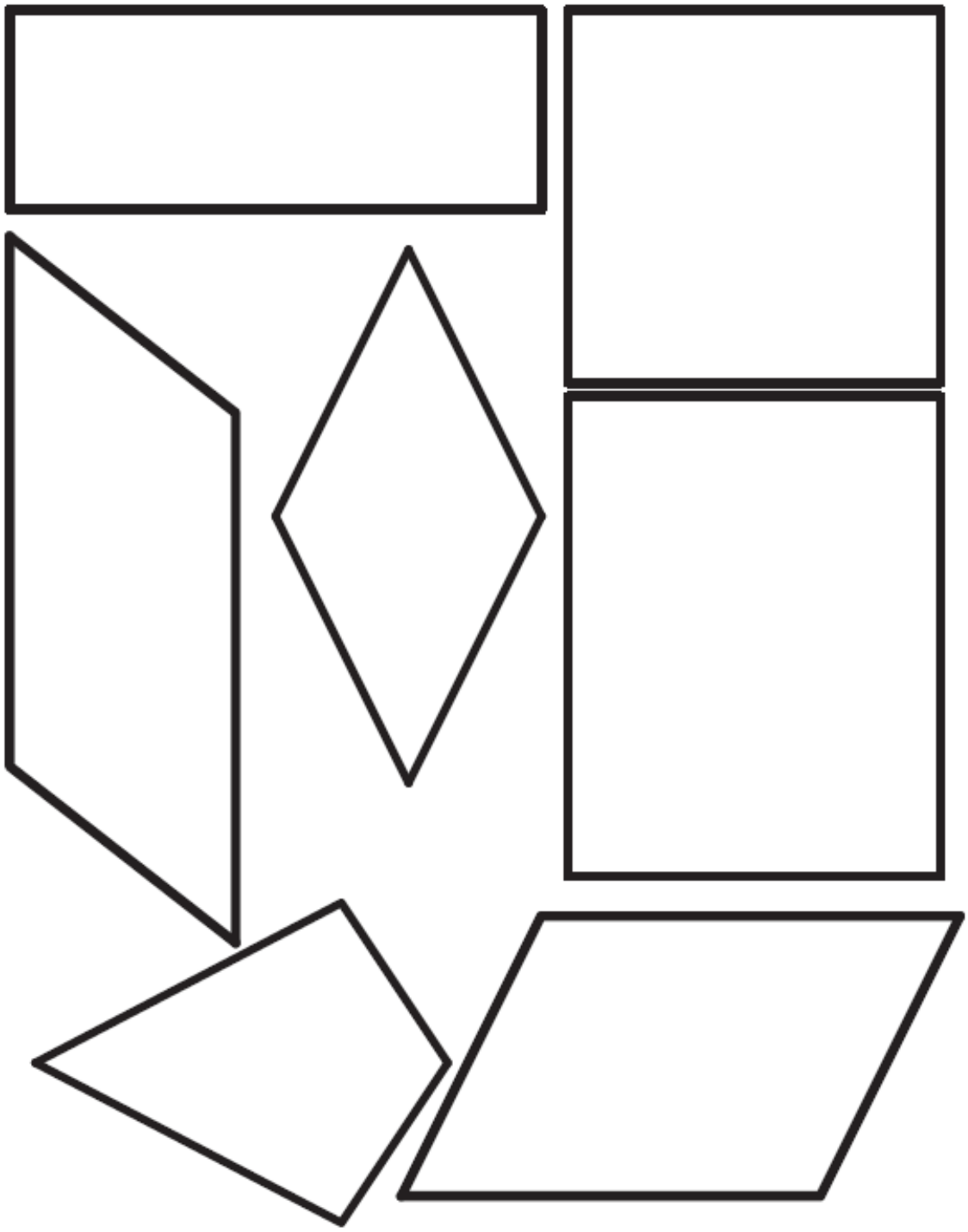
2-D Shapes – BLM 1



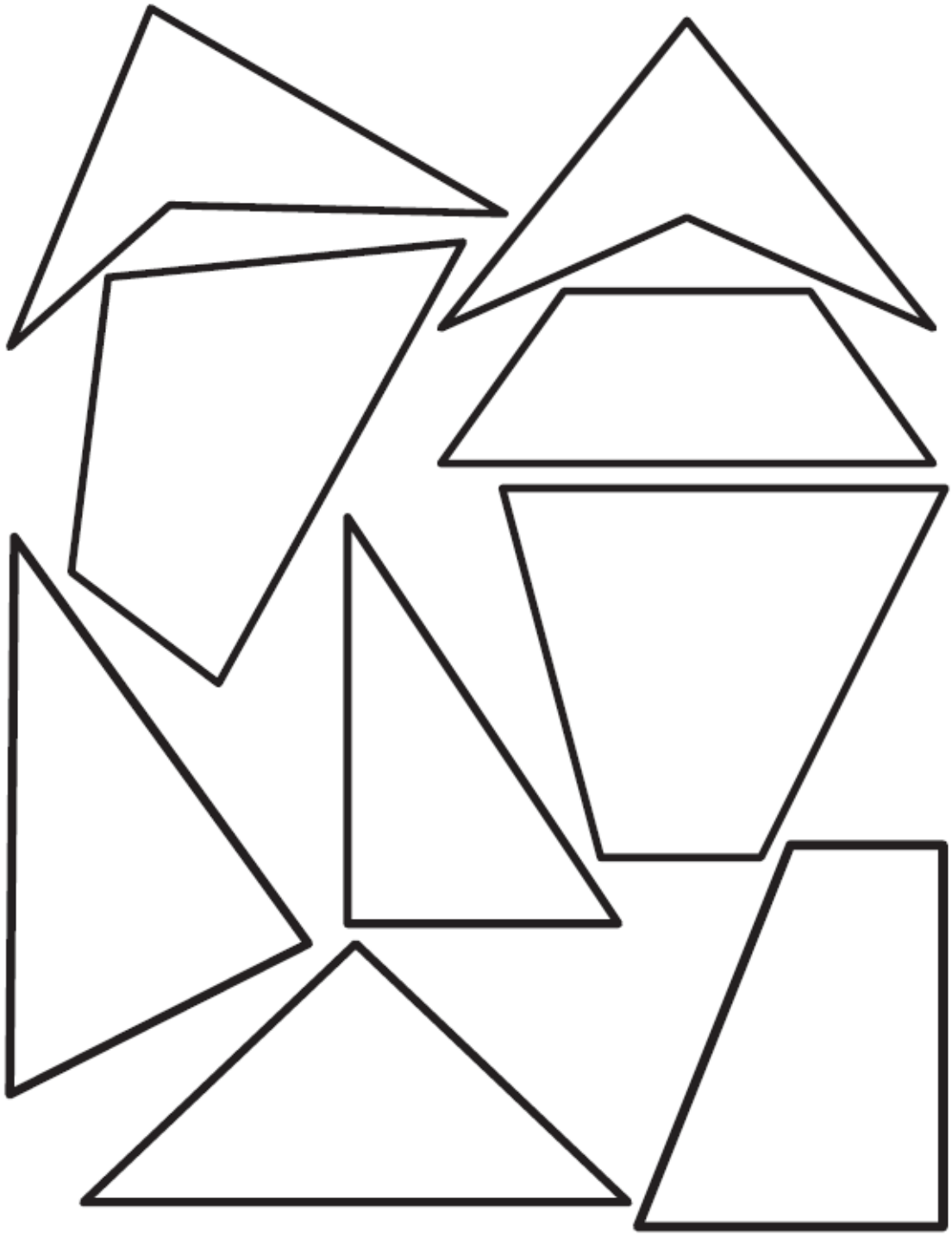
2-D Shapes – BLM 2



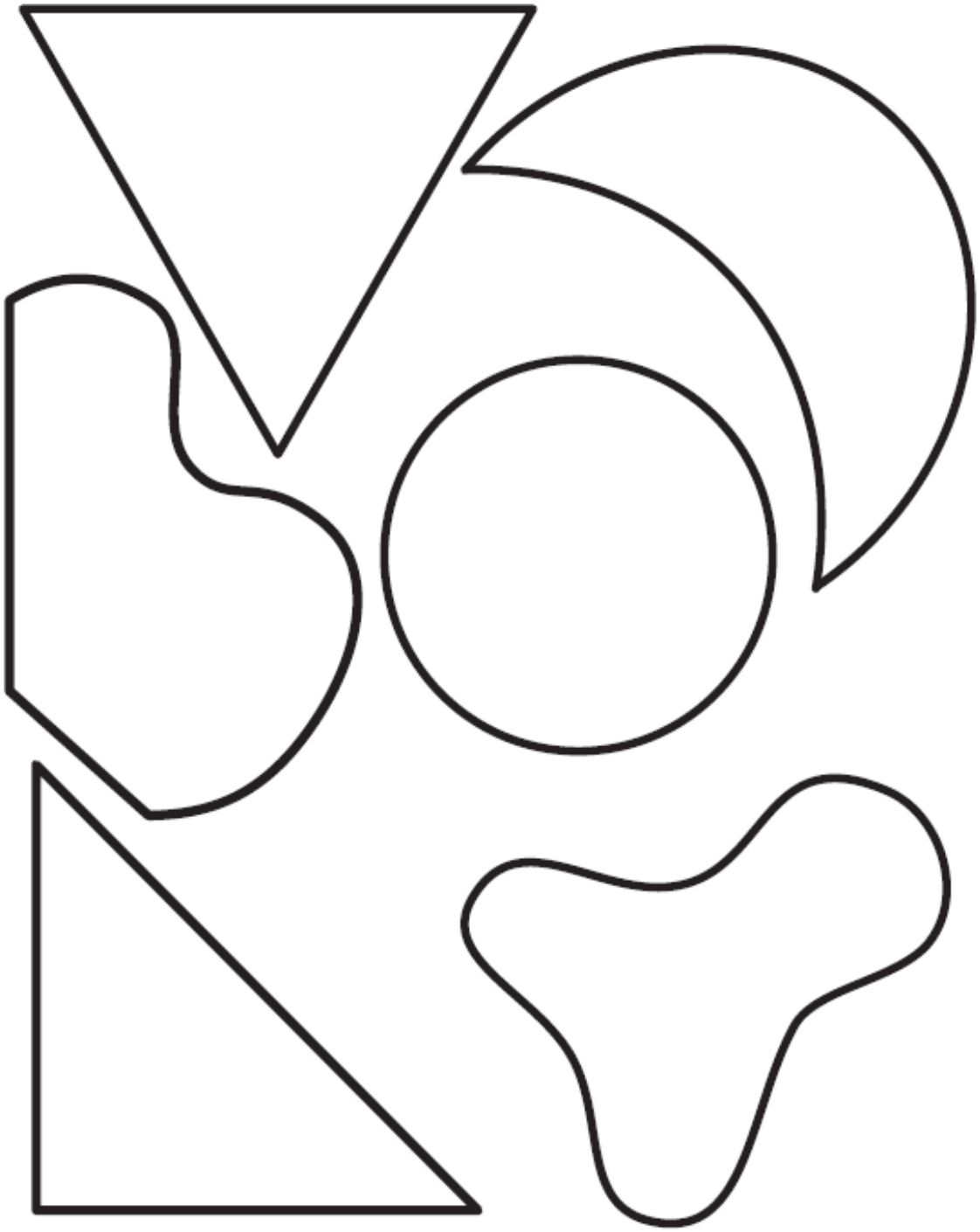
2-D Shapes – BLM 3



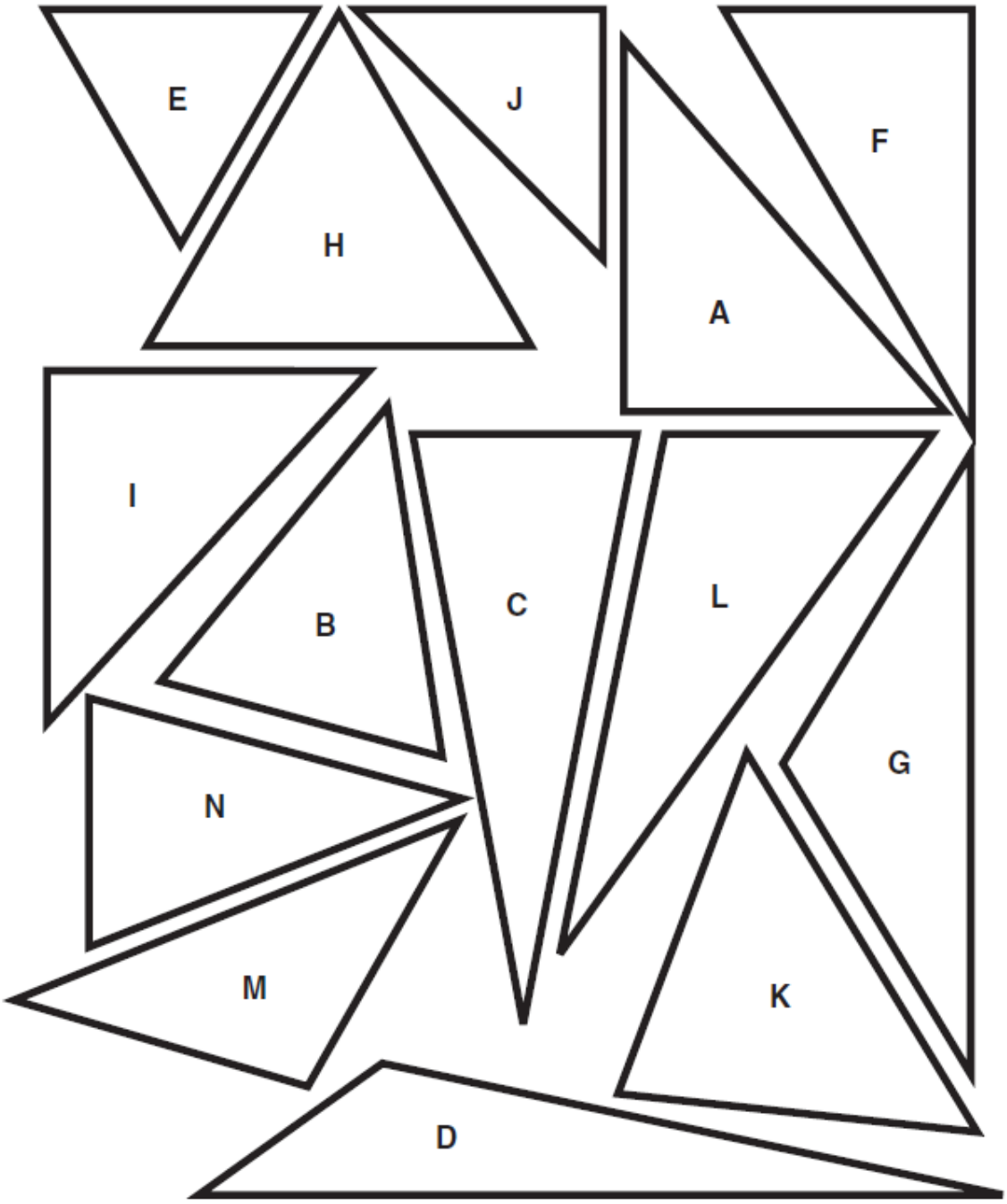
2-D Shapes – BLM 4



2-D Shapes – BLM 5



2-D Shapes – BLM 6



Sample Activity 3: Connections among Geometric Concepts

Extend the sorting activity in Sample Activity 2 (see p. 15) and have students discover and explain relationships among the various geometric concepts.

Provide students with a variety of 2-D shapes, including shapes that are polygons (regular and irregular as well as triangles of all kinds) and those that are not polygons. *Use BLM 1–6 from Sample Activity 2 on pages 16–21*.

Look For …

Do students:

- ☐ apply what they learned in the concept attainment activity to write mathematically correct statements about geometric concepts?
- ☐ critique statements written by others and explain their reasoning?
- ☐ apply their knowledge of angles in making connections between polygons and triangles and among triangles?
- ☐ use cross-classification to determine the best name for a given triangle?
- ☐ draw triangles when given specific labels?
- ☐ explain why it is impossible to draw some triangles when given specific labels?

Connecting Polygons and Triangles

Ask guiding questions, such as:

- how are equilateral triangles related to polygons?
- how are right triangles related to polygons?

Examples connecting polygons and triangles:

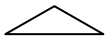
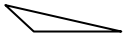
- All equilateral triangles are regular polygons because all sides are congruent and all interior angles are congruent. They are the only triangles that are regular polygons.
