Mathematics

Planning Guide

Grade 6
Representing Relationships

Patterns and Relations (Patterns)
Specific Outcomes 1 and 2

Statistics and Probability (Data Analysis)
Specific Outcome 3

This Planning Guide can be accessed online at:
http://www.learnalberta.ca/content/mepg6/html/pg6_representingrelationships/index.html

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Planning Guide: *Grade 6 Representing Relationships*

This *Planning Guide* addresses the following outcomes from the Program of Studies:

Strand: Patterns and Relations (Patterns)	
Specific Outcomes:	1. Represent and describe patterns and relationships, using graphs and tables. [C, CN, ME, PS, R, V] [ICT: C6–2.3] 2. Demonstrate an understanding of the relationships within tables of values to solve problems. [C, CN, PS, R] [ICT: C6–2.3]
Strand: Statistics and Probability (Data Analysis)	
Specific Outcome:	3. Graph collected data, and analyze the graph to solve problems. [C, CN, PS, R, T] [ICT: C6–2.5, C7–2.1, P2–2.1, P2–2.2]

Curriculum Focus

The changes to the curriculum targeted by this sample include:

- The general outcome in the Pattern and Relations (Patterns) strand focuses on using patterns to describe the world and to solve problems, which is the same as the previous mathematics curriculum.
- The specific outcomes in the Pattern and Relations (Patterns) strand focus on using graphs and tables to represent and describe patterns and relationships and solve problems by demonstrating an understanding of relationships within tables of values, whereas the previous mathematics curriculum focused on visually representing patterns, creating rules to describe patterns and finding approximate number values from a graph.
- The general outcome in the Statistics and Probability (Data Analysis) strand focuses on collecting, displaying and analyzing data to solve problems, whereas the previous mathematics curriculum focused on collecting, displaying and analyzing data to make predictions about a population.
- The specific outcome in the Statistics and Probability (Data Analysis) strand focuses on graphing collected data, and analyzing the graph to solve problems. The previous mathematics curriculum focused on constructing and interpreting double bar graphs to draw conclusions. Constructing and interpreting double bar graphs to draw conclusions is now a Grade 5 learning outcome.

What Is a Planning Guide?

Planning Guides are a tool for teachers to use in designing instruction and assessment that focuses on developing and deepening students' understanding of mathematical concepts. This tool is based on the process outlined in *Understanding by Design* by Grant Wiggins and Jay McTighe.

Planning Steps

The following steps will help you through the Planning Guide:

Step 1: Identify Outcomes to Address

Guiding Questions

- What do I want my students to learn?
- What can my students currently understand and do?
- What do I want my students to understand and be able to do based on the Big Ideas and specific outcomes in the program of studies?

Big Ideas

Mathematics is often referred to as the science of patterns. Patterns permeate every mathematical concept and are found in everyday contexts. Students translate among various representations of patterns using concrete materials, diagrams, graphs and tables of values. As students analyze the structure of patterns and organize the information systematically, they "use their analysis to develop generalizations about the mathematical relationships in the patterns" (National Council of Teachers of Mathematics 2000, p. 159).

Algebraic reasoning is directly related to patterns because this reasoning focuses on making generalizations based on mathematical experiences and recording these generalizations (Van de Walle and Lovin 2006, p. 281).

Some characteristics of patterns include the following (most adapted from Van de Walle and Lovin 2006, pp. 265–268).

- "A pattern must involve some repetition or regularity" (Small 2009, p. 3).
- There are different types of patterns that can be modelled in a variety of ways, including concrete materials, diagrams, charts and graphs.
- Patterns include repetitive patterns and growth patterns.
- Patterns using concrete and pictorial representations can be translated into patterns using numbers to represent the quantity in each step of the pattern. The steps in a pattern are often translated as the sequence of items in the pattern.
- Growth patterns are evident in a wide variety of contexts including arithmetic and geometric situations. Arithmetic patterns are formed by adding or subtracting the same number each time. Geometric patterns are formed by multiplying or dividing by the same number each time.
- Pattern rules are used to generalize relationships in patterns. These rules can be recursive and functional.
- A recursive relationship describes how a pattern changes from one step to another step. It describes the evolution of the pattern by stating the first element in the pattern together with an expression that explains what you do to the previous number in the pattern to get the next one.
- A functional relationship is a rule that determines the number of elements in a step by using the number of the step; i.e., a rule that explains what you do to the step number to get the value of the pattern for that step. In other words, for every number input there is only one output using the function rule.

