Mathematics

Planning Guide

Grade 6
Factors and Multiples

Number
Specific Outcome 3

This Planning Guide can be accessed online at:
http://www.learnalberta.ca/content/mepg6/html/pg6_factorsmultiples/index.html

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Planning Guide: *Grade 6 Factors and Multiples*

Strand: Number
Specific Outcome: 3

This *Planning Guide* addresses the following outcomes from the program of studies:

Strand: Number	
Specific Outcome:	3. Demonstrate an understanding of factors and multiples by: • determining multiples and factors of numbers less than 100 • identifying prime and composite numbers • solving problems using multiples and factors.

Curriculum Focus

This sample targets the following changes to the curriculum:

- The general outcome focuses on number sense, whereas the previous program of studies specified demonstrating a number sense for decimals and common fractions, exploring integers and showing number sense for whole numbers.
- The specific outcome includes determining multiples and factors of numbers less than 100, identifying prime and composite numbers, and solving problems using multiples and factors. The previous program of studies introduced factors, multiples, primes and composites in Grade 5 and continued to work with them in Grade 6, including common multiples, common factors, least common multiple, greatest common factor and prime factorization using numbers 1 to 100.

What Is a Planning Guide?

Planning Guides are a tool for teachers to use in designing instruction and assessment that focuses on developing and deepening students' understanding of mathematical concepts. This tool is based on the process outlined in *Understanding by Design* by Grant Wiggins and Jay McTighe.

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Planning Steps

The following steps will help you through the Planning Guide:

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Step 1: Identify Outcomes to Address

Guiding Questions

- What do I want my students to learn?
- What can my students currently understand and do?
- What do I want my students to understand and be able to do based on the Big Ideas and specific outcomes in the program of studies?

Big Ideas

- Factors, multiples, primes and composites are all natural numbers. Since zero is included in the set of whole numbers, zero cannot be a factor, a multiple, a prime number or a composite number.

The following definitions are available at http://learnalberta.ca/content/memg/index.html.

Natural Numbers
"A natural number is any number in the list {1, 2, 3, 4, … }.
The natural numbers are also called the counting numbers."

Factors
"A factor is a natural number that exactly divides a given natural number."

For example, the complete set of factors of 18 is 1, 2, 3, 6, 9 and 18.
The set of factors of any natural number is always a finite set of numbers.

Factors are used in number facts. For example, 3 × 6 = 18; therefore, 3 and 6 are both factors of 18. Factors are also used in the area of rectangles and the volume of right rectangular prisms. For example, the area of a rectangle that is 3 m wide and 3 m long is 15 square metres; therefore, 3 and 5 are both factors of 15. Similarly, the volume of a right rectangular prism that is 2 m wide, 3 m long and 4 m high is 24 cubic metres; therefore 2, 3 and 4 are all factors of 24.

Multiples
"A multiple is a natural number that is the product of a given natural number and another natural number … Alternately, a multiple is a natural number that can be exactly divided by a given natural number."

For example, the complete set of multiples of 3 is 3, 6, 9, 12, 15, 18, …
The set of multiples of any natural number is always an infinite set of numbers.

Multiples are used in number facts as skip counting. For example, 3 × 6 = 18 or skip count 6, 12 and 18; therefore, 18 is a multiple of both 3 and 6. Multiples are also used in the area of rectangles and the volume of right rectangular prisms. For example, the area of a rectangle that is 3 m wide and 3 m long is 15 square metres; therefore, 15 is a multiple of both 3 and 5. Similarly,

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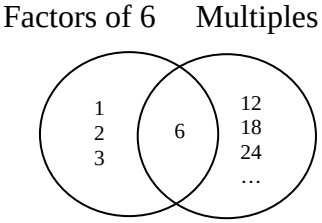
the volume of a right rectangular prism that is 2 m wide, 3 m long and 4 m high is 24 cubic metres; therefore, 24 is a multiple of 2, 3 and 4.

Connecting Factors and Multiples
Any natural number has factors and multiples.
For example, the factors of 6 are 1, 2, 3 and 6. The multiples of 6 are 6, 12, 18, 24 and so on. Therefore, 6 is a factor of itself as well as a multiple of itself as shown in the following Venn diagram.



This leads to the generalization that any natural number is a factor and a multiple of itself.

Classes of Numbers
As students study numbers, they "should identify classes of numbers and examine their properties" (NCTM 2000, p. 151).
Some classes of numbers are described next.

Prime Numbers
"A prime number is a natural number that has exactly two factors: 1 and itself."

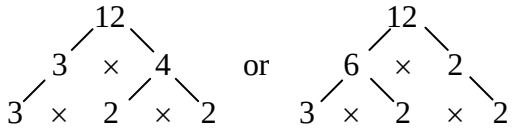
All prime numbers, except 2, are odd numbers. Examples of prime numbers are 2, 3, 5 and 7. In each case, the factors include only the number itself and 1.

The number 1 is not a prime number because it has only one factor.

Composite Numbers
"A composite number is a natural number that has three or more factors."

Composite numbers include even and odd numbers. Examples of composite numbers are 4 and 15. The factors of 4 are 1, 2 and 4. The factors of 15 are 1, 3, 5 and 15.

A composite number can always be expressed as the product of prime numbers. These prime numbers can be found by using a factor tree to find the prime factorization.
Example:



Therefore, 12 can be written as the product of prime numbers: $3 \times 2 \times 2$.

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Sequence of Outcomes from the Program of Studies

See http://www.education.alberta.ca/teachers/program/math/educator/progstudy.aspx for the complete program of studies.

Grade 5	Grade 6	Grade 7
Specific Outcomes	**Specific Outcomes**	**Specific Outcomes**
3. Apply mental mathematics strategies and number properties, such as: • skip counting from a known fact • using doubling or halving • using patterns in the 9s facts • using repeated doubling or halving to determine, with fluency, answers for basic multiplication facts to 81 and related division facts.	3. Demonstrate an understanding of factors and multiples by: • determining multiples and factors of numbers less than 100 • identifying prime and composite numbers • solving problems using multiples and factors.	1. Determine and explain why a number is divisible by 2, 3, 4, 5, 6, 8, 9 or 10, and why a number cannot be divided by 0. 5. Demonstrate an understanding of adding and subtracting positive fractions and mixed numbers, with like and unlike denominators, concretely, pictorially and symbolically (limited to positive sums and differences).

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Step 2: Determine Evidence of Student Learning

Guiding Questions

- What evidence will I look for to know that learning has occurred?
- What should students demonstrate to show their understanding of the mathematical concepts, skills and Big Ideas?

Using Achievement Indicators

As you begin planning lessons and learning activities, keep in mind ongoing ways to monitor and assess student learning. One starting point for this planning is to consider the achievement indicators listed in *The Alberta K–9 Mathematics Program of Studies with Achievement Indicators*. You may also generate your own indicators and use them to guide your observation of students.

The following indicators may be used to determine whether or not students have met this specific outcome. Can students:

- identify multiples for a given number and explain the strategy used to identify them?
- determine all the whole number factors of a given number, using arrays?
- identify the factors for a given number and explain the strategy used; e.g., concrete or visual representations, repeated division by prime numbers, factor trees?
- provide an example of a prime number and explain why it is a prime number?
- provide an example of a composite number and explain why it is a composite number?
- sort a given set of numbers as prime and composite?
- graph the factors of prime and composite numbers? Make inferences about factors, multiples, primes and composites from the graph?
- solve a given problem involving factors or multiples?
- explain why 0 and 1 are neither prime nor composite?
- apply the concept of factors and multiples to prime and composite numbers?