- All right triangles are irregular polygons because right triangles must have a 90^o angle and the sum of the angles in any triangle must be 180^o. Therefore, all the angles cannot be the same size.

Connecting Different Classifications of Triangles

Have students name the triangles in different ways and then decide on the best name for the shape, using cross-classification, if possible. Provide students with index cards to check right angles and compare lengths. The corner of the index card will fit perfectly into a right angle. Students should also use a protractor to measure the angle and determine if it is 90^o, greater than 90^o or less than 90^o.

Discuss different ways to record the data collected. One way might be to use a chart.

Examples of connections between triangles:

Triangle	Classified by Sides	Classified by Angles	Best Name
	Isosceles triangle	Right triangle	Right isosceles triangle
	Scalene triangle	Right triangle	Right scalene triangle
	Equilateral triangle	Acute triangle (Equiangular triangle)	Equilateral or Equiangular triangle
	Isosceles triangle	Acute triangle	Acute isosceles triangle
	Scalene triangle	Acute triangle	Acute scalene triangle
	Isosceles triangle	Obtuse triangle	Obtuse isosceles triangle
	Scalene triangle	Obtuse triangle	Obtuse scalene triangle

As a follow-up activity, provide students with a chart as shown below.

	Equilateral	Isosceles	Scalene
Right			
Acute			
Obtuse			

Challenge students to sketch a triangle in each of the nine cells.

Then, discuss the following problem:
