Mathematics

Planning Guide

Grade 6

Equations with Letter Variables

Patterns and Relations (Variables and Equations)

Specific Outcomes 3 and 4

Shape and Space (Measurement)

Specific Outcome 3

This Planning Guide can be accessed online at:
http://www.learnalberta.ca/content/mepg6/html/pg6_equationswithlettervariables/index.html

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Planning Guide: *Grade 6 Equations with Letter Variables*

Strand: Patterns and Relations (Variables and Equations)
Specific Outcomes: 3 and 4

Strand: Shape and Space (Measurement)
Specific Outcome: 3

This *Planning Guide* addresses the following outcomes from the Program of Studies:

Strand: Patterns and Relations (Variables and Equations)	
Specific Outcomes:	3. Represent generalizations arising from number relationships, using equations with letter variables. [C, CN, PS, R, V] 4. Express a given problem as an equation in which a letter variable is used to represent an unknown number. [C, CN, PS, R]
Strand: Shape and Space (Measurement)	
Specific Outcome:	3. Develop and apply a formula for determining the: • perimeter of polygons • area of rectangles • volume of right rectangular prisms. [C, CN, PS, R, V]

Curriculum Focus

The changes to the curriculum targeted by this sample include:

- The general outcome in the Patterns and Relations (Variables and Equations) strand focuses on representing algebraic expressions in multiple ways, which is the same as the previous mathematics curriculum.
- The general outcome in the Shape and Space (Measurement) strand focuses on using direct and indirect measurement to solve problems, whereas the previous mathematics curriculum focused on describing and comparing everyday phenomena, using either direct or indirect measurement.
- The specific outcome in the Shape and Space (Measurement) strand focuses on developing and applying a formula for determining the perimeter of polygons, area of rectangles and volume of right rectangular prisms, which is the same as the previous math curriculum. The previous mathematics curriculum also included finding the surface area of right rectangular prisms with and without a formula.

What Is a Planning Guide?

Planning Guides are a tool for teachers to use in designing instruction and assessment that focuses on developing and deepening students' understanding of mathematical concepts. This tool is based on the process outlined in *Understanding by Design* by Grant Wiggins and Jay McTighe.

Planning Steps

The following steps will help you through the Planning Guide:

- **Step 1: Identify Outcomes to Address** (p. 5)
- **Step 2: Determine Evidence of Student Learning** (p. 10)
- **Step 3: Plan for Instruction** (p. 11)
- **Step 4: Assess Student Learning** (p. 44)
- **Step 5: Follow-up on Assessment** (p. 52)

Step 1: Identify Outcomes to Address

Guiding Questions

- What do I want my students to learn?
- What can my students currently understand and do?
- What do I want my students to understand and be able to do based on the Big Ideas and specific outcomes in the program of studies?

Big Ideas

Patterns are used to develop mathematical concepts and are found in everyday contexts. The various representations of patterns, including symbols and variables, provide valuable tools in making generalizations of mathematical relationships. Some characteristics of patterns include the following (adapted from Van de Walle and Lovin 2006, pp. 265, 268).

- "A pattern must involve some repetition or regularity" (Small 2009, p. 3).
- Patterns using concrete and pictorial representations can be translated into patterns using numbers to represent the quantity in each step of the pattern. The steps in a pattern are often translated as the sequence of items in the pattern.
- Pattern rules are used to generalize relationships in patterns. These rules can be recursive and functional.
- A recursive relationship describes how a pattern changes from one step to another step. It describes the evolution of the pattern by stating the first element in the pattern together with an expression that explains what you do to the previous number in the pattern to get the next one.
- A functional relationship is a rule that determines the number of elements in a step by using the number of the step; i.e., a rule that explains what you do to the step number to get the value of the pattern for that step. In other words, for every number input, there is only one output using the function rule.
- Variables are used to describe generalized relationships in the form of an expression or an equation (formula).
- "Functional relationships can be expressed in real contexts, graphs, algebraic equations, tables and words. Each representation for a given function is simply a different way of expressing the same idea. Each representation provides a different view of the function. The value of a particular representation depends on its purpose" (Van de Walle and Lovin 2006, p. 284).

Algebraic reasoning is directly related to patterns because this reasoning focuses on making generalizations based on mathematical experiences and recording these generalizations using symbols or variables (Van de Walle and Lovin 2006, p. 281).

"A variable is a symbol that can stand for any one of a set of numbers or objects" (Van de Walle and Lovin 2006, p. 274). Variables are used in different ways to generalize concepts as mathematical literacy is developed. They can be used (Cathcart, Pothier and Vance 1994, p. 368):

- in equations as unknown numbers; e.g., $4 + x = 6$
- to describe mathematical properties; e.g., $a + b = b + a$
- to describe functions; e.g., Input (n): 1, 2, 3, 4 …; and
  Output ($3n$): 3, 6, 9, 12 …
- in formulas to show relationships; e.g., $A = L \times W$.

By investigating patterns, students:

- solve problems
- develop understandings of important mathematical concepts and relationships
- investigate the relationships among quantities (variables) in a pattern
- generalize patterns using words and variables
- extend and connect patterns
- construct understandings of functions.

(National Council of Teachers of Mathematics 1991, p. 1.)

As students analyze the structure of patterns and organize the information systematically, they "use their analysis to develop generalizations about the mathematical relationships in the patterns" (National Council of Teachers of Mathematics 2000, p. 159).

Students use patterns to develop understanding of measurement concepts, including perimeter of polygons, area of rectangles and volume of right rectangular prisms. They generalize the patterns using words and variables that can be written as a formula, "a special algebraic equation that shows a relationship between two or more different quantities" (Small 2009, p. 9).

Formulas for finding the perimeter and area of 2-D shapes and volume of 3-D objects provide a method of measuring by using only measures of length (Van de Walle and Lovin 2006, p. 230).

In using any type of measurement such as length, area or volume, it is important to discuss the similarities in developing understanding of the different measures; e.g., first identify the attribute to be measured, then choose an appropriate unit and finally compare that unit to the object being measured (National Council of Teachers of Mathematics 2000, p. 171). It is important to understand the attribute (perimeter, area or volume) before measuring.

Definitions

Perimeter

"Perimeter is the distance around a polygon.
Perimeter is the sum of the lengths of all of the sides of a polygon"
(http://learnalberta.ca/content/memg/index.html).
The standard units for measuring perimeter are linear units such as "mm," "cm," "m" and "km."

Area
Area is "a measure of the space inside a region or how much it takes to cover a region" (Van de Walle and Lovin 2006, p. 234).

The standard units for measuring area are square units such as "mm^2," "cm^2," "m^2" and "km^2." To calculate the area of any rectangle, multiply the length by the width.

Volume
"Volume is the amount of space that an object takes up" (Van de Walle and Lovin 2006, p. 244). The standard units for measuring volume are cubic units such as "mm^3," "cm^3," "m^3" and "km^3." To calculate the volume of any right prism, multiply the area of the base by the height of the prism. Another way to calculate the volume of a right rectangular prism is to multiply the length by the width by the height.

Key ideas in understanding the attribute of area are described below. Many of these ideas also apply to perimeter and volume.

- Conservation—an object retains its size when the orientation is changed or it is rearranged by subdividing it in any way.
- Iteration—the repetitive use of identical non-standard or standard units of area to entirely cover the surface of the region.
- Tiling—the units used to measure the area of a region must not overlap and must completely cover the region, leaving no gaps.
- Additivity—add the measures of the area for each part of a region to obtain the measure of the entire region.
- Proportionality—there is an inverse relationship between the size of the unit used to measure area and the number of units needed to measure the area of a given region; i.e., the smaller the unit, the more you need to measure the area of a given region.
- Congruence—comparison of the area of two regions can be done by superimposing one region on the other region, subdividing and rearranging, as necessary.
- Transitivity—when direct comparison of two areas is not possible, a third item is used that allows comparison; e.g., to compare the area of two windows, find the area of one window using non-standard or standard units and compare that measure with the area of the other window, especially if $A = B$ and $B = C$, then $A = C$; similarly for inequalities.
- Standardization—using standard units for measuring area such as "cm^2" and "m^2" facilitates communication of measures globally.
- Unit/unit-attribute relations—units used for measuring area must relate to area; e.g., "cm^2" must be used to measure area and not "cm" or "ml."

(Alberta Education 2006, Research section pp. 2–4.)

Sequence of Outcomes from the Program of Studies

See http://www.education.alberta.ca/teachers/program/math/educator/progstudy.aspx for the complete program of studies.