Sample behaviours to look for related to these indicators are suggested for some of the activities listed in **Step 3, Section C: Choosing Learning Activities** (p. 11).

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Step 3: Plan for Instruction

Guiding Questions

- What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should I use?
- How will I meet the diverse learning needs of my students?

A. Assessing Prior Knowledge and Skills

Before introducing new material, consider ways to assess and build on students' knowledge and skills related to counting. For example:

1. State whether each of the following statements is true or false. Explain your thinking.
   a) If $4 \times 8 = 32$, then $6 \times 8 = 32 + 8 + 8$.
   b) If $6 \times 9 = 54$, then $7 \times 9 = 54 + 9 + 9$.
   c) If $8 \times 8 = 64$, then $6 \times 8 = 64 - 8 - 8$.
2. Continue each of the following patterns by placing a number in each blank provided. Explain your thinking.
   a) 7, 14, 21, 28, 35, _____, _____, _____.
   b) 54, 48, 42, 36, _____, _____, _____.
3. Explain why division by zero is not possible or is undefined.
4. Explain why multiplication by zero results in an answer of zero.

If a student appears to have difficulty with these tasks, consider further individual assessment, such as a structured interview, to determine the student's level of skill and understanding. See **Sample Structured Interview: Assessing Prior Knowledge and Skills** (p. 9).

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Sample Structured Interview: Assessing Prior Knowledge and Skills

Directions	Date:	
	Not Quite There	**Ready to Apply**
Place each of the following statements before the student. a. If $4 \times 8 = 32$, then $6 \times 8 = 32 + 8 + 8$. b. If $6 \times 9 = 54$, then $7 \times 9 = 54 + 9 + 9$. c. If $8 \times 8 = 64$, then $6 \times 8 = 64 - 8 - 8$ Say, **"State whether each of the statements is true or false. Explain your thinking."**	Does not correctly identify one or more of the statements as true or false. Correctly identifies statements (a) and (c) as true and also correctly identifies statement (b) as false but is unable to explain why one or more of them is true or false.	Correctly answers that statements (a) and (c) are true and statement (b) is false and provides explanations, such as the following: • 6×8 is six groups of eight which is two groups of eight more than 4×8 or 32. • 7×9 is seven groups of nine which is only one group of nine more than 6×9 or 54, not two more groups as shown. • 6×8 is six groups of eight which is two groups of eight less than 8×8 or 64.
Place each of the following patterns before the student. • 7, 14, 21, 28, 35, _____, _____, _____. • 54, 48, 42, 36, _____, _____, _____. Say, **"Continue each of the patterns by placing a number in each blank provided. Explain your thinking."**	Does not correctly continue one or both of the patterns. Correctly continues one or both of the patterns but is unable to explain the pattern.	Correctly continues both patterns: • 42, 49, 56 • 30, 24, 18 Describes the patterns correctly, such as the following: • skip counting up by 7 starting at 7 • skip counting down by 6 starting at 54.

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"Explain why division by zero is not possible or is undefined."	Does not explain why division by zero is not possible or is undefined.	Explains clearly why division by zero is not possible or is undefined by providing an explanation such as the following: Any division statement can be written as a multiplication statement. Therefore, $\frac{5}{0} = ?$ can be written as $0 \times ? = 5$. Since there is no number that can be multiplied by 0 to get 5, then this is an impossible situation. Therefore, division by zero is undefined.
"Explain why multiplication by zero results in an answer of zero."	Does not explain why multiplication by zero results in an answer of zero.	Explains clearly why multiplication by zero results in an answer of zero by providing an explanation such as the following: One meaning of multiplication is repeated addition or taking equal groups of a given quantity. For example, 2×3 means two groups of 3 resulting in 6. Therefore, 0×3 means no groups of 3, resulting in an answer of zero.

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B. Choosing Instructional Strategies

Consider the following instructional strategies for teaching factors, multiples, primes and composites.

- Introduce and reinforce the concepts by using problem-solving contexts.
- Provide students with a variety of problems that apply the concepts of multiples and factors. Encourage them to solve the problems in different ways and explain the process. Also, provide time for students to share their solutions with others. Stimulate class discussion to evaluate critically the various procedures. Emphasize understanding, flexibility and efficiency when students select problem-solving strategies.
- Connect the new concepts to prior knowledge such as odd and even numbers, skip counting, multiplication and division number facts, area of rectangles and volume of rectangular prisms.
- Integrate patterns as students explore the concepts.
- "Create a classroom environment that encourages student exploration, questioning, verification and sense making" (NCTM 1991, p. 5).
- Provide opportunities for students to explore the relationships among factors, multiples, primes and composites.
- Encourage students to communicate their thinking by connecting manipulatives, diagrams and symbols to illustrate the concepts.
- Provide a variety of ways to illustrate the concepts.
- Encourage students to make and critique generalizations related to the concepts.
- Provide extensions that build on the concepts. Examples may include least common multiple and greatest common factor.

C. Choosing Learning Activities

The following learning activities are examples of activities that could be used to develop student understanding of the concepts identified in Step 1.

Sample Activities:

Teaching Factors and Multiples of Numbers

1. **Arrays and Area of Rectangles** (p. 13)
2. **Volume of Rectangular Prisms** (p. 15)
3. **Anticipation/Reaction Guide** (p. 17)
4. **Common Multiples and Common Factors** (p. 19)

Teaching Prime and Composite Numbers by Connecting to Multiples and Factors of Numbers

1. **Prime and Composite Numbers—Arrays and Graphing** (p. 22)
2. **Sieve of Eratosthenes** (p. 27)
3. **Factor Trees** (p. 29)
4. **Open and Closed Sorts** (p. 30)

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Teaching Factors and Multiples of Numbers

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Sample Activity 1: Arrays and Area of Rectangles

Review the concept of area and build on students' prior knowledge of the concept.

Provide students with square tiles and centimetre grid paper.

Problem
A rectangular garden has an area of 18 square metres.
a. Using natural numbers, what could the length and width of this garden be?
b. Find all the possible lengths and widths of this garden.

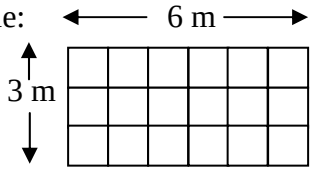
Have students represent the problem with the square tiles. Each tile is one square metre.

Have students draw diagrams on the centimetre grid paper to show the different rectangles created.

Encourage students to share their answers and justify them.

Look For …
Do students:
- ☐ construct arrays and describe them appropriately; e.g., an array of 12 items could be three rows with four items in each row?
- ☐ apply their knowledge of area of rectangles to new problems?
- ☐ connect concrete, pictorial and symbolic representations of factors and multiples?
- ☐ transfer data from diagrams to a chart by using appropriate headings for the chart?
- ☐ organize data in a chart and interpret this data?
- ☐ make generalizations about factors and multiples?

a. Example:



The rectangle has an area of 18 square metres. Three rows with six square metres in each row make up the total area or the array. The width is 3 m and the length is 6 m.

b. To verify that they have included all the possible lengths and widths, suggest that students record their results in an organized chart, such as the following:

Width (m)	1	2	3	
Length (m)	18	9	6	
Area (m^2)	18	18	18	

Discuss that a 3 by 6 array represents the same rectangle as a 6 by 3 array by reorienting the array.