"Of the nine cells in the chart, two of them are impossible to fill. Can you tell which ones they are and why?" (Van de Walle and Lovin 2006, p. 197)

Bulletin Board Activity

Use the chart with the nine empty cells as a bulletin board activity. Ask students to each make a triangle and place it in the appropriate cell on a large chart posted on the bulletin board. Have them label each triangle as an irregular or regular polygon.

Sample Activity 4: Tree Diagrams

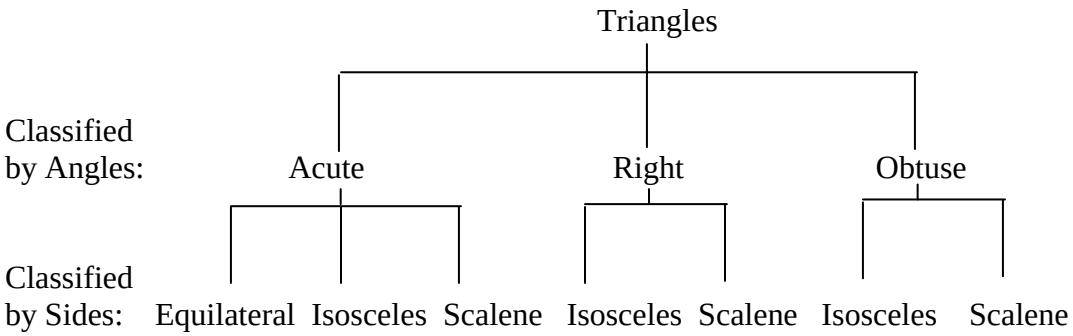
Instruct students to show the connections among geometric concepts by drawing a tree diagram. Explain that these diagrams show the relationships among concepts. See Step 1 (p. 6) for an example of a tree diagram showing the relationship among regular and irregular polygons and the different kinds of triangles.

A tree diagram showing the relationship among different triangles could be the following.

Look For …

Do students:

- ☐ demonstrate their understanding of the relationships among regular and irregular polygons and triangles by drawing a tree diagram?
- ☐ demonstrate their understanding of the relationship among different triangles by drawing and interpreting tree diagrams?
- ☐ interpret and critique tree diagrams drawn by others?



Sample Activity 5: Property Lists for Regular Polygons and Different Kinds of Triangles

Provide students with three or four examples of a given polygon; e.g., a regular polygon or a specific type of triangle. Have students work in groups to find as many properties of the given polygon as they can. Explain that each property listed must apply to all the diagrams of the polygon provided to them.

Encourage students to share their property lists with one another and adjust them as needed.

