Mathematics

Planning Guide

Grade 6

Improper Fractions and Mixed Numbers

Number

Specific Outcome 4

This Planning Guide can be accessed online at:
http://www.learnalberta.ca/content/mepg6/html/pg6_improperfractionsmixednumbers/index.html

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Planning Guide: *Grade 6 Improper Fractions and Mixed Numbers*

Strand: Number
Specific Outcome: 4

This *Planning Guide* addresses the following outcomes from the program of studies:

Strand: Number	
Specific Outcome:	4. Relate improper fractions to mixed numbers and mixed numbers to improper fractions.

Curriculum Focus

This sample targets the following changes to the curriculum:

- The general outcome focuses on number sense, whereas the previous program of studies specified demonstrating a number sense for decimals and common fractions, exploring integers and showing number sense for whole numbers.
- The specific outcome focuses on relating improper fractions to mixed numbers and mixed numbers to improper fractions, while the achievement indicators emphasize connecting the concrete, pictorial and symbolic forms. The previous program of studies outcome focused on demonstrating and explaining the meaning of improper fractions and mixed numbers concretely, pictorially and symbolically.

What Is a Planning Guide?

Planning Guides are a tool for teachers to use in designing instruction and assessment that focuses on developing and deepening students' understanding of mathematical concepts. This tool is based on the process outlined in *Understanding by Design* by Grant Wiggins and Jay McTighe.

Planning Steps

The following steps will help you through the Planning Guide:

Step 1: Identify Outcomes to Address

Guiding Questions

- What do I want my students to learn?
- What can my students currently understand and do?
- What do I want my students to understand and be able to do based on the Big Ideas and specific outcomes in the program of studies?

Big Ideas

Since improper fractions and mixed numbers are included in the set of fractions, it is important that connections are made to prior knowledge about the meaning of fractions and comparing fractions. Students develop and demonstrate understanding of fractions by connecting the concrete, pictorial and symbolic representations. By observing students, asking questions and listening to their explanations, the teacher probes for deeper understanding. *Professional Standards for School Mathematics* states that it is beneficial for students to create their own models and allow for some confusion in order to reveal "what students do and don't understand" (NCTM 1991, p. 163).

Meaning of Fractions
In developing understanding of fractions, students should represent them with physical materials to show "part of a unit whole or of a collection. Teachers should help students understand fractions as division of numbers" (NCTM 2000, p. 33). The unit whole is often referred to as the whole region. The collection is referred to as the whole set.

Van de Walle and Lovin (2006) suggest that previous knowledge about fractions be transferred first to improper fractions and then to mixed numbers. Improper fractions, like proper fractions, include a numerator or top number that *counts* and a denominator or bottom number that tells *what is being counted* (pp. 68–69).

The authors go on to provide the following definitions of a numerator and a denominator of a fraction:

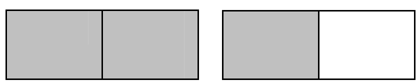
> The denominator of a fraction indicates by what number the whole has been divided in order to produce the type of part under consideration. Thus, the denominator is a divisor. In practical terms, the denominator names the kind of fractional part that is under consideration. The numerator of a fraction counts or tells how many of the fraction parts (or the type indicated by the denominator) are under consideration. Therefore, the numerator is a multiplier—it indicates a multiple of the given fractional part (p. 66).

For example, $\frac{2}{3}$ is twice what you get when you divide the whole region or whole set into three equal parts, with each equal part making up $\frac{1}{3}$ of the whole.

"In every question, the unit fraction plays a significant role. If you have $\frac{5}{3}$ and want to find the whole, you first need to find $\frac{1}{3}$" (Van de Walle and Lovin 2006, p. 75).

Fractions as Part of a Region

An example of fractions as part of a unit whole or a whole region is representing $\frac{3}{2}$ cakes, where the whole region is one cake. The unit fraction is $\frac{1}{2}$ of a cake. A visual representation could be:



1 whole cake $\quad$ $\frac{1}{2}$ of a whole cake

$\frac{1}{2}$ $\quad$ $\frac{2}{2}$ $\quad$ $\frac{3}{2}$

(counting by halves)

1 cake + $\frac{1}{2}$ of a cake = $1\frac{1}{2}$ cakes

$\frac{3}{2}$ of a cake is equivalent to $1\frac{1}{2}$ cakes.

Note: The two rectangular cakes each represent the same size cake—one is entirely shaded while the other one has only $\frac{1}{2}$ shaded.

Fractions as Part of a Set

An example of fractions as part of a whole set or collection is representing $\frac{3}{2}$ of a set of shapes, where the whole set is two shapes. The unit fraction is $\frac{1}{2}$ of the set of shapes.

1 whole set $\quad$ $\frac{1}{2}$ of the whole set

$\frac{1}{2}$ $\quad$ $\frac{2}{2}$ $\quad$ $\frac{3}{2}$

(counting by halves)

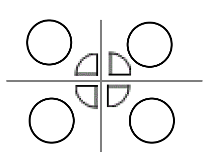
1 whole set + $\frac{1}{2}$ of a set = $1\frac{1}{2}$ sets

$\frac{3}{2}$ of a set is equivalent to $1\frac{1}{2}$ sets.

Note: The two sets each represent the same size sets, each with two items—one is entirely shaded while the other one has only $\frac{1}{2}$ shaded.

Fractions as the Division of Two Numbers

An example of fractions as the division of two numbers is sharing five cookies equally among four people. A visual representation could be:



When you divide five cookies equally among four people ($5 \div 4$), each person receives $\frac{5}{4}$ or $1\frac{1}{4}$ cookies (Small 2009, p. 43).

Comparing Fractions

Small (2009, pp. 48–49) suggests these ideas regarding comparing fractions.

- Fractions can be compared only if the whole is known in each case.
- If two fractions have the same denominator, the one with the greater numerator is greater.
- If two fractions have the same numerator, the one with the greater denominator is less.
- Some fractions can be compared by relating them to benchmark numbers such as 0, 1 and $\frac{1}{2}$. If one fraction is greater than $\frac{1}{2}$ and the other is less than $\frac{1}{2}$, it is easy to compare them.
- Fractions can be compared by renaming them with common denominators or by renaming them with common numerators.
- No matter what two different fractions are selected, there is a fraction between them.

The following definitions are available at http://learnalberta.ca/content/memg/index.html.

Fractions

Fractions form a number system that includes whole numbers and also an infinite set of numbers between every whole number. They are numbers that can be written in the form $\frac{a}{b}$, where *a*, the numerator, and *b*, the denominator, are whole numbers and $b \neq 0$. In working with fractions, the whole region or the whole set must be stated. Then the fraction represents part of the given whole. The denominator is the divisor and tells how many equal parts are in the whole. The numerator is the multiplier and tells how many of those equal parts the fraction stands for.

Denominator

The denominator is "the number of equal parts into which the whole is divided."
It is also the divisor and tells what is being counted; e.g., 4 is the denominator in $\frac{3}{4}$ (Van de Walle and Lovin 2006, p. 68).

Numerator

The numerator is "the number of equal parts in a set to be considered [or] of a whole to be considered."

It is also the multiplier and indicates a multiple of the given fractional part; e.g., 3 is the numerator in $\frac{3}{4}$ (Van de Walle and Lovin 2006, pp. 66, 68).

Proper Fraction

"A proper fraction is a fraction in which the numerator is less than the denominator"; e.g., $\frac{3}{4}$.

A proper fraction is less than 1.

Improper Fraction
"An improper fraction is a fraction in which the numerator is greater than or equal to the denominator"; e.g., $\frac{3}{3}$, $\frac{5}{4}$.
When the numerator is greater than the denominator, the fraction is greater than 1.
When the numerator is a multiple of the denominator, the fraction is equal to a whole number; e.g., $\frac{9}{3} = 3$.

Mixed Number
"A mixed number is a number written as a whole number (not including zero) and a fraction"; e.g., $2\frac{3}{4}$.
A mixed number is greater than 1.

Sequence of Outcomes from the Program of Studies

See http://www.education.alberta.ca/teachers/program/math/educator/progstudy.aspx for the complete program of studies.

Grade 5	Grade 6	Grade 7
Specific Outcomes	**Specific Outcomes**	**Specific Outcomes**
7. Demonstrate an understanding of fractions by using concrete, pictorial and symbolic representations to: • create sets of equivalent fractions • compare fractions with like and unlike denominators.	4. Relate improper fractions to mixed numbers and mixed numbers to improper fractions.	5. Demonstrate an understanding of adding and subtracting positive fractions and mixed numbers, with like and unlike denominators, concretely, pictorially and symbolically (limited to positive sums and differences).

Step 2: Determine Evidence of Student Learning

Guiding Questions

- What evidence will I look for to know that learning has occurred?
- What should students demonstrate to show their understanding of the mathematical concepts, skills and Big Ideas?