Factors
Introduce the term *factor* by saying that all of the factors of 18 are included in the chart. Have students list the factors of 18: 1, 2, 3, 6, 9 and 18.

Through discussion, have students generalize that the set of factors of any number is a finite set; i.e., there are a limited number of factors of any number.

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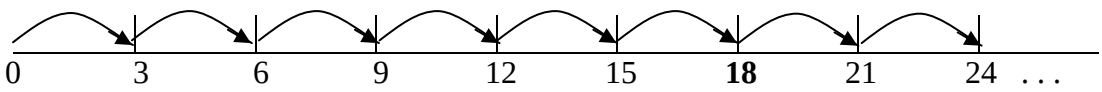
Have students suggest a definition for *factor*; e.g., "a factor is a natural number that exactly divides a given natural number."

Provide students with other context problems using areas of rectangles. Have them list all the factors of the given areas by relating these factors to arrays or the length and width of rectangles.

Multiples

After students understand the concept of *factor*, introduce the term, *multiple*. Refer to the chart above and say that 18 is a multiple of 1, 2, 3, 6, 9 and 18.

- Connect multiples to skip counting. Have students use their tiles made into arrays. For example, the 3 by 6 array pictured above shows skip counting by 3 as you count the number of squares in each column in the array. Similarly, this array shows skip counting by 6 as you count the number of squares in each row in the array.
- Use the number line to show skip counting. Example: skip count by 3 as shown on the number line below.



- Connect multiples to multiplication; e.g., 18 is a multiple of 3 because you multiply 3 by 6 to get 18 or $6 \times 3 = 18$.

Encourage the students to list the multiples of 3 in a chart so that they can see how multiplication is connected to the multiples.

Example: Multiples of 3

Number of the Multiple	1st multiple	2nd multiple	3rd multiple	4th multiple	5th multiple	6th multiple	…
Multiples of 3 (Skip count by 3)	3	6	9	12	15	**18**	…

Integrate patterns by encouraging students to describe the relationship between the two rows in the chart; i.e., you multiply the number of the multiple by 3 to get the value of that multiple; therefore, the 100th multiple of 3 is $100 \times 3 = 300$.

Connect multiples to division; e.g., 18 is a multiple of 3 because 3 divides evenly into 18.

Have the students list the multiples of 3: 3, 6, 9, 12, 15, **18**, 21, 24 … Through discussion, have students generalize that the set of multiples of any number is an infinite set; i.e., there is no end to the number of multiples of any number. Then have students create a definition for *multiple*; e.g., "a multiple is a natural number that is the product of a given natural number and another natural number … Alternately, a multiple is a natural number that can be exactly divided by a given natural number."

Provide students with other problem contexts using multiples, area of rectangles and arrays.

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Sample Activity 2: Volume of Rectangular Prisms

Review the concept of volume and build on students' prior knowledge of the concept.

Provide students with centicubes.

Problem
A rectangular prism has a volume of 12 cubic centimetres.
a. Using natural numbers, what could the length, width and height of this rectangular prism be?
b. Find all the possible lengths, widths and heights of this rectangular prism.

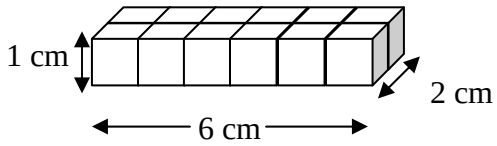
Have students represent the problem with the centicubes. Each centicube has a volume of one cubic centimetre.

Have students draw diagrams to show the different rectangular prisms created.

Encourage students to share their answers and justify them.

Look For …
Do students:
- ☐ apply their knowledge of volume of rectangular prisms to new problems?
- ☐ connect concrete, pictorial and symbolic representations of factors and multiples?
- ☐ transfer data from diagrams to a chart by using appropriate headings for the chart?
- ☐ organize data in a chart and interpret this data?
- ☐ make generalizations about multiples by interpreting patterns?

a. Example:



The rectangular prism is 6 cm long, 2 cm wide and 1 cm high. The total volume is 12 cubic centimetres.

b. To verify that they have included all the possible widths, lengths and heights, suggest that students record their results in an organized chart, such as the following:

Length (cm)	Width (cm)	Height (cm)	Volume (cm^3)
6	2	1	12
4	3	1	12
3	2	2	12
12	12	12	12

Discuss that a 3 by 2 by 2 rectangular prism is the same as a 2 by 2 by 3 rectangular prism, by reorienting the prism.

Factors
Introduce the term *factor* by saying that all the factors of 12 are included in the chart. Have students list the factors of 12: 1, 2, 3, 4, 6 and 12.

Review the definition for *factor*; e.g.,"a factor is a natural number that exactly divides a given natural number."

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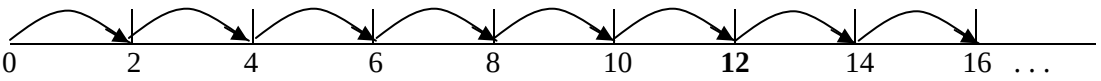
Through discussion, have students review that the set of *factors* of any number is a finite set; i.e., there is a limited number of factors of any number.

Provide students with other context problems using the volume of a rectangular prism. Have them list all the factors of the given volumes by relating these factors to the length, width and height of the rectangular prisms.

Multiples

After students understand the concept of *factor*, introduce the term *multiple*. Refer to the chart above and say that 12 is a multiple of 1, 2, 3, 4, 6 and 12.

- Connect multiples to skip counting. Have students use their centicubes made into rectangular prisms to show skip counting and the various multiples of a given number; e.g., the 6 cm by 2 cm by 1 cm prism pictured above shows skip counting by 2 as you count the number of centicubes in each double column in the prism. Similarly, this prism shows skip counting by 6 as you count the number of centicubes in each row in the prism.
- Use the number line to show skip counting. For example, skip count by 2 as shown on the number line below.



- Connect multiples to multiplication; e.g., 12 is a multiple of 2 because you multiply 2 by 6 to get 12 or $6 \times 2 = 12$. Encourage students to list the multiples of 2 in a chart so that they can see how multiplication is connected to the multiples.

Example: Multiples of 2

Number of the Multiple	1st multiple	2nd multiple	3rd multiple	4th multiple	5th multiple	6th multiple
Multiples of 2 (Skip count by 2)	2	4	6	8	10	**12**

Integrate patterns by encouraging students to describe the relationship between the two rows in the chart; i.e., you multiply the number of the multiple by 2 to get the value of that multiple; therefore, the 100th multiple of 2 is $100 \times 2 = 200$.

Connect multiples to division; e.g., 12 is a multiple of 2 because 2 divides evenly into 12.

Have the students list the multiples of 2: 2, 4, 6, 8, 10, **12**, 14, 16, 18 …

Review the set of multiples of any number as an infinite set; i.e., there is no end to the number of multiples of any number. Also review the definition for *multiple*; e.g., "a multiple is a natural number that is the product of a given natural number and another natural number … Alternately, a multiple is a natural number that can be exactly divided by a given natural number."

Provide students with other problem contexts using multiples and volumes of rectangular prisms.

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Sample Activity 3: Anticipation/Reaction Guide

Use Blackline Master – Anticipation/Reaction Guide (p. 18) for this activity.

