Mathematics

Planning Guide

Grade 6
Ratio and Percent

Number
Specific Outcomes 5 and 6

This Planning Guide can be accessed online at:
http://www.learnalberta.ca/content/mepg6/html/pg6_ratiopercent/index.html

Table of Contents

Planning Guide: *Grade 6 Ratio and Percent*

Strand: Number
Specific Outcomes: 5 and 6

This *Planning Guide* addresses the following outcomes from the program of studies:

Strand: Number	
Specific Outcomes:	5. Demonstrate an understanding of ratio, concretely, pictorially and symbolically. 6. Demonstrate an understanding of percent (limited to whole numbers), concretely, pictorially and symbolically.

Curriculum Focus

This sample targets the following changes to the curriculum:

- The general outcome focuses on number sense, whereas the previous program of studies specified demonstrating a number sense for decimals and common fractions, exploring integers and showing number sense for whole numbers.
- The specific outcomes include demonstrating an understanding of ratio and percent (limited to whole numbers), concretely, pictorially and symbolically, which is very similar to the related outcomes in the previous program of studies.

What Is a Planning Guide?

Planning Guides are a tool for teachers to use in designing instruction and assessment that focuses on developing and deepening students' understanding of mathematical concepts. This tool is based on the process outlined in *Understanding by Design* by Grant Wiggins and Jay McTighe.

Planning Steps

The following steps will help you through the Planning Guide:

Step 1: Identify Outcomes to Address

Guiding Questions

- What do I want my students to learn?
- What can my students currently understand and do?
- What do I want my students to understand and be able to do based on the Big Ideas and specific outcomes in the program of studies?

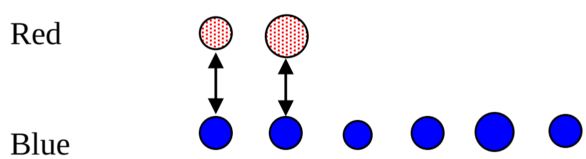
Big Ideas

Ratios
Ratios are an extension of one-to-one matching and one-to-many matching.

For example, when comparing two red marbles to six blue marbles, the following statements could be made:

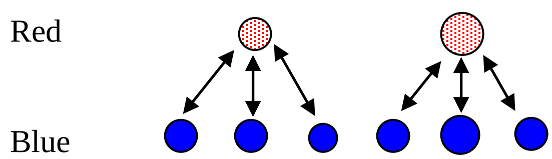
a. There are four more blue marbles than red marbles.
b. There are $\frac{2}{6}$ or $\frac{1}{3}$ as many red marbles as blue marbles.
c. There are $\frac{6}{2}$ or three times as many blue marbles as red marbles.

Statement (a) uses one-to-one matching along with subtraction. The diagram below shows this matching:



Every red marble is matched with a blue marble and there are four blue marbles left. 6 – 2 = 4.

Statement (b) uses one-to-many matching. The diagram below shows this matching:

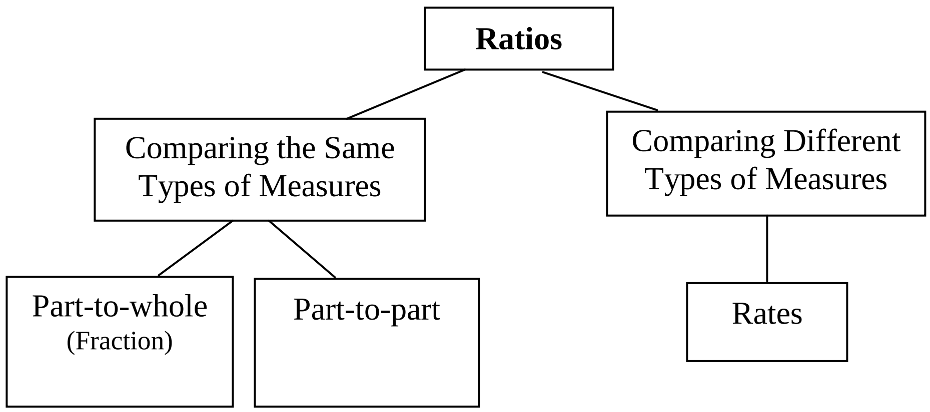


Every red marble is matched with three blue marbles and there are no marbles left over. The ratio of the number of red marbles to the number of blue marbles is 2 to 6 or 1 to 3. Other ways to write the ratio include $\frac{1}{3}$ or 1:3. This ratio compares the two amounts; i.e., there are $\frac{1}{3}$ as many red marbles as blue marbles.

Statement (c) uses many-to-one matching.
(Lilly 1999, p. 178)

Definition of Ratio
Small (2009, p. 74) states, "a ratio compares quantities with the same unit; for example, three boys to two girls (the unit being children)."

Van de Walle and Lovin (2006, p. 154) state, "a ratio is a comparison of any two quantities. A key developmental milestone is the ability of a student to begin to think of a ratio as a distinct entity, different from the two measures that made it up." The authors go on to say that ratios include rates as shown by the tree diagram below (2006, p. 155).



Part-to-part Ratios
"A ratio can compare one part of a whole to another part of the same whole. For example, the number of girls in a class can be compared to the number of boys" (Van de Walle and Lovin 2006, p. 155).

In the previous example that compares the number of red marbles to the number of blue marbles, a part-to-part ratio is used; i.e., 2 to 6 or 1 to 3.

Part-to-whole Ratios
"Ratios can express comparisons of a part to a whole; for example, the ratio of girls to all students in the class. Because fractions are also part-to-whole ratios, it follows that every fraction is also a ratio" (Van de Walle and Lovin 2006, pp. 154–155).

In the previous example with marbles, the whole set is eight marbles. Therefore, the ratio of the number of red marbles to all the marbles in the whole set is 2 to 8 or $\frac{2}{8}$ or 2:8. Other ways to write this ratio include 1 to 4, $\frac{1}{4}$ or 1:4. The red marbles make up $\frac{1}{4}$ of the set of marbles.

Within and Between Ratios

A *within* ratio is "a ratio of two measures in the same setting" (Van de Walle and Lovin 2006, p. 169).

In the previous example with marbles, a within ratio would be the ratio of the number of red marbles to the number of blue marbles in the given set of eight marbles.

A *between* ratio is "a ratio of two corresponding measures in different situations. In the case of similar rectangles, the ratio of the length of one rectangle to the length of another is a between ratio, that is, it is 'between' the two rectangles" (Van de Walle and Lovin 2006, p. 169).

The focus of Grade 6 is *within* ratios.

Proportional Thinking

Proportional reasoning is used in ratios and percents. Van de Walle and Lovin (2006, p. 154) state, "It is the ability to think about and compare multiplicative relationships between quantities. These relationships are represented symbolically as ratios." They go on to define *proportion* as "a statement of equality between two ratios" (2006, p. 156).

Although students will not use proportional reasoning to solve problems until Grade 8, they need to use proportional thinking to understand the relationship between ratios and percent.

Definition of Percent

Percent is a part-to-whole ratio that compares a quantity to 100. It means "out of 100" and is written as % (http://learnalberta.ca/content/memg/index.html).

"The term *percent* is simply another name for *hundredths*" (Van de Walle and Lovin 2006, p. 119).

Small (2009, p. 80) states, "The actual amount that a percent represents is based on the whole of which it is a percent." She goes on to say, "It is not possible to interpret a percent meaningfully without knowing what the whole is" (2009, p. 81).

Since percent is a part-to-whole ratio, then every percent can be written as a fraction or a decimal. Conversely, every fraction or decimal can be written as a percent.

Principles and Standards for School Mathematics states:

> Percent, which can be thought about in ways that combine aspects of both fractions and decimals, offer students another useful form of rational number. Percents are particularly useful when comparing fractional parts of sets or numbers of unequal size, and they are also frequently encountered in problem-solving situations that arise in everyday life (NCTM 2000, p. 217).

Van de Walle and Lovin (2006, p. 175) connect percents to fractions when they state, "all percent problems are exactly the same as the equivalent fraction examples. They involve a part and a whole measured in some unit and the same part and whole measured in hundredths, that is, in percents."

Sequence of Outcomes from the Program of Studies

See http://www.education.alberta.ca/teachers/program/math/educator/progstudy.aspx for the complete program of studies.

Grade 5	Grade 6	Grade 7
Specific Outcomes	**Specific Outcomes**	**Specific Outcomes**
7. Demonstrate an understanding of fractions by using concrete, pictorial and symbolic representations to: • create sets of equivalent fractions • compare fractions with like and unlike denominators.	5. Demonstrate an understanding of ratio, concretely, pictorially and symbolically. 6. Demonstrate an understanding of percent (limited to whole numbers), concretely, pictorially and symbolically.	3. Solve problems involving percents from 1% to 100%.

Step 2: Determine Evidence of Student Learning

Guiding Questions

- What evidence will I look for to know that learning has occurred?
- What should students demonstrate to show their understanding of the mathematical concepts, skills and Big Ideas?

Using Achievement Indicators

As you begin planning lessons and learning activities, keep in mind ongoing ways to monitor and assess student learning. One starting point for this planning is to consider the achievement indicators listed in *The Alberta K–9 Mathematics Program of Studies with Achievement Indicators*. You may also generate your own indicators and use them to guide your observation of the students.

The following indicators may be used to determine whether or not students have met the specific outcomes. Can students:

- provide a concrete or pictorial representation for a given ratio?
- write a ratio from a given concrete or pictorial representation?
- express a given ratio in multiple forms, such as 3:5, $\frac{3}{5}$ or 3 to 5?
- identify and describe ratios from real-life contexts and record them symbolically?
- explain the part-to-whole and part-to-part ratios of a set; e.g., for a group of three girls and five boys, explain the ratios 3:5, 3:8 and 5:8?
- explain that comparison of ratios requires comparison of the same size whole?
- solve a given problem involving ratio?
- explain that "percent" means "out of 100"?
- explain that percent is a ratio out of 100?
- use concrete materials and pictorial representations to illustrate a given percent?
- record the percent displayed in a given concrete or pictorial representation?
- express a given percent as a fraction and a decimal by connecting concrete, pictorial and symbolic representations?
- express a given fraction or a given decimal as a percent by connecting concrete, pictorial and symbolic representations?
- identify and describe percents from real-life contexts and record them symbolically?
- solve a given problem involving percents?
- explore the comparison of percents?
- solve part–whole–percent problems in which two out of three are provided and the third must be found?