Grade 5	Grade 6	Grade 7
Patterns and Relations (Variables and Equations) **Specific Outcomes** 2. Express a given problem as an equation in which a letter variable is used to represent an unknown number (limited to whole numbers). [C, CN, PS, R] 3. Solve problems involving single-variable, one-step equations with whole number coefficients and whole number solutions. [C, CN, PS, R]	**Patterns and Relations (Variables and Equations)** **Specific Outcomes** 3. Represent generalizations arising from number relationships, using equations with letter variables. [C, CN, PS, R, V] 4. Express a given problem as an equation in which a letter variable is used to represent an unknown number. [C, CN, PS, R]	**Patterns and Relations (Variables and Equations)** **Specific Outcomes** 5. Evaluate an expression, given the value of the variable(s). [CN, R] 7. Model and solve, concretely, pictorially and symbolically, problems that can be represented by linear equations of the form: • $ax + b = c$ • $ax = b$ • $\frac{x}{a} = b, a \neq 0$ where *a*, *b* and *c* are whole numbers. [CN, PS, R, V]
Shape and Space (Measurement) **Specific Outcomes** 2. Design and construct different rectangles, given either perimeter or area, or both (whole numbers), and make generalizations. [C, CN, PS, R, V]	**Shape and Space (Measurement)** **Specific Outcomes** 3. Develop and apply a formula for determining the: • perimeter of polygons • area of rectangles • volume of right rectangular prisms. [C, CN, PS, R, V]	**Shape and Space (Measurement)** **Specific Outcomes** 2. Develop and apply a formula for determining the area of: • triangles • parallelograms • circles. [CN, PS, R, V]

Grade 5	Grade 6	Grade 7
4. Demonstrate an understanding of volume by: • selecting and justifying referents for cm^3 or m^3 units • estimating volume, using referents for cm^3 or m^3 • measuring and recording volume (cm^3 or m^3) • constructing right rectangular prisms for a given volume. [C, CN, ME, PS, R, V]		

Step 2: Determine Evidence of Student Learning

Guiding Questions

- What evidence will I look for to know that learning has occurred?
- What should students demonstrate to show their understanding of the mathematical concepts, skills and Big Ideas?

Using Achievement Indicators

As you begin planning lessons and learning activities, keep in mind ongoing ways to monitor and assess student learning. One starting point for this planning is to consider the achievement indicators listed in the *Mathematics Kindergarten to Grade 9 Program of Studies with Achievement Indicators*. You may also generate your own indicators and use them to guide your observation of students.

The following indicators may be used to determine whether or not students have met the specific outcomes. Can students:

- write and explain the formula for finding the perimeter of any given rectangle?
- write and explain the formula for finding the area of any given rectangle?
- develop and justify equations using letter variables that illustrate the commutative property of addition and multiplication; e.g., $a + b = b + a$ or $a \times b = b \times a$?
- describe the relationship, in a given table, using a mathematical expression?
- represent a pattern rule, using a simple mathematical expression such as $4d$ or $2n + 1$?
- identify the unknown in a problem where the unknown could have more than one value, and represent the problem with an equation?
- create a problem for a given equation with one unknown?
- identify the unknown in a problem, represent the problem with an equation, and solve the problem concretely, pictorially or symbolically?
- explain, using models, how the perimeter of any polygon can be determined?
- generalize a rule (formula) for determining the perimeter of polygons, including rectangles and squares?
- explain, using models, how the area of any rectangle can be determined?
- generalize a rule (formula) for determining the area of rectangles?
- explain, using models, how the volume of any right rectangular prism can be determined?
- generalize a rule (formula) for determining the volume of right rectangular prisms?
- solve a given problem involving the perimeter of polygons, the area of rectangles and/or the volume of right rectangular prisms?

Sample behaviours to look for related to these indicators are suggested for some of the activities listed in **Step 3, Section C: Choosing Learning Activities** (p. 15).

Step 3: Plan for Instruction

Guiding Questions

- What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should I use?
- How will I meet the diverse learning needs of my students?

A. Assessing Prior Knowledge and Skills

Before introducing new material, consider ways to assess and build on students' knowledge and skills related to counting. For example:

Provide students with centimetre grid paper and concrete materials such as square tiles.

- Given the equation $24 \div n = 4$, have the students:
  a. draw a diagram for the given equation
  b. create a problem for the given equation
  c. solve the problem.

- The area of a rectangular dog pen is 24 m^2. Have the students:
  a. draw all the possible dog pens using only whole numbers for the length and the width and explain how they know they have included all possible dog pens
  b. identify which rectangular dog pen would require the least fencing and explain their thinking.

If a student appears to have difficulty with these tasks, consider further individual assessment, such as a structured interview, to determine the students' level of skill and understanding. See **Sample Structured Interview: Assessing Prior Knowledge and Skills** (p. 12).

Sample Structured Interview: Assessing Prior Knowledge and Skills

Directions	Date:	
	Not Quite There	**Ready to Apply**
Provide the student with concrete materials such as square tiles. Present the student with the following problem. Read the problem, one part at a time. **"Given the equation:** **24 ÷ *n* = 4** **Draw a diagram for the given equation.** **Create a problem for the given equation.** **Solve the problem you created."**	Attempts to draw a diagram but makes errors in the number of equal groups or the size of each equal group. Has difficulty creating a division problem with clarity or may create a 'take-away' subtraction problem instead. Does not solve the problem correctly or solves a problem that does not represent the given equation.	Draws an accurate diagram showing clearly the number of equal groups and the size of each group; e.g., ☺☺ ☺☺ ☺☺ ☺☺ ☺☺ ☺☺ ☺☺ ☺☺ ☺☺ ☺☺ ☺☺ ☺☺ Four equal groups with six in each group. Creates a division problem with clarity, using equal grouping or equal sharing; e.g., 24 students are placed into four equal groups. How many students are in each group? Solves the problem correctly; e.g., There are six students in each group.

<table>
<tr><th rowspan="2">Directions</th><th colspan="2">Date:</th></tr>
<tr><th>Not Quite There</th><th>Ready to Apply</th></tr>
<tr>
<td>
Provide the student with centimetre grid paper.

Present the student with the following problem. Read the problem, one part at a time.

"The area of a rectangular dog pen is 24 m^2.

Draw all the possible dog pens using only whole numbers for the length and the width. Explain how you know you have included all possible dog pens.

Which rectangular dog pen would require the least fencing? Explain your thinking."
</td>
<td>
Draws some but not all the possible dog pens. Provides little or no explanation as to how he or she knows that all possible dog pens are included in the answer.

Attempts to find the perimeters of each rectangular pen drawn but makes mistakes and provides little or no explanation of the process used.
</td>
<td>
Draws all the possible dog pens. Provides a detailed explanation as to how he or she knows that all possible dog pens are included in the answer. Example:

Completes a chart showing where repetition begins with the 6 m by 4 m rectangle:

<table>
<tr><td>W</td><td>1</td><td>2</td><td>3</td><td>4</td><td>6</td></tr>
<tr><td>L</td><td>24</td><td>12</td><td>8</td><td>6</td><td>4</td></tr>
</table>
Length and width are in metres.

OR

Uses square tiles to build the rectangles in an organized way, recording the length and width for each rectangle constructed.

Explains that the 4 m by 6 m pen would require the least amount of fencing because the perimeter of this pen, 20 m, is the least perimeter of any of the possible pens. Completes the chart, including the perimeter of each dog pen:

<table>
<tr><td>W</td><td>1</td><td>2</td><td>3</td><td>4</td><td>6</td></tr>
<tr><td>L</td><td>24</td><td>12</td><td>8</td><td>6</td><td>4</td></tr>
<tr><td>P</td><td>50</td><td>28</td><td>22</td><td>20</td><td>20</td></tr>
</table>
Length, width and perimeter are in metres.

To find the perimeter add the length and width twice for each dog pen; e.g.,
4 + 6 + 4 + 6 = 20.
</td>
</tr>
</table>

B. Choosing Instructional Strategies

Consider the following instructional strategies for teaching generalizations using variables, including formulas for finding the perimeter of polygons, area of rectangles and volume of rectangular prisms.