Using Achievement Indicators

As you begin planning lessons and learning activities, keep in mind ongoing ways to monitor and assess student learning. One starting point for this planning is to consider the achievement indicators listed in *The Alberta K–9 Mathematics Program of Studies with Achievement Indicators*. You may also generate your own indicators and use them to guide your observation of students.

The following indicators may be used to determine whether or not students have met this specific outcome. Can students:

- demonstrate, using models, that a given improper fraction represents a number greater than 1?
- apply mixed numbers and improper fractions to real-world situations?
- apply mixed numbers and improper fractions to part of a region and part of a set?
- solve problems using improper fractions as the division of numbers?
- translate a given improper fraction between concrete, pictorial and symbolic forms?
- translate a given mixed number between concrete, pictorial and symbolic forms?
- express improper fractions as mixed numbers?
- express mixed numbers as improper fractions?
- place a given set of fractions, including mixed numbers and improper fractions, on a number line and explain strategies used to determine position?
- provide more than one way to write a given mixed number or improper fractions by using equivalent fractions?
- solve part–whole–fraction problems, including improper fractions, in which two out of three are provided and the third must be found?

Sample behaviours to look for related to these indicators are suggested for some of the activities listed in **Step 3, Section C: Choosing Learning Activities** (p. 15).

Step 3: Plan for Instruction

Guiding Questions

- What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should I use?
- How will I meet the diverse learning needs of my students?

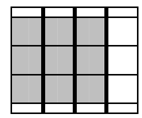
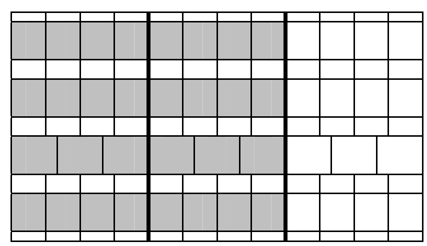
A. Assessing Prior Knowledge and Skills

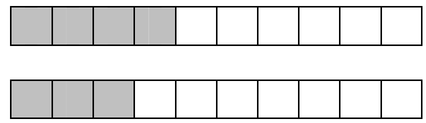
Before introducing new material, consider ways to assess and build on students' knowledge and skills related to fractions. Provide manipulatives such as fraction strips, pattern blocks or fraction circles for students to use, as needed. For example:

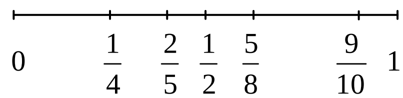
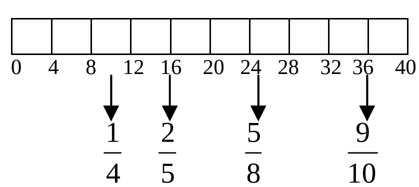
- The area of a room is 12 square metres. If a rug covers $\frac{3}{4}$ of the room, what is the area of the rug? Explain by using diagrams and symbols.
- Write four fractions that are equivalent to $\frac{2}{3}$. Explain by using diagrams and symbols.
- Jack eats $\frac{2}{5}$ of a cake. Jill eats $\frac{3}{10}$ of the same cake. Who eats more cake or do they each eat the same amount of cake? Explain by using diagrams and symbols.
- Place the following fractions on a number line and explain how you decided on the order: $\frac{5}{8}, \frac{9}{10}, \frac{2}{5}, \frac{1}{8}, \frac{1}{4}$.

If a student appears to have difficulty with these tasks, consider further individual assessment, such as a structured interview, to determine the student's level of skill and understanding. See **Sample Structured Interview: Assessing Prior Knowledge and Skills** (p. 11).

Sample Structured Interview: Assessing Prior Knowledge and Skills

Directions	Date:	
	Not Quite There	**Ready to Apply**
Provide manipulatives such as fraction strips, pattern blocks or fraction circles, as needed. Place the following problem before the student: **The area of a room is 12 square metres. If a rug covers $\frac{3}{4}$ of the room, what is the area of the rug? Explain using diagrams and symbols.**	Does not understand the problem as an application of equivalent fractions. States the correct answer but is unable to explain the process by using diagrams and symbols.	Draws a diagram and uses symbols to explain the process in the solution, such as the following:  $\frac{3}{4}=\frac{9}{12}$ "There are 3 square metres in each quarter; therefore, three of the quarters have an area of 9 square metres." Answers the problem correctly, such as the following: "The area of the rug is 9 square metres."
Provide manipulatives such as fraction strips, pattern blocks or fraction circles, as needed. Say, **"Write four fractions that are equivalent to $\frac{2}{3}$. Explain using diagrams and symbols."**	Writes no fractions equivalent to $\frac{2}{3}$. Writes one or two fractions equivalent to $\frac{2}{3}$ with no explanation. Writes four fractions equivalent to $\frac{2}{3}$ and states a rule but neglects to use diagrams in the explanation.	Writes four fractions that are equivalent to $\frac{2}{3}$ and explains why they are equivalent.  Sample: For each third, there are two-sixths, three-ninths and four-twelfths. Therefore, two-thirds is equivalent to twice as much as one-third, namely, four-sixths, six-ninths and eight-twelfths. The equivalent fractions are: $\frac{2}{3}=\frac{4}{6}=\frac{6}{9}=\frac{8}{12}$.

Provide manipulatives such as fraction strips or fraction circles, as needed. Place the following problem before the student: **Jack eats $\frac{2}{5}$ of a cake. Jill eats $\frac{3}{10}$ of the same cake. Who eats more cake or do they each eat the same amount of cake? Explain by using diagrams and symbols.**	Answers incorrectly with little or no attempt at an explanation. Answers correctly but lacks clarity in the explanation or provides no explanation.	Answers correctly and clearly explains using diagrams and symbols. Sample:  The diagram shows the same size cake but the first cake is cut into five equal pieces and the second cake is cut into 10 equal pieces. The diagram shows that two out of five equal pieces of cake is greater that three out of 10 equal pieces of the same size cake. Therefore, Jack eats more of the cake than Jill.

Provide manipulatives such as fraction strips, pattern blocks or fraction circles, as needed. Place the following fractions before the student: $\frac{5}{8}, \frac{9}{10}, \frac{2}{5}, \frac{1}{4}$ Say, **"Place these fractions on a number line and explain how you decided on the order."**	Places the fractions in random order on a number line. Places some of the fractions in the correct order on a number line with limited or no explanation. Places all of the fractions in the correct order on the number line with inaccurate location and with limited or no explanation.	Places all of the fractions in the correct order with accurate location on the number line and explains how the order was decided. Sample Answer 1:  • Student used benchmarks of 0, $\frac{1}{2}$ and 1. • $\frac{1}{4}$ is halfway to $\frac{1}{2}$. • $\frac{2}{5}$ is just less than $\frac{1}{2}$ because 2.5 is half of 5. • $\frac{5}{8}$ is just more than $\frac{1}{2}$ because 5 is a little more than half of 8. • $\frac{9}{10}$ is close to 1, only $\frac{1}{10}$ away from 1. Sample Answer 2: Student converted all the fractions to equivalent fractions with a denominator of 40. $\frac{1}{4}=\frac{10}{40}$, $\frac{2}{5}=\frac{16}{40}$, $\frac{5}{8}=\frac{25}{40}$, $\frac{9}{10}=\frac{36}{40}$. 

B. Choosing Instructional Strategies

Consider the following instructional strategies for teaching improper fractions and mixed numbers.

- Introduce and reinforce the mixed numbers and improper fractions by using a variety of real-world contexts that include whole regions and whole sets.
- Connect the mixed numbers and improper fractions to prior knowledge about the meaning of fractions and equivalent fractions.
- Provide students with a variety of problems that apply the concept of mixed numbers and improper fractions. Encourage them to solve the problems in different ways and explain the process. Also, provide time for students to share their solutions with others. Stimulate class discussion to critically evaluate the various procedures. Emphasize understanding, flexibility and efficiency when students select problem-solving strategies.
- Explore problems that develop understanding of fractions as division of numbers (NCTM 2000, p. 33).
- Emphasize the importance of establishing what the whole region or the whole set is before finding fraction names for regions or sets.
- Encourage students to communicate their thinking by connecting manipulatives, diagrams and symbols to represent the concepts.
- First introduce improper fractions as an extension of proper fractions on the number line. Explore the similarities and differences. Then, connect improper fractions to mixed numbers.
- "Create a classroom environment that encourages student exploration, questioning, verification and sense making” (NCTM 1992, p. 5).
- Provide opportunities for students to explore the relationship between mixed numbers and improper fractions.
- To promote flexible thinking, provide a variety of problems in which two out of the following three are given and the third must be found: the whole, the part and the fraction (NCTM 2000, p. 215).
- Encourage students to make and critique generalizations related to mixed numbers and improper fractions.
- When ordering mixed numbers and improper fractions, make connections to prior knowledge of benchmarks and equivalent fractions. Encourage flexibility in choosing strategies to order fractions.