Sample behaviours to look for related to these indicators are suggested for some of the activities listed in **Step 3, Section C: Choosing Learning Activities** (p. 12).

Step 3: Plan for Instruction

Guiding Questions

- What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should I use?
- How will I meet the diverse learning needs of my students?

A. Assessing Prior Knowledge and Skills

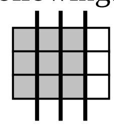
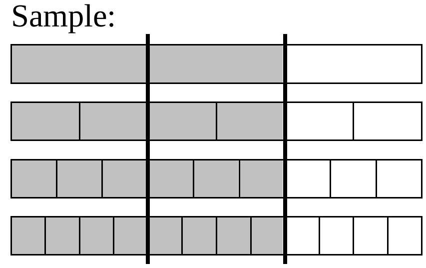
Before introducing new material, consider ways to assess and build on students' knowledge and skills related to fractions and decimals. Provide manipulatives such as fraction strips, pattern blocks or fraction circles for the students to use, as needed.

For example:

- The area of a room is 12 square metres. If a rug covers $\frac{3}{4}$ of the room, what is the area of the rug? Explain by using diagrams and symbols.
- Write four fractions that are equivalent to $\frac{2}{3}$. Explain by using diagrams and symbols.
- Jack eats $\frac{2}{5}$ of a cake. Jill eats $\frac{3}{10}$ of the same cake. Who eats more cake or do they each eat the same amount of cake? Explain by using diagrams and symbols.
- Place the following fractions on a number line and explain how you decided on the order.
  $\frac{5}{8}, \frac{9}{10}, \frac{2}{5}, \frac{1}{8}, \frac{1}{4}$
- Write the following fractions as decimals:
  a. $\frac{3}{10}$ b. $\frac{5}{100}$ c. $\frac{12}{1000}$

If a student appears to have difficulty with these tasks, consider further individual assessment, such as a structured interview, to determine the student's level of skill and understanding. See **Sample Structured Interview: Assessing Prior Knowledge and Skills** (p. 9).

Sample Structured Interview: Assessing Prior Knowledge and Skills

Directions	Date:	
	Not Quite There	**Ready to Apply**
Provide manipulatives such as fraction strips, pattern blocks or fraction circles, as needed. Place the following problem before the student: **The area of a room is 12 square metres. If a rug covers $\frac{3}{4}$ of the room, what is the area of the rug? Explain by using diagrams and symbols.**	Does not understand the problem as an application of equivalent fractions. States the correct answer but is unable to explain the process by using diagrams and symbols.	Draws a diagram and uses symbols to explain the process in the solution, such as the following:  $\frac{3}{4}=\frac{9}{12}$ "There are 3 square metres in each quarter; therefore, 3 of the quarters have an area of 9 square metres." Answers the problem correctly, such as the following: "The area of the rug is 9 square metres."
Provide manipulatives such as fraction strips, pattern blocks or fraction circles, as needed. Say, **"Write four fractions that are equivalent to $\frac{2}{3}$. Explain by using diagrams and symbols."**	Writes no fractions equivalent to $\frac{2}{3}$. OR Writes one or two fractions equivalent to $\frac{2}{3}$ with no explanation. OR Writes four fractions equivalent to $\frac{2}{3}$ and states a rule but neglects to use diagrams in the explanation.	Writes four fractions that are equivalent to $\frac{2}{3}$ and explains why they are equivalent. Sample:  For each third, there are two-sixths, three-ninths and four-twelfths. Therefore, two-thirds is equivalent to twice as much as one-third, namely, four-sixths, six-ninths and eight-twelfths. The equivalent fractions are: $\frac{2}{3}=\frac{4}{6}=\frac{6}{9}=\frac{8}{12}$.

<table>
<tr><th rowspan="2">Directions</th><th colspan="2">Date:</th></tr>
<tr><th>Not Quite There</th><th>Ready to Apply</th></tr>
<tr><td>Provide manipulatives such as fraction strips or fraction circles, as needed.

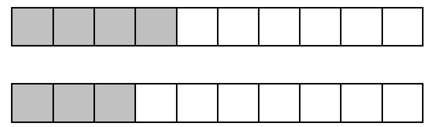
Place the following problem before the student:

Jack eats $\frac{2}{5}$ of a cake. Jill eats $\frac{3}{10}$ of the same cake. Who eats more cake or do they each eat the same amount of cake? Explain by using diagrams and symbols.</td><td>Answers incorrectly with little or no attempt at an explanation.

OR

Answers correctly but lacks clarity in the explanation or provides no explanation.</td><td>Answers correctly and clearly explains using diagrams and symbols.

Sample:



The diagram shows the same size cake but the first cake is cut into 5 equal pieces and the second cake is cut into 10 equal pieces. The diagram shows that 2 out of 5 equal pieces of cake is greater than 3 out of 10 equal pieces of the same size cake. Therefore, Jack eats more of the cake than Jill.</td></tr>
<tr><td>Place the following problem before the student:

Write the following fractions as decimals:
a) $\frac{3}{10}$ b) $\frac{5}{100}$
c) $\frac{12}{1000}$</td><td>Answers some or all of the parts incorrectly.

Does not connect the fraction to the equivalent decimal notation consistently.

May or may not include the zero in the ones' place.</td><td>Answers all of the parts correctly as follows:
a) 0.3 b) 0.05 c) 0.012</td></tr>
</table>

B. Choosing Instructional Strategies

Consider the following instructional strategies for teaching ratio and percent.

- Introduce and reinforce the ratio and percent by using a variety of real-world contexts that include whole regions and whole sets.
- Connect the ratio and percent to prior knowledge about the meaning of fractions and equivalent fractions. Explore the similarities and differences between ratios and fractions.
- Emphasize the importance of establishing what the whole region or the whole set is before finding ratios or percents related to regions or sets.
- Encourage students to communicate their thinking by connecting manipulatives, diagrams and symbols to represent the concepts.
- Introduce ratios first, including part-to-whole and part-to-part ratios, and then introduce percent as a specific ratio (part-to-whole) out of 100. Explore the similarities and differences between ratios in general and percent.
- "Create a classroom environment that encourages student exploration, questioning, verification and sense making" (NCTM 1992, p. 5).
- Provide students with a variety of problems that apply the concepts of ratio and percent. Encourage them to solve the problems in different ways and explain the process. Also, provide time for students to share their solutions with others. Stimulate class discussion to critically evaluate the various procedures. Emphasize understanding, flexibility and efficiency when students select problem-solving strategies.
- To promote flexible thinking, provide a variety of problems in which two out of the following three are given and the third must be found: the whole, the part and the percent (NCTM 2000, p. 215).
- "Use the terms part, whole, and percent (or fraction). Fraction and percent are interchangeable. Help students see … percent exercises as the same types of exercises they did with simple fractions" (Van de Walle and Lovin 2006, p. 121).
- "Require students to use models or drawings to explain their solutions. It is better to assign three problems requiring a drawing and an explanation than to give 15 problems requiring only computation and answers. Remember that the purpose is the exploration of relationships, not computational skill" (Van de Walle and Lovin 2006, p. 121).
- Explore ratio patterns and encourage students to make and critique generalizations.

C. Choosing Learning Activities

The following learning activities are examples of activities that could be used to develop student understanding of the concepts identified in Step 1.

Sample Activities:

Teaching Ratios

1. **Meaning of Ratio: Part-to-whole and Part-to-part** (p. 14)
2. **Equivalent Ratios** (p. 18)
3. **Comparing Ratios** (p. 21)
4. **Ratio Problems** (p. 22)

Teaching Percent

1. **Meaning of Percent and Connections to Fractions and Decimals** (p. 24)
2. **Connecting Fractions and Percent** (p. 27)
3. **Comparing Percents** (p. 28)
4. **Part-and-whole-percent Tasks** (p. 29)
5. **Frayer Model** (p. 35)

Teaching Ratios

Sample Activity 1: Meaning of Ratio: Part-to-whole and Part-to-part

A. Part of a Set

1. Part-to-part

Look For …

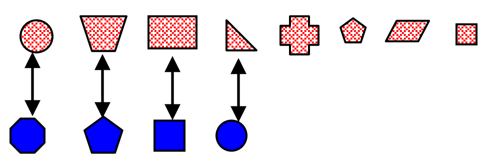
Do students:
- ☐ compare quantities with and without ratios?
- ☐ distinguish between part-to-part ratios and part-to-whole ratios?
- ☐ connect concrete, pictorial and symbolic representations for ratios?
- ☐ explain that order matters when writing ratios?
- ☐ write ratios using three different forms?
- ☐ use ratios in real-world situations including part of a set and part of a region?

Provide students with counters and have them solve the following problem.

Problem
You have eight red counters and four blue counters. Compare the number of red counters to the number of blue counters in as many ways as you can. Explain your thinking.

Have students share their solutions. Include an answer that uses one-to-one matching along with subtraction.
Example 1:

Red

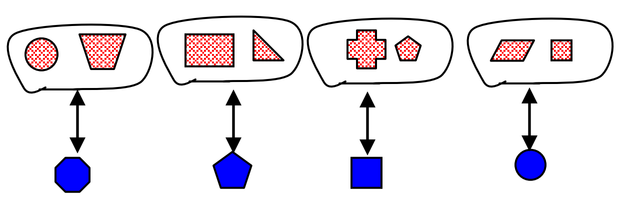


Blue
Every blue counter is matched with a red counter and there are four red counters left.
8 – 4 = 4.