- Build on understanding patterns from Grade 5—connecting the concrete, pictorial and symbolic representations of patterns and developing rules for patterns.
- Build on understanding measurement (perimeter, area and volume) from Grade 5—using patterns and connecting the concrete, pictorial and symbolic representations to construct formulas.
- Provide experiences with various models for patterns and the translations among the models; i.e., concrete materials, diagrams, table of values and pattern rules or formulas.
- Encourage students to describe patterns and rules, orally and in writing, before using algebraic symbols.
- Provide opportunities to connect the concrete and pictorial representations to symbolic representations as well as connecting the symbolic representations to pictorial and concrete representations.
- Use real-world contexts in solving problems using generalizations and formulas.
- Provide a variety of pattern problems using real-world contexts. Encourage students to solve the problems in different ways and explain the process. Also, provide time for students to share their solutions with others. Stimulate class discussion to critically evaluate the various procedures. Emphasize understanding, flexibility and efficiency when students select problem-solving strategies.
- Encourage students to draw diagrams to assist them in visualizing the relationship. Drawing diagrams will help students to construct an equation using a variable for the unknown value and known values.
- Provide pictorial examples of patterns in which students formulate pattern rules or formulas.
- In creating a functional relationship or formula for a given problem, have students represent and extend the problem in a table, describe the pattern shown in the table and use this pattern to write a functional relationship or a formula in terms of the step number. Have students use the created formula or the functional relationship to solve the problem (Van de Walle and Lovin 2006, pp. 269–270).

C. Choosing Learning Activities

The following learning activities are examples that could be used to develop student understanding of the concepts identified in Step 1.

Sample Activities:

Teaching Pattern Rules Using Variables (Recursive and Functional Relationships)

1. **Growing Patterns (What Changes, What Stays the Same)** (p. 18)
2. **Growing Patterns (What Changes: Variables, What Stays the Same: Constant)** (p. 19)
3. **Problem Solving with Patterns** (p. 23)

Teaching Formulas for Perimeter of Polygons

1. **Developing and Applying the Formula for Perimeter of Rectangles)** (p. 26)
2. **Perimeters for Polygonal Trains** (p. 28)

Teaching Formulas for Area of Rectangles

1. **Developing the Formula for the Area of Rectangles** (p. 32)

Teaching Formulas for the Volume of Right Rectangular Prisms

1. **Developing the Formula for the Volume of Rectangular Prisms** (p. 36)

Consolidating and Applying Formulas for Perimeter, Area and Volume

1. **Frayer Model for Perimeter, Area, Volume or Patterns** (p. 40)
2. **Solving Perimeter, Area and Volume Problems** (p. 42)

Teaching Pattern Rules Using Variables (Recursive and Functional Relationships)

Sample Activity 1: Growing Patterns (What Changes, What Stays the Same)

Focus on recursive relationships; i.e., pattern rules that show how patterns change from one step to the next.

Build on students' use of mathematical language to describe a pattern rule to show how a pattern changes from one step to the next. Focus on patterns that increase or decrease by the same amount with each step in the pattern; i.e., linear patterns. Review the role of variables and explain that variables can be used to describe pattern rules.

Look For …

Do students:

- ☐ apply their use of mathematical language in creating pattern rules using variables?
- ☐ examine the patterns to determine what changes and what stays the same in each successive step?
- ☐ apply their knowledge of variables to represent the numbers that change in each step with a letter?
- ☐ always include the first number in a pattern as part of the pattern rule for a recursive relationship?
- ☐ justify why a particular pattern rule using a variable describes what you do to the previous number in the pattern to get the next one?
- ☐ transfer their learning to other patterns?

Provide students with concrete materials such as square tiles.

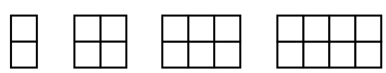
Example:

Rectangle Problem

Write a pattern rule to describe the following pattern by:

- stating the number of squares in the first step
- writing an expression using a variable and a constant to represent what is added to each succeeding number of squares to get the next number of squares.

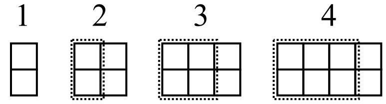
Step Number: 1 2 3 4



Number of Squares: 2 4 6 8
(or area of rectangle)

Sample Solution:
The first step in the pattern has two squares. The pictorial pattern shows that two squares are added to the previous diagram in the pattern to get the next diagram. The dotted box shows the squares from the previous step in each diagram below.

Step Number: 1 2 3 4



Number of squares: 2 4 6 8
(or area of rectangle)

Therefore, the constant that is added to the number of squares in one step to get the number of squares in the next step is the number 2.

The pattern rule can be written as follows:
The first step in the pattern has two squares. The expression, $n + 2$, is the number of squares in a step where n = the number of squares in the previous step.

Sample Activity 2: Growing Patterns (What Changes: Variables, What Stays the Same: Constant)

Focus on functional relationships; i.e., pattern rules that explain what you do to the step number to get the value of the pattern for that step.

- Provide students with concrete materials such as square tiles.
- Explain that students will continue work on translating patterns using pattern rules with variables but will explore a different pattern rule to efficiently find the number of elements in any given step number; e.g., the number of elements in the one-hundredth step.
- Have students examine a variety of linear pictorial patterns and determine what changes and what remains the same in each successive step of the pattern.

Look For …

Do students:

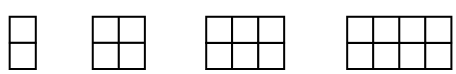
- ☐ examine the patterns to determine what stays the same and what changes in each successive step?
- ☐ apply their knowledge of variables to represent the numbers that change in each step with letters?
- ☐ use the recursive relationship to determine the functional relationship of a given pattern?
- ☐ justify why a particular pattern rule describes a given pattern?
- ☐ transfer their learning to other patterns?
- ☐ solve problems by applying pattern rules that represent functional relationships?

Use the **Rectangle Problem** pattern from the previous activity and guide the discussion to translate the pattern into a functional relationship; i.e., the relationship between the step number and the number of squares in each step.

Rectangle Problem

- Study the following pictorial pattern for 2, 4, 6, 8, ….
- Describe what changes and what stays the same.
- Write a pattern rule using variables that can be used to find the number of squares in the one-hundredth step.

Step Number:	1	2	3	4



Number of squares: (or area of rectangle)	$2 \times \boxed{1}$	$2 \times \boxed{2}$	$2 \times \boxed{3}$	$2 \times \boxed{4}$

Guided Solution

In deciding what stays the same and what changes, guide the discussion to include the dimensions of the composite rectangle in each step; i.e., the width stays the same at two units but the length increases by one unit with each new step in the pattern. The length matches the step number; e.g., the rectangle in the first step has a length of one unit, the rectangle in the second step has a length of two units.

Label the numbers that change in each step in some way; e.g., putting a box around them, as shown above. Review the meaning of a variable with students—a symbol that can stand for any one of a set of numbers or objects.

Have students suggest a symbol to use for the variable in the pattern. Then have them write a pattern rule (or functional relationship) using variables to represent the pattern by connecting the step number to the number of elements in each step; e.g., $A = 2n$, where A equals the number of squares in each step (or the area of the complete rectangle) and n represents the length of the rectangle or the number of squares in the bottom row of each step or the step number.

Ask students what the area of the one-hundredth rectangle is. They should use the functional relationship to answer that the area of the one-hundredth rectangle is 2×100 or 200 square units.

Connect the functional pattern rule to the formula for the area of a rectangle. Ask students how the pattern rule could be changed to represent the area of any rectangle. Through discussion, have them generalize that the area of a rectangle is the length multiplied by the width, with both the length and width as variables.

Comparing Pattern Rules

Explain that there are different pattern rules to describe patterns. Review the pattern rule that describes how a pattern changes from one step to another step and compare it to the pattern rule that describes how the step number relates to the number of elements in each step. Use a chart to summarize the pattern and show the different pattern rules:

Step Number	1	2	3	4	…	100
Number of Squares	2	4	6	8	…	200

Explain that the horizontal arrow focuses on the recursive relationship which continues the pattern along a row: 2, 4, 6, 8, …; i.e., pattern rule is to start at 2 and add 2 to each successive term.

Discuss that the vertical arrow focuses on the functional relationship, i.e., the relationship between the two rows and the pattern rule is $A = 2n$, where A is the number of squares and n is the step number.

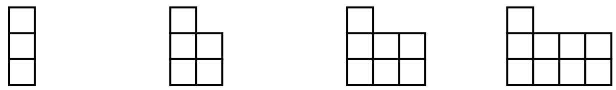
Have students draw diagrams to show other similar patterns with the coefficient of the variable being 3 or 4 as in the following examples:
3, 6, 9, 12, … $3n$
4, 8, 12, 16, … $4n$.

More Strategies for Describing Pattern Rules (Functional Relationships)
To guide students in writing a broader range of functional relationships (pattern rules) using variables for patterns that increase or decrease by a constant greater than one, adjust the pattern with the rectangles as shown below.

Odd Number Problem 1
Draw a pictorial representation of the pattern 3, 5, 7, 9, ….
Write a pattern rule using variables that can be used to find the one-hundredth number in the pattern.