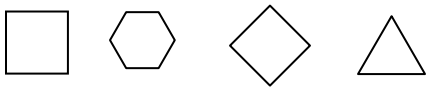
A sample is provided below (Van de Walle and Lovin 2006, p. 226).

Look For …

Do students:

- ☐ list properties for a given polygon by examining the similarities of a given group of diagrams?
- ☐ explain how each property listed applies to all the diagrams in a given group?

Regular Polygons



Properties of sides:

Properties of angles:

Properties of symmetry:

Properties of diagonals:

Sample Activity 6: Minimal Defining Lists

This activity is a follow-up to the sample Activity 5: Property Lists for Regular Polygons and Different Kinds of Triangles.

After the property lists have been completed, discussed and corrected through class discussion, post them in the classroom or duplicate them for distribution to students.

Discuss the meaning of *defining* and *minimal*.

- Defining: necessary properties that describe a particular polygon; e.g., a necessary property of a regular polygon is that all sides are congruent.
- Minimal: the shortest list possible of the necessary properties that are sufficient to completely describe a particular polygon; e.g., sufficient properties of a regular polygon are that it is a polygon with all sides congruent and all angles congruent.

Have students discuss and write the minimal defining lists for regular polygons and each type of triangle
(Van de Walle and Lovin 2006, p. 230).

Look For …

Do students:

- ☐ apply the property lists for triangles and polygons from Sample Activity 5 to create minimal defining lists for a given 2-D shape?
- ☐ choose defining or necessary properties for a given triangle or polygon and explain their thinking?
- ☐ choose minimal or the least number of properties that are sufficient in describing a given triangle or polygon and explain their thinking?

Sample Activity 7: What Is My Shape?

Draw various polygons, including a variety of triangles, one on each sticky label. Place one label on the back of each student. Present the following problem to students.

Problem
Students identify the shape on the label on their backs by asking questions with the following guidelines:

- Ask only two questions of any one student. Then move on to another student.
- The questions must have a "yes" or "no" answer.
- There are four possible answers to a question:
  - Yes
  - No
  - I don't understand; please ask it another way.
  - I don't know how to answer that.

Instruct students to move the label from the back to the front when they have correctly identified the triangle.

Look For …
Do students:
- ☐ ask questions about the properties of their mystery shape?
- ☐ use an answer from one question to guide the next question?
- ☐ correctly answer the questions asked of them?
- ☐ draw a diagram of their mystery shape to apply the knowledge they obtained through questioning their classmates?

Model some questioning strategies, as needed, and provide guidance, as necessary.

Extensions
- Ask students to find a partner whose triangle matches theirs in some way. Share the matches with the class and have students explain their thinking.
- Instruct students to form groups and explain their rule for grouping. For example, one group may say, "Our triangles all have two sides congruent."

(Alberta Education 2006, pp. 31–33)

Sample Activity 8: Frayer Model

Provide students with a template for the Frayer Model and have them fill in the sections, individually or as a group, to consolidate their understanding of a geometric concept such as a regular polygon or an obtuse scalene triangle. A sample of a Frayer Model is as follows.

Look For …

Do students:

- ☐ define a regular polygon in their own words?
- ☐ describe the essential characteristics of a regular polygon?
- ☐ provide examples of a regular polygon by drawing diagrams?
- ☐ provide non-examples of a regular polygon by drawing diagrams?

Frayer Model for a Regular Polygon

(Barton and Heidema 2002, pp. 68–71)

Definition

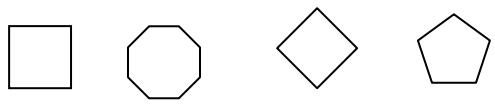
A regular polygon is a polygon that has all sides congruent and all interior angles congruent.

Characteristics

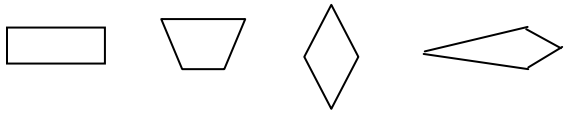
- a polygon: a closed 2-D shape with sides and no curves
- has three or more sides
- all sides are congruent
- has as many lines of symmetry as there are sides on the polygon
- diagonals are congruent if they exist (an equilateral triangle has no diagonals)
- diagonals bisect each other
- diagonals intersect

Regular Polygon

Examples



Non-examples



Sample Activity 9: True or False

Provide students with statements that pertain to the relationship among polygons, including triangles.

Have students explain their reasoning and change the false statements to make them true.

Sample statements:

- All equilateral triangles are regular polygons.
- All polygons are regular polygons.
- All equilateral triangles are isosceles triangles.
- All right triangles are either isosceles or scalene.
- All acute triangles are either isosceles or scalene.
- All scalene triangles are either obtuse triangles or right triangles.
- All right isosceles triangles have angles that measure 90^o, 45^o and 45^o.

Look For …

Do students:

- ☐ apply their knowledge of geometric concepts to decide whether the statements are true or false and explain their thinking?
- ☐ correct the false statements to make them true and explain their thinking?
- ☐ create other statements for students to answer true or false?

Sample Activity 10: Semantic Feature Chart

Explain the purpose of the semantic feature chart to students.

- The semantic feature chart is a graphic organizer that helps you understand a concept by comparing its features to those of other concepts that fall into that same category. You reason and communicate about similarities and differences of the various concepts. The completed chart provides a visual reminder of how certain concepts are alike or different. By using this strategy, you are led to discover both the shared and unique characteristics of the concepts and associated vocabulary words.

Look For …

Do students:

- ☐ apply their knowledge of properties of triangles and polygons to complete the chart?
- ☐ demonstrate understanding of the properties listed across the top row of the semantic feature chart?
- ☐ distinguish among the different triangles by identifying all the appropriate properties?

Provide students with a template of a semantic feature chart and tell them to write the names of different triangles in the first column. Have them also include polygons and regular polygons. Then have them suggest some properties of regular polygons and triangles that can be placed in the top row of the chart.

Have students complete the chart by using an "X" to indicate when a property applies to the term. Encourage students to explain the rationale for their choices.

Note: Explain that the property listed must apply to all types of any given polygon before they mark the "X" to indicate that it applies to the term.

Example:

Properties / **Terms**	**Has at least two acute angles**	**Sum of the interior angles $\geq 180^o$**	**Is a 2-D shape**	**All sides are congruent**	**All angles are congruent**
Polygon		X	X		
Regular Polygon		X	X	X	X
Scalene Triangle	X	X	X		
Obtuse Triangle	X	X	X		
Equilateral Triangle	X	X	X	X	X
Right Triangle	X	X	X		

(Alberta Education 1990, p. 139)

Sample Activity 11: Congruency

Provide students with a variety of cutout polygons, including triangles, some congruent and some not congruent. Have them sort the polygons into groups and explain their sorting rule. Encourage students to sort the shapes in more than one way and to share their sorting rules.