Connect this solution to comparison problems using subtraction. The wording might be, "how many more red counters are there than blue counters?"

Include answers that use many-to-one matching along with multiplication.
Example 2:

Red



Blue

There are twice as many red counters as blue counters.

Introduce the term *ratio*. Explain that the comparison of two quantities related multiplicatively is used in ratios. Note that the definition of ratio as "the comparison of two quantities" has limitations because this comparison could use subtraction, as shown in the first example of one-to-one matching (NCTM 1994, p. 50).

Through discussion, establish that two parts of the whole set are being compared in the given problem; i.e., it is a part-to-part ratio.

Write the ratio of the number of red counters to the number of blue counters in three ways: 8 to 4, 8:4 and $\frac{8}{4}$. Say that the ratio is read as "eight to four." Encourage other ways to write the ratio: 4 to 2, 4:2, $\frac{4}{2}$, 2 to 1, 2:1 and $\frac{2}{1}$.

Emphasize that order is important in writing ratios. Have students write the ratio that compares the number of blue counters to the number of red counters. Encourage students to write the basic ratio to show the comparison; i.e., 1 to 2, 1:2 and $\frac{1}{2}$. Summarize that there are half as many blue counters as red counters.

Provide other examples of part-to-part ratios for part of a set. Have students connect the concrete, pictorial and symbolic representations for each example.

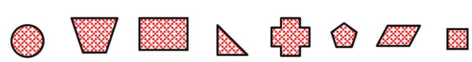
2. **Part-to-whole**

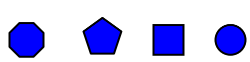
Provide students with counters and have them solve the following problem.

Problem
You have eight red counters and four blue counters. Write a ratio to compare the number of blue counters to the number of counters in the whole set in different ways. Explain your thinking.

Have students share their solutions.

Include an example that shows the part-to-whole ratio: 4 to 12.
Example:

Red 

Blue 

Four out of the 12 counters are blue.
Four-twelfths of the set of counters is blue.

Through discussion, establish that a part is being compared to the whole in the problem; i.e., it is a part-to-whole ratio.

Have students write the ratio of the number of blue counters to the number of all the counters in three ways: 4 to 12, 4:12 and $\frac{4}{12}$. Say that the ratio is read as "four to twelve."

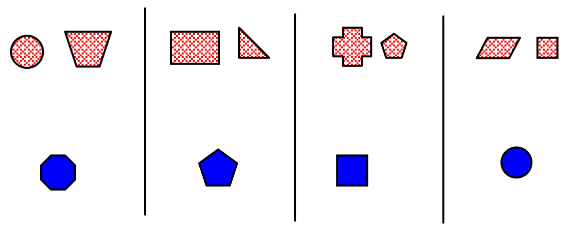
Explain that part-to-whole ratios connect directly to fractions because the denominator of a fraction always refers to the number of equal parts in the whole set. In this case, there are 12 equal parts, each with one item. Review that the equal parts of a set means that each part has the same size; i.e., the same number of items.

Emphasize that the comparison of two quantities using multiplication is used in ratios. Say,
"Four-twelfths of the set of counters is blue."

Include an example that shows the part-to-whole ratio: 1 to 3.
Example:

Red

Blue



One out of every three counters in the set is blue.

The ratio of the number of blue counters to all the counters in the set is 1 to 3, 1:3 or $\frac{1}{3}$.

One-third of the whole set of counters is blue.

Provide other examples of part-to-whole ratios for part of a set. Have students connect the concrete, pictorial and symbolic representations for each example.

B. Part of a Region: Part-to-part and Part-to-whole

Explain that ratios, including part-to-part and part-to-whole, can be used to compare parts of a region as well as parts of a set.

Provide students with concrete materials representing regions such as fraction circles, fraction strips and geoboards. Have students solve the following problem and explain their thinking by connecting concrete, pictorial and symbolic representations. Remind students that order counts when they are writing ratios. They must specify what they are comparing and in what order.

Review the requirements for equal parts of a region; i.e., each of the equal parts must have the same size but not necessarily the same shape.

Problem
A pizza is divided equally into 12 pieces: eight are pepperoni and four are vegetarian. Draw diagrams and write ratios to show comparisons of part-to-part and part-to-whole. Explain your thinking.

Have students share their solutions.

Sample Solution of a Part-to-part Ratio:

V	V	V	V
P	P	P	P
P	P	P	P

The ratio of the number of vegetarian pieces to the number of pepperoni pieces is 4 to 8 or 1 to 2. For every vegetarian piece, there are two pepperoni pieces. There are half as many vegetarian pieces as pepperoni pieces.

Through discussion, have the students generalize that order is very important when writing ratios; e.g., a 4 to 8 ratio compares the number of vegetarian pieces to the number of pepperoni pieces, whereas an 8 to 4 ratio compares the number of pepperoni pieces to the number of vegetarian pieces.

Sample Solution of a Part-to-whole Ratio:

V	V	V	V
P	P	P	P
P	P	P	P

The ratio of the number of vegetarian pieces to the number of pieces in the whole pizza is 4 to 12 or 1 to 3. One out of every three pieces is vegetarian. One-third of the pizza is vegetarian.

C. Ratios in Simplest Form or Basic Ratios

Discuss that 4:12 and 1:3 are equivalent ratios similar to equivalent fractions; i.e., they represent the same comparison of two quantities. Encourage students to write ratios in their simplest or basic form whenever possible. The basic ratio for 4:12 is 1:3. Connect basic ratios to basic fractions in work done with equivalent fractions.

Sample Activity 2: Equivalent Ratios

In addition to the work on equivalent ratios from the last activity, provide other problem-solving situations for students to reinforce the concept of equivalent ratios by connecting concrete, pictorial and symbolic representations. Connect the diagrams to ratio tables as another way to solve the problems.

Remind students that ratios focus on how two amounts are related multiplicatively.

Provide students with counters such as double-sided tiles that are red on one side and white on the other side.

Look For …

Do students:

- ☐ solve a given problem involving ratios and explain the process?
- ☐ connect equivalent ratios to equivalent fractions?
- ☐ connect concrete, pictorial and symbolic representations in solving problems with ratios?
- ☐ distinguish between problems using part-to-part and part-to-whole ratios?
- ☐ use patterns with ratio tables in solving ratio problems?
- ☐ create ratio problems?

Problem 1

A jellybean mixture of red and white jellybeans has three red for every two white jellybeans.

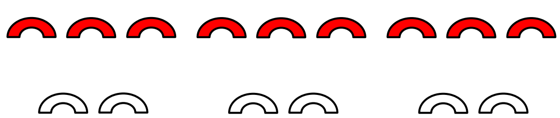
a. What is the ratio of the number of white jellybeans to the number of red jellybeans? Explain.
b. What does the ratio 3:5 mean?
c. How many red jellybeans would be in this mixture with six white jellybeans? Explain your thinking.
d. How many white jellybeans would be in this mixture with 12 red jellybeans? Explain your thinking.

Sample Solution:

a. The ratio of white to red jellybeans is 2 to 3 or 2:3 or $\frac{2}{3}$. There are $\frac{2}{3}$ as many white jellybeans as red jellybeans.
b. The ratio 3:5 means the ratio of the number of red jellybeans to the total number of red and white jellybeans. The red jellybeans make up $\frac{3}{5}$ of all jellybeans.
c. Using a diagram together with a number sentence, triple the number of white jellybeans to make six. Then triple the number of red jellybeans to keep the ratio equivalent.

Red Jellybeans

White Jellybeans



$\frac{\text{red}}{\text{white}} \quad \frac{3}{2} = \frac{\mathbf{9}}{6}$

There are nine red jellybeans in a mixture with six white jellybeans.

Using a ratio table:

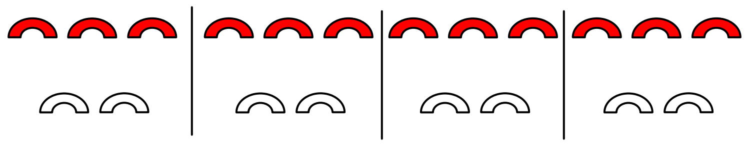
Number of red jellybeans	3	6	**9**
Number of white jellybeans	2	4	6

There are nine red jellybeans in a mixture with six white jellybeans.

d. Using a diagram together with a number sentence, multiply the number of red jellybeans by four to make 12. Then multiply the number of white jellybeans by four to keep the ratio equivalent.

Red Jellybeans

White Jellybeans



$$\frac{\text{red}}{\text{white}} \quad \frac{3}{2} = \frac{12}{8}$$

There are eight white jellybeans in a mixture with 12 red jellybeans.

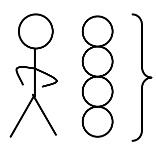
Using a ratio table:

Number of red jellybeans	3	6	9	12
Number of white jellybeans	2	4	6	**8**

There are eight white jellybeans in a mixture with 12 red jellybeans.

Provide other problems to reinforce equivalent ratios and suggest that students use a ratio table if they do not suggest it themselves.

Problem 2
Shorty is four buttons high.
All the buttons have the same width.



Shorty

Stilts, his friend, is six buttons high. If Shorty is six paperclips high, then how many paperclips high will Stilts be? Explain your thinking.

Sample Solution:

Using a ratio table, simplify the 4:6 ratio to make it a basic 2:3 ratio. Then multiply each part of the ratio by 3 because Stilts is 6 buttons high.

	Shorty	Divide each part of the ratio by 2	Multiply each part of the ratio by 3
Number of buttons high	4	2	6
Number of paperclips high	6	3	9

$$\frac{\text{buttons}}{\text{paperclips}} \quad \frac{4}{6} = \frac{2}{3} = \frac{6}{9}$$

Stilts will be nine paperclips high.
(Cathcart, Pothier and Vance 1994, p. 338)

Problem 3
You give 25 cookies to two friends in this way—each time Joshua gets two, Tristian gets three. Predict how many cookies each person will get and then test to check your prediction. Explain your thinking. (Small 2009, p. 75)

Encourage students to create ratio problems.