Guided Solution:

Step: 1 2 3 4



Number of squares: $2 \times \boxed{1} + 1$ $2 \times \boxed{2} + 1$ $2 \times \boxed{3} + 1$ $2 \times \boxed{4} + 1$

Discuss what changes and what stays the same in the pattern. Have students write an expression for the number of squares in each step, using a box and then a variable to represent what changes and using constants to represent what stays the same.

Through discussion, have students generalize a pattern rule (functional relationship) in the form of an equation such as $B = 2n + 1$, where B = the number of squares in a given step and n = the step number.

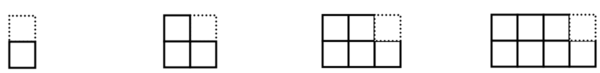
Have students use the pattern rule to find the number of squares in the one-hundredth step; i.e., $2 \times 100 + 1 = 201$.

Odd Number Problem 2

Draw a pictorial representation for the pattern 1, 3, 5, 7, ….
Write a pattern rule using variables that can be used to find the one-hundredth number.

Guided Solution

Step: 1 2 3 4



Number of squares: $2 \times \boxed{1} - 1$ $2 \times \boxed{2} - 1$ $2 \times \boxed{3} - 1$ $2 \times \boxed{4} - 1$

Discuss what changes and what stays the same in the pattern. Have students write an expression for the number of squares in each step, using a box and then a variable to represent what changes and using constants to represent what stays the same.

Through discussion, have students generalize a pattern rule (functional relationship) in the form of an equation such as $B = 2n - 1$, where B = the number of squares in a given step and n = the step number.

Have students use the pattern rule to find the number of squares in the one-hundredth step; i.e., $2 \times 100 - 1 = 199$.

Have students draw graphs with discrete elements to represent the various pattern rules created. The graphs provide a visual tool to represent the functional relationships.

Consolidate the learning by having students examine the recursive relationships of a variety of linear patterns to determine if the patterns increase by 1, 2, 3 and so on. Their explorations should lead them to conclude that a pattern that increases by 1 has the variable in the pattern rule (functional relationship) with a coefficient of 1, a pattern that increases by 2 has the variable in the pattern rule with a coefficient of 2 and so on. Once the coefficient of the variable is determined, then by substituting the step number for the variable it logically follows whether a constant must be added or subtracted to obtain the required number of elements for that step.

For example, provide students with the following problem.

Odd Number Problem 3
Find the one-hundredth term in the pattern 7, 9, 11, 13, ….

Guided Solution

Suggest that the pattern be represented in a table of values showing the step number and the number of elements in each step.

Step Number (n)	1	2	3	4	…	100
Number of Elements in the Step (E)	7	9	11	13	…	?

Through discussion, have students verbalize the following explanation.

> Since the pattern in the bottom row of the chart increases by two each time, then the pattern rule that relates the two rows in the table (functional relationship) will have $2n$ included in it. Substituting 1 for n in the first step, you get $2 \times 1 = 2$. In order to get the required number of elements (7) for this first step, you must add five. Therefore, the pattern rule can be written as $E = 2n + 5$, where E is the number of elements in each step and n is the step number.
>
> The one-hundredth step would have $2 \times 100 + 5 = 205$ elements.

To conclude, review that the recursive relationships for patterns are needed to determine their functional relationships. It is the functional relationships written as pattern rules that show the power of algebra, providing a general rule that can be used to find the number of elements in any step when given the step number.

Have students discuss the similarities and differences between recursive and functional relationships, recognizing that both are included in patterns but each of them has a different role.

- A recursive relationship describes the pattern between successive numbers in one of the rows in a table of values.
- A functional relationship is a general rule to describe the relationship between two rows of numbers in a table of values.

Sample Activity 3: Problem Solving with Patterns

Provide students with problems using everyday contexts in which they can apply their understanding of functional relationships.

Pizza Problem

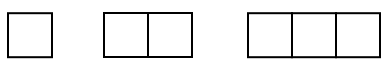
Pete's Pizza Parlour has square tables that each seats four people. If you push two tables together, six people can be seated. If you push three tables together, eight people can be seated.

- Write a pattern rule that can be used to calculate the number of people that can be seated given any number of tables put end-to-end.
- Use your pattern rule to find how many people can be seated if 50 tables are put end-to-end.

Guided Solution

Build on students' knowledge of creating charts for patterns and have them suggest how the information in the problem can be represented in a chart. Encourage students to draw diagrams to represent the pattern and place the data in a chart.

For example:
Number of Tables: 1 2 3



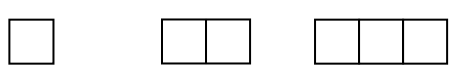
Number of People: 4 6 8

Number of Tables (n)	1	2	3	4	…	50
Number of People (P)	4	6	8	?	…	?

Have students describe the recursive relationship of the pattern of numbers in the bottom row of the chart; i.e., each succeeding number increases by 2.

Build on the students' understanding of patterns in writing functional relationships that connect the step number with the number of elements in each step (see previous activity). Provide scaffolding for students, if necessary, by having them examine the diagrams in the pattern and notice what changes and what stays the same.

Number of Tables: 1 2 3



$2 \times \boxed{1} + 2$ $2 \times \boxed{2} + 2$ $2 \times \boxed{3} + 2$

Discuss that the constant, 2, is added in each expression because two people sit at the ends in each diagram.

Discuss that the constant, 2, multiplies each step number because for each table there are two people seated on the sides that are not the ends; i.e., for one table, two people can be seated at the sides that are not the ends; for two tables, $2 \times 2 = 4$ people can be seated at the sides that are not the ends; for three tables, $2 \times 3 = 6$ people can be seated at the sides that are not the ends, and so on. See the expressions written below the diagrams.

Instruct students to write a pattern rule using an equation with variables; e.g., $2n + 2 = P$, where n is the number of tables placed end-to-end and P is the number of people.

Have students use their pattern rule to find the number of people that can be seated with 50 tables placed end-to-end by substituting 50 for n; i.e., $2 \times 50 + 2 = 102$. Have students write a sentence to answer the question asked in the problem; e.g., "When 50 tables are placed end-to-end, 102 people can be seated."

Provide other real-world problems for students to write pattern rules (functional relationships) with variables and use them to solve the problems. Remind them to use diagrams and charts to represent the problems so that they are better able to write the pattern rules.

Look For …

Do students:

- ☐ transfer the information in the problem to another model such as a chart or a diagram?
- ☐ apply their understanding of recursive relationships (how a pattern changes from step-to-step) to find a functional relationship (the relationship between two rows of numbers in a chart) using variables?
- ☐ use the pattern rule with variables for the functional relationship to solve the problem using larger numbers?

Create Problems for a Given Equation

Reverse the procedure. Provide students with an equation and have them create a problem for the given equation.

Teaching Formulas for Perimeter of Polygons

Sample Activity 1: Developing and Applying the Formula for Perimeter of Rectangles

Review the concept of perimeter and build on students' prior knowledge of the concept.

Provide students with centicubes, 30-centimetre rulers, centimetre grid paper and a variety of rectangles. Blackline Masters 25 and 27 can be downloaded from the website http://www.ablongman.com/vandewalleseries.

Have students predict the perimeter of each rectangle.

Then ask students to measure the lengths and widths of the different rectangles using the centicubes and/or the 30-centimetre rulers and record the data in a chart, as shown below. Have them compare the actual perimeters to the predicted perimeters. Ask students to look for patterns in the chart and suggest a rule or formula that could be used to find the perimeter of any rectangle. Have students share their ideas and choose the formula that works best for them.

Look For …

Do students:

- ☐ predict the perimeters of polygons prior to measuring them?
- ☐ describe perimeter patterns using concrete, pictorial and symbolic representations?
- ☐ apply their knowledge of two-dimensional figures to explain the difference between regular and irregular polygons?
- ☐ generalize formulas by examining perimeter patterns for polygons?
- ☐ apply their knowledge of variables in formulating a rule for perimeters of polygons?
- ☐ demonstrate flexibility in creating formulas for the perimeter of polygons?
- ☐ apply perimeter formulas to solve problems?

Rectangle	Predicted Perimeter (cm)	Length (cm)	Width (cm)	Actual Perimeter (cm)
A				
B				
C				
D				

Through discussion, have students generalize the following formulas for the perimeter of rectangles:

- $P = L + L + W + W$
- $P = 2L + 2W$
- $P = 2(L + W)$.

Formula for the Perimeter of a Square

Review that a square is a special rectangle with all sides congruent. Explain that the formulas for the perimeter of rectangles can also be used to find the perimeter of squares; however, the perimeter of a square can be found more efficiently with a different formula that they will develop.