Highlight the examples in which students match congruent polygons by superimposing one over the other. Explain that the term used to describe two polygons that fit perfectly over each other is congruent. Through discussion, have students verbalize the essential characteristics of congruent shapes; i.e., the shapes must have the same size and shape, which means that the corresponding sides are the same length and the corresponding angles are the same size.

Look For …

Do students:

- ☐ demonstrate flexibility by sorting the 2-D shapes in more than one way?
- ☐ communicate clearly their sorting rules?
- ☐ critique sorting rules described by others?
- ☐ focus on highlighted sorting rules that involve congruency and superimpose the 2-D shapes to determine congruency?
- ☐ apply the concept of congruency to other geometric situations?

Sample Activity 12: Using Technology to Draw 2-D Shapes and Determine Congruency

Have students use a variety of technology tools to draw 2-D shapes and decide if the shapes are congruent.

Examples of technology:

- *Microsoft Word* has autoshapes, including 2-D shapes along with a draw and paint program. Superimpose one shape on another shape to determine congruency (same size and shape).
- *Geometer's Sketchpad* can be used to draw 2-D shapes and record the measures of the sides and the angles. These measures can be used to determine congruency of the 2-D shape drawn (Key Curriculum Press 2001).

Look For …

Do students:

- ☐ use technology appropriately to draw 2-D shapes?
- ☐ use technology to decide if two shapes are congruent?
- ☐ use technology to change the orientation of various 2-D shapes and note that the properties of the shape do not change?
- ☐ change the length of a side or the size of an angle and determine the effect on the 2-D shape?

Sample Activity 13: Constructing 2-D Shapes without Technology

Provide students with centimetre grid paper and triangular or isometric grid paper *(use BLM 7 and 8 on pages 34 and 35).*

1. Have students draw two congruent regular polygons in different orientations on square dot paper or triangular dot paper. Ask them to compare the lengths of the sides of the polygons. Have them measure the angles of the polygons using their protractors and compare the size of the angles.

2. Instruct students to use their rulers, square dot paper and triangular dot paper to construct 2-D shapes that fit the given conditions or terms. Encourage them to compare their polygons with others and discuss similarities and differences.

    a. three sides and one right angle
    b. six sides with all angles congruent and all sides congruent
    c. five sides
    d. three sides with two of equal length
    e. four sides with exactly two sides parallel
    f. right scalene triangle
    g. obtuse isosceles triangle
    h. two congruent equilateral triangles.

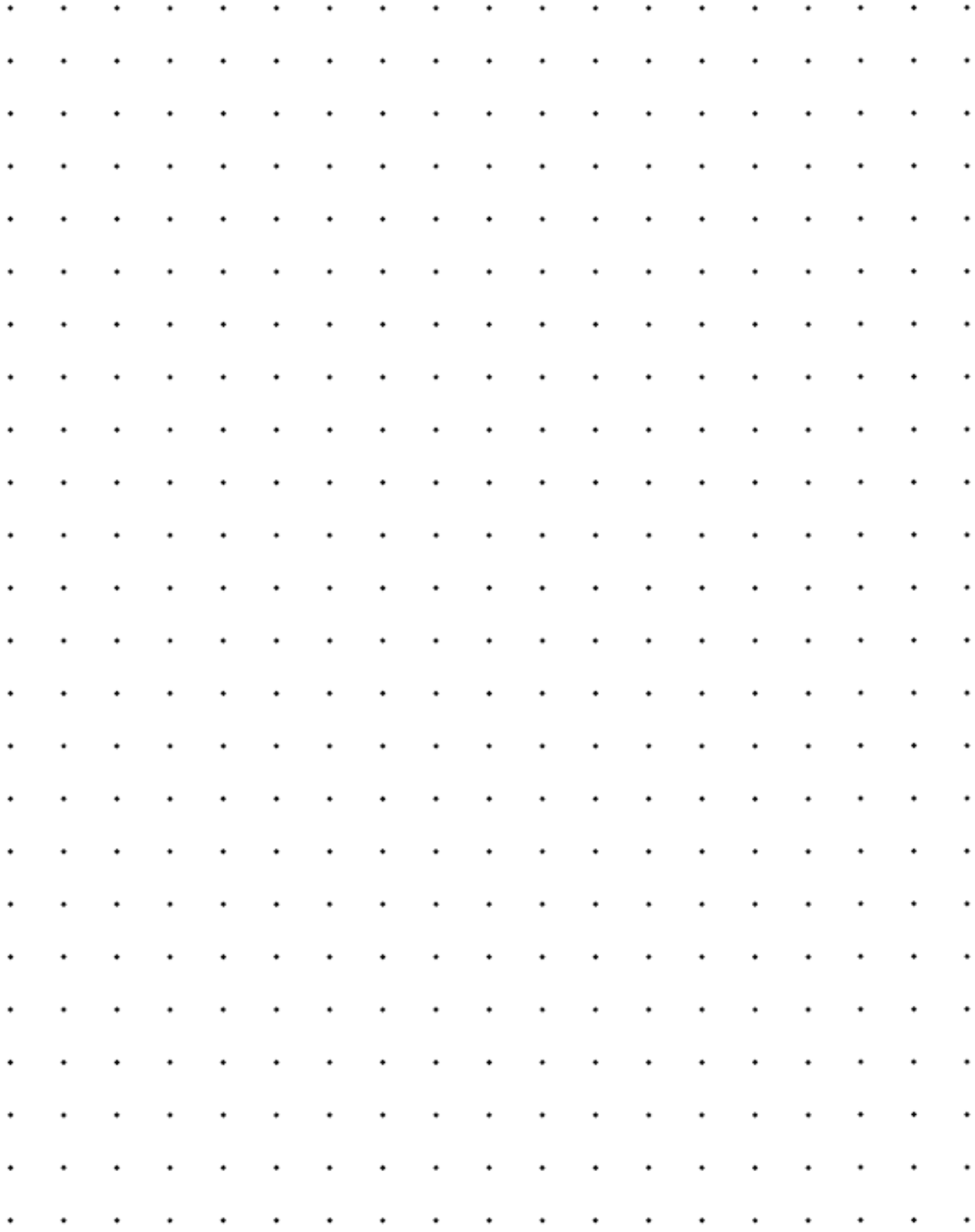
Look For …

Do students:

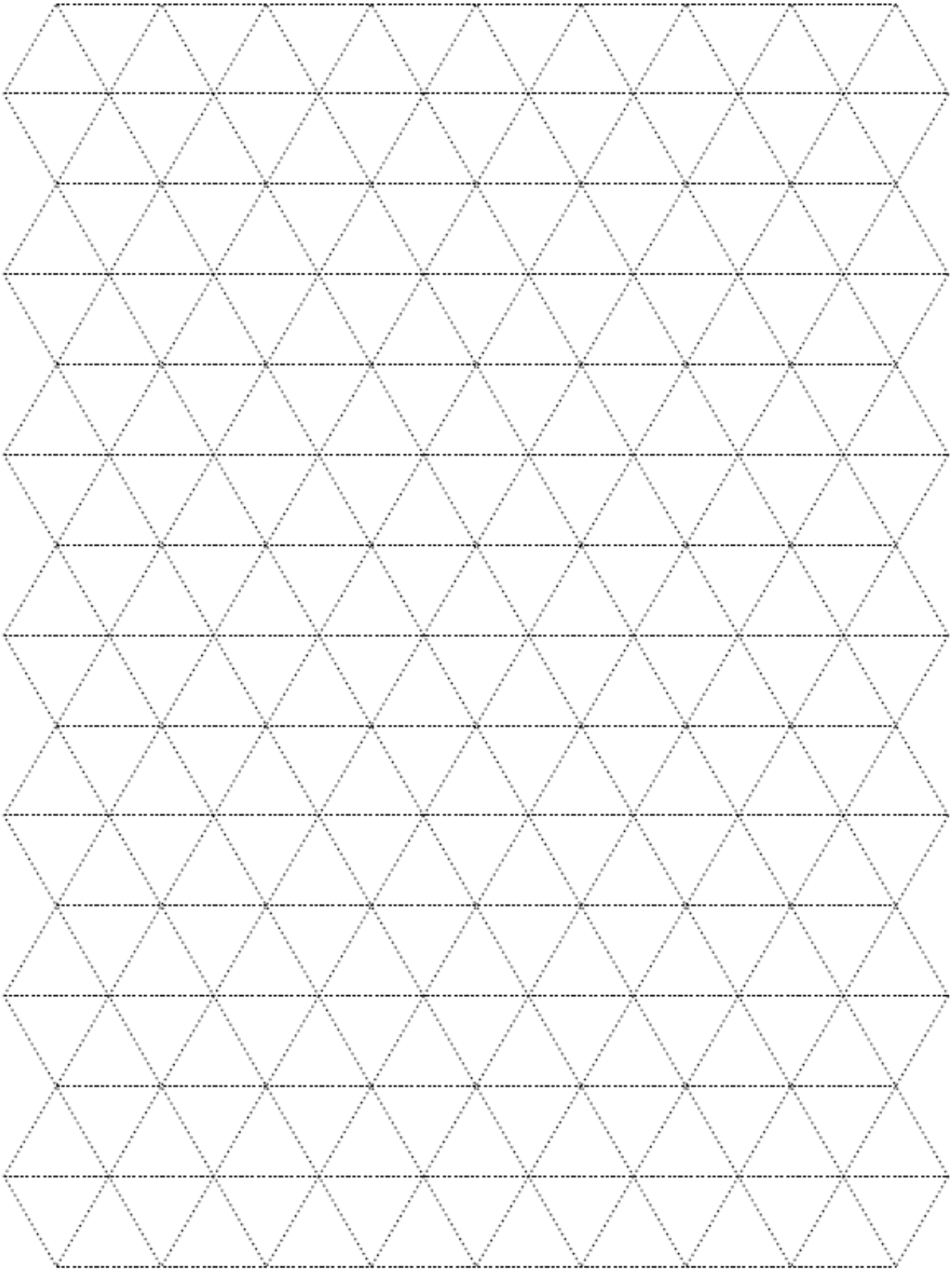
- ☐ use their ruler and dot paper to draw 2-D shapes accurately?
- ☐ apply their knowledge of congruency when drawing congruent 2-D shapes?
- ☐ demonstrate flexibility by drawing more than one shape that satisfies the given conditions or terms?
- ☐ critique the diagrams drawn by others to determine if they satisfy the given conditions or terms?

(Alberta Education 1990, pp. 141–144)

Centimetre grid paper – BLM 7



Isometric grid paper – BLM 8



Step 4: Assess Student Learning

Guiding Questions

- Look back at what you determined as acceptable evidence in Step 2.
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

In addition to ongoing assessment throughout the lessons, consider the following sample activities to evaluate students' learning at key milestones. Suggestions are given for assessing all students as a class or in groups, individual students in need of further evaluation, and individual or groups of students in a variety of contexts.

A. Whole Class/Group Assessment

The following student assessment task, Types of Polygons and Triangles (p. 39), could be used with a whole group or class. It includes a marking rubric to be used with the assessment.

Types of Polygons and Triangles

In this assessment task, students will demonstrate their understanding of different types of triangles as well as regular and irregular polygons. This assessment task is only a sample of concepts in specific outcomes 4 and 5 related to 2-D shapes.

Students will sort a given set of triangles in two different ways, label the groups and describe the sorting rule. Then they will draw a specified triangle and provide the best name for the shape. Finally, they will compare two polygons by describing similarities and differences and labelling them appropriately.

Place Question 1 on a separate page from questions 2 and 3. Administer Question 1 and collect the answers prior to administering questions 2 and 3 because Question 2 provides a hint for the classification in Question 1.

Materials required:

- rulers or straight edges
- protractors or index cards for drawing angles or checking the type of angles
- square dot paper and triangular dot paper.

By sorting a wide variety of triangles into three different groups in two ways, students identify the six types of triangles. When triangles are classified by angles, the labels are right, obtuse and acute. When triangles are classified by sides, the labels are scalene, isosceles and equilateral.

Protractors may be used to draw the right, obtuse and acute triangles but students may wish to use an index card as a referent to decide the approximate size of the angles. Some students may wish to use the square dot paper and/or the triangular dot paper to draw the triangles. Students

should explain why their drawings are correct. In naming the constructed triangles, students should cross-classify and include the name for each triangle that classifies by sides and by angles; e.g., a right triangle with two congruent sides is called a right isosceles triangle.

When comparing the regular polygon and the irregular polygon in Question 3, students list similarities and differences.

Examples of similarities

- Both polygons have:
  - six sides
  - six angles
  - angles that are all greater than or equal to 90^o
  - at least two pairs of parallel sides
  - the name of hexagon because they each have six sides
  - the same sum of the interior angles.

Examples of differences

- The regular polygon:
  - has all sides congruent, whereas the irregular polygon does not have all sides congruent
  - has all interior angles congruent, whereas the irregular polygon does not have all interior angles congruent
  - has all obtuse angles, whereas the irregular polygon has five right angles and one reflex angle
  - is convex, because it has no interior angles that are reflex angles, whereas the irregular polygon is concave because it has an interior angle that is reflex; i.e., greater than 180^o and less than 360^o.

Task-specific Criteria

Each student will:

- identify the characteristics of a given set of triangles according to their sides and/or their interior angles
- sort a given set of triangles and explain the sorting rule
- identify a specified triangle from a given set of triangles; e.g., isosceles
- draw a specified triangle; e.g., scalene
- demonstrate congruence (sides-to-sides and angles-to-angles) in a regular polygon by superimposing
- demonstrate congruence (sides-to-sides and angles-to-angles) in a regular polygon by measuring
- demonstrate that the sides of a given regular polygon are of the same length and that the angles of a regular polygon are of the same measure

- sort a given set of polygons as regular or irregular and justify the sorting
- name triangles and polygons to describe their essential properties.