Sample Activity 3: Comparing Ratios

Provide the following problems to students to stimulate thinking about the need to have the same whole when comparing ratios just as there is a need to have the same whole when comparing fractions. Do not prompt students. Let them explore the situations, discuss the answers and then make a generalization.

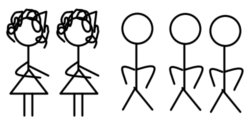
Look For …

Do students:
- ☐ critique real-world ratio situations and justify their answers?
- ☐ connect comparing fractions with comparing ratios; i.e., the whole must be the same when making comparisons?
- ☐ connect concrete, pictorial and symbolic representations when comparing ratios?
- ☐ create real-world situations in which ratios are compared?

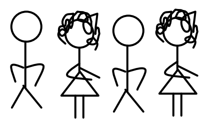
Problem 1 (Van de Walle and Lovin 2006, pp.158–159)

Situation 1: Which class team has more girls compared to the entire class? Explain your thinking.

The Meteors



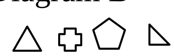
The Comets



Situation 2: Which diagram has more triangles compared to all the shapes in the diagram?

Diagram A

Diagram B



Problem 2 (Cathcart, Pothier and Vance 1994, p. 338)

Which of the following mixtures will be lighter in colour?
Mixture A: six tablespoons of white sugar mixed with eight tablespoons of brown sugar.
Mixture B: four tablespoons of white sugar mixed with three tablespoons of brown sugar.
Explain your thinking.

Problem 3 (Van de Walle and Lovin 2006, p. 162)

Farmer Jones collects four brown eggs for every 10 white eggs in the large hen house. In the smaller hen house, the ratio of brown to white is 1 to 3. In which hen house do the hens lay more brown eggs or is it the same in each?

Encourage students to create other real-world problems that compare ratios.

Sample Activity 4: Ratio Problems

Provide a variety of problems including part-to-whole and part-to-part comparisons. Use problem-solving contexts that include both part of a region and part of a set. Have students create ratio problems for others to solve. Encourage students to explain their thinking by using diagrams, ratio tables and number sentences.

The following problem applies ratios to probability. Review that probability is the ratio of the number of favorable outcomes to the number of total outcomes in a probability experiment.

Look For …

Do students:

- ☐ apply ratios to probability experiments?
- ☐ record the data in an organized way?
- ☐ interpret the data collected to solve the problem?
- ☐ justify the process used to solve the problem?

Problem: The M&M's Experiment

Materials: M&M's for each group of students.
Provide each group of students with a random sample of M&M's from a large bag of M&M's.
Have them examine the M&M's in their samples and answer the following questions:

The M&M's Experiment

a. Estimate the number of M&M's in the large bag.
b. Which colour do you expect to find most often?
c. Which colour do you expect to find least often?
d. Write and describe two ratios using the M&M's in your sample.

Have students count the number of each colour of M&M's.
Record the results in a class tally chart to find out how many M&M's are in the whole bag, what colour appeared most often and what colour appeared least often.

Class Tally Chart

Colour of M&M's	**Tallies**	**Total**	**Ratio of colour total to total number of M&M's**
Brown			
Red			
Yellow			
Green			
Tan			
Orange			
Total # of M&M's: ________			

Ask students to compare their estimates with the actual count of M&M's. Encourage students to explain how they used ratios to help them estimate which colour appeared most often and which colour appeared least often.

Have students compare their ratios with the ratio of the number of each colour of M&M's to the total number of M&M's. (NCTM 1994, pp. 49–56)

Teaching Percent

Sample Activity 1: Meaning of Percent and Connections to Fractions and Decimals

Draw on prior knowledge by reviewing fractions and decimals using hundredths:

- Have students share their experiences in using percent and their understanding of percent.
- Through discussion of various real-world examples, have students generalize that percent is a special part-to-whole ratio that compares a quantity to one hundred.
- Encourage students to describe percent as another name for hundredths so that the connection with fractions is understood.

Look For …

Do students:

- ☐ provide real-world examples of percent?
- ☐ apply their understanding of hundredths to percent?
- ☐ explain that percent is a part-to-whole ratio that is out of 100?
- ☐ connect concrete, pictorial and symbolic representations for percent?
- ☐ write percent as a fraction and a decimal?

Take an example of percent provided by a student and write it using the numeral and the symbol %. By examining the symbol for percent, have students discover that it is a shortcut for writing 100—the slanted one and the two zeros are included in the symbol.

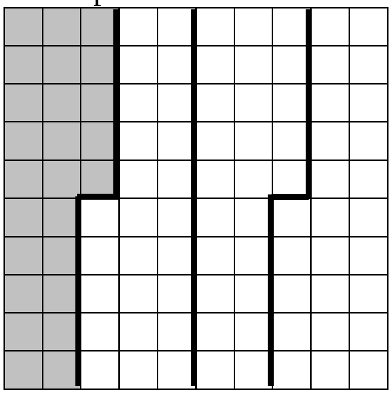
Use the same concrete materials for percent that were used for teaching hundredths, such as base ten materials, place value charts and fraction circles divided into hundredths.

Base Ten Materials, the Hundredths Grid and the Place Value Chart

First, connect the concrete to the pictorial and symbolic representations. Provide the students with base ten materials and hundredths grids.

Review that the flat in the base ten materials represents the whole region or 100%, each of the small units to represent one hundredth or 1% and each long represents one tenth or 10%. Have students place the longs and the units to cover part of the whole flat. Then have them shade in a hundredths grid to show the part of the whole flat that was covered. Finally, have students write the percent, decimal and fraction representations.

Example:



The diagram shows 25% of the whole region shaded. 25% is equivalent to the fraction $\frac{25}{100}$ or $\frac{1}{4}$, and the decimal 0.25. The decimal is shown in the place value chart.

Place Value Chart

ones	tenths	hundredths
0	2	5

The dark line segments in the diagram show that 25% is $\frac{1}{4}$ of the whole.

Encourage students to shade the whole in a variety of ways to show 25% and compare the results. Review that each of the four quarters making up a whole region must have the same size but not necessarily the same shape.

Through a variety of examples, have students generalize how to express any percent as a decimal. For example, percent means hundredths so read the percent as hundredths and then write it as a decimal showing hundredths; e.g., 4% is four hundredths and is written as the decimal 0.04.

Reverse the procedure and connect the symbolic representation of percent to the concrete and pictorial representations. Provide students with a percent such as 75%. Have them represent this percent with the base ten materials and shade in the hundredths grid. Then have them write the percent as a fraction and a decimal.

Fraction Circles Divided into Hundredths

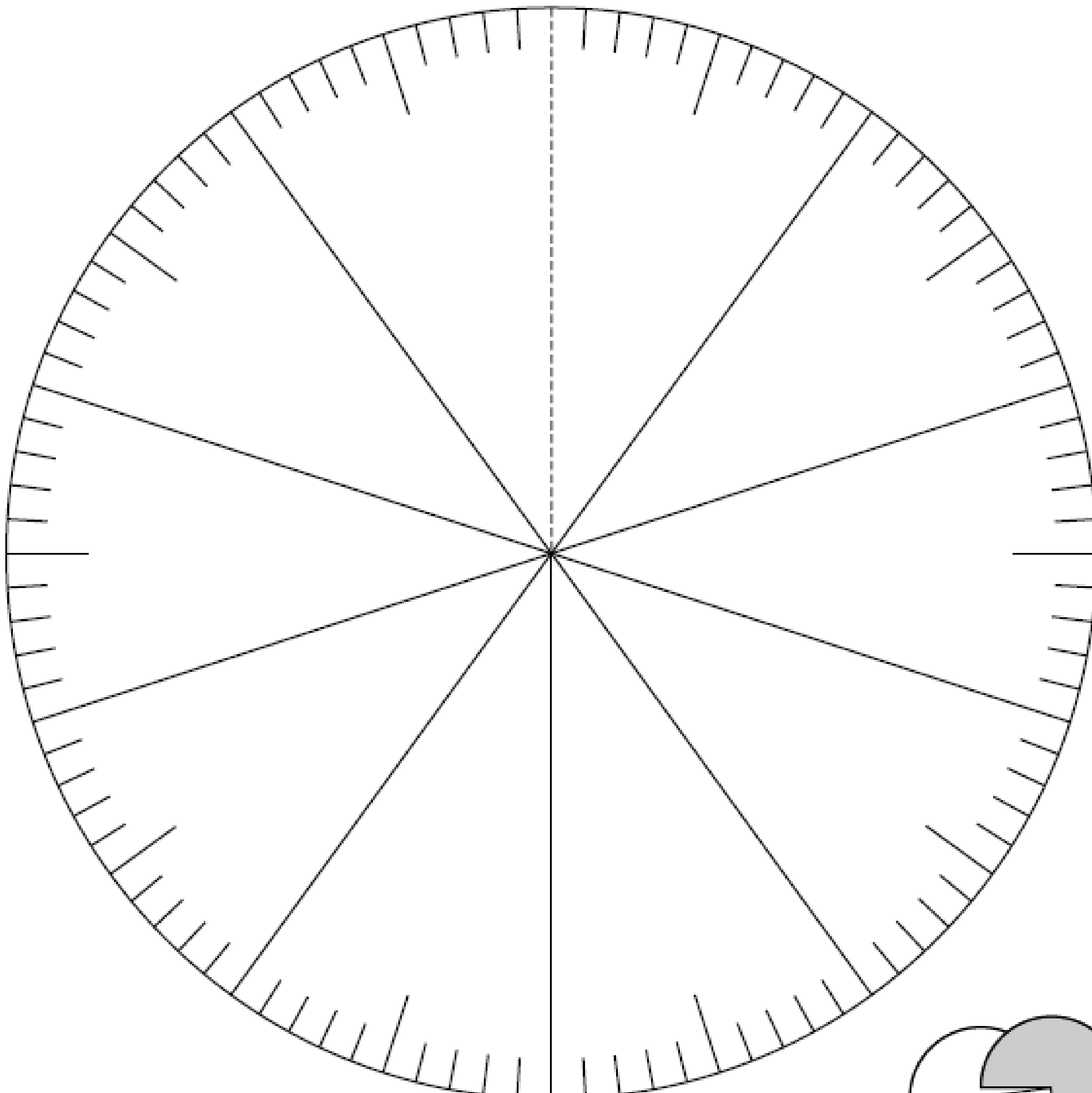
Use BLM 1 on page 26.