Have them draw squares of different sizes; e.g., the length of one side could be 2 cm, 2.5 cm, 3 cm, 3.5 cm and so on.

Have students find the perimeter of each square and record the data in a chart such as the one shown below.

Square	A	B	C	D	E
Side Length (cm)	2	2.5	3	3.5	
Perimeter (cm)	8	10			

Through discussion of the patterns shown in the chart, have students generalize that the formula for the perimeter of a square could be written as $P = 4n$, where P = the perimeter of the square and n = length of one side of the square.

Have students use the formula for the perimeter of a square to find the length of one side of a square with a perimeter of 36 cm.

Different Rectangles with the Same Perimeter

Have students draw on centimetre grid paper as many rectangles as possible with whole number dimensions that have a perimeter of 20 cm. Encourage them to apply the formula for the perimeter of rectangles to show that the sum of the length and the width is half the total perimeter of the rectangle. This information is useful in recording the data in a chart using patterns:

Perimeter (cm)	Length + Width (cm)	Length (cm)	Width (cm)
20	10	9	1
		8	2
		7	3
		.	.

See the *Diagnostic Mathematics Program: Division II: Measurement*, pp. 133–137, for more detailed instructions and Blackline Masters (Alberta Education 1990).

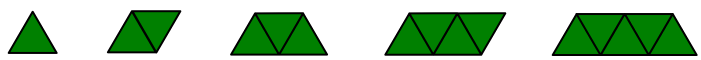
Sample Activity 2: Perimeters for Polygonal Trains

Provide students with pattern blocks, triangular dot paper and centimetre grid paper. Present students with the following problem and review the vocabulary; e.g., a regular polygon is a two-dimensional figure with all sides congruent and all angles congruent.

Problem
Investigate the perimeter of trains formed by using one or more of the same congruent regular polygons. Sides must correspond exactly. Use equilateral triangles, squares, pentagons and hexagons as the regular polygons.

Example:

Step Number: 1 2 3 4 5



Perimeter: 3 4 5 6 7

(Each unit is the length of one side of the equilateral triangle in Step 1.)

Suggest that students record their data in a chart such as the following:

Number of Cars Step Number	Perimeter			
	Triangle	Square	Pentagon	Hexagon
1				
2				
3				
4				
n				
100				

Sample Solution:

Number of Cars Step Number	Perimeter			
	Triangle	Square	Pentagon	Hexagon
1	3 units	4 units	5 units	6 units
2	4 units	6 units	8 units	10 units
3	5 units	8 units	11 units	14 units
4	6 units	10 units	14 units	18 units
n	$n + 2$ units	$2n + 2$ units	$3n + 2$ units	$4n + 2$ units
100	102 units	202 units	302 units	402 units

Encourage students to observe the patterns such as the patterns in the columns or the rows. Have students suggest a formula or a pattern rule that could be used to find the perimeter of a train with *n* cars for each of the polygons.

Ask students to apply the formula to find the perimeter of 100 cars for each of the polygons.

Problem
Challenge students to generalize the perimeter of a polygon with *m* sides and with *n* being the number of cars in the train.

Sample Solution

- multiply the number of polygons in the train by 2 less than the number of sides in the given polygon and add 2
- $n(m - 2) + 2$, where n = number of cars in the train and m = the number of sides on the given polygon.

Applying the formula to a train made out of equilateral triangles we have the perimeter of a train with 100 cars to be: $100(3 - 2) + 2 = 100 + 2 = 102$.

(National Council of Teachers of Mathematics 1991, pp. 49–50; *Patterns and Pre-Algebra, Grades 4–6*, Alberta Education 2007, pp. 118–121.)

Have students define perimeter in their own words; e.g., the perimeter of any polygon is the sum of the lengths of all the sides of the polygon.

Teaching Formulas for Area of Rectangles

Sample Activity 1: Developing the Formula for the Area of Rectangles

Review the concept of area and build on students' prior knowledge of the concept. See the end-to-end plan for factors and multiples that connects the area of rectangles to factors and multiples.

Provide students with centicubes, 30-centimetre rulers, centimetre grid paper and a variety of rectangles. Blackline Masters 25 and 27 can be downloaded from the website http://www.ablongman.com/vandewalleseries.

Ask students to predict the area of each rectangle.

Then have students measure the lengths and widths of the different rectangles using the 30-centimetre rulers and record the data in a chart, as shown below. Encourage students to use the centicubes to make the rectangles if needed (each face of the rectangular prism made with centicubes is a rectangle). Have students compare the actual areas to the predicted areas.

Ask students to look for patterns in the chart and suggest a rule or formula that could be used to find the area of any rectangle. Have students share their ideas and choose the formula that works best for them.

Look For …

Do students:

- ☐ predict the areas of rectangles prior to measuring them?
- ☐ describe area patterns using concrete, pictorial and symbolic representations?
- ☐ connect area of rectangles to arrays?
- ☐ generalize formulas by examining area patterns for rectangles, including arrays?
- ☐ apply their knowledge of variables in formulating a rule for the area of rectangles?
- ☐ demonstrate flexibility in creating formulas for the area of rectangles, including squares?
- ☐ apply the formula for the area of a rectangle to solve problems?

Rectangle	Predicted Area (cm^2)	Length (cm)	Width (cm)	Actual Area (cm^2)
A				
B				
C				
D				

Through discussion, have students generalize the following formulas for the area of rectangles:

- A = length × width
- $A = L \times W$.

Formula for the Area of a Square

Review that a square is a special rectangle with all sides congruent. Explain that the formulas for the area of rectangles can also be used to find the area of squares; however, the area of a square can be found more efficiently with a different formula that the students will develop.

Have students draw squares of different sizes or construct them using square tiles; e.g., the length of one side could be 2 cm, 4 cm, 6 cm, 8 cm and so on. Have students find the area of each square and record the data in a chart such as the one shown below.

Square	A	B	C	D	E
Side Length (cm)	2	4	6	8	
Area (cm^2)	4	16	36		

Through discussion of the patterns shown in the chart, have students generalize that the formula for the area of a square can be written as $A = n \times n$, where A = the area of the square and n = the length of one side.

Have students use the formula for the area of a square to find the area of a square that has a side length of 24 cm.

Different Rectangles with the Same Area

Have students draw on centimetre grid paper as many rectangles as possible with whole number dimensions that have an area of 20 cm^2. Some students may wish to use centicubes or square tiles to create the different rectangles. Encourage them to apply the formula for the area of rectangles, use patterns and record the data in a chart.

Remind students that the values for the length and width of the rectangle are factors of 20.

Area (cm^2)	Length (cm)	Width (cm)
20	20	1
	10	2
	5	4

(Alberta Education 1990, pp. 149–155.)

Teaching Formulas for the Volume of Right Rectangular Prisms

Sample Activity 1: Developing the Formula for the Volume of Rectangular Prisms

Review the concept of volume and build on students' prior knowledge of the concept. See the end-to-end plan for factors and multiples that connects the volume of right rectangular prisms to factors and multiples.

Provide students with centicubes, 30-centimetre rulers and a variety of rectangular prisms, including a cube, labelled A, B, C and D. (See the Blackline Master for rectangular prisms in the *Diagnostic Mathematics Program, Division II: Measurement*, Alberta Education 1990, p. 164.)

Have students predict the volume of each rectangular prism.

Then have students measure the lengths and widths of the different rectangular prisms using the 30-centimetre rulers and record the data in a chart as shown below. Encourage students to use the centicubes to fill or make the rectangular prisms. Have students compare the actual volumes to the predicted volumes.

Ask students to look for patterns in the chart and suggest a rule or formula that could be used to find the volume of any rectangular prism. Have students share their ideas and choose the formula that works best for them.

Look For …

Do students:

- ☐ predict the volumes of rectangular prisms prior to measuring them?
- ☐ describe volume patterns using concrete, pictorial and symbolic representations?
- ☐ generalize formulas by examining volume patterns for right rectangular prisms?
- ☐ apply their knowledge of variables in formulating a rule for the volume of right rectangular prisms?
- ☐ demonstrate flexibility in creating formulas for the volume of right rectangular prisms, including cubes?
- ☐ apply the formula for the volume of right rectangular prisms to solve problems?

Rectangle	Predicted Volume (cm^3)	Length (cm)	Width (cm)	Height (cm)	Volume (cm^3)
A					
B					
C					
D					

"When students build cube models of prisms, many will notice that instead of counting each cube to calculate the volume, they can multiply the number of cubes in each layer, which is represented by the area of the base when dealing with the volume formula, by the number of layers, which is represented by the height when dealing with the formula."
(Small 2009, p. 155.)