Early finishers can:

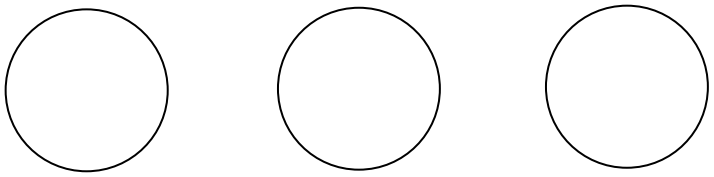
- draw an obtuse isosceles triangle and justify their answer
- draw two congruent regular pentagons and justify their answer
- draw two different regular polygons with obtuse angles
- explain why all equilateral triangles are regular polygons
- explain why it is impossible to have an equilateral triangle with a right angle.

Types of Polygons and Triangles—
Student Assessment Task

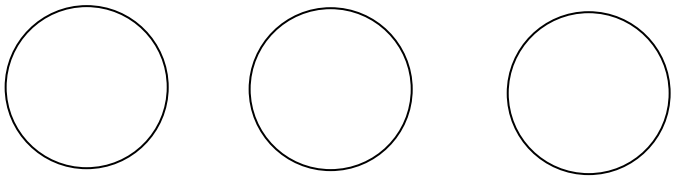
Do Question 1 first and hand it in. Then you will be given questions 2 and 3 to complete.

1. Given the following triangles:

   a. Sort the following triangles into three groups so that no triangle belongs to two groups. Place the number of each triangle in one of the circles below. Label each group. Describe the sorting rule.



   b. Sort triangles into three groups different than those used in 1 (a). No triangle should belong to two groups. Place the number of each triangle in one of the circles below. Label each group. Describe the sorting rule.



(Van de Walle and Lovin 2006, p. 197)

(continued)

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2. Draw the following triangles. Provide the **best** name for each triangle, using two criteria. Explain your thinking.

   a. right triangle with two sides congruent

   b. obtuse triangle with no sides congruent

   c. acute triangle with all sides congruent

3. Compare the polygons below by listing at least three similarities and at least three differences related to the sides and angles. Label each polygon as a regular polygon or an irregular polygon. Explain your thinking.

SCORING GUIDE:
Types of Polygons and Triangles

Student: _____

Level / Criteria	4 Excellent	3 Proficient	2 Adequate	1 Limited*	Insufficient / Blank*
Sorts a given set of triangles and explains the sorting rule. Question 1	The student sorts the triangles into three distinct groups in two different ways, labels each group correctly and explains the sorting rules using precise mathematical language.	The student sorts the triangles into three distinct groups in two different ways, labels each group correctly and clearly explains the sorting rules.	The student sorts the triangles into three distinct groups in two different ways, provides most of the labels for each group and provides a limited explanation of the sorting rules.	The student attempts to sort the triangles into three groups only one way but the groups may overlap. The student has difficulty with the labels, providing little or no explanation of the sorting rule.	No score is awarded because there is insufficient evidence of student performance, based on the requirements of the assessment task.
Draws a specified triangle, using two criteria, and names the triangle. Question 2	The student draws each triangle very accurately, clearly justifies each drawing using precise mathematical language and provides the best name for each triangle using the two criteria provided.	The student draws each triangle accurately, justifies each drawing and provides the best name for each triangle using the two criteria provided.	The students draws each triangle with some inaccuracy, partially justifies each drawing and provides the best name for most of the triangles using the two criteria provided.	The student draws at least one of the triangles with inaccuracies, provides no justification for the drawings and has difficulty naming the triangles, using a maximum of one of the criteria.	No score is awarded because there is insufficient evidence of student performance, based on the requirements of the assessment task.
Compares the sides and angles of regular and irregular polygons. Question 3	The student describes, using precise mathematical language, more than three similarities and three differences between the regular and irregular polygons, and labels each diagram appropriately along with a sophisticated justification.	The student clearly describes three similarities and three differences between the regular and irregular polygons, and labels each diagram appropriately along with a clear justification.	The student provides a limited description of one or two similarities and one or two differences between the regular and irregular polygons and labels each diagram appropriately along with an acceptable explanation.	The student attempts to describe one similarity and/or one difference between the regular and irregular polygons, but the description is incorrect or confusing. The student may or may not label the diagrams appropriately and provides no explanation.	No score is awarded because there is insufficient evidence of student performance, based on the requirements of the assessment task.

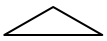
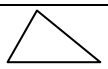
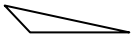
***** When work is judged to be limited or insufficient, the teacher makes decisions about appropriate interventions to help the student improve.

B. One-on-one Assessment

Provide the student with a protractor, a ruler and/or an index card to check for right angles (or angles greater than or less than a right angle) and compare lengths. Place the chart before the student and read the instructions. Instruct the student to complete the chart and explain his or her reasoning. The first column is completed as an example but you may wish to remove the check marks before showing the chart to the student.

Instructions

Look at the column under Obtuse Triangle. Check the figures in the column for obtuse triangles. Do the same for each of the 2-D shapes named in the top row of the chart (Alberta Education 1990, p. 73).

	Obtuse Triangle	Right Triangle	Acute Triangle	Scalene Triangle	Isosceles Triangle	Equilateral Triangle	Regular Polygon	Irregular Polygon
								
								
								
	✓							
								
								
	✓							

If the student has difficulty including more than one check mark for certain 2-D shapes, remind the student of the properties of the shape being discussed. For example, the obtuse triangle must have an obtuse angle but the lengths of the sides may vary; i.e., it could be an obtuse isosceles triangle with two sides congruent or an obtuse scalene with no sides congruent.

If the student has difficulty understanding the terms across the top row of the chart, review these terms by using examples. For example, draw an obtuse triangle. Use the index card or a protractor to show that one angle in the triangle is greater than a right angle or 90^o but less than a straight line or 180^o. Then have the student choose an obtuse triangle from the shapes provided in the chart and explain why it is an obtuse triangle. Use a similar procedure to review any of the other terms that the student does not understand.

When the chart is completed, have the student focus on the diagram of the first triangle. Have the student look across the row containing this triangle and note which names were checked off: right triangle, isosceles triangle and irregular polygon. Explain that "irregular polygon" is a general term that applies to any polygon that does not have all sides congruent and all angles congruent, whereas, the terms right and isosceles are more specific to the given triangle. Suggest that the student name the triangle using both terms: right isosceles. Have the student name the other triangles using two terms: one that classifies by angles and one that classifies by sides.