- On the template, write the major ideas about factors and multiples. Include true statements and also any misconceptions students may have about the concepts.
- Have students fill in the guide under the heading "Before," by stating whether each statement is true or false based on background knowledge.
- Discuss student predictions and anticipations listed on the guide as part of the ongoing learning activities. This discussion provides important information for assessment *for* learning.
- Choose learning strategies to address any misconceptions that are revealed by using the Anticipation/Reaction Guide.
- As students are involved in further learning activities, have them evaluate the statements in terms of what they have learned and place a check mark in the "After" column for any statement with which they now agree.
- Have students contrast their predictions with what they have learned and discuss the similarities and differences.

Look For …

Do students:

- ☐ ask questions to clarify any of the statements provided in the anticipation guide?
- ☐ explain their reasoning for agreeing or disagreeing with a statement from the anticipation guide?
- ☐ exhibit any misconceptions as they respond to the statements?
- ☐ develop understanding of concepts for which there were misconceptions initially?

Sample statements to use in the Anticipation/Reaction Guide:

1. Every number has 1 as a factor.
2. Every number is a multiple of itself.
3. Zero is a factor of every number.
4. Zero is a multiple of every number.
5. Multiples of 2 are called even numbers.
6. Multiples of 3 are called odd numbers.
7. Every multiple of a given number is always greater than any factor of that number.
8. There are always more multiples than factors of any number.

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Blackline Master – Anticipation/Reaction Guide

(Barton and Heidema 2002, pp. 95–96)

Directions: In the column labelled "Before," write true (T) or false (F) beside each statement. After learning more about the topic that may include reading a text selection, place a check mark in the "After" column next to any statement with which you now agree. Compare your original opinions with those you developed as you learned more about the topic.

Topic: _____

Before (T or F)	Statements	After
	1.	
	2.	
	3.	

Adapted with permission from Mary Lee Barton and Clare Heidema, *Teaching Reading in Mathematics: A Supplement to Teaching Reading in the Content Areas Teacher's Manual* (2nd ed.) (Aurora, Co: McREL (Mid-continent Research for Education and Learning, 2002), p. 113.

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Sample Activity 4: Common Multiples and Common Factors

Provide students with a variety of problems that apply the concepts of common multiples and common factors. Encourage them to solve the problems in different ways and explain the process. Also, provide time for students to share their solutions with others. Stimulate class discussion to evaluate critically the various procedures. Emphasize understanding, flexibility and efficiency when students select problem-solving strategies.

Look For …

Do students:

- ☐ understand the problem?
- ☐ use a variety of strategies to solve the problem?
- ☐ distinguish between factors and multiples of numbers?
- ☐ transfer data from a problem to a chart by using appropriate headings for the chart?
- ☐ organize data in a chart and interpret this data?
- ☐ explain and justify the process in solving the problem?
- ☐ apply the concepts to similar problems?
- ☐ create similar problems?

Problem—Common Multiples

During the past two years, Brooklyn wore out a pair of runners every 12 months, a pair of socks every six months and a T-shirt every nine months. How often, at those rates, will she have to replace all three items at once?

Sample Solutions:

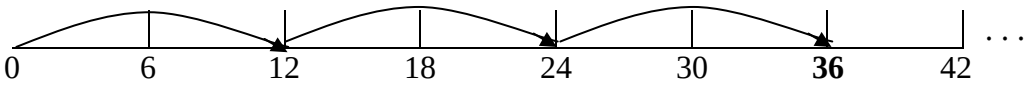
Type A: Skip Counting Using a Chart

Runners	12	24	**36**	48	60	72	…
Socks	6	12	18	24	30	**36**	…
T-shirt	9	18	27	**36**	45	54	…

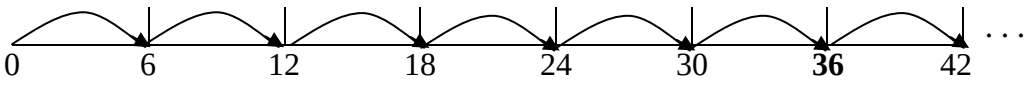
36 is a common multiple of 6, 9 and 12.
Brooklyn will have to replace all three items at once every 36 months or three years.

Type B: Skip Counting on the Number Line

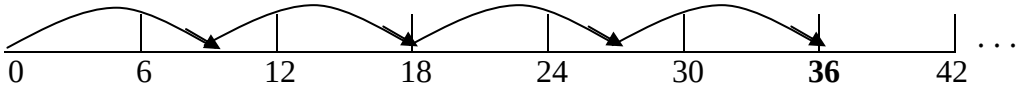
Runners—skip count by 12



Socks—skip count by 6



T-shirt – skip count by 9



36 is a common multiple of 6, 9 and 12.

Brooklyn will have to replace all three items at once every 36 months or three years.

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Problem—Common Factors
Maren bought a number of marbles for 45 cents. Nicholas bought a number of marbles for 75 cents. If each of the marbles costs the same, how many marbles could they each buy?

Sample Solution:

List the possible factors for each person in a chart. Highlight the factors that show the same cost per marble in each chart.

Maren

# of marbles	1	3	5	9	15	45
Cost per marble	45 cents	**15 cents**	9 cents	**5 cents**	**3 cents**	**1 cent**

Nicholas

# of marbles	1	3	5	15	25	75
Cost per marble	75 cents	25 cents	**15 cents**	**5 cents**	**3 cents**	**1 cent**

The common factors of 45 and 75 are 15, 5, 3 and 1.
Therefore, the same cost for each marble could be 15 cents, 5 cents, 3 cents or 1 cent.

If each marble costs 15 cents, Maren can buy 3 marbles and Nicholas can buy 5 marbles.
If each marble costs 5 cents, Maren can buy 9 marbles and Nicholas can buy 15 marbles.
If each marble costs 3 cents, Maren can buy 15 marbles and Nicholas can buy 25 marbles.
If each marble costs 1 cent, Maren can buy 45 marbles and Nicholas can buy 75 marbles.

Encourage students to create other problems using multiples and factors. Have students write their problems on sticky notes and put the notes into groups on the board. Encourage students to explain how the problems grouped together are alike.

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Teaching Prime and Composite Numbers by Connecting to Multiples and Factors of Numbers

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Sample Activity 1: Prime and Composite Numbers—Arrays and Graphing

Provide students with square tiles, centimetre grid paper, a chart to record the arrays and a grid for graphing the factors for each array. See below for the chart and grid for graphing.

Encourage students to relate factors to arrays in the following problem.

Look For …

Do students:

- ☐ construct arrays and describe them appropriately?
- ☐ connect arrays to area?
- ☐ connect concrete, pictorial and symbolic representations of factors, multiples, primes and composites?
- ☐ transfer data from diagrams to a chart by using appropriate headings for the chart?
- ☐ organize data in a chart and interpret this data?
- ☐ transfer data efficiently and correctly from a chart to a graph?
- ☐ make generalizations about prime and composite numbers by interpreting patterns in charts and graphs?

Problem 1

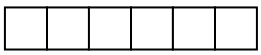
Create all the possible rectangular arrays for each of the numbers 1 to 20. Graph the factors shown by these arrays.

Sample Solution:

Use the square tiles to make the rectangular arrays representing each of the numbers 1 to 20.

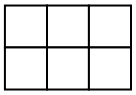
For example, the following rectangular arrays can be made using six tiles.

The first array has one row and six columns. It is a 1 by 6 array.
The second array has six rows and one column. It is a 6 by 1 array.
The third array has two rows and three columns. It is a 2 by 3 array.
The fourth array has three rows and two columns. It is a 3 by 2 array.