Through discussion, have students generalize the following formulas for the volume of right rectangular prisms:

- V = length x width × height
- $V = L \times W \times H$
- V = area of the base × height or $V = B \times h$, where B = the area of the rectangular base.

Formula for the Volume of a Cube

Review that a cube is a special right rectangular prism with all faces congruent. Explain that the formulas for the volume of right rectangular prisms can also be used to find the volume of cubes; however, the volume of a cube can be found more efficiently with a different formula that they will develop.

Provide students with some cubes of different sizes; e.g., one edge could be 2 cm, 4 cm, 6 cm, 8 cm and so on. Alternately, have students construct cubes of different sizes out of centicubes or multilink cubes.

Have students find the volume of each cube and record the data in a chart such as the one shown below.

Cube	A	B	C	D	E
Edge Length (cm)	2	4	6	8	?
Volume (cm^3)	8	64	216	?	?

Through discussion of the patterns shown in the chart, have students generalize that the formula for the volume of a cube can be written as $V = n \times n \times n$, where V = the volume of the cube and n = the length of one edge.

Have students use the formula for the volume of a cube to find the volume of a cube that has an edge length of 10 cm.

Different Right Rectangular Prisms with the Same Volume

Have students use centicubes to construct as many rectangular prisms as possible with whole number dimensions that have a volume of 24 cm^3. Encourage them to apply the formula for the volume of rectangular prisms, use patterns and record the data in a chart. Remind students that the values for the length, width and height of the rectangular prism are factors of 24.

Volume (cm^3)	Length (cm)	Width (cm)	Height (cm)
24	24	1	1
	12	2	1
	8	3	1
	6	4	1
	6	2	2
	4	3	2

(Alberta Education 1990, pp. 162–164.)

Consolidating and Applying Formulas for Perimeter, Area and Volume

Sample Activity 1: Frayer Model for Perimeter, Area, Volume or Patterns

Provide students with a template for the Frayer Model and have them fill in the sections individually or as a group to consolidate their understanding of perimeter, area, volume or patterns.

A sample of a Frayer Model for area follows.

Look For …

Do students:

- ☐ apply their knowledge of the concept and write a definition in their own words that is mathematically correct?
- ☐ create and justify examples and non-examples of the concept?
- ☐ use visual and numeric representations for the concept?
- ☐ create a problem that applies the concept?

Frayer Model for Area

(Barton and Heidema 2002, pp. 68–71)

Area

Definition

Area is the amount of surface. The surface may be plane or curved, but, in either case, it is two-dimensional. Area is measured in mm^2, cm^2, m^2, km^2, etc.

Examples

- tiles on a floor
- sod for a lawn
- carpet for a floor

Non-examples

- fencing around a garden
- edging around a tablecloth
- baseboards for a room

Visual and Numeric Representation

Step: 1 2 3 4

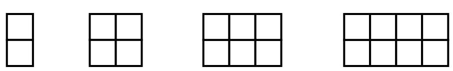
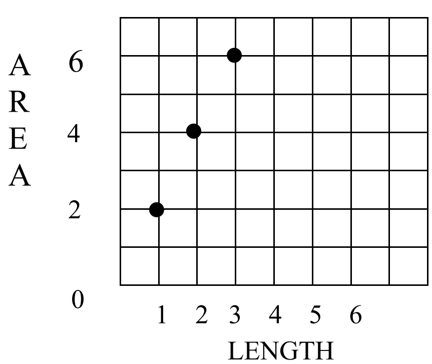


Table of Values:

Length (units)	1	2	3	4
Area (square units)	2	4	6	8

Formula: $A = L \times W$. Since the width is always the same in each step (2 units), the symbolic representation is $A = 2 \times L$.



Real-life Applications

- You have 36 metres of fencing for a rectangular dog pen. What dimensions of the pen would provide the greatest area?
- If the length of a pig's rectangular pen is 10 m and the width is 5 m, how much room does the pig have to run around?

Sample Activity 2: Solving Perimeter, Area and Volume Problems

Provide students with problems that apply the concepts of perimeter, area and volume. Encourage students to estimate the answer before calculating it. The estimate will help them decide if their answer is reasonable. Have them solve the problems by applying the appropriate formulas. Encourage students to use a calculator when necessary. Suggest that students use smaller numbers in the problem initially, if they have difficulty applying the formulas.

Sample Problems

a. Estimate and then find the width of a rectangular plot of land with an area of 1536 m^2 and a width of 32 m. Explain your thinking.

Sample Solution

- Estimate first. 1536 is about 1500. 32 is about 30.
- Since both numbers are less than the original numbers, then the estimated quotient will be a good estimate (using the constant quotient estimation strategy).
- Since the area of a rectangle is found by multiplying the length by the width ($A = L \times W$), then the estimated length is found as follows: $30 \times ? = 1500 \rightarrow 30 \times \mathbf{50} = 1500$.
- The estimated length of the rectangle is 50 m.

Sample Solution

- Calculate the length by using $A = L \times W$.
- Substituting into the formula, the equation becomes $1536 = 32 \times L$.
- To find the length, divide 1536 by 32 because division is the opposite of multiplication; i.e., it is used to find the missing factor when the product is given.
- Using a calculator: $1536 \div 32 = \mathbf{48}$.

Check for reasonableness.
The calculated answer of 48 m is very close to the estimated answer of 50 m.

Answer the problem.
The length of the rectangular plot of land is 48 m.

Look For …
Do students:
- ☐ ask questions to clarify their understanding of the problems?
- ☐ use simpler numbers, if needed, to decide how to solve the problems?
- ☐ use appropriate estimation strategies prior to calculating the answer?
- ☐ apply the appropriate formulas to solve the problems?
- ☐ communicate clearly the solution process, using appropriate mathematical language?
- ☐ use the calculator appropriately?
- ☐ check the reasonableness of the answer by comparing the calculated answer to the estimated answer?
- ☐ answer the problem using appropriate units in a sentence that answers the question asked in the problem?

b. Estimate and then find the length of fencing required to go all the way around a rectangular field that is 136 m wide and 425 m long. Explain your thinking.
c. The area of the base of a paper tissue box (right rectangular prism) is 264 cm^2. If the height of the box is 7 cm, estimate and then find the volume. Explain your thinking.
d. The area of a rectangular floor is 35 m2. What could the volume of the room be? Explain your thinking.
e. Estimate and then find the least perimeter for a rectangular garden with an area of 64 m2. Explain your thinking.
f. Estimate and then find the greatest area for a rectangular garden with a perimeter of 356 m.

Step 4: Assess Student Learning

Guiding Questions

- Look back at what you determined as acceptable evidence in Step 2.
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Sample Assessment Tasks

In addition to ongoing assessment throughout the lessons, consider the following sample activities to evaluate students' learning at key milestones. Suggestions are given for assessing all students as a class or in groups, individual students in need of further evaluation, and individual or groups of students in a variety of contexts.

A. Whole Class/Group Assessment

The following is a performance assessment that could be used with a whole group or class. It includes a marking rubric to be used with the assessment.

Gardening with Formulas

In this assessment, students will solve problems to demonstrate their understanding of patterns and formulas related to the perimeter and area of rectangles and the volume of right rectangular prisms by using concrete materials, as needed, diagrams and symbols. They will create pictorial representations of rectangles and explain why the formula for the area of a rectangle is A = length × width.

Then students will explain how to find the largest area of a rectangular garden for a given perimeter. By using larger numbers, students will need to apply the formula for the area of a rectangle and the perimeter of a rectangle.

Finally, students will apply the volume formula for right rectangular prisms to find different dimensions for the floor of a garden shed, given the volume and height of the shed.

Materials required: square tiles, centicubes.

Students could explain why the formula for the area of a rectangle is A = length × width by using the square tiles to set up different arrays. They draw the arrays and show that the area of each the rectangular array is found by multiplying the number of rows (width) by the length of each row (length). The data for the arrays could be shown in a chart. The pattern in the chart can be generalized as the formula for the area of rectangles.

When given the perimeter of a rectangle and asked to find the rectangle with the largest area, students could use the formula for the perimeter of a rectangle, $P = 2L + 2W$ and take half of it. If the entire perimeter is 40 m, then half the perimeter is 20 m. Therefore, $L + W = 20$ m. Students could list the different dimensions that add up to 20 in a chart and include the area for each. When they get to 10 m by 10 m, the numbers are repeated so they do not have to be listed in the chart.