- "Functional relationships can be expressed in real contexts, graphs, algebraic equations, tables and words. Each representation for a given function is simply a different way of expressing the same idea. Each representation provides a different view of the function. The value of a particular representation depends on its purpose" (Van de Walle and Lovin 2006, p. 284).
- Graphs provide an effective model for patterns by "describing and representing relationships between various quantities visually" (Small 2009, p. 13). These visual displays "quickly reveal information about data" (Small 2009, p. 179). Van de Walle and Lovin (2006, p. 287) state, "The context provides meaning to the graph, and the graph adds understanding to the context."

Sequence of Outcomes from the Program of Studies

See http://education.alberta.ca/teachers/program/math/educator/progstudy.aspx for the complete program of studies.

Grade 5	Grade 6	Grade 7
Patterns and Relations (Patterns)	**Patterns and Relations (Patterns)**	**Patterns and Relations (Patterns)**
Specific Outcomes	**Specific Outcomes**	**Specific Outcomes**
1. Determine the pattern rule to make predictions about subsequent elements. [C, CN, PS, R, V]	1. Represent and describe patterns and relationships, using graphs and tables. [C, CN, ME, PS, R, V] [ICT: C6–2.3]	1. Demonstrate an understanding of oral and written patterns and their equivalent linear relations. [C, CN, R]
	2. Demonstrate an understanding of the relationships within tables of values to solve problems. [C, CN, PS, R] [ICT: C6–2.3]	2. Create a table of values from a linear relation, graph the table of values, and analyze the graph to draw conclusions and solve problems. [C, CN, PS, R, V] [ICT: C7–3.1]
Statistics and Probability (Data Analysis)	**Statistics and Probability (Data Analysis)**	**Statistics and Probability (Data Analysis)**
Specific Outcomes	**Specific Outcomes**	**Specific Outcomes**
2. Construct and interpret double bar graphs to draw conclusions. [C, PS, R, T, V] [ICT: C6–2.2, P5–2.3]	3. Graph collected data, and analyze the graph to solve problems. [C, CN, PS, R, T] [ICT: C6–2.5, C7–2.1, P2–2.1, P2–2.2]	3. Construct, label and interpret circle graphs to solve problems. [C, CN, PS, R, T, V] [ICT: P2–3.3]

Step 2: Determine Evidence of Student Learning

Guiding Questions

- What evidence will I look for to know that learning has occurred?
- What should students demonstrate to show their understanding of the mathematical concepts, skills and Big Ideas?

Using Achievement Indicators

As you begin planning lessons and learning activities, keep in mind ongoing ways to monitor and assess student learning. One starting point for this planning is to consider the achievement indicators listed in the *Mathematics Kindergarten to Grade 9 Program of Studies with Achievement Indicators*. You may also generate your own indicators and use these to guide your observation of the students.

The following indicators may be used to determine whether or not students have met this specific outcome. Can students:

- translate a pattern to a table of values and graph the table of values (limited to linear graphs with discrete elements)?
- create a table of values from a given pattern or a given graph?
- describe, using everyday oral and written language, the relationship shown on a graph?
- generate values in one column of a table of values, given values in the other column and a pattern rule?
- state, using mathematical language, the relationship in a given table of values?
- create a concrete or pictorial representation of the relationship shown in a table of values?
- predict the value of an unknown term, using the relationship in a table of values, and verify the prediction?
- formulate a rule to describe the relationship between two columns of numbers in a table of values?
- identify missing elements in a given table of values?
- identify errors in a given table of values?
- describe the pattern within each column of a given table of values?
- create a table of values to record and reveal a pattern to solve a given problem?
- determine an appropriate type of graph for displaying a set of collected data, and justify the choice of graph?
- solve a given problem by graphing data and interpreting the resulting graph?

Sample behaviours to look for related to these indicators are suggested for some of the activities listed in **Step 3, Section C: Choosing Learning Activities** (p. 12).

Step 3: Plan for Instruction

Guiding Questions

- What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should I use?
- How will I meet the diverse learning needs of my students?

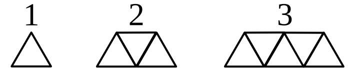
A. Assessing Prior Knowledge and Skills

Before introducing new material, consider ways to assess and build on students' knowledge and skills related to patterns, perimeter and area.

Provide students with centimetre grid paper.
For example:

- Darcy is making a pattern using toothpicks as shown below.

  Diagram Number: 1 2 3

  Number of Toothpicks: 3 7

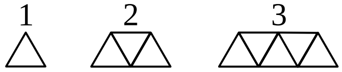
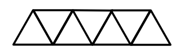
  a. Predict how many toothpicks will be used to make the fourth diagram in the pattern. Explain your thinking.
  b. Draw the next diagram in this pattern and explain your thinking.
  c. Describe a pattern rule to determine how many toothpicks will be used in each succeeding diagram.
  d. Use your pattern rule to find the number of toothpicks in the fifth diagram.