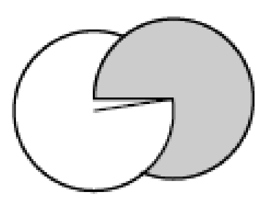
Photocopy two copies of the fraction circles for each student, each in a different colour. Using the design in the corner of the blackline master, intersect the two circles as shown.

Provide each student with the intersected fraction circles. Have them use their circles to show a part of the whole. Then have them explain what percent, fraction and decimal that the part represents. This can be done in small groups or with the whole class.

Reverse the procedure. Provide students with a given percent, fraction or decimal and have them show it on their fraction circles. As they display their fraction circles, have them justify why their circles show the given percent, fraction or decimal. Use this activity as assessment for learning. You can quickly see which students are holding up the circles that correctly show the given percent, fraction or decimal.

Hundreds disk – BLM 1





Sample Activity 2: Connecting Fractions and Percent

Provide students with base ten materials and hundredths grids. Use transparent base ten materials and/or transparent hundredths, tenths and fifths grids on the overhead projector to guide discussion.

See the Diagnostic Mathematics Program: Numeration: Division II for the Blackline Masters for hundredths and tenths (Alberta Education 1990, p. 250).

Look For …

Do students:

- ☐ apply knowledge of equivalent fractions in solving percent problems?
- ☐ connect concrete, pictorial and symbolic representations in solving percent problems?
- ☐ always compare a quantity to 100 when using percent?

Problem

You eat $\frac{3}{5}$ of the cake. What percent of the cake do you eat? Explain your thinking.

Provide guidance, as needed, in solving this problem by connecting the concrete, pictorial and symbolic representations. Have students use the flats from the base ten materials to represent the cake. Discuss what they could use to show $\frac{1}{5}$ of the cake. Answers might include two longs or a strip of paper 2 cm wide and 10 cm long. Have students cover the flat to show $\frac{3}{5}$ or 60%. Encourage a student to show $\frac{3}{5}$ or 60% on the overhead projector using transparent base ten materials or transparencies superimposed on one another (the fifths superimposed on the hundredths).

Tell students to shade in the hundredths grid to show $\frac{3}{5}$ or 60%.

Then have students write a number sentence showing the equivalent ratios:

$\frac{3}{5} = \frac{60}{100}$.

Have students answer the problem: You eat 60% of the cake.

Reverse the procedure and have students write a percent as a fraction in simplest form, using the base ten materials and the hundredths grid to explain their reasoning. Use percents such as 10%, 20%, 25%, 40% and 50%. Have the student generalize how to change a percent to a fraction using a number sentence with equivalent ratios.

Sample Activity 3: Comparing Percents

Provide students with hundredths grids.
Give the following problem to students to stimulate thinking about the need to have the same whole when comparing percents just as there is a need to have the same whole when comparing fractions. Do not prompt students. Let them explore the situation, discuss the answers and then make a generalization.

Problem
Fifty percent of Sam's pets are fish. Twenty percent of Susie's pets are fish. Who has more fish? Explain.

Sample Solution:

It depends on how many pets each person has. If each person has the same number of pets, Sam has more fish. (Small 2009, p. 81)

Encourage students to create real-world problems that compare percents.

Look For …

Do students:

- ☐ critique real-world percent situations and justify their answers?
- ☐ connect comparing fractions with comparing percents; i.e., the whole must be the same when making comparisons?
- ☐ connect concrete, pictorial and symbolic representations when comparing percents?
- ☐ create real-world situations in which percents are compared?

Sample Activity 4: Part-and-whole-percent Tasks

Refer to the part-whole-fraction tasks that are provided in the Planning Guide for Grade 6: Improper Fractions and Mixed Numbers, Number, Specific Outcome 4. Review how to find the third component when two of the part–whole–fraction components are given. Use a similar process for percent to help students make the connection between fractions and percent. Use the word *percent* instead of *fraction* as shown in the following activities.

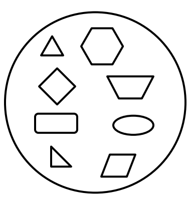
1. Given the Whole and Percent, Find the Part

a. Part of a Whole Set

Provide students with a set of counters, e.g., pattern blocks, and sheet of paper on which a circle is drawn. Instruct them to take eight of the counters and place them in the circle to represent the whole set.

Problem
If eight counters are the whole set, how many counters are in 75% of the set? Explain your thinking by using diagrams and symbols.

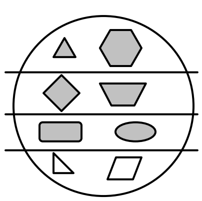
Sample Solution:



This is the whole set.

$75\% = \frac{75}{100} = \frac{3}{4}$

Divide the set into four equal parts with two counters in each part.
Shade three of the equal parts.



The shaded part is 75% or $\frac{3}{4}$ of the whole set of counters.

There are six counters in 75% of the set.

Look For …

Do students:

- ☐ think flexibly about percent as they explore various problems using the whole, the percent and the part?
- ☐ connect pictorial and symbolic forms for percent?
- ☐ identify the whole, the percent and the part for each problem?
- ☐ solve problems using the whole as a region and as a set?
- ☐ communicate clearly, with precise mathematical language, the process used in solving the different types of problems that apply the meaning of percent?
- ☐ create other similar problems that apply the meaning of percent?

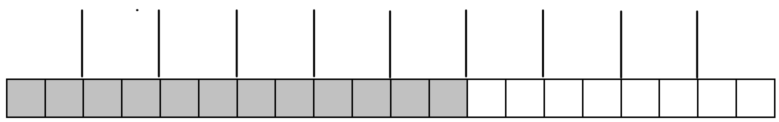
b. Part of a Whole Region

Provide students with strips of paper, each 20 cm long.

Sample Problem
A ribbon is 20 cm long. You cut off 60% of the ribbon. What is the length of the ribbon that you cut off?
Explain your thinking using diagrams and symbols.

Sample Solution
$60\% = \frac{60}{100} = \frac{6}{10}$ Six out of 10 equal parts of the ribbon are cut off.
Since the ribbon is 20 cm long, then each equal part is 2 cm. Therefore the part cut off is 6×2 cm = 12 cm long.

$\frac{6}{10} = \frac{12}{20}$ The shaded part shows the section of the ribbon that is cut off.



cm 0 2 4 6 8 10 12 14 16 18 20
10% 20% 30% 40% 50% 60% 70% 80% 90% 100%

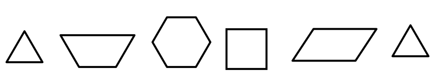
2. Given the Part and Percent, Find the Whole

a. Part of a Whole Set

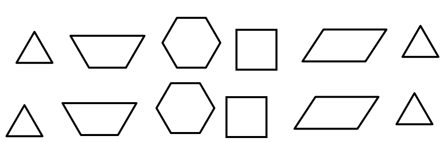
Provide students with a set of counters; e.g., pattern blocks.

Problem 1
If six counters are 50% of the whole set, how big is the whole set? Explain using counters, diagrams and symbols.

The following diagram has six counters and is 50% or one-half of the whole set.

 This diagram shows 50% or $\frac{1}{2}$ of the set.

To find the whole set, double the number of counters to make 12 counters.



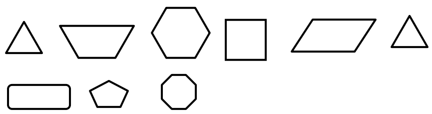
This diagram shows 100% or $\frac{2}{2}$ of the set or the whole set.

The whole set has 12 counters.

Problem 2
If nine counters are 75% of a whole set, how big is the whole set? Explain using counters, diagrams and symbols.

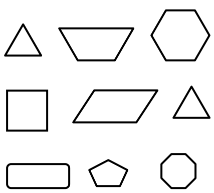
Sample Solution:

The following diagram has nine counters and is 75% or $\frac{3}{4}$ of the whole set. This means that three out of four equal parts are shown.



This diagram is 75% or $\frac{3}{4}$ of the whole set.

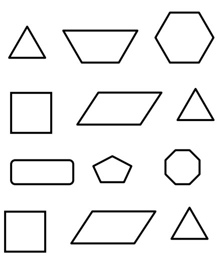
The numerator *counts* and the denominator *tells what is being counted*. Therefore, this set shows three of the four equal parts that make up the whole set. Divide the set into three equal parts as shown below. Each equal part has the same number of items; i.e., three counters in each equal part. Each of the equal parts represents one-fourth or 25% of a whole set.



This diagram is 75% or $\frac{3}{4}$ of the whole set.

$$\frac{3}{4}=\frac{9}{?}$$

To make the whole set, 100% or four-fourths are needed so we must add another equal part consisting of three counters.



$$\frac{3}{4}=\frac{9}{12}$$

This diagram is 100% or $\frac{4}{4}$ of the whole set.

The whole set has 12 counters in all.