Length (m)	19	18	17	16	15	14	13	12	11	**10**
Width (m)	1	2	3	4	5	6	7	8	9	**10**
Area (m^2)	19	38	51	64	75	84	91	96	99	**100**

Some students may know that the square provides the greatest area for a given perimeter of a rectangle. They may use the formula for the perimeter of a square, $P = 4 \times n$, where P = the perimeter of the square and n = the length of one side of the square. Substituting into the formula: $40 = 4 \times n$. Therefore, $n = 10$.

Students must answer the question asked in a sentence with the correct number and unit for the measurement. Sample answer: "The dimensions of the garden that would provide the most space to plant the vegetables are 10 m by 10 m. It would be a square garden."

The formula for the volume of a right rectangular prism can be written as V = area of the rectangular base × height or $V = B \times h$. Because the volume and the height are provided in the problem, the student can use the formula to write the equation: $24 = 2 \times B$, where 24 m^3 is the volume of the rectangular prism, 2 m is the height and B is the area of the rectangular base. $2 \times \mathbf{12} = 24$. Therefore, the area of the rectangular base is 12 cm^2. Some students may need to use the centicubes to build the rectangular prisms with different bases.

Using the formula for the area of rectangles, $A = L \times W$, students can write the factor pairs for 12 and may include them in a chart. When they get to 4 m by 3 m, the numbers are repeated so they do not have to be listed in the chart because students apply the commutative property of multiplication; i.e., $4 \times 3 = 3 \times 4$.

Length (m)	12	6	4	3
Width (m)	1	2	3	4
Area of the base (m^2)	12	12	12	12

Students must answer the question asked in a sentence with the correct number and unit for the measurement. Sample answer: "The dimensions of the rectangular base of the shed could be 6 m by 2 m or 4 m by 3 m. If the dimensions of the base are 12 m by 1 m, the shed would likely be too narrow."

Task-specific Criteria

Each student will:

- write and explain the formula for finding the perimeter of any given rectangle
- write and explain the formula for finding the area of any given rectangle
- identify the unknown in a problem where the unknown could have more than one value, and represent the problem with an equation
- identify the unknown in a problem, represent the problem with an equation, and solve the problem concretely, pictorially or symbolically
- explain, using models, how the area of any rectangle can be determined
- generalize a rule (formula) for determining the area of rectangles
- explain, using models, how the volume of any right rectangular prism can be determined
- generalize a rule (formula) for determining the volume of right rectangular prisms
- solve a given problem involving the perimeter of polygons, the area of rectangles and/or the volume of right rectangular prisms.

Early finishers can:

- find the rectangle with the largest area that has a perimeter of 24 cm
- find the rectangle with the least perimeter that has an area of 144 cm^2
- find all the possible right rectangular prisms that have a volume of 64 cm^3, using only whole number dimensions
- find the one-hundredth term in the pattern 8, 12, 16, 20, ….

Gardening with Formulas—Student Assessment Task

Materials: square tiles, centicubes

You are getting ready to plant a vegetable garden. In making decisions about the garden, you will use some formulas for perimeter, area and volume.

1. Explain, using concrete materials and at least two different diagrams, why the formula for the area of a rectangle is A = length × width.

2. You have 40 metres of fencing to enclose your rectangular garden. Find the dimensions (length and width) of the garden using all 40 m of fencing that will provide the most room to plant the vegetables. Explain your thinking using appropriate formulas.

3. You and your dad are constructing a garden shed in the shape of a right rectangular prism with a volume of 24 m^3. If the height of the shed is 2 m, what could the dimensions (length and width) of the floor be? Provide two examples using whole numbers. Explain your thinking using an appropriate formula.

Student: ____________________________________

SCORING GUIDE:
Gardening with Formulas

Level / Criteria	4 Excellent	3 Proficient	2 Adequate	1 Limited *	Insufficient / Blank *
Explains, using models and diagrams, why the formula for the area of rectangles is $A = L \times W$ Question 1	The student uses the square tiles to represent at least two rectangles, draws and labels very accurate diagrams that represent the concrete materials, and uses precise mathematical language to explain why the formula for the area of rectangles is $A = L \times W$.	The student uses the square tiles to represent two rectangles, draws and labels accurate diagrams that represent the concrete materials, and explains clearly why the formula for the area of rectangles is $A = L \times W$.	The student uses square tiles to represent a rectangle, draws but may not label a diagram that represents the concrete materials, and provides a limited explanation of why the formula for the area of rectangles is $A = L \times W$.	The student may have difficulty using the square tiles to represent a rectangle, draws an inaccurate diagram with no labels to represent the concrete materials and does not explain why the formula for the area of rectangles is $A = L \times W$.	No score is awarded because there is insufficient evidence of student performance, based on the requirements of the assessment task.
Connects perimeter and area of rectangles to solve a problem Question 2	The student describes clearly, using precise mathematical language with the use of formulas, the solution process, and answers the problem correctly using appropriate units.	The student describes clearly with the use of formulas, the solution process, and answers the problem correctly using appropriate units.	The student may or may not use formulas to provide a limited explanation of the solution process, may have some errors in solving the problem and may not use appropriate units.	The student uses no formulas, has difficulty providing any explanations, makes many errors in solving the problem and does not use the appropriate units.	No score is awarded because there is insufficient evidence of student performance, based on the requirements of the assessment task.
Applies the formula for the volume of right rectangular prisms to solve a problem Question 3	The student describes clearly, using precise mathematical language with the use of formulas, how to find two examples for the base of the right rectangular prism, and uses appropriate units.	The student describes clearly, with the use of formulas, how to find two examples for the base of the rectangular prism, and uses appropriate units.	The student may or may not use formulas to provide a limited explanation of how to find an example for the base of the rectangular prism and may not use appropriate units.	The student uses no formulas, has difficulty providing any explanation of how to find an example for the base of the rectangular prism, makes many errors in solving the problem and does not use the appropriate units.	No score is awarded because there is insufficient evidence of student performance, based on the requirements of the assessment task.

* When work is judged to be limited or insufficient, the teacher makes decisions about appropriate interventions to help the student improve.

B. One-on-one Assessment

Provide the student with triangular pattern blocks. Present the following problem to the student. Provide guidance, as necessary, to connect this problem to the perimeter of polygons.

Harry's Hamburger Haven has triangular tables that each seat three people. If you push two tables together, four people can be seated. If you push three tables together, five people can be seated and so on.

a. Draw a diagram to show the pattern in the problem.

b. Write a general rule, using variables, which can be used to calculate the number of people that can be seated given any number of tables put end-to-end. Explain your thinking.

c. Use your general rule to find how many people can be seated if 30 tables are put end-to-end. Show your work.

If the student has difficulty understanding the problem, have him or her read the problem orally and state the problem in his or her own words. Follow up by asking some questions to clarify the essential parts of the problem; e.g., "What shape is each table? How many people can be seated at one table? Two tables?"

Use a similar process for each part of the problem to ensure the student understands the problem.

a. If the student has difficulty drawing a diagram, have him or her use the triangular pattern blocks to show the pattern, using one table with three people, two tables with four people and so on. Ask him or her to draw the pattern shown by the concrete representation. Show the student that the number of people seated at each step is the same as the perimeter of each polygon drawn, where the unit is the length of one side of the triangle.

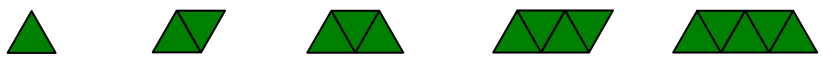
b. If the student has difficulty writing the general rule for the pattern, ask the student what changes and what stays the same in the pattern. The student may say that there is always one more person seated at each successive table. Remind the student that this pattern rule (recursive relationship) cannot be used to find how many people are seated at any number of tables.

Focus attention on the relation between the number of tables and the number of people that can be seated; i.e., the functional relationship of the pattern. As the student examines the pictorial pattern, guide him or her to observe that there are always two people seated at the slanted sides of the table(s). Therefore, 2 is the constant. Then focus attention on the horizontal sides of the table. Guide the student to observe that the number of people seated on the horizontal sides changes with each step and is always the same as the step number; e.g., in the first step, one person is seated on the horizontal side of the table, in the second step, two people are seated on the horizontal sides of the table.

Encourage the student to describe the general rule (functional relationship) in his or her own words before using variables to write the equation. Through discussion, have the student describe, in his or her own words, the following connection: "Three people can be seated with one triangular table, four people can be seated with two triangular tables, five people can be seated with three triangular tables and so on. Then the number of people seated is always two more than the number of triangular tables."