C. Choosing Learning Activities

The following learning activities are examples of activities that could be used to develop student understanding of the concepts identified in Step 1.

Sample Activities:

Teaching Improper Fractions

Teaching Improper Fractions and Mixed Numbers

Teaching Improper Fractions

Sample Activity 1: Part of a Whole Region (Area Model)

Look For …

Do students:

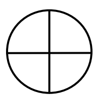
- ☐ apply knowledge of proper fractions to improper fractions?
- ☐ use regions and sets to represent the whole?
- ☐ explain the similarities and differences between part of a region and part of a set?
- ☐ describe the role of the numerator and the denominator?
- ☐ describe the similarities and differences between proper fractions and improper fractions?
- ☐ solve problems using improper fractions as the division of two numbers?

Review the meaning of a proper fraction as part of a whole region and a whole set. Discuss the role of the numerator and the denominator.

Provide students with a collection of fractional parts in which the whole region is established.

Example:

Establish that the whole region is a whole pizza, as shown below. Display it on the overhead projector or the white board as a reference for students.



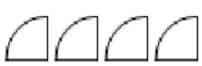
Discuss the meaning of the top (numerator) and bottom (denominator) of each fraction.

For each collection of fractional parts of pizza, ask students:

- to count the parts and write the correct fraction with a top (numerator) and a bottom (denominator) separated by a horizontal bar
- if the fraction represents less than one whole pizza, the same as one whole pizza or greater than one whole pizza.

Sample Solutions

a.  Answer: The fractional parts represent $\frac{3}{4}$ of a pizza.
It is less than one whole pizza.

b.  Answer: The fractional parts represent $\frac{4}{4}$ of a pizza.
It is the same as one whole pizza.

c.  Answer: The fractional parts represent $\frac{5}{4}$ of a pizza.
It is more than one whole pizza.

Provide other fractional parts of pizzas such as thirds, sixths or eighths and repeat the process.

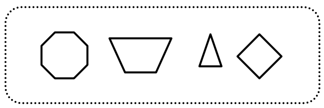
Introduce the term *improper fraction*. Explain that all fractions in which the numerator is greater than or equal to the denominator are called improper fractions. Have students sort the fractions that they have written into proper fractions and improper fractions (Van de Walle and Lovin 2006, p. 67).

Sample Activity 2: Part of a Whole Set

Provide students with a collection of fractional parts in which the whole set is established; e.g., a set of counters, such as pattern blocks.

Example:

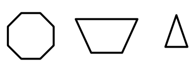
Establish that the whole set is a set of four counters as shown below. Display them on the overhead projector or the white board as a reference for the students.

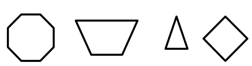


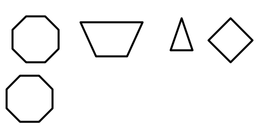
Review the meaning of the top (numerator) and the bottom (denominator) of each fraction. For each collection of fractional parts of a set, ask students:

- to count the parts and write the correct fraction with a top (numerator) and a bottom (denominator) separated by a horizontal bar
- if the fraction represents less than one whole set, the same as one whole set or greater than one whole set.

Sample Solutions

a.  Answer: The counters represent $\frac{3}{4}$ of the whole set.
It is less than one whole set.

b.  Answer: The counters represent $\frac{4}{4}$ of the whole set.
It is same as one whole set.

c.  Answer: The counters represent $\frac{5}{4}$ of the whole set.
It is greater than one whole set.

Provide other fractional parts of sets such as thirds, sixths or eighths and repeat the process.

Compare the terms, *proper fractions* and *improper fractions*, discussing the similarities and differences. Have students sort the fractions that they have written into proper fractions and improper fractions.

Teaching Improper Fractions and Mixed Numbers

Sample Activity 1: Fractions as the Division of Numbers (Expressing Improper Fractions as Mixed Numbers)

Build on students' prior knowledge of proper fractions as division.

Provide problems that involve the division of two numbers involving equal sharing in which the quotient can be written as an improper fraction and then as a mixed number.

Provide concrete materials for students to use, as needed, such as fraction circles, fraction strips, pattern blocks and/or money.

Problems

a. Five cookies are shared equally by four people. How many cookies does each person receive?
b. $9 is shared equally by four people. How much money does each person receive?

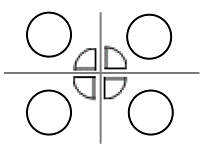
Look For …

Do students:

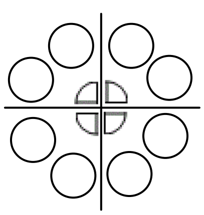
- ☐ understand that fractions may represent the division of numbers?
- ☐ connect division problems to improper fractions?
- ☐ connect the concrete, pictorial and symbolic representations for improper fractions?
- ☐ use a visual to show the connection between an improper fraction and a mixed number?

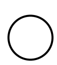
Have students solve the problems. Guide discussion of the solution process, connecting the concrete, pictorial and symbolic representations. Through discussion, have students generalize that the answer to the problem can be represented as an improper fraction or a mixed number.

Sample Solutions:

a. 

When you divide five cookies equally among four people ($5 \div 4$), each person receives $\frac{5}{4}$ or $1\frac{1}{4}$ cookies (Small 2009, p. 43).

b. 

 represents $1 or one loonie.

 represents one loonie divided into four equal parts with each part being one quarter of a dollar.

When you divide $9 equally among four people ($9 \div 4$), each person receives $\frac{9}{4}$ or $2\frac{1}{4}$ dollars. Each person's share can also be written as $2.25.

Sample Activity 2: Fractions as the Division of Numbers Using Patterns (Expressing Improper Fractions as Mixed Numbers)

Provide counters for students to use, as needed.

Group students according to the number of people in their family. Bring cookies to class to eat after this problem-solving activity!

Problem
You bought a baker's dozen (13) of cookies that you want to share equally with your family. How many cookies will each person get?

After each group has solved the problem, encourage students to share their solutions with the whole class.

Suggest that students put their answers in a chart as shown below.

Number of People in the Family	Number of Cookies for Each Member of the Family
2	$\frac{13}{2}$ or $6\frac{1}{2}$
3	$\frac{13}{3}$ or $4\frac{1}{3}$
4	
5	

Discuss the patterns that can be seen from the numbers in the chart. Have students generalize that the number of cookies for each person is found by dividing the total number of cookies by the number of people sharing them.

Have students suggest a rule to describe the number of cookies that each person gets if 13 cookies are shared by *n* people. Each person gets $\frac{13}{n}$ cookies, which is the quotient of the two numbers.

Extend the problem and have the student use 15 cookies instead of 13 cookies (NCTM 1994, pp. 17, 18).

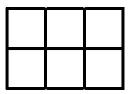
Look For …
Do students:
- ☐ solve the problem and explain the process?
- ☐ describe patterns of improper fractions and mixed numbers shown in the chart?
- ☐ generalize the meaning of the improper fractions and mixed numbers created in the solution of the problem?
- ☐ apply the patterns to other similar problems?

Sample Activity 3: Expressing Improper Fractions as Mixed Numbers (Part of a Region)

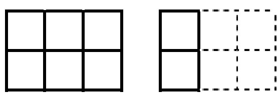
Take an example of an improper fraction along with the concrete and pictorial representations. Ask students to suggest another way to write the same amount as the improper fraction and justify their answer.

Problem

The following diagram represents a whole cake divided into six equal parts.



The following diagram represents $\frac{8}{6}$ of the cake.



What is another way to express eight-sixths of a cake?

Sample Solution 1:

The diagram shows one whole cake and $\frac{2}{6}$ of another cake. This amount could be written as one whole and two-sixths more or 1 and $\frac{2}{6}$ or $1\frac{2}{6}$.

Since $\frac{2}{6}$ is equivalent to $\frac{1}{3}$, then $1\frac{2}{6}$ is equivalent to $1\frac{1}{3}$.

Therefore, $\frac{8}{6} = 1\frac{2}{6}$ and $1\frac{2}{6} = 1\frac{1}{3}$.

Sample Solution 2:

Students who apply the understanding that the denominator of a fraction is the divisor may divide to show another way to write $\frac{8}{6}$.

$\frac{8}{6}$ can be written as the division, $\begin{array}{r} 1 \\ 6\overline{)\ 8} \\ \underline{-6} \\ 2 \end{array}$. $\frac{8}{6}$ can be written as $1\frac{2}{6}$ or $1\frac{1}{3}$.

Provide other examples using whole regions and whole sets.