- Construct a double bar graph to represent the following data on attendance at a circus. Use the graph to answer the questions.
  In the year 2007, attendance at the circus was 4000 on Monday, 4500 on Tuesday, 3000 on Wednesday and 3500 on Thursday.
  In the year 2008, attendance at the circus was 3000 on Monday, 3500 on Tuesday, 4000 on Wednesday and 3500 on Thursday.

  a. How many more people attended the circus on Tuesday than Wednesday?
  b. What general statement could be made about the data in your graph?

If a student appears to have difficulty with these tasks, consider further individual assessment, such as a structured interview, to determine the students' level of skill and understanding. See **Sample Structured Interview: Assessing Prior Knowledge and Skills** (p. 9).

Sample Structured Interview: Assessing Prior Knowledge and Skills

Directions	Date:	
	Not Quite There	**Ready to Apply**
Present the student with the following problem. Read the problem one part at a time, if necessary. **Darcy is making a pattern using toothpicks as shown below.** **Diagram Number (top)** 1 2 3  3 7 **Number of toothpicks (bottom)** **Predict how many toothpicks will be used to make the fourth diagram in the pattern. Explain your thinking.** **Draw the next diagram in this pattern and explain your thinking.** **Describe a pattern rule to determine how many toothpicks will be used in each succeeding diagram.** **Use your pattern rule to find the number of toothpicks in the fifth diagram.**	Guesses the number of toothpicks in the fourth diagram but is unable to explain his or her thinking. Attempts to the draw the next diagram in the pattern but makes some errors. Does not describe a pattern rule that represents the pattern. Does not find the correct answer or finds the number of toothpicks in the fifth diagram but does not explain the process using a pattern rule.	Predicts correctly that there are 15 toothpicks in the fourth diagram and explains clearly why this prediction is made; e.g., describes the diagram or the number pattern. Draws correctly the next diagram, explaining that 2 more triangles must be made using 4 more toothpicks:  Describes a pattern rule using mathematical language such as: "Each succeeding diagram has 4 more toothpicks than the previous diagram. The first diagram has 3 toothpicks." OR Describes a pattern rule using a mathematical expression such as: "$N + 4$, where N is the number of toothpicks in the previous diagram; $N = 3$ in the first diagram." Uses the pattern rule to find the number of toothpicks in the fifth diagram. Example: "The pattern is made by starting at 3 and adding 4 to get the next number: 3, 7, 11, 15, 19, …. Therefore, there are 19 toothpicks in the fifth diagram."

<table>
<tr><th rowspan="2">Directions</th><th colspan="2">Date:</th></tr>
<tr><th>Not Quite There</th><th>Ready to Apply</th></tr>
<tr><td>Provide the student with centimetre grid paper. Present the student with the following problem. Read the problem one part at a time, if necessary.

Construct a double bar graph to represent the following data on attendance at a circus. Use the graph to answer the questions.

In the year 2007, attendance at the circus was 4000 on Monday, 4500 on Tuesday, 3000 on Wednesday and 3500 on Thursday.
In the year 2008, attendance at the circus was 3000 on Monday, 3500 on Tuesday, 4000 on Wednesday and 3500 on Thursday.

How many more people attended the circus on Tuesday than Wednesday?

What general statement could be made about the data in your graph?</td><td>Has difficulty organizing the data to represent it in on a double bar graph. Requires assistance in deciding on the labels and the scale for the graph.

Has difficulty answering one or both of the questions.

Compares only one year when answering the question or compares both years but reads data incorrectly from the graph and/or makes computational errors.

Provides an incorrect or no general statement about the data in the graph.</td><td>Constructs a double bar graph accurately, using appropriate scale and labels for the graph.

Answers correctly that there were 1000 more people attending the circus on Tuesday than on Wednesday and justifies his or her answer clearly.

Provides a general statement that describes the overall graph; e.g., more people attended the circus in 2007 than in 2008, the total attendance on each of the four days was at least 7000.</td></tr>
</table>

B. Choosing Instructional Strategies

Consider the following instructional strategies for teaching patterns and relationships, including translating patterns using concrete, pictorial and symbolic representations.