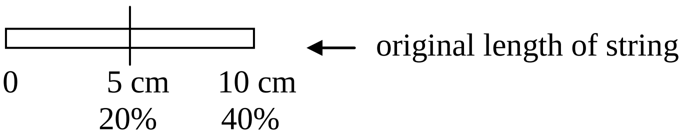
b. Part of a Whole Region

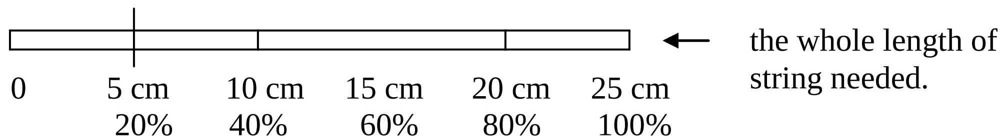
Provide students with the longs from the base ten blocks; each one is 10 cm long. Alternately, provide students with strips of paper, each 10 cm long.

Problem 1
You have 10 cm of string. It is 40% of the length of string you need. What is the length of string that you need? Explain your thinking.

Sample Solution:

I have 40% of the string and I need 100%. If 10 cm is 40%, then 5 cm is 20%. I need five groups of 20% to make 100%. Therefore, I need five groups of 5 cm or 25 cm to make 100% or the whole length of the string.





$$\frac{40}{100} = \frac{10}{25}$$

I need a string that is 25 cm long.

3. Given the Whole and the Part, Find the Percent

a. Part of a Whole Set

Problem

The set of animals below is the whole set.

The birds make up what percent of this set of animals? Explain.

Sample Solution:

Three out of the five animals are birds. Therefore, birds make up $\frac{3}{5}$ of the set of animals.

To change $\frac{3}{5}$ to percent, write an equivalent ratio with the second ratio out of a hundred to show percent.

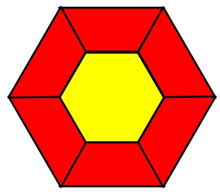
$$\frac{3}{5} = \frac{60}{100}$$

The birds make up 60% of the whole set of animals.

b. Part of a Whole Region

Provide pattern blocks and triangular dot paper (*use BLM 2 on page 34*).

Problem
The design below is from pattern blocks.



What percent of this design does the yellow hexagon represent? Explain.

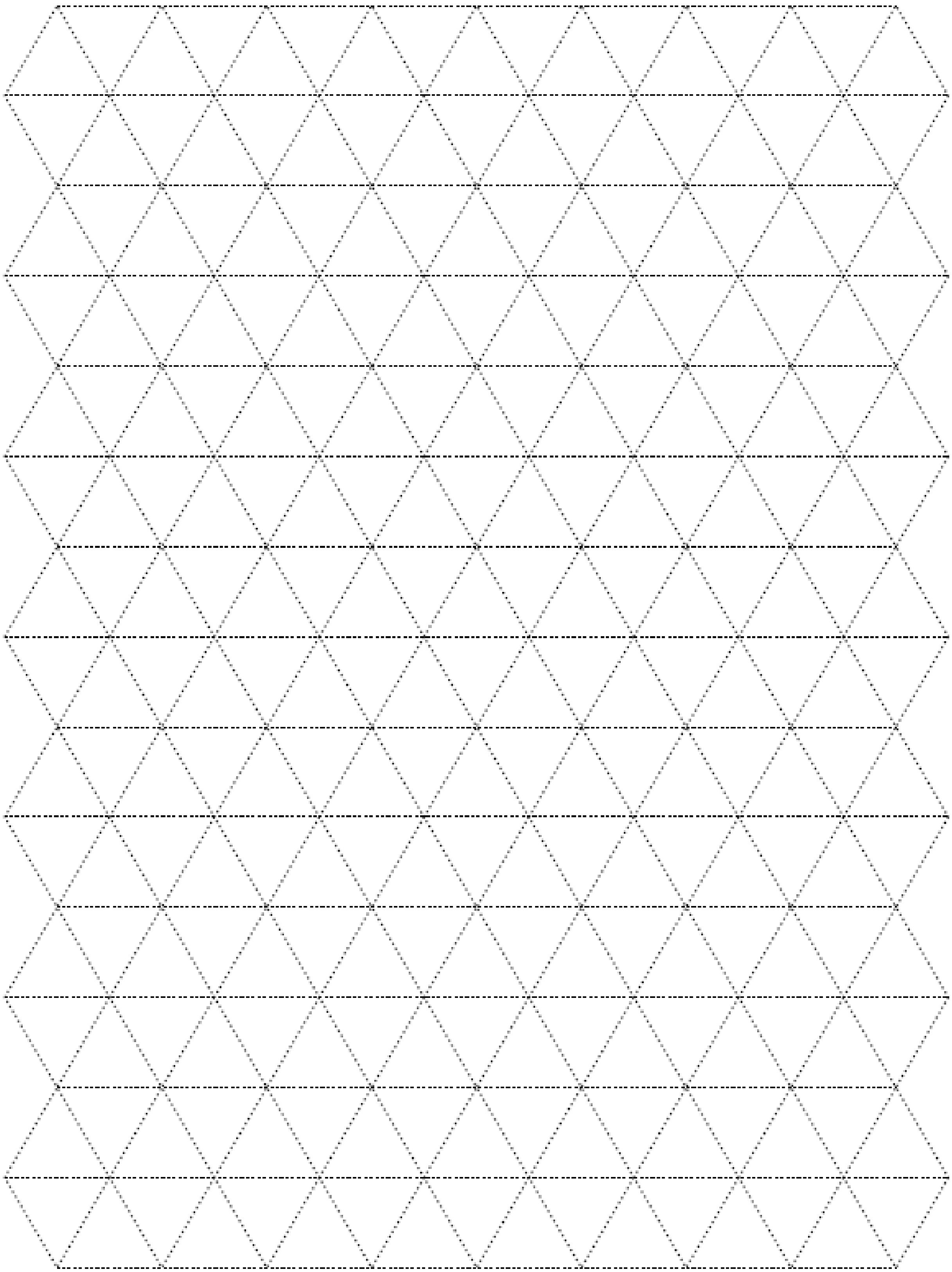
Sample Solution:

By rearranging the six trapezoids, three hexagons can be made. Therefore, the design has four congruent hexagons in all. The yellow hexagon is $\frac{1}{4}$ of all the hexagons; i.e., $\frac{1}{4}$ of the whole design.

$\frac{1}{4} = \frac{25}{100}$ The yellow hexagon represents 25% of the entire design.

(NCTM 2000, p. 215 and Van de Walle and Lovin 2006, p. 120)

Isometric grid paper – BLM 2



Sample Activity 5: Frayer Model

Have students summarize their understanding of ratios by completing a Frayer Model, such as the following example. If students are not familiar with using the Frayer Model, this strategy for consolidating understanding of concepts should be modelled first (*I do*) and then done with students (*We do*) before having them complete one on their own (*You do*). Encourage students to complete a Frayer Model for percent as well.

Look For …

Do students:

- ☐ apply their knowledge of ratios and write a definition and the characteristics in their own words that is mathematically correct?
- ☐ create and justify examples of ratios using regions and sets?
- ☐ create a problem that applies ratios?
- ☐ create and justify non-examples of ratios using regions and sets?
- ☐ create a problem that does not use ratios?

Frayer Model for Ratios

(Barton and Heidema 2002, pp. 68–71)

Definition

A ratio is a comparison of any two quantities (Van de Walle and Lovin 2006, p. 154).

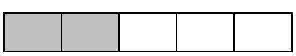
Characteristics

- a ratio compares two quantities multiplicatively
- ratios include percents and rates
- a ratio compares part-to-part or part-to-whole
- in writing the ratio of one quantity to another quantity, order is important
- ratios can be written in different ways; e.g., 2:3, 2 to 3 or $\frac{2}{3}$
- percent is always a part-to-whole ratio
- percent is out of 100; the percent sign is %
- percents are useful when comparing fractional parts of sets or numbers of unequal size
- percent problems are the same as equivalent fraction problems

Ratios

Examples

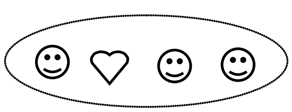
Region



The ratio of the area of the shaded part to the area of the unshaded part is 2 to 3, 2:3 or $\frac{2}{3}$.

Two-fifths of the region is shaded.

Set



The ratio of the number of happy faces to the number of hearts is 3 to 1, 3:1 or $\frac{3}{1}$.

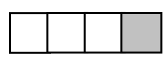
Three-quarters of the set is happy faces.

Problem

How much money is 25% of $12?

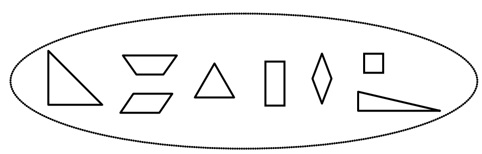
Non-examples

Region



The area of the unshaded part is 2 square units more than the area of the shaded part.

Set



There are two more quadrilaterals than triangles in the set.

Problem

How much more money must be added to $12 to make a total of $20?

Step 4: Assess Student Learning

Guiding Questions

- Look back at what you determined as acceptable evidence in Step 2.
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

In addition to ongoing assessment throughout the lessons, consider the following sample activities to evaluate students' learning at key milestones. Suggestions are given for assessing all students as a class or in groups, individual students in need of further evaluation, and individual or groups of students in a variety of contexts.

A. Whole Class/Group Assessment

The following student assessment task, Giving to Charity Using Ratios and Percent (p. 40), could be used with a whole group or class. It includes a marking rubric to be used with the assessment.

Evidence the Student Has Achieved the Outcomes

Each student will:

- provide a concrete or pictorial representation for a given ratio
- write a ratio from a given concrete or pictorial representation
- express a given ratio in multiple forms, such as 3:5, $\frac{3}{5}$ or 3 to 5
- identify and describe ratios from real-life contexts and record them symbolically
- explain the part-to-whole and part-to-part ratios of a set; e.g., for a group of three girls and five boys, explain the ratios 3:5, 3:8 and 5:8
- solve a given problem involving ratio
- explain that "percent" means "out of 100"
- explain that percent is a ratio out of 100
- use concrete materials and pictorial representations to illustrate a given percent
- record the percent displayed in a given concrete or pictorial representation
- express a given percent as a fraction and a decimal
- identify and describe percents from real-life contexts, and record them symbolically solve a given problem involving percents.