Look For …

Do students:

- ☐ interpret diagrams of improper fractions, recognizing what region or set represents the whole?
- ☐ express improper fractions as mixed numbers and explain the process used?
- ☐ explain that a mixed number is the sum of a whole number and a proper fraction?
- ☐ demonstrate flexibility in converting improper fractions into mixed numbers?
- ☐ classify improper fractions according to those that can be represented by mixed numbers and those that are represented only by whole numbers?

Sorting Improper Fractions

Provide examples of improper fractions. Have students sort them into groups and justify the sorting rule. For example, students may sort them into a group that can be written as a mixed number and a group that can be written only as a whole number.

Sample Activity 4: Expressing Mixed Numbers as Improper Fractions

Reverse the procedure from the previous activity and have students write mixed numbers as improper fractions.

Take an example of a mixed number along with the concrete and pictorial representations. Ask students to write an improper fraction that represents the same amount as the mixed number and justify their answer.

Look For …

Do students:

- ☐ interpret diagrams of mixed numbers by focusing on how many equal parts the whole region or whole set is divided into?
- ☐ express mixed numbers as improper fractions and explain the process used?
- ☐ explain that a mixed number is the sum of a whole number and a proper fraction?
- ☐ demonstrate flexibility in converting mixed numbers into improper fractions?
- ☐ make generalizations regarding an efficient and effective method to express mixed numbers as improper fractions?

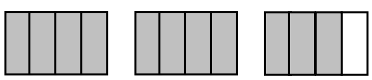
Problem

The shaded part of the following diagram represents $2\frac{3}{4}$ cakes.

Write an improper fraction that represents the same amount as $2\frac{3}{4}$ cakes.

Solution 1:

Divide each cake into **quarters** because three-**quarters** is the fractional part of the mixed number.



Count the quarters that make up the shaded part of the diagram. There are 11 quarters that are shaded. $\frac{11}{4}$ is an improper fraction because the numerator is greater than the denominator.

Solution 2:

Change the whole number into quarters because the fractional part of the mixed number is three-quarters. There are four quarters in each whole; therefore, there are eight quarters in two wholes. The eight quarters plus the three quarters total 11 quarters. The improper fraction, $\frac{11}{4}$, represents the same amount as $2\frac{3}{4}$.

Have students share their strategies in converting a mixed number into an improper fraction. Provide other examples using whole regions and whole sets.

Sample Activity 5: Naming Amounts Greater than One in Different Ways

Provide students with concrete materials, such as pattern blocks, square tiles, fraction circles and counters. Also provide them with square dot paper and triangular dot paper. (*Use BLM 1 and 2 on pages 25 and 26.*)

1. Naming a Mixed Number as an Improper Fraction

Give students a mixed number, such as $2\frac{2}{3}$.

a. Have them represent the mixed number using concrete materials and draw a corresponding diagram.
b. Tell them to find an improper fraction that represents the same amount and to justify their answer.

2. Naming an Improper Fraction as a Mixed Number

Give students an improper fraction, such as $\frac{12}{5}$.

a. Have them represent the improper fraction using concrete materials and draw a corresponding diagram.
b. Tell them to find a mixed number that represents the same amount and justify their answer (Van de Walle and Lovin 2006, p. 69).

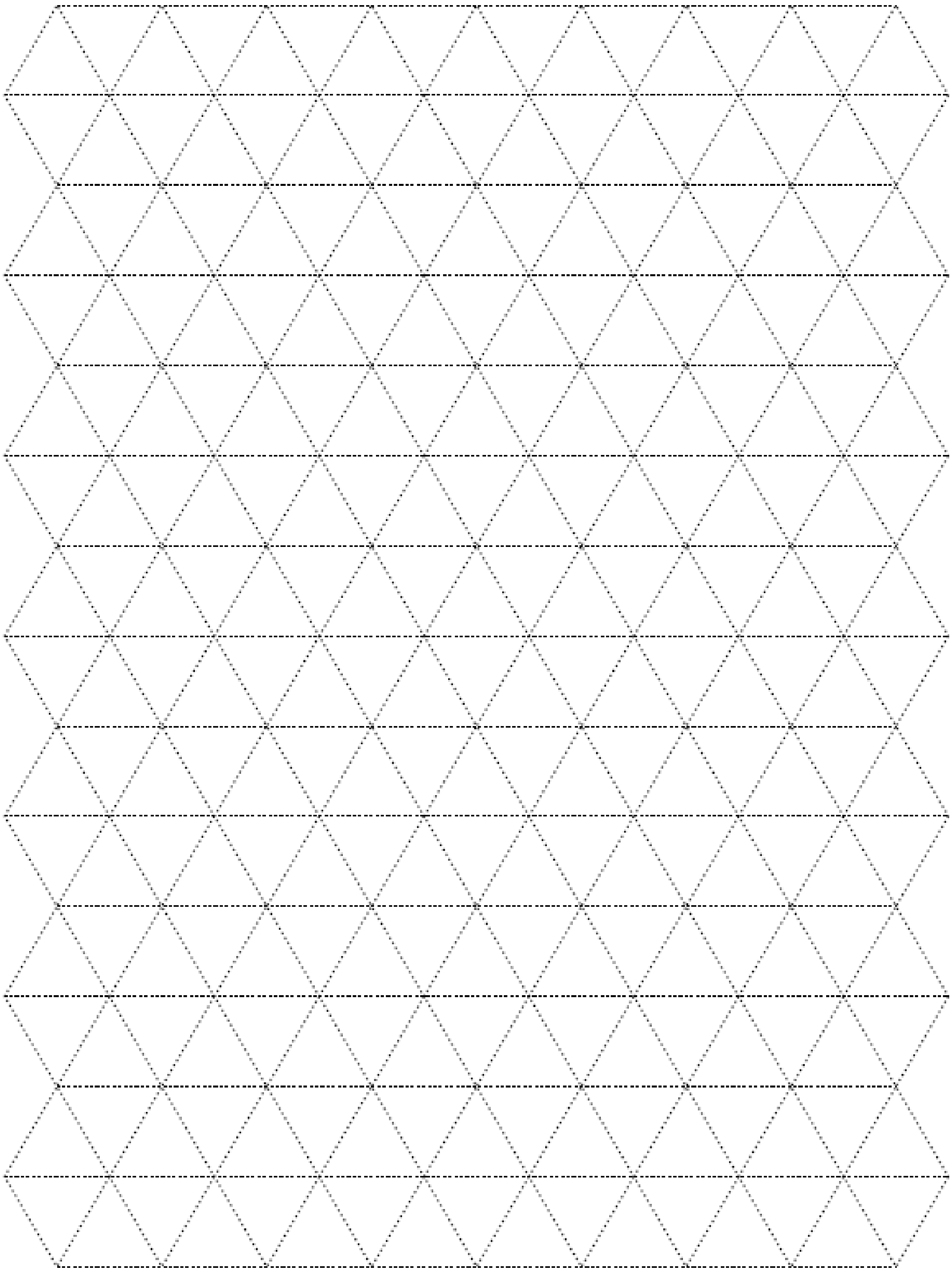
Look For …

Do students:

- ☐ translate a given improper fraction or mixed number between concrete, pictorial and symbolic forms?
- ☐ use a variety of ways to express an improper fraction as a mixed number?
- ☐ use a variety of ways to express a mixed number as an improper fraction?
- ☐ clearly communicate the connections between mixed numbers and improper fractions using correct mathematical language?

2-cm square grid – BLM 1

Isometric grid paper – BLM 2



Sample Activity 6: Parts–Whole–Fraction Tasks

1. Given the Whole and Fraction, Find the Part

a. Part of a Whole Set

Provide students with a set of counters, e.g., pattern blocks, and a sheet of paper on which a circle is drawn. Instruct them to take six of the counters and place them in the circle to represent the whole set.

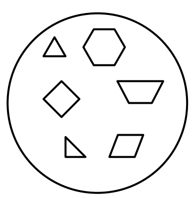
Problem
If six counters are the whole set, how many counters are in:

i. two-thirds of the set?
ii. six-sixths of the set?
iii. eight-sixths of the set?

Explain your thinking by using diagrams and symbols.

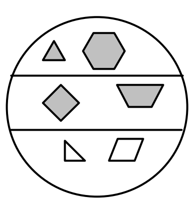
Look For …
Do students:
- ☐ think flexibly about fractions as they explore various problems using the whole, the fraction and the part?
- ☐ connect pictorial and symbolic forms for proper and improper fractions?
- ☐ identify the whole, the fraction and the part for each problem?
- ☐ solve problems using the whole as a region and as a set?
- ☐ communicate clearly, with precise mathematical language, the process used in solving the different types of problems that apply the meaning of fractions?
- ☐ create other similar problems that apply the meaning of fractions?

Sample Solution:



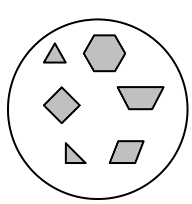
This set is the whole set.