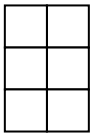
1 by 6

6 by 1



2 by 3

3 by 2



These arrays are recorded in the following chart.

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Chart to Record the Arrays and Factors for Numbers 1 to 20

Number of Tiles	Size of Each Array	Number of Arrays Formed	Factors
1			
2			
3			
4			
5			
6	1 by 6, 6 by 1, 2 by 3, 3 by 2	4	1, 2, 3, 6
7			
8			
9			
10			
11			
12			
13			
14			
15			
16			
17			
18			
19			
20			

Continue creating arrays with the square tiles and filling in the chart.

Then use the information from the chart to complete the following graph.

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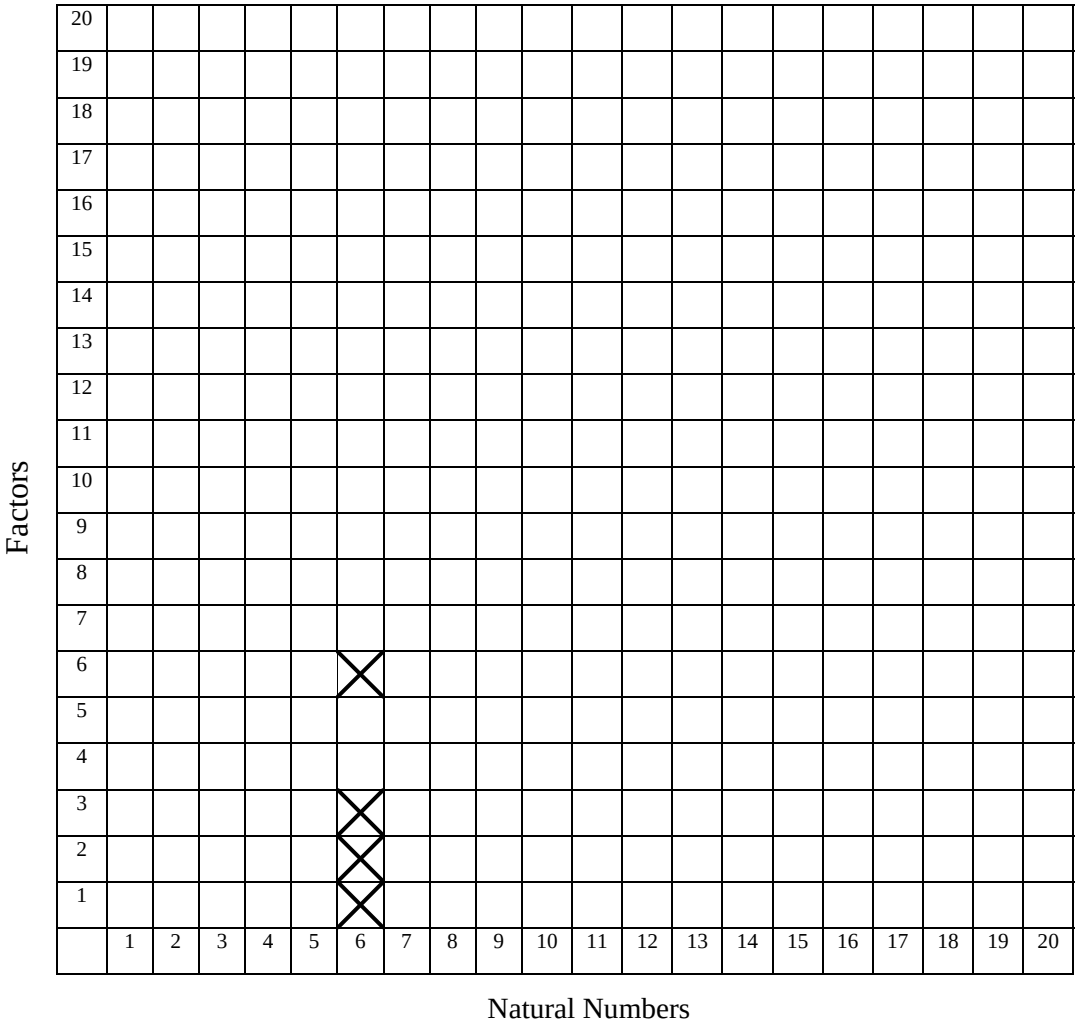
Factors of Numbers



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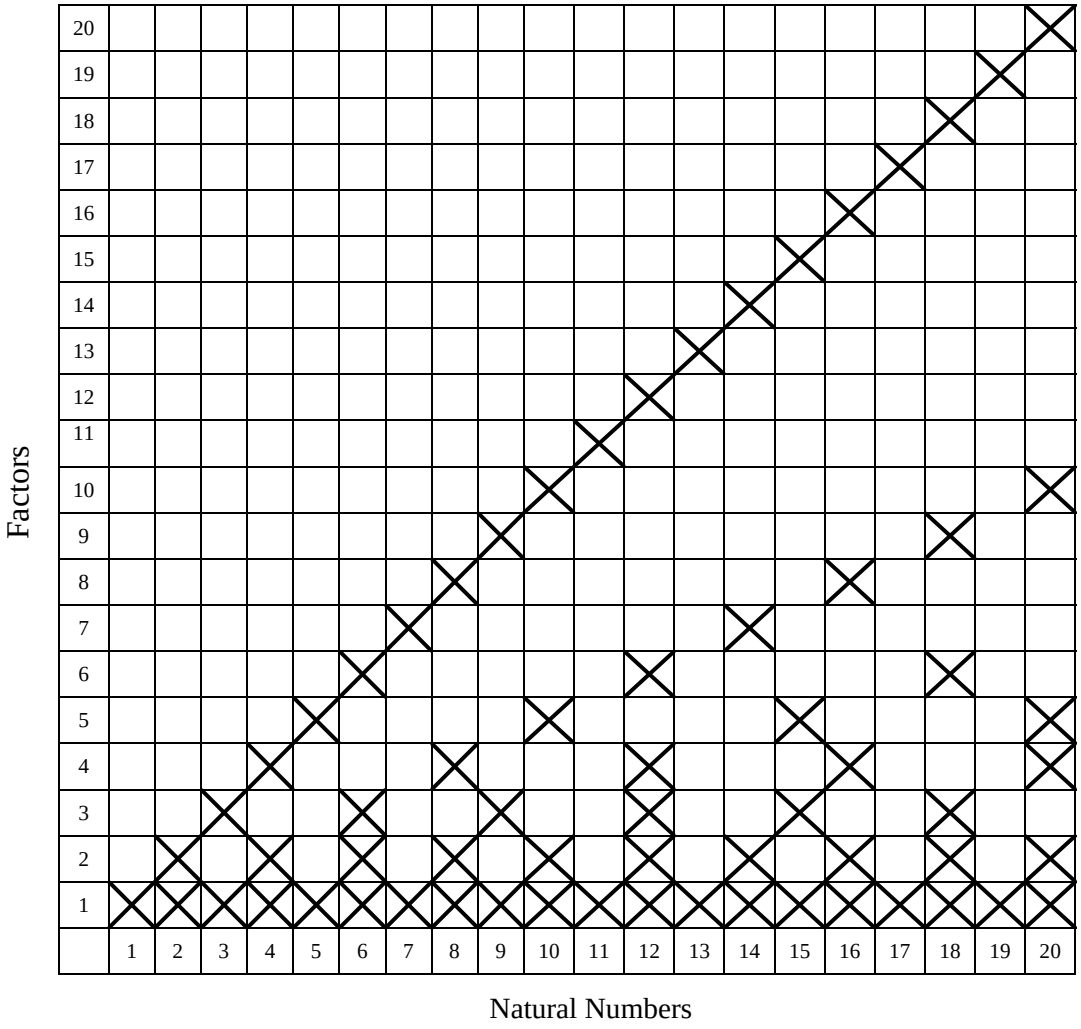
Completed Graph: Factors of Numbers



Problem 2
What observations can you make by looking at patterns in the graph you completed for Problem 1?