Task Specific Criteria

Each student will:

- provide a concrete or pictorial representation for a given ratio
- write a ratio from a given concrete or pictorial representation
- express a given ratio in multiple forms, such as 3:5, $\frac{3}{5}$ or 3 to 5

- identify and describe ratios from real-life contexts, and record them symbolically
- explain the part-to-whole and part-to-part ratios of a set
- solve a given problem involving ratio
- explain that percent is a ratio out of 100
- use concrete materials and pictorial representations to illustrate a given percent
- express a given percent as a fraction
- identify and describe percent from real-life contexts, and record them symbolically
- solve a given problem involving percents.

Teacher Notes

Summary of Task

In this assessment task, students will solve problems to demonstrate their understanding of ratios and percent by using concrete materials, as needed, diagrams and symbols. They will express ratios in the different ways, including part-to-part and part-to-whole, and will explain why certain comparisons are ratios. By matching subsets, students will write a ratio in the simplest form, using multiple forms. Students will provide different ways to show 25% when given the percent and will be asked to find the part and the whole. Then they will critique a statement made about percent.

Materials required: counters (two-sided counters that are white on one side and red on the other side work very well).

Students first learn to compare two sets of objects by using one-to-one matching and subtraction. Later, they learn that ratios can also be used to compare two sets of objects; e.g., dolls and outfits. Statements of comparison using ratios could include:

- the ratio of the number of outfits to the number of dolls is 12 to 8 (part-to-part)
- the ratio of the number of dolls to the number of outfits is 8 to 12 (part-to-part)
- there are $\frac{8}{12}$ as many dolls as outfits (part-to-part)
- since $\frac{8}{12}$ is equivalent to $\frac{2}{3}$, there are $\frac{2}{3}$ as many dolls as outfits (part-to-part)
- the dolls make up $\frac{8}{20}$ or $\frac{2}{5}$ of the set of items given away (part-to-whole)
- the outfits make up $\frac{12}{20}$ or $\frac{3}{5}$ of the set of items given away (part-to-whole)
- there are $1\frac{1}{2}$ times as many outfits as dolls (part-to-part).

The reason that these statements use ratios is because they all compare quantities multiplicatively rather than by using subtraction.

Students must match the smallest possible subsets when describing ratios in the simplest form. The diagram for matching the number of dolls to the number of outfits in the simplest form could be:

Dolls: ☺☺ ☺☺ ☺☺ ☺☺
Outfits: ○○○ ○○○ ○○○ ○○○

The ratio of the number of dolls to the number of outfits in the simplest form is 2 to 3, $\frac{2}{3}$ or 2:3.

Part-whole-percent problems require students to understand that percent is a part-to-whole ratio that compares a quantity to 100. An open-ended problem in which only the percent is given and the student must find the part and the whole provides multiple entry points so all students can experience some success. For example, 25% could represent 25 out of 100 cars given away to charity, where 25 is the part and 100 is the whole. The diagram could be the following:

25% is 25 out of 100.

25% is $\frac{25}{100}$ or $\frac{1}{4}$ of the whole.

25 cars given away	25 cars kept
25 cars kept	25 cars kept

25 out of a total of 100 cars are given away.

Other answers could include any ratio of a part to a whole that is equivalent to $\frac{25}{100}$ or $\frac{1}{4}$, such as $\frac{2}{8}$, $\frac{3}{12}$ and $\frac{50}{200}$. Ratio tables can be used to list some of the ratios:

Number of cars given away	1	2	3	4	…
Total number of cars	4	8	12	16	…

Early finishers can:

- find the amount of money that is 75% of $36
- find the number of coins in the whole set of coins if 12 coins are 40% of the whole set
- use diagrams to explain what each of the following ratios mean:

  a) 1:3 b) 1:2 c) 3 to 2 d) $\frac{4}{9}$.

Giving to Charity Using Ratios and Percent—Student Assessment Task

Brooklyn and Nicholas are deciding to give away some of their things to charity.

1. Brooklyn is giving away 12 doll outfits together with eight dolls.

   Use counters of two different kinds—one kind to show the outfits and the other kind to show the dolls.

   a. What statements could Brooklyn make when she compares the dolls and the outfits? Include at least three statements.

   b. Which of these comparisons use ratios? Explain.

   c. Brooklyn uses a ratio in the simplest form to decide how to package the dolls with the outfits. Using a diagram and a related number sentence, find the ratio of the number of dolls to the number of outfits in the simplest form. Write the ratio in three different forms.

2. Nicholas is giving away 25% of his toy cars.

   a. How many cars could Nicholas have to begin with and how many could he be giving away? Describe at least three different ways. Explain your thinking using diagrams, symbols and words.

   b. Brooklyn said that 25% of 12 is 4. Is Brooklyn correct? Justify your answer using diagrams and symbols.

SCORING GUIDE:
Giving to Charity Using Ratios and Percent

Student: ___________________________________

Level / Criteria	4 Excellent	3 Proficient	2 Adequate	1 Limited*	Insufficient / Blank*
Writes comparison statements using ratios in different ways and explains why they are ratios. Question 1 (a) and (b)	The student writes at least three comparison statements, correctly identifies the ratios and explains why they are ratios using precise mathematical language. ☐	The student writes three comparison statements, correctly identifies the ratios and explains clearly why they are ratios. ☐	The student writes two comparison statements, correctly identifies the ratios and provides a limited explanation as to why they are ratios. ☐	The student writes one or two comparison statements but has difficulty identifying ratios or explaining why they are ratios. ☐	No score is awarded because there is insufficient evidence of student performance, based on the requirements of the assessment task. ☐
Simplifies a ratio using diagrams, symbols and words. Question 1 (c)	The student draws a very accurate diagram of the ratio with an appropriate number sentence and writes the ratio in the simplest form: 2:3, 2 to 3 and $\frac{2}{3}$. ☐	The student draws an accurate diagram of the ratio with an appropriate number sentence and writes the ratio in the simplest form: 2:3, 2 to 3 and $\frac{2}{3}$. ☐	The student draws a diagram of the ratio with or without a number sentence and writes the ratio in the simplest form using some but not all of 2:3, 2 to 3 and $\frac{2}{3}$. ☐	The student draws an inaccurate or no diagram of the ratio, provides no number sentence and attempts to write the ratio but not in the simplest form, using some but not all three of the required forms. ☐	No score is awarded because there is insufficient evidence of student performance, based on the requirements of the assessment task. ☐
Solves a part-whole-percent problem when given the percent and critiques a percent statement. Question 2 (a)	The student describes clearly, using precise mathematical language, accurate diagrams and appropriate symbols, at least three different ways to write the part and the whole when given the percent. ☐	The student describes clearly, using accurate diagrams and appropriate symbols, three different ways to write the part and the whole when given the percent. ☐	The student describes, using diagrams with some symbols, two different ways to write the part and the whole when given the percent. ☐	The student attempts to describe, with an inaccurate or no diagram, one way to write the part and the whole when given the percent. ☐	No score is awarded because there is insufficient evidence of student performance, based on the requirements of the assessment task. ☐
Critiques a percent statement. Question 2 (b)	The student describes clearly, using precise mathematical language, accurate diagrams and appropriate symbols, why the percent statement provided is incorrect. ☐	The student describes clearly, using accurate diagrams and appropriate symbols, why the percent statement provided is incorrect. ☐	The student describes, using diagrams with some symbols, why the percent statement provided is incorrect. ☐	The student attempts to describe, with an inaccurate or no diagram, why the percent statement provided is incorrect (or correct). ☐	No score is awarded because there is insufficient evidence of student performance, based on the requirements of the assessment task. ☐

* When work is judged to be limited or insufficient, the teacher makes decisions about appropriate intervention to help the student improve.

Student Learning Goals

- Area of need (what's hard for me): ___________________________________
  - Action
- Strength to strengthen: ___________________________________
  - Action

B. One-on-one Assessment

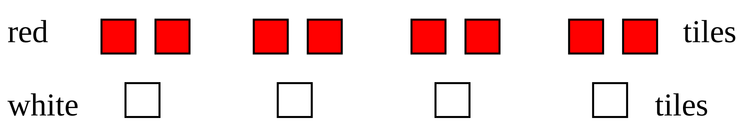
To help students solve problems related to the outcomes, review the meaning of ratios and percent, connect the concrete, pictorial and symbolic representations, and use real-world contexts.

Meaning of Ratios
Present the following problem to the student. Provide double-sided counters, red on one side and white on the other side.

Problem
You have twice as many red tiles as white tiles.
a. How many red tiles do you have if you have four white tiles?
b. How many white tiles do you have if you have four red tiles?
c. How many red tiles do you have if you have six tiles in all?

i. Have the student solve part (a). If the student has difficulty, ask the student to continue using the tiles to show another two red tiles for every one white tile. Have the student continue the pattern until there are four white tiles in all as shown below.

Encourage the student to use the tiles to show the meaning of the ratios. Review what the words 'twice as many' mean and guide the student to write the ratio as 2 to 1, 2:1 or $\frac{2}{1}$. Ask what the numbers in the ratio represent; i.e., the 2 is the number of red tiles; the 1 is the number of white tiles.



Ask the student to continue the pattern in the ratio table:

red tiles	2	4	6	?
white tiles	1	2	3	?