Encourage the student to put a box around the number that changes with each step as shown below. Then suggest that the student use a variable to represent the box and write the general rule; e.g., $N + 2 = P$, where N = the step number or the number of triangular tables and P = the number of people seated for that step.

Step Number: 1 2 3 4 5



Number of People: 3 4 5 6 7

OR $\boxed{1} + 2$ $\boxed{2} + 2$ $\boxed{3} + 2$ $\boxed{4} + 2$ $\boxed{5} + 2$

Perimeter (each unit is the length of one side of the equilateral triangle).

To reinforce the student's understanding of the pattern rule (functional relationship), suggest that the student put the data into a chart (or table of values) with the headings: Number of Tables and Number of People. This will help the student see the pattern in its symbolic form and can be used as a strategy for other problems. Focus attention on the relationship between the two rows of numbers in the chart to find the pattern rule that describes the functional relationship; i.e., you must add two to each step number (or the number of tables) to get the number of people seated for that step.

Number of Tables (N)	1	2	3	4	…	30
Number of People Seated (P)	3	5	7	9	…	?

c. If the student wants to extend the pattern all the way to 30, suggest that he or she use the pattern rule created; e.g., $N + 2 = P$, where N = the step number and P = the number of people seated for that step. Ask the student to replace the N with 30 to get the number of people that can be seated at 30 tables.

C. Applied Learning

Provide opportunities for students to apply the formulas for perimeter, area and volume in a practical situation and notice whether or not the understanding transfers. For example, have the student estimate and then calculate the volume of soil removed from a hole in the shape of a right rectangular prism that is 25 cm long, 16 cm wide and 9 cm deep. Does the student …

- demonstrate understanding of the problem?
- apply the formula for finding the volume of a rectangular prism; i.e., $V = L \times W \times H$?
- estimate the volume using appropriate estimation strategies?
- use appropriate strategies to calculate the answer to the problem such as a paper-and-pencil, personal strategies or a calculator?
- correctly answer the problem using the correct number and units?
- apply the process to answer other similar problems?
- use formulas to solve problems about perimeter of polygons or area of rectangles?

Step 5: Follow-up on Assessment

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction?

A. Addressing Gaps in Learning

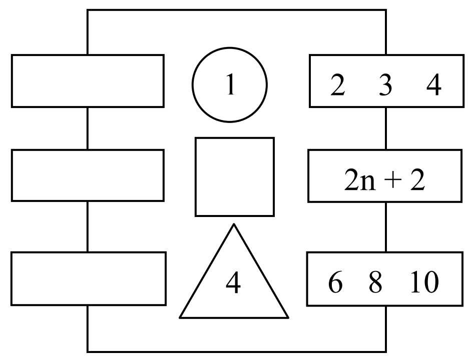
- Build on students' understanding of patterns, area, perimeter and volume from previous grades.
- Use everyday contexts so students understand the purpose of the equations and formulas and are motivated to complete the tasks.
- Encourage students to use manipulatives to represent various patterns and then draw corresponding diagrams along with charts. For example, use square tiles to create arrays to show area and develop the area formula, $A = L \times W$. Another example is to use centicubes to construct rectangular prisms to show how the length and width are used make one layer and the height is used to show the number of layers, thereby laying the foundation for the volume formula, $V = L \times W \times H$ or V = area of the base × the height.
- Have students explain their thinking and provide scaffolding to overcome any misconceptions or misunderstandings.
- Use numbers in the patterns that can be translated readily into diagrams. Then larger numbers can be used as the formula is applied.
- Explain that there are two types of pattern rules or relationships: a rule that describes how a pattern changes from one step to another step (recursive relationship); a rule that explains what you do to the step number to get the value of the pattern for that step (functional relationship). Emphasize that the rule connecting the step number to the number of elements in that step is used to find how many elements are in any given step number such as the one-hundredth step. Provide examples of each type of rule for a given pattern.
- In creating functional relationships for a given pattern, have students explain the recursive relationship among the numbers in the individual rows in the table of values and use this relationship in establishing the functional relationship that relates the two rows or columns of numbers in a table of values.
- Emphasize the importance of drawing diagrams and/or creating a table of values when representing a problem with an equation. Provide scaffolding in labelling the table of values, if necessary.
- Reinforce that variables stand for what changes in a pattern while constants stand for what remains the same in the pattern.
- Reinforce understanding of patterns by integrating patterns in every strand and emphasize the power of patterns in mathematics.
- Encourage students to bring examples from newspapers and magazines that apply patterns, perimeter, area and volume.

B. Reinforcing and Extending Learning

Students who have achieved or exceeded the outcomes will benefit from ongoing opportunities to apply and extend their learning. Consider strategies such as the following to support students in developing a deeper understanding of the concept.

- Provide tips for parents on applying various patterns, including formulas for perimeter, area and volume at home or in the community.
  - Predict and then calculate how much you will save on the 20th day if you save \$5 on the first day, \$8 on the second day, \$11 on the third day and so on.
  - Estimate and then calculate the area of a room that is 15 m long and 8 m wide.
  - Estimate and then calculate the length of baseboards required in a square room that is 4.8 m on each side.
  - Estimate and then calculate the volume of a cereal box that is 24 cm long, 10 cm wide and 33 cm high.

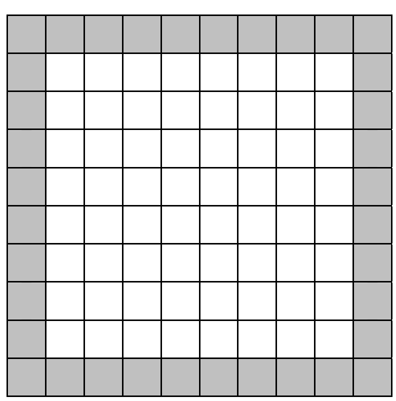
- Reinforce the functional relationship of pattern rules by using the Function Machine. To make a Function Machine, cut three holes (circle, square and triangle) in a vertical alignment from a blank piece of paper. The circle is the input, the triangle is the output and the square is the rule or linear relation. Cut a vertical slit on each side of the shapes. Cut transparent slips that are wide enough to fit through the slits. On two of the strips, write numbers to represent the input and the output. On the third strip, write the pattern rule.



Transparent strip

  Place the template with the three shapes on the overhead. Place two strips so that they are exposed through two shapes and have students find the appropriate response for the third shape; e.g., if the input and output are provided, students must find the function rule; if the input and the function rule are provided, students must find the output for each input. (National Council of Teachers of Mathematics 1991, pp. 67–68.)

- Provide students with a 10-by-10 grid to solve the following border problem.

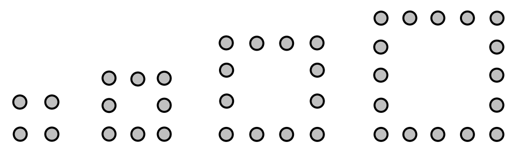


– How many squares are there in the border of the 10-by-10 grid? Explain how you know.
– Describe a different way to get your answer.
– Write a pattern rule (using variables) or a formula that could be used to find the number of squares in the border of a square grid of any size. Explain your thinking.

(Burns 1989, p. 27; National Council of Teachers of Mathematics 2000, p. 185.)

- Provide students with the following post problem.

  The dots represent the posts needed to enclose square fields of various sizes.

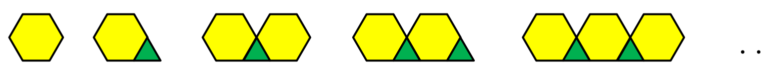


– Write a pattern rule (using variables) or a formula that could be used to find the number of posts needed if you are given the number of posts on one side. Explain your thinking.
– Use your rule to find the number of posts needed if you have 100 posts on one side of the square field. Explain your thinking.

- Provide students with pattern blocks and triangular dot paper. Have them use the pattern blocks to form a pattern with trains of different polygons and ask them to describe and extend the pattern for the perimeter of each train. Ask students to draw the diagrams on triangular dot paper.

  Example:

Train



Number of Polygons	1	2	3	4	5	6
Perimeter	6	7	11	12	16	?

a. What are the next few perimeters? Explain your thinking.
b. Write the pattern of numbers to describe each of the following:
   - odd-numbered trains
   - even-numbered train.

(National Council of Teachers of Mathematics 1991, p. 50.)

- Have students solve real-world problems such as the following.

– Find the dimensions of a rectangular garden with an area of 72 m^2 and a perimeter of 36 m. Explain your thinking.
– Find the dimensions of a rectangular plot of land with an area of 64 m^2 that will require the least amount of fencing. Explain your thinking.
– Find the volume of air in a room shaped like a right rectangular prism that is 35 m long, 25 m wide and 4 m high.

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