Sample Solutions:

- 1 is a factor of every number.
- Every even number has a factor of 2.
- Every other number has a factor of 3.
- 4 is a factor of every fourth number.
- Every number is a factor of itself.
- The number 6 has the first three counting numbers as factors.

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Problem 3
Circle the numbers with only two arrays (or two factors) in the chart you completed in Problem 1. These numbers are called prime numbers because they have exactly two factors. Find a number greater than 20 that is prime. Explain your thinking.

Problem 4 (NCTM 1990, pp. 33–35, 42–43)
Colour the numbers with more than two arrays (or more than two factors) in the chart you completed in Problem 1. These numbers are called composite numbers because they have more than two factors. Find a number greater than 20 that is composite. Explain your thinking.

Note: Guide the discussion to include the observation that the number 1 is neither prime nor composite because it has only one factor.

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Sample Activity 2: Sieve of Eratosthenes (NCTM 1990, p. 36)

Provide each student with a copy of the hundreds chart (*use BLM 1 on page 28*).

Explain what a sieve is. Tell students that Eratosthenes, a Greek mathematician who was born around 230 BC, used a sieve to find all the prime numbers from 1 to 100. The prime numbers stay in the sieve and are shown by circling these numbers. The rest of the numbers fall through the sieve and are shown by crossing them out.

Review the meaning of prime and composite from the previous activity. Have students explain why the number 1 is neither prime nor composite.

Provide the following instructions to students and guide the discussion with a transparency of the hundreds chart on the overhead projector.

Look For …

Do students:

- ☐ follow directions carefully?
- ☐ apply their knowledge of multiples of numbers?
- ☐ recognize common multiples as they cross out the multiples in the hundreds chart?
- ☐ successfully identify all the prime numbers less than 100?
- ☐ explain how prime numbers are connected to factors of numbers?

- Cross out number 1 because it is neither prime nor composite.
- Circle 2 because it is the first prime number. Cross out all the multiples of 2 except the number 2 because they are composite numbers.
- Circle 3 because it is the next prime number. Cross out all the multiples of 3 except the number 3 because they are composite numbers.
- Circle 5 and cross out all the multiples of 5 except the number 5.
- Circle 7 and cross out all the multiples of 7 except the number 7.
- Circle the remaining numbers that are not crossed out.
- Count the prime numbers.

Have students share the results of completing their sieves. Ensure that all students have all the prime numbers circled on their sieves and all the composite numbers crossed out. Remind them that 1 must be crossed out because it is neither prime nor composite.

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Hundreds Chart – BLM 1

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

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Sample Activity 3: Factor Trees

Provide students with the following problem and have them share their solutions, using a variety of natural numbers greater than 1 but less than 100. Review the meaning of natural numbers and prime numbers. Through discussion, have students generalize that there is one and only one prime factorization for any natural number greater than 1 even though there are different ways to construct the factor trees.

Look For …

Do students:

- ☐ understand the terms used in the problem, such as composite number, product and prime numbers?
- ☐ use their background knowledge to find the prime factorization of a composite number and explain the process clearly with appropriate mathematics language?
- ☐ critique processes used by others to the solve the problem?
- ☐ show flexibility in solving the problem in a variety of ways?

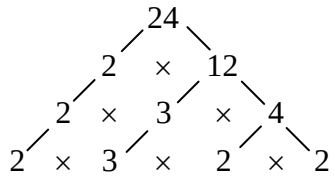
Problem

Write any composite number as the product of prime numbers. This is called the prime factorization of the number.

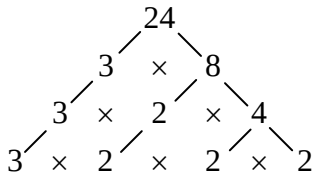
Sample Solution 1 (Lilly 1999, p. 56):

Use a factor tree to display the product of prime numbers for any composite number such as 24.

Factor Tree 1



Factor Tree 2



⇐ Write the number you are factoring at the top of the tree.

⇐ Choose any pair of factors as branches. If either of these is not prime, factor again.

⇐ Keep factoring until all the factors are prime numbers.

Sample Solution 2:

Use repeated division by prime numbers to find the prime factorization of 24.

Repeated Division 1

$\frac{24}{2} = 12, \quad \frac{12}{2} = 6, \quad \frac{6}{2} = 3$

$24 = 2 \times 2 \times 2 \times 3$

Repeated Division 2

$\frac{24}{3} = 8, \quad \frac{8}{2} = 4, \quad \frac{4}{2} = 2$

$24 = 3 \times 2 \times 2 \times 2$

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Sample Activity 4: Open and Closed Sorts

Provide students with some numbers from the set of whole numbers, 0 to 100.

Open Sort
Have students sort the numbers into groups and explain the sorting rule. Encourage students to show the relationships among the numbers by using Venn diagrams.

Closed Sort
Provide the sorting categories for students, such as primes and composites. Have students sort the numbers into the categories provided and explain why each number belongs to a certain category.

Look For …

Do students:

- ☐ correctly classify the numbers and explain the rules for sorting?
- ☐ exhibit flexibility in classifying the numbers in more than one way?
- ☐ correctly explain why a given number belongs in a specific category?

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Step 4: Assess Student Learning

Guiding Questions

- Look back at what you determined as acceptable evidence in Step 2.
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

In addition to ongoing assessment throughout the lessons, consider the following sample activities to evaluate students' learning at key milestones. Suggestions are given for assessing all students as a class or in groups, individual students in need of further evaluation, and individual or groups of students in a variety of contexts.

A. Whole Class/Group Assessment

The following student assessment task, Dog Pens: Factors, Multiples, Primes and Composites (p. 33), could be used with a whole group or class. It includes a marking rubric to be used with the assessment.

Evidence the Student Has Achieved the Outcomes

Each student will:
- determine all the factors of a given number, using arrays
- identify all the factors for a given number and explain the strategy used
- distinguish between multiples and factors
- distinguish between prime and composite numbers

solve a given problem involving factors or multiples.

Task Specific Criteria

Each student will:
- determine all the factors of a given number, using arrays
- identify all the factors for a given number and explain the strategy used
- distinguish between multiples and factors
- distinguish between prime and composite numbers
- solve problems using multiples and factors
- apply factors and multiples of numbers, as well as prime and composite numbers, to rectangles with the same area but different dimensions.

Teacher Notes

Summary of Task

In this assessment, students will solve a problem to demonstrate their understanding of factors and multiples by using square tiles, diagrams and symbols to find all the possible dimensions of rectangles with a given area. They will then explain which rectangle would be best for a dog pen. Finally, they will explain how factors and multiples of numbers, as well as prime and composite numbers, are used in the problem and the solution.

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Materials required: 24 square tiles for each student; centimetre grid paper, 18 cm by 24 cm; 30 centimetre rulers.