- Build on understanding patterns, including perimeter and area, from Grade 5—connecting the concrete, pictorial and symbolic representations of patterns and developing rules for patterns.
- Provide experiences with various models for patterns and the translations among the models; i.e., concrete materials, diagrams, table of values, graphs (limited to linear graphs with discrete elements) and pattern rules.
- Encourage students to describe patterns and rules, orally and in writing, before using algebraic symbols.
- Provide an opportunity to connect the concrete and pictorial representations to symbolic representations as well as to connect the symbolic representations to pictorial and concrete representations.
- Provide examples of growth patterns that are arithmetic (the same number added or subtracted each time) and geometric (the same number is multiplied or divided each time).
- In creating tables of values and graphs for patterns, use real-world contexts.
- In creating a table of values for a given pattern, encourage students to state the step number in one column or row and the corresponding number of elements in each step in the other column or row.
- In writing pattern rules for oral or written patterns, encourage students to draw diagrams to assist them in visualizing the relationship.
- Provide pictorial examples of patterns in which students formulate rules that are recursive (describing how a pattern changes from one step to another, given the first element in the pattern) and functional (describing the relationship between two columns or rows of numbers in a table of values).
- In creating a functional relationship for a given pictorial pattern, have students represent and extend the pattern in a table of values, describe the pattern shown in the table and use this pattern to write a functional relationship or a formula in terms of the step number. Have students use the created formula or the functional relationship to calculate the 20^{th} or 50^{th} entry in the table (Van de Walle and Lovin 2006, pp. 269–270).
- Provide students with a variety of pattern problems using real-world contexts. Encourage them to solve problems in different ways and explain the process. Also, provide time for students to share their solutions with others. Stimulate class discussion to critically evaluate the various procedures. Emphasize understanding, flexibility and efficiency when students select problem-solving strategies.

C. Choosing Learning Activities

The following learning activities are examples that could be used to develop student understanding of the concepts identified in Step 1.

Sample Activities:

Teaching the Representation of Patterns

1. **Sorting Patterns—Similarities and Differences** (p. 14)
2. **Creating Patterns** (p. 16)
3. **Translating Patterns (Concrete, Pictorial, Table of Values, Graph)** (p. 17)

Teaching Pattern Rules (Recursive and Functional Relationships)

1. **Growing Patterns (What Changes, What Stays the Same)** (p. 22)
2. **Growing Patterns (What Changes: Variables, What Stays the Same: Constant)** (p. 24)
3. **Input–Output** (p. 29)
4. **Problem Solving with Patterns** (p. 31)

Teaching the Representation of Patterns

Sample Activity 1: Sorting Patterns—Similarities and Differences

Provide students with different patterns. Encourage students to construct patterns using concrete materials such as square tiles and toothpicks. Have them sort patterns and explain their sorting rule. Then have them sort the concrete materials another way and explain the sorting rule.

Look For …

Do students:

- ☐ sort patterns in different ways and explain the sorting rule?
- ☐ explain the similarities among all patterns?
- ☐ explain the meaning of repeating patterns and growing patterns?
- ☐ explain the difference between increasing and decreasing patterns?
- ☐ explain the difference between growing patterns that increase by a constant amount and those that increase by different amounts following a pattern?

Differences among Patterns

Guide the discussion to generalize that some patterns increase by the same amount each time, whereas other patterns increase by an amount that changes but forms a pattern. Continue the discussion on differences among patterns by asking students to suggest ways to convert the increasing patterns into decreasing patterns.

Similarities among Patterns

Review the similarities among the patterns and ask students to describe the essential characteristics. Generalize that patterns "must involve some repetition or regularity" (Small 2009, p. 3).

Examples of Patterns:

Handshake Problem

Students shake hands with one another and count the number of handshakes for a given number of people. The pattern can be represented in a chart as follows:

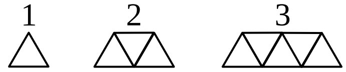
Number of People	2	3	4	5	6	7
Number of Handshakes	1	3	6	10	?	?

Act out this situation to verify the pattern and continue the pattern for 6 and 7 people.

Toothpick Problem

You are making a pattern using toothpicks as shown below.

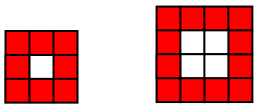
Diagram Number: 1 2 3



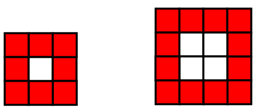
Number of Toothpicks: 3 7

Continue the pattern to find the number of toothpicks in the sixth diagram. Explain your thinking.

Pool Problem 1
You are building a square pool. You will surround the pool with square patio tiles. The red or shaded squares represent the patio tiles. Find the number of square patio tiles needed for the next two pools in the pattern below. Explain your thinking.

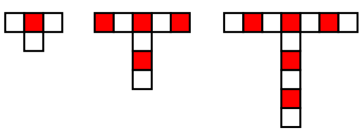


Pool Problem 2
You are building a square pool. You will surround the pool with square patio tiles. The red or shaded squares represent the patio tiles. Find the area of the fifth pool in this pattern. Explain your thinking.



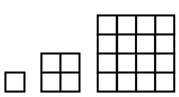
(Ferrini-Mundy, Lappan and Phillips 1997, p. 283)

Growing T's
Andy is making a pattern by placing square tiles together to make the letter T. Describe the pattern that he is making. Find the number of square tiles in the fifth diagram. Explain your thinking.