C. Applied Learning

Provide opportunities for students to use their understanding of different types of triangles as well as regular and irregular polygons. For example, have the students find a 3-D object in the real world that has a face shaped like a regular polygon. Does the student:

- find a suitable object?
- explain why the face on the object is a regular polygon?
- find a congruent regular polygon, if asked, and explain why it is congruent?
- include an equilateral triangle as a regular polygon, if asked?
- provide a more descriptive name for the regular polygon if asked; e.g., regular hexagon?
- find an example of an irregular polygon and compare it to the regular polygon, if asked?

Step 5: Follow-up on Assessment

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction?

A. Addressing Gaps in Learning

If the student has difficulty describing and providing examples of the regular and irregular polygons, including the different types of triangles, have the student construct 2-D shapes using a variety of manipulatives such as geoboards, straws and string.

By using straws and string construction, the focus is on the sides of 2-D shapes and provides a visual for students to see the essential properties; e.g., a regular polygon has all sides congruent. Have the students trace and superimpose the angles to determine congruency. Suggest that students use the corner of an index card or a protractor to determine whether triangles have obtuse, right or acute angles.

Encourage the students to use real-world objects to describe and provide examples of faces of 3-D objects that are regular and irregular polygons, including triangles of all types.

Use concept attainment (see **Step 3, Section C: Choosing Learning Activities**, **Sample Activity 1**, p. 13) to aid students in describing properties of geometric concepts. It provides an opportunity for students to construct understanding of the geometric concepts and also the relationship among the concepts. Since this activity focuses attention on the properties of a given quadrilateral, students are better able to identify and sort polygons and justify their answers using sound mathematical reasoning.

If students are having difficulty identifying and sorting the polygons, have them construct one polygon, such as an equilateral triangle, out of straws or paper strips with brass fasteners. By changing the lengths of the straws or paper strips, e.g., making one side shorter or longer, students see visually the relationship between an equilateral triangle and an isosceles triangle. Have them focus on how the angles change when the side length changes (Alberta Education 1990, pp. 137–140).

Provide students with time to construct the different triangles using their protractor and ruler or straight edge so that the focus is on the size of the angles as well as the lengths of the sides; e.g., construct a right isosceles triangle. Since the lengths of the congruent sides may vary, students will have a variety of right isosceles triangles to explore as they share their constructions.

Ask guiding questions that aid students in connecting regular and irregular polygons with the different types of triangles. For example, have students sort a set of triangles into two groups, regular and irregular polygons, and justify their sorting. Also, have students construct tree diagrams to show relationships (see the tree diagram in **Step 1, Big Ideas**, p. 6, and in **Step 3, Section C: Choosing Learning Activities, Sample Activity 4**, p. 24).

Have students create a bulletin board display of the different types of triangles. Each student could create a triangle and place it in an appropriate cell in a chart, such as the one shown below. This helps students discover the cross-classifications that are possible when sorting triangles; i.e., they must classify each triangle by sides and by angles. They will also discover which cells in the chart are not possible to fill with any triangle.

Students could label each triangle as a regular or an irregular polygon.

Types of Triangles

	Equilateral	**Isosceles**	**Scalene**
Right			
Acute			
Obtuse			

See **Step 3, Section C: Choosing Learning Activities, Sample Activity 3: Connections among Geometric Concepts**, pp. 22–23, for more information.

B. Reinforcing and Extending Learning

Students who have achieved or exceeded the outcomes will benefit from ongoing opportunities to apply and extend their learning. Consider strategies such as the following to support students in developing a deeper understanding of the concept.

Parent Tips
Provide tips for parents on helping their children reinforce understanding of regular and irregular polygons as well as the different kinds of triangles.

- Capitalize on outings with the children and have them find examples in the real world of 3-D objects with faces that are regular or irregular polygons or different kinds of triangles.
- Use real-world objects and have the children describe the faces of these objects, paying particular attention to different kinds of triangles; e.g., different side lengths and different angle sizes.
- In magazines, commercial drawings or drawings created by children, have children identify different kinds of triangles as well as regular and irregular polygons.

Explorations Using a Geoboard
Provide each pair of students with a geoboard and a set of cards on which different lists of properties are written and that apply to the various triangles, regular polygons or irregular polygons. One student reads the list of properties and evaluates the work done by the other student as he or she creates a geometric shape that has the properties listed. Encourage students to create as many different shapes as possible for a given set of properties. Have students exchange roles periodically.

If necessary, model this activity by reading a set of properties from a card while students create appropriate geometric shapes on their transparent geoboards. Have students share their ideas about which shapes are described by the given properties by placing on the overhead their geoboards with the shapes outlined. Encourage students to evaluate one another's work as it is presented to the class.

Mystery Definition
Present students with three groups of triangles or other polygons on the overhead, chalkboard or in a handout.

First Group – Choose examples of the concept that allow for all the possible variations of the mystery definition or properties.

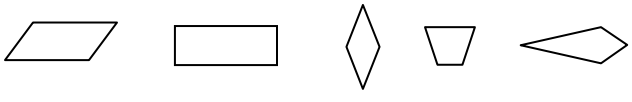
Second Group – Choose non-examples to be as close to the examples as necessary to provide the needed information for an accurate definition.

Third Group – Choose a mixture of examples and non-examples of the concept. Include non-examples that most likely confuse students.

Example (Van de Walle 2006, p. 225):

First Group – All of these have something in common.

Second Group – None of these have it.



Third Group – Which of these have it? Explain your thinking.

Have students create other mystery definitions and share them with the class.

Make a Pair
Provide students with two decks of cards. The first deck of cards has a triangle drawn on each card. The second deck of cards has some properties listed on each card.

Instructions:

- Play this game with a partner.
- Place each of the two decks of cards face down.
- Decide on who starts the game by rolling a die. The one who rolls the highest number starts the game.
- The first player turns over a card from each pile of cards. If the cards match, the player takes the two cards and has another turn. If the cards do not match, the cards are left, as is, and the second player turns over one card from each pile. The cards match if the properties listed on one card describe the triangle drawn on the other card.
- The winner is the player who has the most pairs of cards at the end of the game.

Adaptations:

- Write the name of a triangle, e.g., right scalene triangle or obtuse isosceles triangle, on each card instead of drawing a diagram of a triangle.
- Include regular and irregular polygons on the first deck of cards and include corresponding descriptions of properties on the second deck of cards.
- Play Concentration with the two decks of cards.

Constructing Triangles and Making Generalizations
Provide students with straws and rulers. Have them cut the straws to represent each set of lengths and use these straws to construct triangles. Present students with the following problem.

Problem:

Decide which of the following sets of side lengths can be used to construct triangles. Name each triangle constructed. Explain why some of the examples cannot be constructed as triangles.

a. 8 cm, 5 cm and 5 cm
b. 5 cm, 4 cm and 3 cm
c. 4 cm, 4 cm and 4 cm
d. 6 cm, 3 cm and 2 cm
e. 10 cm, 4 cm and 4 cm

Sample explanation as to why (d) and (e) cannot be drawn as triangles:
"In order to form a triangle, the sum of the lengths of the two shorter sides must be greater than the longer side" (Small 2009, p. 97).

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