Divide the six counters into three equal parts because the denominator of two-thirds is 3. Then shade two of the equal parts because the numerator of two-thirds is 2.



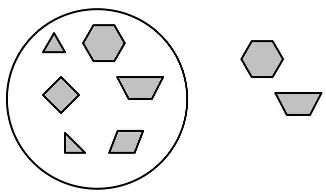
The shaded part is $\frac{2}{3}$ or $\frac{4}{6}$ of the whole set of counters.

Shade six of the six counters to show six-sixths of the set.



The shaded part is $\frac{6}{6}$ of the whole set of counters, which is the same as the whole set.

To show eight-sixths of the set, draw the whole set of six counters and add two more counters to make eight counters. The numerator counts to eight and the denominator shows that you are counting sixths. This means that you have more than a whole set. The two added counters make two-sixths of the set, which when added to the whole set makes $1 + \frac{2}{6}$ or $1\frac{2}{6}$ of the whole set of counters.

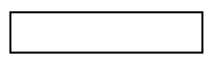


The shaded part is $\frac{8}{6}$ or $1\frac{2}{6}$ or $1\frac{1}{3}$ of the whole set of counters.

b. Part of a Whole Region

Provide students with strips of poster board that are each double the length and width of Cuisenaire rods; e.g., the light green rod (1 cm by 3 cm) becomes a strip 2 cm by 6 cm.

Problem

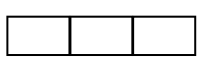
If the strip shown at the right is the whole ,
draw a strip that is:

i. two-thirds of the whole
ii. five-thirds of the whole.

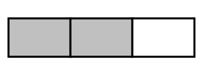
Explain your thinking.

Sample Solution:

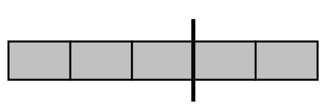
i. Divide the whole into three equal parts to show thirds. This shows three-thirds.



Shade two of the three equal parts to show two-thirds of the whole strip shaded.



ii. Add two more parts to the whole strip to show five-thirds of the whole strip shaded.

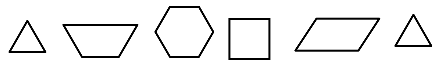


2. Given the Part and Fraction, Find the Whole

a. Part of a Whole Set

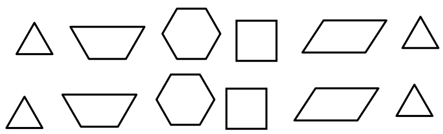
Provide students with a set of counters; e.g., pattern blocks.

Problem 1
If six counters are one-half of the whole set, how big is the whole set? Explain using counters, diagrams and symbols.
The following diagram has six counters and is one-half of the whole set.



This diagram shows $\frac{1}{2}$ of the set.

To find the whole set, double the number of counters to make 12 counters.



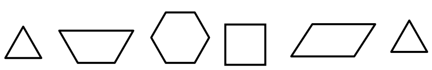
This diagram shows $\frac{2}{2}$ of the set or the whole set.
The whole set has 12 counters.

Symbolically: $\frac{1}{2}=\frac{6}{?} \longrightarrow \frac{1}{2}=\frac{6}{12}$

Problem 2
If six counters are two-thirds of a whole set, how big is the whole set? Explain using counters, diagrams and symbols.

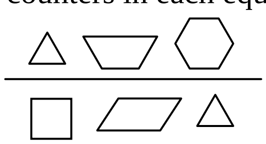
Sample Solution:

The following diagram has six counters and is two-thirds of the whole set. This means that two out of three equal parts are shown.



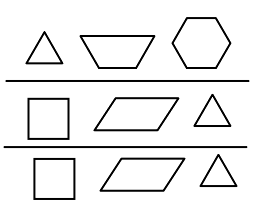
This diagram is $\frac{2}{3}$ of the whole set.

The numerator *counts* and the denominator *tells what is being counted*. Therefore, this set shows two of the three equal parts that make up the whole set. Divide the set into two equal parts as shown below. Each equal part has the same number of items; i.e., three counters in each equal part. Each of the equal parts represents one-third of a whole set.



This diagram is $\frac{2}{3}$ of the whole set.

To make the whole set, three-thirds are needed so we must add another equal part consisting of three counters.



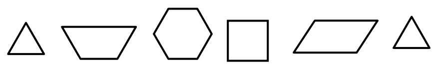
This diagram is $\frac{3}{3}$ of the whole set.
The whole set has nine counters in all.

Symbolically: $\frac{2}{3}=\frac{6}{?}$ $\quad \frac{2}{3}=\frac{6}{9}$

Problem 3
If six counters are three-halves of a whole set, how big is the whole set? Explain using counters, diagrams and symbols.

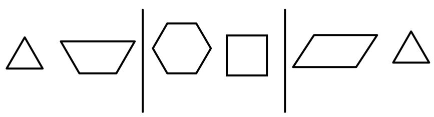
Sample Solution:

The following diagram has six counters and is three-halves of the whole set. This means that three equal parts are shown, but the whole set has only two of these equal parts.



This diagram is $\frac{3}{2}$ of the whole set.

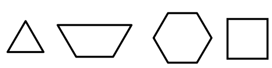
The numerator *counts* and the denominator *tells what is being counted*. Therefore, this diagram shows three of the halves. Divide the counters into three equal parts as shown. Each of the equal parts represents one-half of a whole set.



This diagram is $\frac{3}{2}$ of the whole set.

Counting by halves: $\frac{1}{2}$ $\frac{2}{2}$ $\frac{3}{2}$

Take two of the halves to make the whole set.



This diagram is $\frac{2}{2}$ of the whole set.

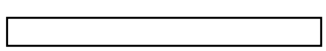
The whole set has four counters in all.

Symbolically: $\frac{3}{2}=\frac{6}{?} \longrightarrow \frac{3}{2}=\frac{6}{\mathbf{4}}$

b. Part of a Whole Region

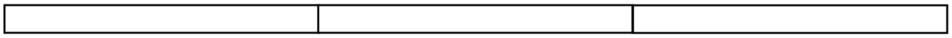
Provide the students with strips of poster board that are each double the length and width of Cuisenaire rods; e.g., the light green rod (1 cm by 3 cm) becomes a strip 2 cm by 6 cm.

Problem 1
If the strip shown below is one-third of the whole, draw the whole strip. Explain your thinking.



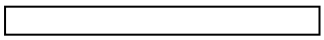
Sample Solution:

To find the whole strip, triple the length, because three groups of one-third make one whole.



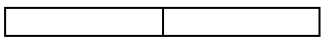
This diagram represents the whole strip.

Problem 2
If the strip shown below is two-thirds of the whole, draw the whole strip.



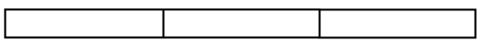
Sample Solution:

The numerator *counts* and the denominator *tells what is being counted*. Therefore, the strip shows two of the thirds. Divide the strip into two equal parts as shown. Each of the equal parts represents one-third of a whole strip.



This diagram is $\frac{2}{3}$ of the whole strip.

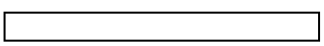
To make the whole strip, three-thirds are needed so we must add another equal part to the previous diagram that shows two equal parts.



This diagram is $\frac{3}{3}$ of the whole strip.

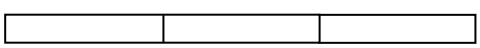
In other words, this diagram represents the whole strip.

Problem 3
If the strip shown below is three-halves of the whole, draw the whole strip.



Sample Solution:

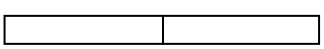
The numerator *counts* and the denominator *tells what is being counted*. Therefore, the strip shows three of the halves. Divide the strip into three equal parts as shown. Each of the equal parts represents one-half of a whole strip.



This diagram is $\frac{3}{2}$ of the whole strip.

Counting by halves: $\frac{1}{2}$ $\frac{2}{2}$ $\frac{3}{2}$

Take two of the halves to show the whole strip.



This diagram is $\frac{2}{2}$ of the whole strip.

In other words, this diagram represents the whole strip.

3. Given the Whole and the Part, Find the Fraction

a. Part of a Whole Set

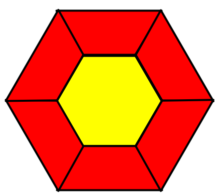
Problem
The set of animals below is the whole set.

i. The birds make up what fraction of this set of animals? Explain.
ii. Three more animals are added to make a total of nine animals. These nine animals are what fraction of the whole set of six animals? Explain.

b. Part of a Whole Region

Provide pattern blocks and triangular dot paper. (*Use BLM 2 on p. 26 from Sample Activity 5*.)