Students should be able to find all the factors of a given number by using arrays (rows and columns) with square tiles. Some students will show the arrays (or rectangles) only by drawing the diagrams and may not need to use the square tiles. Students should justify that they have included all the possible rectangles in their solution.

Note: An array with four rows of six square metres is considered the same as an array with six rows of four square metres because the corresponding rectangles have the same dimensions.

In relating factors and multiples to the problem, students should recognize that the dimensions of all the different rectangles are factors of the given area of the rectangles. In turn, the area is a multiple of each factor. They should also explain that in some cases, the length is a multiple of the width of the rectangle (e.g., 12 m is a multiple of 2 m), or, in other words, the width is a factor of the length (e.g., 2 m is a factor of 12 m).

Students should explain that the area is a composite number because it has more than two factors. Some of the dimensions are composite numbers (4 m, 6 m, 8 m, 12 m, 24 m) while others are prime numbers (2 m, 3 m) because prime numbers have exactly two factors. It is important that students recognize that the number 1 is neither prime nor composite but it is a factor of 24.

Early finishers can:

- solve a similar problem by changing the area of the rectangle from 24 square metres to 36 square metres
- explain why the whole number dimensions of all the rectangles for a given area are not always prime and composite numbers
- find all the prime number dimensions of a rectangle in which the sum of the width and length is 24 m.

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Dog Pens: Factors, Multiples, Primes and Composites—Student Assessment Task

The area of a rectangular dog pen is 24 square metres.

1. Draw and label diagrams to show all the possible rectangular dog pens that could be made. Explain your thinking. Which of the pens would you choose for a dog? Why?

2. How are factors and multiples used in this problem? Explain the problem and solution.

3. How are prime and composite numbers used in this problem? Explain the problem and solution.

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SCORING GUIDE:
Dog Pens: Factors, Multiples, Primes and Composites

Student: _____

Level / Criteria	4 Excellent	3 Proficient	2 Adequate	1 Limited*	Insufficient / Blank*
Finds all the factors of a given composite number and represent them visually. Question 1	The student draws very accurate representations of all the possible rectangular arrays, justifies the solution using precise mathematical language and explains clearly which pen is best for a dog. ☐	The student draws accurate representations of all the possible rectangular arrays and provides a clear explanation of the solution, as well as which pen is best for a dog. ☐	The student draws representations of most of the possible rectangular arrays and provides a limited explanation of the solution, as well as which pen is best for a dog. ☐	The student draws an inaccurate or confusing representation of only one rectangular array and provides no explanation of the solution or which pen is best for a dog. ☐	No score is awarded because there is insufficient evidence of student performance, based on the requirements of the assessment task. ☐
Explains how factors and multiples are used in a real-world problem. Question 2	The student provides clear conceptual understanding of how factors and multiples are used in the problem and the relationship between them. The student identifies all the factors and multiples in the problem and the solution. ☐	The student provides some conceptual understanding of how factors and multiples are used in the problem and a limited description of the relationship between them. The student identifies most of the factors and multiples in the problem and the solution. ☐	The student provides limited understanding of how factors and multiples are used in the problem with little description of the relationship between them. The student identifies some of the factors and multiples in the problem and the solution. ☐	The student provides very little or no evidence of understanding how factors and multiples are used in the problem. The student does not correctly identify the factors and multiples in the problem and the solution. ☐	No score is awarded because there is insufficient evidence of student performance, based on the requirements of the assessment task. ☐
Explains how prime and composite numbers are used in a real-world problem. Question 3	The student provides clear conceptual understanding of how prime and composite numbers are used in the problem and connects them to factors. The student identifies all the prime and composite numbers in the problem and the solution. ☐	The student provides some conceptual understanding of how prime and composite numbers are used in the problem and connects them to factors. The student identifies most of the prime and composite numbers in the problem and the solution. ☐	The student provides very limited understanding of how prime and composite numbers are used in the problem and has some difficulty connecting them to factors. The student identifies some of the prime and composite numbers in the problem and the solution. ☐	The student provides little or no understanding of how prime and composite numbers are used in the problem and does not connect them to factors. The student does not correctly identify the prime and composite numbers in the problem and the solution. ☐	No score is awarded because there is insufficient evidence of student performance, based on the requirements of the assessment task. ☐

***** When work is judged to be limited or insufficient, the teacher makes decisions about appropriate interventions to help the student improve.

Student Learning Goals

- Area of need: _____
  - Action
- Strength to reinforce: _____
  - Action

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B. One-on-one Assessment

For students having difficulty with factors and multiples of numbers, as well as prime and composite numbers, begin with smaller numbers in a problem-solving context that is of interest to students. Provide manipulatives to use and guide students by asking questions that build on their previous knowledge and stimulate thinking and discussion. Guiding questions may be needed to help students understand the relationships between factors and multiples of numbers and how they connect to prime and composite numbers. Reinforce the meaning of the terms as necessary.

Guide students through the series of activities described below, providing clarification where needed. For example, you may have to create the first rectangle for the student. Then ask how many rows there are and how many tiles are in each row. This focuses attention on the factors of 12 and also on labelling the length and width of the rectangle that is drawn on grid paper.

When the student sorts the numbers into two groups in *part f* (see below), it may be necessary to use the tiles to represent each number and make the necessary arrays. As the student develops understanding, encourage efficiency by asking the student if he or she can list the factors without using the tiles; i.e., by dividing a number into the given number to see if it divides evenly without a remainder. In doing so, the student is reinforcing understanding of number facts.

Sample Problem

Make a rectangular dog pen using 12 square tiles. Each square tile is one square metre.

a. Use the tiles to make all the possible dog pens.
b. Draw and label a diagram of each rectangle on the centimetre grid paper. Pick the dog pen that you think a dog would like the best. Why do you think so?
c. Write a list of the numbers used for the length and width of the pens. How many are there? These numbers are called factors of 12 because they divide evenly into 12.
d. Find all the factors of 18 by using 18 square tiles to make all the possible rectangles. How many factors are there?
e. Find all the factors of 11 by using 11 square tiles to make all the possible rectangles. How many factors are there?
f. Sort the numbers 2, 3, 4, 5, 6, 7, 8 into two groups: those that have exactly two factors and those that have more than two factors.
g. Numbers with exactly two factors are called prime numbers. Numbers with more than two factors are called composite numbers. Give another example of each. Explain.
h. What about the number 1? How many factors does it have? Is it a prime number? Why or why not? Is it a composite number? Why or why not?

To develop a deeper understanding of the concepts, see the activities using square tiles and arrays listed in **Step 3, Section C: Choosing Learning Activities** (p. 11), such as:

- Sample Activity 1: Arrays and Area of Rectangles (p. 13)
- Sample Activity 1: Prime and Composite Numbers—Arrays and Graphing (p. 22).

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C. Applied Learning

Provide opportunities for students to use strategies for multiples, factors, primes and composites in a practical situation and notice whether or not these strategies transfer. For example, ask the student to solve the following problem and explain the thinking for solving this problem.

You jog every fourth day and swim every sixth day. On what days do you jog and swim?

Does the student:

- use skip counting to find the multiples of 4 and 6?
- understand that the common multiples of 4 and 6 are 12, 24, 36, 48 … because both 4 and 6 divide evenly into each of these numbers?
- multiply 4 by 6 to get 24 but miss the answer of 12?
- apply multiples to other real-world problems? For example, Jimmy works every third day and Susie works every fifth day. On what days do they work together?