(Lee and Freiman 2006, pp. 428–430)

Square Problem
Describe the pattern shown by the squares. Find the perimeter of the sixth square in this pattern. Explain your thinking.



(National Council of Teachers of Mathematics 1997, p. 51)

Sample Activity 2: Creating Patterns

After students have had experience sorting different types of patterns and reviewing their essential characteristics, ask them to extend a pattern that begins with two numbers, e.g., 4, 8, …, in at least three different ways. Share the different patterns created.

Ask students if only one pattern could be made if three numbers were provided; e.g., 4, 8, 12.

Share ideas including: 4, 8, 12, 16, …
4, 8, 12, 4, 8, 12, …
4, 8, 12, 20, 28, 32, 36, 44, 52, ...
4, 8, 12, 17, 22, 28, 34, 41, 48, …
(Small 2009, p. 3)

Look For …

Do students:

- ☐ exhibit flexible thinking in the creation of different patterns?
- ☐ describe a pattern using clear, concise mathematical language?
- ☐ critically analyze a sequence of numbers to determine whether or not they form a pattern?

Sample Activity 3: Translating Patterns (Concrete, Pictorial, Table of Values, Graph)

Explain to the students that there are many different types of patterns but the focus for representing a given pattern in many different ways will be on patterns that increase/decrease by the same amount.

i. Have students translate a concrete or pictorial pattern to a table of values and graph (limited to linear graphs with discrete elements). Do the first example with students and explain the vocabulary; e.g., step number, table of values, graph.

   - The step number refers to the number of each diagram in the pattern.
   - The table of values is a chart that includes the step number and the number of elements in each step.
   - The graph will represent ordered pairs of numbers and will consist of individual points that are not connected; i.e., discrete elements. Encourage students to verbalize that the ordered pairs they graph are directly connected to the numbers in the table of values. Remind students that the first member of each ordered pair must be the number from the horizontal axis.

Look For …

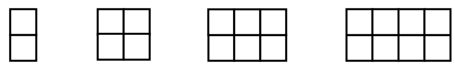
Do students:

- ☐ provide appropriate headings for a table of values to represent a given concrete, pictorial or graph pattern?
- ☐ use a variety of manipulatives to translate patterns from a table of values or a graph to a concrete representation?
- ☐ accurately represent patterns on a graph and use appropriate labels?
- ☐ interpret a graph and describe the relationship it represents?

Example:
Provide students with square tiles and centimetre grid paper.

Rectangle Problem
Describe the following pattern.
Represent the pattern in a table of values and a graph. Find the number of squares in the fifth step.

Step Number: 1 2 3 4



Number of squares: 2 4 6 8
(or area of rectangle)

Sample Solution

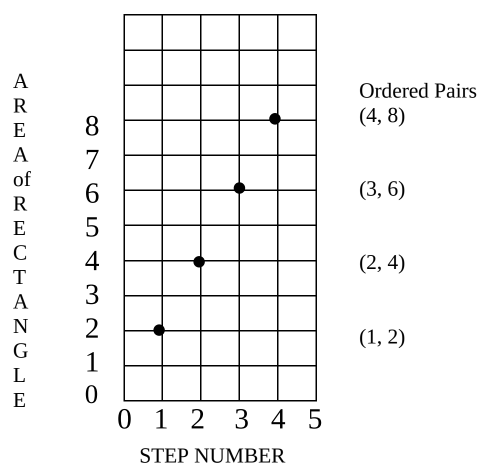
For each succeeding rectangle, the number of squares increases by 2, i.e., the area of each succeeding rectangles increases by 2 square units.

Table of Values

Step Number	1	2	3	4
Area of Rectangle	2	4	6	8

Graph
The horizontal axis is the step number. The vertical axis is the area of the rectangle.

RECTANGLES IN THE PATTERN



By extending the table of values or the graph, the number of squares for the fifth step is 10 squares.

Provide other concrete and pictorial linear patterns for students to translate into tables of values and graphs.

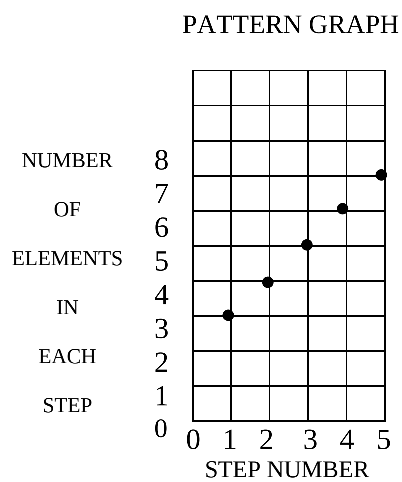
ii. Reverse the procedure and have students translate a given pattern in a graph (limited to linear graphs with discrete elements) to a table of values and then to a concrete or pictorial representation. Provide students with concrete materials such as square tiles and pattern blocks.