Have the student answer the question asked in part (a). Encourage the student to write the number sentence to show the equivalent ratios and relate it to equivalent fractions.

$$\frac{\text{red tiles}}{\text{white tiles}} = \frac{2}{1} = \frac{8}{4}$$

ii. Ask the student to solve part (b). If necessary, guide the student to use the tiles, diagram and chart. The student should recognize that the first two sets of 2 to 1 ratios include four red tiles. Have the student show you the four red tiles and the two white tiles. Ask the student if the 2:1 ratio shows a part-to-part ratio or a part-to-whole ratio. Have the student justify his or her answer.

iii. Ask the student to solve part (c). If necessary, guide the student to use the tiles, diagram and chart. Remind the student that the six tiles represent the whole; i.e., the red and white tiles combined. If necessary, remind the student that part (b) had six tiles in all. The student should then be able to say that there are four red tiles when there are six tiles in all.

Change the ratio and have the student answer similar problems. Also, change the context to money; e.g., you have three times as much money as I do. How much money do you have if I have $4?

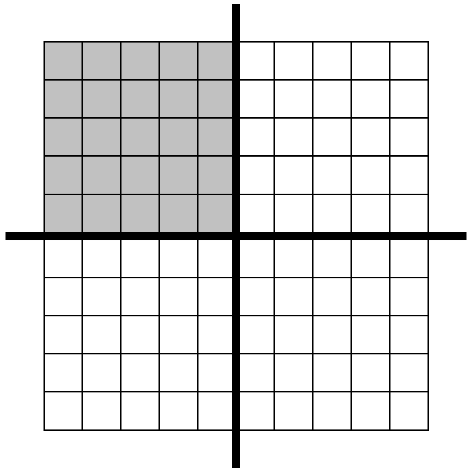
Ask the student to compare two girls to eight boys without using subtraction. If the student has difficulty, suggest that he or she find the ratio of girls to boys or boys to girls. Ask, "How many times as many boys as girls are there?" Also, suggest that the student find the fraction of the group of children that are girls. Then have him or her find the fraction of the children that are boys.

Meaning of Percent

Provide the student with hundredths grids and review that percent is a part-to-whole ratio that compares a quantity to 100. Connect percent to prior knowledge about hundredths.

Shade in part of a hundredths grid. Ask the student what percent is shaded and have him or her justify the answer. Have the student write the percent using the percent sign. If the student has difficulty, remind the student that each little square in the hundredths grid represents one-hundredth or 1%. Have the student count the little squares, reminding the student that each column represent ten-hundredths or 10%.

Reverse the procedure. Provide the student with a percent such as 25%. Ask the student to shade in the hundredths grid to show 25%. If the student has difficulty, remind the student that each little square in the grid represents 1%; therefore, 25 of these squares must be shaded to show 25%. Ask the student to write the fraction and the decimal that represents the shaded part. If the student has difficulty writing the percent as a fraction, suggest that the student shade 25 squares to show one-quarter of the whole grid shaded, as shown in the following example.



If the student has difficulty writing the decimal for the percent, connect percent to hundredths; e.g., $25\% = \frac{25}{100} = 0.25$.

Ask the student to solve the following problem.

Problem
You read 50% of a book. How many pages could you have read and how many pages could there be in the book? Explain.

If the student has difficulty, remind the student that 50% is 50 out of 100. Ask, "What does the 50 mean related to the book? What does the 100 mean related to the book?" Guide the student to understand that 50 represents the pages read and the 100 represents the total number of pages in the book.

If the student has difficulty finding other answers to the problem, ask, "What fraction is the same as 50%?" If necessary, suggest that 50 is one-half of 100.

Then suggest that the student use a ratio table:

Number of pages read	1	2	3	…
Total number of pages on the book	2	4	6	…

Connect these equivalent ratios to prior knowledge about equivalent fractions.

See **Step 3, Section C, Choosing Learning Activities, Sample Activity 1: Meaning of Ratio: Part-to-whole and Part-to-part** (p. 14) and **Sample Activity 1: Meaning of Percent and Connections to Fractions and Decimals** (p. 24) for detailed descriptions of activities to promote understanding of these concepts.

C. Applied Learning

Provide opportunities for students to use their understanding of ratio and percent in a practical situation and notice whether or not this understanding transfers.

For example, ask the student to solve the following problem and explain the thinking done to solve this problem.

You are making a mixture of red and green candies that has three red candies for every two green candies. What percent of the mixture is made up of red candies?

Does the student:

- explain that the three red candies and two green candies each represent a part of the ratio?
- know that the total of five candies represents the whole in the ratio?
- compare the number of red candies to the total number of candies?
- explain the part-to-whole ratio as $\frac{3}{5}$, 3 to 5 or 3:5?
- change the part-to-whole ratio to percent by providing a ratio out of 100?
- explain that $\frac{3}{5}$ is equivalent to $\frac{60}{100}$ or 60%?
- apply ratio and percent to other real-world problems?

Step 5: Follow-up on Assessment

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction?

A. Addressing Gaps in Learning

Students who have difficulty solving the basic facts using strategies will enjoy more success if one-on-one time is provided in which there is open communication to diagnose where the learning difficulties lie. Assessment by observing a student solving problems will provide valuable data to guide further instruction. Success in problem solving depends on a positive climate in which the students are confident in taking risks. By building on the existing understandings of each student and accommodating the individual learning styles, success will follow.

If the difficulty lies in solving the basic facts, use the following strategies:

- Draw on the prior knowledge of students, spending time to review proper fractions as part of a region and part of a set. Review the meaning of fractions and how the numerator relates to the part and the denominator to the whole.
- Emphasize the similarities and differences between part of a region and part of a set.
- Provide everyday problem-solving contexts that students can relate to.
- Use a variety of concrete materials such as counters, fraction strips, fraction circles, 11 × 11 geoboards and pattern blocks. Connect the concrete to diagrams and symbols as students develop understanding of ratio and percent.
- Allow students to use concrete materials, as long as necessary, to establish an understanding of the concepts.
- Review equivalent fractions by connecting concrete, pictorial and symbolic representations. Connect equivalent fractions to equivalent ratios.
- Explain the similarities and differences between comparisons using subtraction and comparisons using multiplication. Emphasize that ratios use multiplication in comparing two quantities.
- Provide examples of part-to-part ratios as well as part-to-whole ratios.
- Connect part-to-whole ratios to fractions and to percent.
- Encourage students to integrate patterns with ratios and use a ratio table to illustrate equivalent ratios in solving problems.
- Ask guiding questions to direct student thinking. See the examples provided in Step 4, Part B: One-on-one Assessment (p. 42).
- Provide time for students to explore and construct their own meaning rather than being told.
- Encourage flexibility in thinking as students describe various ways to solve ratio and percent problems.
- Have students share their thinking with others so that students, who are having difficulty, hear (in 'kid' language) how another person thinks about ratio and percent.

B. Reinforcing and Extending Learning

Students who have achieved or exceeded the outcomes will benefit from ongoing opportunities to apply and extend their learning. These activities should support students in developing a deeper understanding of the concept and should not progress to the outcomes in subsequent grades.

Consider the following strategies:

- Provide tips for parents on using ratios and percent at home or in the community.
  - Make mixtures of various kinds using simple ratios such as a mixture of red and white jellybeans that has two red jellybeans for every five white jellybeans. Ask how many red jellybeans are needed in a mixture with 10 white jellybeans. Ask what fraction of the mixture is made up of white jellybeans.
  - Compare two quantities using multiplication instead of subtraction; e.g., there are twice as many boys as girls in a family.
  - Compare percents; e.g., would you rather have 25% of $12 or 30% of $10?
  - Discuss percents that are used in everyday life. For example, the child may spend 50% of his or her allowance and save 50% for a future purchase, such as a bike. Use 50% and one-half interchangeably. Similarly, use 25% and $\frac{1}{4}$ interchangeably in everyday contexts.
- Stacey earns $12 for every $4 that Tim earns.
  - Stacey earns how many times as much as Tim? Explain.
  - If Tim earns $8, how much does Stacey earn?
  - If Stacey earns $15, how much does Tim earn?
- Suppose you made a sample batch of purple clay by mixing two sticks of red and three sticks of blue. You want to make a larger batch of exactly the same shade. You will use eight sticks of red. How many sticks of blue should you use? Explain your thinking (Lilly 1999, p. 184).
- In a garden there are eight pea plants to every six bean plants. Find the ratio of the number of bean plants to the number of pea plants in simplest form. Use diagrams and symbols to show your thinking.
- Use 12 cubes of three different colours so that these ratios compare pairs of colours. Draw and label a diagram to show the cubes: 1:2, 6 to 4; 1 to 3 (Hope 1997, p. 81).
- Marnie has 18 coins in the ratio of one loonie to two quarters. What coins does she spend to give a new ratio of loonies to quarters of 1:3? Give all possible answers. Explain your thinking.
- Susie jogs 3 km for every 2 km that Pete jogs. This week, they jogged a total of 15 km. How many kilometres did they each jog this week?
- Three baseballs cost as much as two basketballs. One baseball and one basketball cost $10. What is the cost of a baseball? (Greenes 1980, Card number 142-D)
- Brandon eats two-fifths of a cake and Rheena eats 30% of the same cake. Who eats more cake or do they eat the same amount? Explain your thinking.
- One-quarter of a jar of candies are red and 40% are black. What percent of the candies are neither red nor black?
- Shane and Jamie each have a bag with the same number of candies. Shane eats 0.4 of his bag and Jamie eats 35% of her bag. Who eats the greater number of candies?

- Bobby has 100 coins totalling $7. He has no nickels. What coins does he have if 60% of the coins were of the same kind?
- Jan and Karen are reading the same book. If Jan has read $\frac{4}{5}$ of the book and Karen has read 60% of the book, who has read more pages?
- Jodi runs three laps in nine minutes. Terry runs two laps in five minutes. Who is the faster runner or are they equally fast? Explain your thinking (Van de Walle and Lovin 2006, p. 162).
- Marnie spends 25% of her allowance and Peter spends 50% of his allowance. Is it possible that 25% of Marnie's allowance could be greater than 50% of Peter's allowance? Explain.

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