Problem
The design below is made out of pattern blocks.



i. What fraction of this design does the yellow hexagon represent? Explain.
ii. Suppose another hexagon is added to this design to make a new design. The new design is what fraction of the original design? Explain.
(NCTM 2000, p. 215; Van de Walle and Lovin 2006, p. 71)

Sample Activity 7: Anticipation/Reaction Guide

Use Blackline Master – Anticipation/Reaction Guide (p. 34) for this activity.

- On the template, write the major ideas about improper fractions and mixed numbers. Include true statements and also any misconceptions students may have about the concepts.
- Have students fill in the guide under the heading "Before," by stating whether each statement is true or false based on background knowledge.
- Discuss student predictions and anticipations listed on the guide as part of the ongoing learning activities. This discussion provides important information for assessment *for* learning.
- Choose learning strategies to address any misconceptions that are revealed by using this Anticipation/Reaction Guide.
- As students are involved in further learning activities, have them evaluate the statements in terms of what they have learned and place a check mark in the "After" column next to any statement with which they now agree.
- Have the students contrast their predictions with what they have learned and discuss the similarities and differences.

Look For …

Do students:

- ☐ ask questions to clarify any of the statements provided in the anticipation guide?
- ☐ explain their reasoning for agreeing or disagreeing with a statement from the anticipation guide?
- ☐ exhibit any misconceptions as they respond to the statements?
- ☐ develop understanding of concepts for which there were misconceptions initially?

Sample statements to use in the Anticipation/Reaction Guide:

1. A proper fraction is a number in which the numerator is less than the denominator.
2. An improper fraction is a number in which the numerator is greater than but not equal to the denominator.
3. The denominator of a given fraction divides the whole into equal parts and the numerator describes the number of parts being considered.
4. Equal parts of a set must have the same size and same shape.
5. Equal parts of a region must have the same size and same shape.
6. The numerator counts and the denominator tells what is being counted.
7. Between any two improper fractions there is another improper fraction.
8. All improper fractions can be written as mixed numbers.
9. All whole numbers can be written as improper fractions.

Blackline Master — Anticipation/Reaction Guide

(Barton and Heidema 2002, pp. 95–96)

Directions: In the column labelled "Before," write true (T) or false (F) beside each statement. After learning more about the topic that may include reading a text selection, place a check mark in the "After" column next to any statement with which you now agree. Compare your original opinions with those you developed as you learned more about the topic.

Topic: ______________________________

Before (T or F)	Statements	After
	1.	
	2.	
	3.	

Adapted with permission from Mary Lee Barton and Clare Heidema, *Teaching Reading in Mathematics: A Supplement to Teaching Reading in the Content Areas Teacher's Manual* (2nd ed.) (Aurora, CO: McREL (Mid-continent Research for Education and Learning, 2002), p. 113.

Sample Activity 8: Frayer Model

Have students summarize their understanding of improper fractions by completing a Frayer Model, such as the following example. If students are not familiar with using a **Frayer Model**, this strategy for consolidating understanding of concepts should be modelled first (*I do*) and then done with students (*We do*) before having them complete one on their own (*You do*). Encourage students to complete a Frayer Model for mixed numbers as well.

For a sample Frayer Model created for proper fractions, see Planning Guide: Grade 4 Fractions and Decimals (http://www.learnalberta.ca/content/mepg4/html/pg4_fractionsanddecimals/index.html). Go to Plan for Instruction: Sample Learning Activity and download activity Teaching the Naming and Recording Fractions for the Parts of a Region or Set.

Look For …

Do students:

- ☐ apply their knowledge of improper fractions and write a definition and the characteristics in their own words that is mathematically correct?
- ☐ create and justify examples of regions and sets for improper fractions?
- ☐ create a problem that applies improper fractions?
- ☐ create and justify non-examples of regions and sets for improper fractions?
- ☐ create a problem that does not use improper fractions?

Frayer Model for an Improper Fraction

(Barton and Heidema 2002, pp. 68–71)

Definition

"An improper fraction is a fraction in which the numerator is greater than or equal to the denominator."

(http://learnalberta.ca/content/memg/index.html)

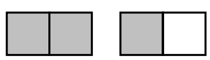
Characteristics

- represents the whole or greater than the whole region or whole set
- the numerator counts
- the denominator shows what is being counted
- the denominator divides the whole into equal parts
- equal parts of a region have the same size but not necessarily the same shape
- equal parts of a set have the same number of objects
- can be written as a mixed number if the numerator is greater than the denominator and is not a multiple of the denominator

Improper Fraction

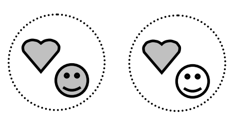
Examples

Region



one whole or $\frac{2}{2}$ is shaded $\frac{3}{2}$ or $1\frac{1}{2}$ is shaded

Set



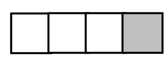
one whole or $\frac{2}{2}$ is shaded $\frac{3}{2}$ or $1\frac{1}{2}$ is shaded

Problem

How much money is $\frac{3}{2}$ of $12?

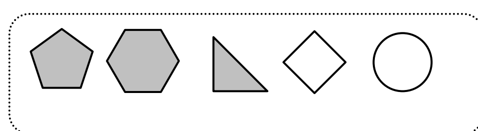
Non-examples

Region



$\frac{1}{4}$ of the region is shaded

Set



$\frac{3}{5}$ of the set is shaded

Problem

How much money is $\frac{2}{3}$ of $12?

Sample Activity 9: Choose, Explain, Test

Provide students with two fractions, such as $\frac{19}{8}$ and $\frac{13}{5}$. Ask them to decide which fraction is greater and to justify their answer by using any strategy they wish to use. Encourage students to use concrete materials, such as fraction strips or fraction circles, rather than drawings because drawings are often inaccurate and may lead to false conclusions. Other strategies could include using benchmarks or equivalent fractions
(Van de Walle and Lovin 2006, p. 77).

Look For …

Do students:

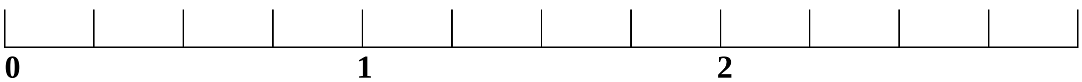
- ☐ apply strategies for ordering proper fractions to ordering mixed numbers and improper fractions?
- ☐ use a variety of strategies to order fractions?
- ☐ communicate clearly which fraction is greater by using precise mathematical language?
- ☐ think flexibly by using strategies to order fractions described by other students?
- ☐ think critically to evaluate strategies used by other students to order fractions?

Sample Activity 10: Ordering Fractions

Provide students with concrete materials, such as fraction strips, square tiles, pattern blocks, fraction circles and counters. Give students a set of fractions, including mixed numbers and improper fractions. Ask them to order the fractions on a number line and explain the strategies used. Have them justify their answers by using models, such as fraction strips.

Problem
Place the following numbers on the number line below.

$\frac{3}{2}$, $\frac{3}{4}$, $2\frac{3}{4}$, $1\frac{1}{4}$, $\frac{7}{4}$, $2\frac{3}{8}$, $\frac{8}{4}$



Sample Solution 1:

Convert all mixed numbers to improper fractions. Then use equivalence to write all the fractions with a denominator of eight. Finally, order the fractions by looking at the numerators. Convert the number line into skip counting sections using eighths instead of quarters and place the fractions on it.

Look For …
Do students:
- ☐ apply strategies for ordering proper fractions on a number line to ordering mixed numbers and improper fractions?
- ☐ use a variety of strategies to order fractions?
- ☐ communicate clearly which strategy is used to order numbers?
- ☐ think flexibly by using strategies to order fractions described by other students?
- ☐ think critically to evaluate strategies used by other students to order fractions?

Sample Solution 2:

Convert all improper fractions to mixed numbers. Then use equivalence to write all the fractional parts with a denominator of eight. Convert the number line into skip counting by eighths instead of quarters and place the fractions on it.

Sample Solution 3:

Convert all improper fractions to mixed numbers. Use the benchmarks of 0, $\frac{1}{2}$, 1, $1\frac{1}{2}$, 2, $2\frac{1}{2}$ to place the fractions on the number line. For example, $2\frac{3}{8}$ is slightly less than $2\frac{1}{2}$ because 3 is a little less than half of 8.

Provide other sets of fractions for students to order on a number line and have them justify their answers. Their explanation may include models or some other reasoning that is mathematically correct.

Step 4: Assess Student Learning

Guiding Questions

- Look back at what you determined as acceptable evidence in Step 2.
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

In addition to ongoing assessment throughout the lessons, consider the following sample activities to evaluate students' learning at key milestones. Suggestions are given for assessing all students as a class or in groups, individual students in need of further evaluation, and individual or groups of students in a variety of contexts.