Provide the following graph and have students write the ordered pair for each point on the graph. Explain that the vertical axis on the graph is labelled "Number of Elements in Each Step" because this label is generic and will apply to any concrete or pictorial representation of this pattern. Remind students that the ordered pairs provide the numbers that are needed to put into the table of values.

Have students share their ideas for the concrete or pictorial representation for the pattern. Through discussion, generalize that a given pattern shown in a table of values or a graph can be represented in different ways using concrete materials or diagrams.

Translation Problem
Use the following graph to:

a. describe the relationship shown on the graph
b. translate the pattern shown on the graph below into a table of values and a concrete or pictorial representation.



Sample Solution (*Patterns and Pre-Algebra, Grades 4–6,* Alberta Education 2007, p. 118)

a. The graph shows that as each step increases, the number of elements in each step increases by one. The first step has 3 elements.

   Another way to describe the relationship shown on the graph is to say that the number of elements in each step is always two more than the step number.

b.

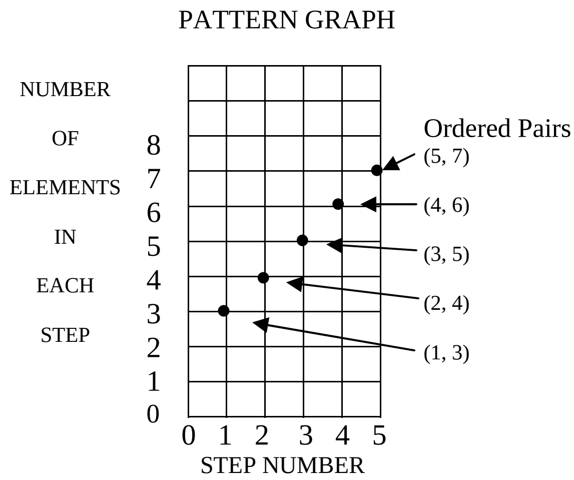
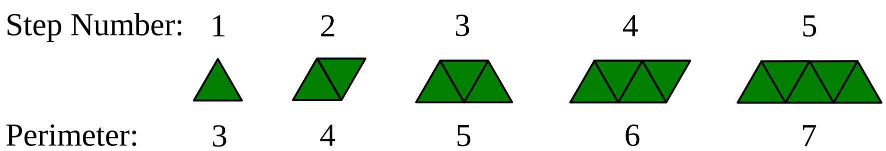


Table of Values

Step Number	1	2	3	4	5
Number of Elements in Each Step	3	4	5	6	7

Sample Pictorial Representation



(Each unit is the length of one side of the equilateral triangle in Step 1.)

Provide other linear graphs (using discrete elements) for students to describe using everyday language and then have them translate these graphs into tables of values and concrete or pictorial representations. Also, have them solve a problem by drawing a graph and interpreting it.

Teaching Pattern Rules
(Recursive and Functional Relationships)

Sample Activity 1: Growing Patterns (What Changes, What Stays the Same)

Focus on recursive relationships; i.e., pattern rules that show how patterns change from one step to the next.

Provide students with concrete materials such as square tiles.

Tell students that they will continue work on translating patterns but this time they will translate a pattern (concrete, pictorial, table of values or graph) into a pattern rule using mathematical language. The focus will be on how a pattern changes from one step to another step. Provide students with examples of patterns in which a constant is added or subtracted each time to produce the next term in the pattern. Choose examples from the first activity on sorting patterns.

Focus student's attention on what changes and what stays the same in each pattern.

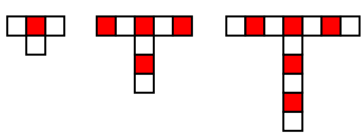
Look For …

Do students:

- ☐ examine the patterns to determine what stays the same and what changes in each successive step?
- ☐ use appropriate mathematical language to describe a pattern rule that explains how a pattern changes from one step to the next?
- ☐ always include the first number in a pattern as part of the pattern rule for a recursive relationship?
- ☐ justify why a particular pattern rule describes a given pattern?
- ☐ transfer their learning to other patterns?

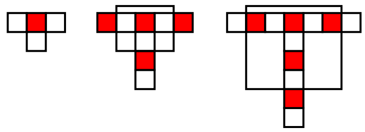
Example:

Growing T's (Lee and Freiman 2006, pp. 428–430)
Andy is making a pattern by placing square tiles together to make the letter *T*. Describe the pattern that he is making. Find the number of square tiles in the fifth diagram. Explain your thinking.