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Step 5: Follow-up on Assessment

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction?

A. Addressing Gaps in Learning

Students who have difficulty solving problems using multiples, factors, primes and composites will enjoy more success if one-on-one time is provided in which there is open communication to diagnose where the learning difficulties lie. Assessment by observing a student solving problems will provide valuable data to guide further instruction. Success in problem solving depends on a positive climate in which the students are confident in taking risks. By building on the existing understandings of each student and accommodating the individual learning styles, success will follow.

Use the following strategies to help students become effective and efficient in solving problems using multiples, factors, primes and composites:

- provide problems that relate to the student's interest; use the student's name in the problem
- initially, use smaller numbers in the problem
- have the student paraphrase the problem
- include problems that focus on only one set of multiples or factors first, then move on to common multiples and common factors
- connect multiples and factors to previous knowledge about number facts and area of rectangles
- have the student connect the concrete, pictorial and symbolic representations
- ensure the student has a solid understanding of multiples and factors before introducing prime and composite numbers
- provide manipulatives such as square tiles for students to represent the problem, as needed
- encourage flexibility in thinking by having the student solve problems in a variety of ways, such as using a number line or patterns in a chart
- ask guiding questions to show connections among multiples, factors, primes and composites
- provide a graphic organizer such as the KNWS chart (see Blackline Master—KNWS Chart, p. 41).

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B. Reinforcing and Extending Learning

Students who have achieved or exceeded the outcomes will benefit from ongoing opportunities to apply and extend their learning. These activities should support students in developing a deeper understanding of the concept and should not progress to the outcomes in subsequent grades.

Consider the following strategies:

- Provide tips for parents on practising multiples and factors of numbers, as well as prime and composite numbers, at home or in the community.
  - Have the children discuss different ways to arrange chairs in a gym or auditorium for a talent night; e.g., have children provide all the different ways 72 chairs could be arranged in rectangular arrays.
  - For some card games, such as Concentration, have children arrange the cards in different arrays on the table or the floor prior to starting the game. Review that the factors of the total number of cards are shown by the rows and columns in the different arrays. Remind children that some numbers are composite and have more than two factors while other numbers are prime and have only two factors.
  - Encourage the use of skip counting for reviewing multiplication facts.
  - Solve problems with common multiples such as the following:
    - You visit your grandma every fourth day and a schoolmate every sixth day. On what days do you visit both your grandma and a schoolmate?
  - Solve problems with common factors such as the following:
    - You have a red ribbon that is 75 cm long and a green ribbon that is 50 cm long. You want to cut each ribbon in pieces the same length. What are the possible lengths you could cut each ribbon?

- Goldbach's Conjecture: every even number greater than 2 can be expressed as the sum of two prime numbers; e.g., $20 = 3 + 17$, $66 = 23 + 43$.
  - Find the sums for all even numbers between 20 and 70.
  - Find more than one way to write chosen even numbers as the sum of two prime numbers.
  - The number 2 is never used in any of the sums. Explain.
    (NCTM 1990, pp. 38–39)

- Least Common Multiple
  - Johnny takes a trip every four years, Lori takes a trip every eight years and Cindy takes a trip every six years. If they all take a trip in 2012, in which year would they all take a trip again?
  - Josh swims every third day, Dylan swims every fifth day and Nicholas swims every tenth day. If they all swim together today, in how many days will they all swim together again?
  - Bus No. 43 leaves the depot once every 18 minutes. Bus No. 74 leaves once every 30 minutes. How often do both buses leave the depot together?

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- Greatest Common Factor
  - Each sticker in a box costs the same. Lauren buys some and pays 30 cents in all. Brandon buys some and pays 45 cents in all. What is the most each sticker can cost?
  - You have a blue ribbon that is 42 cm long and a yellow ribbon that is 28 cm long. You want to cut both ribbons into smaller ribbons that are all the same length. What is the longest length that you could use to cut both of the ribbons?
  - You have a piece of cloth that is 15 cm by 30 cm and you cut it into congruent squares with no cloth left over. What are the dimensions of the largest possible square?

- Carroll Diagrams
  A Carroll diagram is "a chart for sorting that allows for cross-classification" (Small 2009, p. 213).

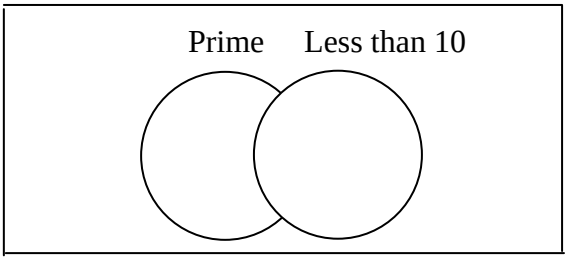
  Place the numbers from 2 to 15 in the following Carroll Diagram:

	Prime	**Composite**
Factor of 15		
Not a Factor of 15		

- Venn Diagrams
  A Venn diagram is a visual system of sorting circles that allows for cross-classification (Small 2009, p. 219).

  Sort whole numbers less than 20 in the following Venn diagram (Small 2009, p. 168):

Whole Numbers Less than 20



- Factor Game

  Materials:
  – *use BLM 1 on page 28 from Sample Activity 2*
  – factor tally chart (shown below).

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Playing the Factor Game

Play the Factor Game with a partner. (Beesey 1994, p. 11)

- Player 1 picks a number on the hundreds chart and crosses it out.
- Player 2 finds the factors of this number on the hundreds chart and crosses them out, then tallies them on the tally chart.
- Switch roles.
- Continue the game until all the numbers on the hundreds chart are crossed out.
- Add up the tallies on the tally chart and record the total.

Tally Chart

Player 1	Player 2

- Explain why the sum of the digits of any multiple of 3 is itself divisible by 3 (NCTM 2000, p. 217).

Sample Solution:

Multiples of 3	3	6	9	12	15	18	21	24	27	30	33	…
Sum of the digits	3	6	9	3	6	9	3	6	9	3	6	…

The pattern in the second row for the sum of the digits of multiples of 3 is:
3, 6, 9, 3, 6, 9, 3, 6, 9, …

Since 3, 6 and 9 are each divisible by 3, the sum of the digits of any multiple of 3 is itself divisible by 3.

Extension

A number in the form of *abc abc* always has several prime number factors. Which prime numbers are always factors of a number of this form? Why?

- An example of a number in the form of *abc abc* is 153 153.
- Using expanded form, write 153 153 as 153 × 1000 + 153.
- Since 153 is a common f
- actor to both numbers in this sum, then 153 × 1000 + 153 = 153 (1000 + 1).
- Since 153 153 = 153 × 1001, then 1001 is a factor of 153 153.
- Therefore, any number of the form *abc abc* will have 1001 as a factor.
- Next, find the prime factors of 1001. They include 7, 11, 13, 77, 91 and 143.
- Therefore, these prime numbers are always factors of numbers in the form of *abc abc*.

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Blackline Master—KNWS

(Barton and Heidema 2002, p. 113)

K What facts do I **KNOW** from the information in the problem?	**N** Which information do I **NOT** need?	**W** **WHAT** does the problem ask me to find?	**S** What **STRATEGY** will I use to solve the problem?

Adapted with permission from Mary Lee Barton and Clare Heidema, *Teaching Reading in Mathematics: A Supplement to Teaching Reading in the Content Areas Teacher's Manual* (2nd ed.) (Aurora, CO: McREL (Mid-continent Research for Education and Learning, 2002), p. 113.

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