A. Whole Class/Group Assessment

The following student assessment task, A Party Using Mixed Numbers and Improper Fractions (p. 42) could be used with a whole group or class. It includes a marking rubric to be used with the assessment.

Evidence the Student Has Achieved the Outcomes

Each student will:

- demonstrate, using models, that a given improper fraction represents a number greater than 1
- apply mixed numbers and improper fractions to real-world contexts, including whole sets and whole regions
- express improper fractions as mixed numbers
- express mixed numbers as improper fractions
- translate a given improper fraction between concrete, pictorial and symbolic forms translate a given mixed number between concrete, pictorial and symbolic forms.

Task-specific Criteria

Each student will:

- draw and label accurate diagrams to show improper fractions and mixed numbers used in the problems
- translate a given improper fraction between concrete, pictorial and symbolic forms
- translate a given mixed number between concrete, pictorial and symbolic forms
- explain clearly, using appropriate mathematical language, the relation between mixed numbers and improper fractions, as well as the role of the numerator and denominator of a fraction
- express an improper fraction as a mixed number
- express a mixed number as an improper fraction
- estimate the value of a mixed number using an appropriate strategy, such as benchmarks
- solve problems using mixed numbers and improper fractions.

Teacher Notes

- Summary of task:
  In this assessment task, students will solve problems to demonstrate their understanding of mixed numbers and improper fractions by using concrete materials, as needed, diagrams and symbols. They will express an improper fraction as a mixed number, using a party bag with four items or a multiple of four items in it as the whole set. Then they will estimate the size of a mixed number and express it as an improper fraction using a pizza as the whole region.
- Materials required: counters, bags or paper cups to represent the party bags, fraction strips and fraction circles.

Students should be able to express mixed numbers as improper fractions as well as express improper fractions as mixed numbers. They should be able to show these relationships using problem-solving contexts that include whole sets and whole regions. Students should explain what the whole is for each problem as well as the meaning of fractions, including the role of the numerator and the denominator.

The numerator *counts* and the denominator *tells what is being counted*. Another explanation is that the denominator is the number of equal parts in the whole and the numerator is the number of those equal parts that the fraction represents. Equal parts of a region have the same area or size but not necessarily the same shape. Equal parts of a set have the same number of items in each part.

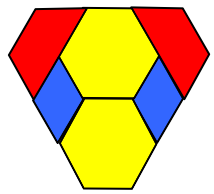
For the problem in which $\frac{15}{4}$ party bags are filled, the denominator tells what is being counted; i.e., quarters. In other words, "the denominator of the fraction indicates by what number the whole has been divided in order to produce the type of part under consideration" (Van de Walle and Lovin 2006, p. 66).

The numerator counts the quarters to a total of 15 quarters. Since the denominator is 4, there could be four treats in each party bag or a multiple of four. Suppose there are eight treats in each party bag, then the improper fraction equivalent to $\frac{15}{4}$ would be $\frac{30}{8}$. The mixed number is $3\frac{3}{4}$ or $3\frac{6}{8}$.

For the problem with the $3\frac{5}{8}$ pizzas, the pizzas can be divided into eight equal pieces or a multiple of eight such as 16. The mixed number equivalent to $3\frac{5}{8}$ would be $3\frac{10}{16}$. The improper fraction is $\frac{29}{8}$ or $\frac{58}{16}$.

Early finishers can:

- find the amount of money that is $\frac{3}{2}$ of \$12
- find the number of coins in the whole set of coins if 12 coins are $\frac{4}{3}$ of the whole set
- find at least three different names for the following design if the hexagon represents the whole.



A Party Using Mixed Numbers and Improper Fractions— Student Assessment Task

You are having a party at your home. You serve pizzas and make party bags for your friends.

1. You put the same number of treats in each party bag. So far, you have filled $\frac{15}{4}$ party bags.

   a. Draw a diagram to show the party bags that you have filled. Show the treats inside each party bag. (You may show this in more than one way.)

   b. Write the number of party bags you have filled as a mixed number. Explain your thinking. (You may show this in more than one way.)

2. At the party, you and your friends eat $3\frac{5}{8}$ pizzas.

   a. About how many pizzas do you and your friends eat in all? Circle the BEST answer. Justify your response.

      i) 3 pizzas ii) $3\frac{1}{2}$ pizzas iii) 4 pizzas

   b. Each pizza is the same size. Draw a diagram to show the pizzas eaten. (You may show this in more than one way.)

   c. Write an improper fraction to show how much pizza was eaten. Explain your thinking. (You may show this in more than one way.)

Scoring Guide:
A Party Using Mixed Numbers and Improper Fractions

Student: ______________________________

Level / Criteria	4 Excellent	3 Proficient	2 Adequate	1 Limited*	Insufficient / Blank*
Expresses an improper fraction as a mixed number using the whole as a set. Question 1	The student draws very accurate diagrams of at least two ways to fill party bags and explains clearly, using precise mathematical language, how improper fractions can be expressed as mixed numbers. ☐	The student draws an accurate diagram of the filled party bags and provides a clear explanation of how an improper fraction can be expressed as a mixed number. ☐	The student draws a diagram of the filled party bags and provides a limited explanation of how an improper fraction can be expressed as a mixed number. ☐	The student draws inaccurate or no diagrams of the filled party bags and provides an incorrect or no explanation of how an improper fraction can be expressed as a mixed number. ☐	No score is awarded because there is insufficient evidence of student performance, based on the requirements of the assessment task. ☐
Estimates the value of a mixed number. Question 2 (a)	The student explains clearly an appropriate strategy, such as benchmarks, to estimate the amount of pizza eaten. ☐	The student explains an appropriate strategy, such as benchmarks, to estimate the amount of pizza eaten. ☐	The student correctly estimates the amount of pizza eaten, but has difficulty explaining the strategy used. ☐	The student incorrectly estimates the amount of pizza eaten. ☐	No score is awarded because there is insufficient evidence of student performance, based on the requirements of the assessment task. ☐
Expresses a mixed number as an improper fraction using diagrams and symbols. Questions 2 (b) and 2 (c)	The student draws very accurate diagrams of the pizzas in at least two ways and explains clearly, using precise mathematical language, how mixed numbers can be expressed as improper fractions. ☐	The student draws an accurate diagram of the pizzas and provides a clear explanation of how a mixed number can be expressed as an improper fraction. ☐	The student draws a diagram of the pizzas and provides a limited explanation of how a mixed number can be expressed as an improper fraction. ☐	The student draws inaccurate or no diagram of the pizzas and provides an inaccurate or no explanation of how a mixed number can be expressed as an improper fraction. ☐	No score is awarded because there is insufficient evidence of student performance, based on the requirements of the assessment task. ☐

* When work is judged to be limited or insufficient, the teacher makes decisions about appropriate intervention to help the student improve.

Student Learning Goals

- Area of need (what's hard for me): ______________________________
  - Action
- Strength to strengthen: ______________________________
  - Action

B. One-on-one Assessment

To help students solve problems related to the outcome:

1. **Review the Meaning of Fractions**
   If the student has difficulty representing fractions concretely, pictorially and symbolically, spend time developing understanding of the meaning of fractions.

   Review the meaning of fractions as part of a whole region or a whole set by using a variety of manipulatives, such as paper strips, fraction strips, fraction circles, pattern blocks and counters. Have the student write the fraction that is represented by the shaded or coloured part (concrete to symbolic). Then have the student draw a diagram to represent the fraction shown by the manipulatives (concrete to pictorial). For example, provide the student with a diagram that shows two out of three equal parts of a circle shaded and have the student write the appropriate fraction and explain his or her thinking.

   Provide real-world contexts that develop understanding of fractions as division of numbers; e.g., if you share three pizzas of the same size equally among four people, each person gets $\frac{3}{4}$ of a pizza. Connect the concrete, pictorial and symbolic forms.

   Review the meaning of the numerator and the denominator using whole sets and whole regions. Remind the student that the denominator is the number of equal parts in the whole. Equal parts of a region have the same area but not necessarily the same shape. Equal parts of a set have the same number of items in each part. The numerator is the number of equal parts the fraction represents. Also, explain that the numerator *counts* and the denominator *tells what is being counted.*

   After students understand fractions by translating from the concrete to the pictorial to the symbolic representations, reverse the process and have students demonstrate understanding by translating from the symbolic to the pictorial to the concrete. For example, have the student use pattern blocks to show that three out of five of a set of blocks are triangles.

2. **Connect Proper Fractions to Improper Fractions**
   Connect the meaning of proper fractions to the meaning of improper fractions through the use of drawings and manipulatives.

3. **Express Improper Fractions as Mixed Numbers**
   Ask the student to draw a diagram to show $\frac{5}{4}$ pizzas and write it as a mixed number. If the student has difficulty, focus on the whole region; i.e., the whole pizza. Explain that it is divided into four equal parts because the denominator is 4. Since the whole has four quarters and you need five quarters, suggest that the student draw one more quarter of a pizza. This might be done by drawing another whole pizza and shading in one quarter of the pizza.