- Make a transparency of the Growing T's.
- Label the step numbers and the number of elements in each step.
- On the overhead projector, place a blank transparency over the first figure in the sequence.
- Trace the figure.
- Slide this tracing over top of the next figure.
- Draw a square around the part of the second figure that stayed the same.
- Continue this process by tracing the previous figure and superimposing it on the next figure in the sequence.
- Have students explain what changes and what stays the same in the pattern.

Step Number: 1 2 3



Number of Squares: 4 8 12

4 + 4 8 + 4

(*Patterns and Pre-Algebra, Grades 4–6*, Alberta Education 2007, p. 125)

- Through discussion, have students generalize that, from step-to-step, four are added each time to the previous number of squares. The step number changes but the 4 is constant; i.e., another group of four is always added to the previous step.
- Explain that a pattern rule that describes the step-by-step growth of a pattern requires two parts:
  - the number of elements in the first step
  - the number of elements added to each successive step to create the next step.
- Have students write the pattern rule for the Growing T's pattern; e.g., first term is 4, and 4 is added to each succeeding term to make the pattern.
- Have students write pattern rules, using mathematical language, for other linear patterns by specifying the first term in the pattern and identifying the constant that is added to or subtracted from each term to get the next term.

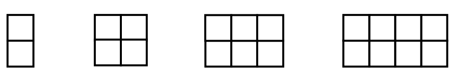
Note: This is a recursive relationship to describe the pattern. Functional relationships will be discussed in the next activity.

Example of another pattern:

Rectangle Problem

Write a pattern rule to describe the following pattern by stating the first term in the pattern and the constant that is added to each succeeding term to get the next term.

Step Number: 1 2 3 4



Number of squares: 2 4 6 8
(or area of rectangle)

Sample Solution

The first term is 2 and two are added to each succeeding number in the pattern to get the next number in the pattern.

Sample Activity 2: Growing Patterns (What Changes: Variables, What Stays the Same: Constant)

Focus on functional relationships; i.e., pattern rules that explain what you do to the step number to get the value of the pattern for that step.

Provide students with concrete materials such as square tiles.

Explain that students will continue work on translating patterns using pattern rules but will explore a different pattern rule to efficiently find the number of elements in any given step number; e.g., the number of elements in the 100th step.

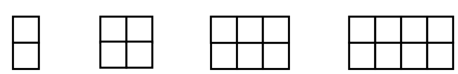
Have the students examine a variety of linear pictorial patterns and determine what changes and what remains the same in each successive step of the pattern.

Use the **Rectangle Problem** pattern from the previous activity and guide the discussion to translate the pattern into a functional relationship, i.e., the relationship between the step number and the number of squares in each step.

Rectangle Problem
- Study the following pictorial pattern for 2, 4, 6, 8, ….
- Describe what changes and what stays the same.
- Write a pattern rule, using mathematical language, that can be used to find the number of squares in the one-hundredth step.

Look For …
Do students:
- ☐ examine the patterns to determine what stays the same and what changes in each successive step?
- ☐ use appropriate mathematical language to describe a pattern rule that explains how a pattern changes from one step to the next?
- ☐ use the recursive relationship to determine the functional relationship of a given pattern?
- ☐ justify why a particular pattern rule describes a given pattern?
- ☐ transfer their learning to other patterns?
- ☐ solve problems by applying pattern rules that represent functional relationships?

Step Number:	1	2	3	4



Number of squares: (or area of rectangle)	2 × [1]	2 × [2]	2 × [3]	2 × [4]

Guided Solution

In deciding what stays the same and what changes, guide the discussion to include the dimensions of the composite rectangle in each step; i.e., the width stays the same at 2 units but the length increases by 1 unit with each new step in the pattern.

Label the numbers that change in each step in some way such as putting a box around them as shown above. Then have students state, using mathematical language, the relationship between the step number and the number of squares in each step; e.g., double the step number to get the number of squares for that step.

Ask the students what the area of the one-hundredth rectangle is. They should use the functional relationship to answer that the area of the one-hundredth rectangle is 2×100 or 200 square units.

Comparing Pattern Rules

Explain that there are different pattern rules to describe patterns. Review the pattern rule that describes how a pattern changes from one step to another step and compare it to the pattern rule that describes how the step number relates to the number of elements in each step. Use a chart to summarize the pattern and show the different pattern rules:

Step Number	1	2	3	4	…	100
Number of Squares	2	4	6	8	…	200

Explain that the horizontal arrow focuses on continuing the pattern: 2, 4, 6, 8, …. ; i.e., the pattern rule is to start at 2 and add two to each successive term.

Discuss that the vertical arrow focuses on the functional relationship; i.e., the relationship between the two rows and the pattern rule is double the step number to get the number of squares for that step.