Then ask the student to write $\frac{5}{4}$ pizzas as a whole number and a fraction; i.e., a mixed number. Point to the whole pizza and have the student write the whole number 1. Then point to the quarter pizza and have the student write one quarter. This can be written as $1 + \frac{1}{4}$ which is the same as $1\frac{1}{4}$.

Remind the student that the amount of pizza written as an improper fraction or a mixed number represents the same amount of pizza.

Connect the previous work with the meaning of fractions as part of a unit whole to the problems that develop understanding of fractions as the division of numbers; e.g., if you divide five pizzas of the same size equally among four people, how much pizza does each person get?

4. Express Mixed Numbers as Improper Fractions

Reverse the procedure. Ask the student to draw a diagram to show $2\frac{2}{3}$ cakes and write it as an improper fraction. If the student has difficulty, focus on the whole cake. Explain that it must be divided into three equal parts because the denominator of the fraction is 3. Suggest that the student draw two whole cakes that are the same size, each divided into three equal parts that are all shaded. Then have the student draw another whole cake the same size divided into three equal parts but shade in only two of the equal parts because the numerator of the fraction is 2.

Have the student count the number of equal parts that are shaded. Remind the student that each equal part is one-third of the whole so count: one-third, two-thirds, three-thirds, four-thirds, five-thirds, six-thirds, seven-thirds and eight-thirds. Finally, have the student write an improper fraction for eight-thirds. Encourage the student to write an equation, $2\frac{2}{3} = \frac{8}{3}$, to show that the mixed number and the improper fraction represent the same amount of cake.

Reinforce the concepts of mixed number and improper fractions by using contexts that include whole sets as well as whole regions.

C. Applied Learning

Provide opportunities for students to use their improper fractions and mixed numbers in a practical situation and notice whether or not the concepts transfer.

For example, ask the student to solve the following problem and explain the thinking done to solve this problem.

Would you rather have $3\frac{2}{5}$ pizzas or $\frac{16}{5}$ pizzas?

Does the student:

- express both amounts as either mixed numbers or improper fractions?
- estimate that both fractions represent about the same amount?
- recognize that both amounts are provided in fifths so only the tops (numerators) of the improper fractions have to be compared?
- explain clearly the strategy used in solving the problem?
- apply these concepts to other real-world problems successfully?

Step 5: Follow-up on Assessment

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction?

A. Addressing Gaps in Learning

Students who have difficulty solving the basic facts using strategies will enjoy more success if one-on-one time is provided in which there is open communication to diagnose where the learning difficulties lie. Assessment by observing a student solving problems will provide valuable data to guide further instruction. Success in problem solving depends on a positive climate in which the students are confident in taking risks. By building on the existing understandings of each student and accommodating the individual learning styles, success will follow.

If the difficulty lies in solving the basic facts using strategies, use the following strategies:

- Draw on the prior knowledge of students, spending time reviewing proper fractions as part of a region and part of a set. Review the meaning of fractions and how they relate to a part and to a whole. Also, review the meaning of fractions as the division of numbers.
- Emphasize the similarities and differences between part of a region and part of a set.
- Provide everyday problem-solving contexts that students can relate to.
- Use a variety of concrete materials, such as folding paper, fraction strips, fraction circles, pattern blocks and geoboards. Connect the concrete to diagrams and symbols as students develop understanding of improper fractions and mixed numbers.
- Allow students to use concrete materials, as long as necessary, to establish an understanding of the concepts.
- Emphasize the similarities and differences between improper fractions and mixed numbers.
- Prior to ordering mixed numbers and improper fractions, review ordering proper fractions using benchmarks (Grade 4, Number, Outcome 8) and equivalent fractions (Grade 5, Number, Outcome 7). Encourage flexibility in choosing strategies to order fractions.
- Connect the number line to concrete fraction strips, recognizing that the number line is very abstract for many students.
- Encourage students to skip count on the number line to locate the position of various fractions; e.g., to find $\frac{5}{4}$ on the number line, skip count by quarters. Remind students that the numerator *counts* and the denominator tells *what is being counted.*
- Ask guiding questions to direct student thinking. See the examples provided in **Step 4, Part B: One-on-one Assessment** (pp. 43–44).
- Provide time for students to explore and construct their own meaning rather than being told.
- Encourage flexibility in thinking as students describe various ways to connect mixed numbers with improper fractions and to order fractions.

- Have students share their thinking with others so that students, who are having some difficulty, hear (in 'kid' language) how another person thinks about improper fractions and mixed numbers.

B. Reinforcing and Extending Learning

Students who have achieved or exceeded the outcomes will benefit from ongoing opportunities to apply and extend their learning. These activities should support students in developing a deeper understanding of the concept and should not progress to the outcomes in subsequent grades.

Consider the following strategies:

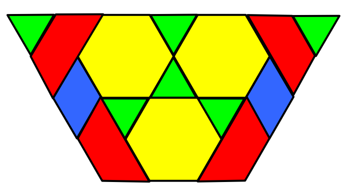
- Provide tips for parents on using mixed numbers and improper fractions at home or in the community.
- Order different lengths of string using fractional measures. Order the symbolic fractions first and then verify the answer by using the strings and placing them side by side, aligning them along a given line.
- Use examples with fractions to compare two quantities, such as comparing $\frac{11}{8}$ pepperoni pizzas with $1\frac{2}{5}$ ham and pineapple pizzas—each whole pizza is the same size.
- In comparing lengths or pizzas, use a variety of different fractions; e.g., same numerator and different denominators, same denominator and different numerators, fractions greater than $1\frac{1}{2}$ and fractions less than $1\frac{1}{2}$.
- Ask for a length of rope that is between two measures, such as $1\frac{3}{4}$ m and $1\frac{9}{10}$ m.
- Determine how many pieces of pizza a family would eat if the family eats $2\frac{1}{4}$ pizzas with each pizza divided into eight equal pieces.
- Provide students with a set of numbers, including whole numbers, proper fractions, improper fractions and mixed numbers. Have them sort the numbers into groups and justify the sorting rule using an open sort. Decide if a closed sort is necessary for some students. Use a closed sort by providing the categories, such as proper fractions, mixed numbers, improper fractions and whole numbers.
- Provide students with a mixed number or improper fraction and ask them to find three fractions that are close in value to the fraction and explain why.
- Have students write at least five fractions that are between $1\frac{9}{10}$ and 1 and encourage them to explain their thinking.
- Provide students with a set of proper and improper fractions in which each fraction has an equivalent fraction. Have the students pair each fraction with the equivalent fraction and explain why they make a pair (Van de Walle and Lovin 2006, p. 116).

- Challenge students to order fractions, such as $1\frac{11}{20}$, $\frac{8}{5}$, $\frac{17}{12}$ and $1\frac{7}{15}$. Encourage them to use a variety of strategies, including changing mixed numbers to improper fractions and then finding the lowest common denominator.
- Present students with the following problem.

  Draw a strip that is 6 cm long. This strip represents $\frac{3}{5}$ of the whole strip. Draw a strip to show each of the following and explain your thinking:
  - the whole strip
  - a strip to show $\frac{1}{2}$
  - a strip to show $\frac{8}{5}$
  - a strip to show $1\frac{1}{5}$.
- Provide students with triangular dot paper or an isometric grid. (*Use BLM 2 on page 26 from Sample Activity 5.*) Present students with the following problem.

  You have $2\frac{2}{3}$ hexagonal cakes.
  - Draw these cakes on dot paper.
  - Write this amount in three different ways.
- Provide students with pattern blocks and the following design. You may wish to draw the design on triangular dot paper or have students draw it on the dot paper. (*Use BLM 2 on page 26 from Sample Activity 5.*)

  Then have students write three different names for the following design. Explain that the hexagon represents one whole.



  Encourage students to create their own designs and challenge other students to find different number names for it (Hope 1997, p. 25).

- Provide students with the following problem.
  If $15 is five-thirds of the money that is in my wallet, how much money do I have in my wallet?

- Proper and Improper Fraction Game (Cathcart, Pothier and Vance 1994, p. 360)
  Materials: two standard dice (one red and one green) and a recording sheet.
  Two players, labelled A and B, play this game.

  Procedure:

  - Roll the dice. Make a fraction by using the number from the red die as the numerator and the number from the green die as the denominator.
  - If the fraction is greater than 1, player A wins. If the fraction is less than 1, player B wins. If the fraction is equal to 1, both players score one point.
  - Repeat this 16 times for one game. The player with the most points after 16 rolls wins that game.
  - Play several games, then answer these questions:

    - Did the same player win each time?
    - Do both players have the same chance of winning?
    - Is this a fair game?

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