Have students describe, using mathematical language, the functional relationship of other similar patterns and draw diagrams to represent them. For example:

- 3, 6, 9, 12, …
- 4, 8, 12, 16, ….

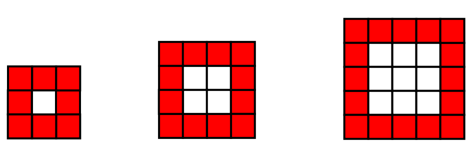
More Strategies for Describing Pattern Rules (Functional Relationships)

To guide students in describing a broader range of functional relationships for patterns that increase or decrease by a constant greater than one, provide a variety of pictorial patterns (as shown below).

Use the Pool Problem from the Sample Activity 1: Sorting Patterns—Similarities and Differences (p. 14) and adapt it to find a functional relationship between the length of the pool and the number of patio tiles needed for that pool.

Pool Problem (Ferrini-Mundy, Lappan and Phillips 1997, p. 283)

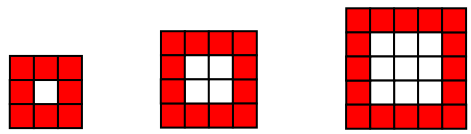
You are building a square pool. You will surround the pool with square patio tiles. The red or shaded squares represent the patio tiles. Describe a pattern rule that can be used to find the number of square patio tiles needed for the 50th pool in this pattern. Explain your thinking.



Guided Solution

Label the pictorial pattern showing the pool numbers or the length of each pool and the number of patio tiles needed for each pool as shown below.

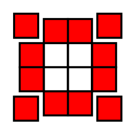
Pool Number: 1 2 3
(length of pool)



Number of Patio Tiles: $4 \times \boxed{1} + 4$ $4 \times \boxed{2} + 4$ $4 \times \boxed{3} + 4$

Discuss what changes and what stays the same in the pictorial representation of the pattern of the pool and patio tiles. Guide the discussion to conclude that there are four corner tiles in each step; therefore, these four tiles stay the same and are added in each succeeding diagram. Similarly, guide the discussion to conclude that there are four equal sides to each pool; therefore the length of the pool is always multiplied by four to get the number of patio tiles that are not corner tiles.

Use the diagrams to illustrate; e.g., the second pool has four corner patio tiles and four groups of two or eight patio tiles that are not corner tiles.



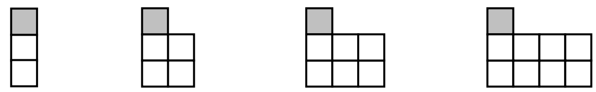
Through discussion, have students explain that the length of the pool changes with each succeeding diagram in the pattern. Have students put a box around the numbers that change in each step.

Encourage students to use mathematical language to describe the functional relationship between the length of the pool and the number of patio tiles; e.g., quadruple the length of the pool or the pool number and add four to get the number of patio tiles for that pool.
Have the student use the pattern rule to find the number of patio tiles needed for the 50^{th} pool in this pattern; i.e., $4 \times 50 + 4 = 204$.

Odd Number Problem 1
Draw a pictorial representation of the pattern 3, 5, 7, 9, ….
Write a pattern rule, using mathematical language, which can be used to find the 100^{th} number in the pattern.

Guided Solution

Step Number: 1 2 3 4



Number of squares: $2 \times \boxed{1} + 1$ $2 \times \boxed{2} + 1$ $2 \times \boxed{3} + 1$ $2 \times \boxed{4} + 1$

Discuss what changes and what stays the same in the pictorial representation of the pattern. Have students put a box around the numbers that change in each step.

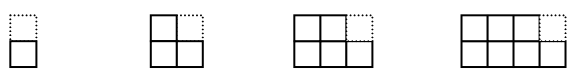
Encourage students to use mathematical language to describe the functional relationship; i.e., double the step number and add one to get the number of squares for that step.

Have student use the pattern rule to find the number of squares in the 100th step; i.e., $2 \times 100 + 1 = 201$.

Odd Number Problem 2
Draw a pictorial representation for the pattern 1, 3, 5, 7, … and write a pattern rule, using mathematical language, that can be used to find the 100th number.

Guided Solution

Step: 1 2 3 4



Number of squares: $2 \times \boxed{1} - 1$ $2 \times \boxed{2} - 1$ $2 \times \boxed{3} - 1$ $2 \times \boxed{4} - 1$

Discuss what changes and what stays the same in the pictorial representation of the pattern. Have students put a box around the numbers that change in each step.

Encourage students to use mathematical language to describe the relationship between the step number and the number of squares for that step; e.g., double the step number and subtract one to get the number of squares for that step.

Have students use this pattern rule to find the number of blocks in the 100th step.

Have students draw graphs with discrete elements to represent the various pattern rules created. The graphs provide a visual tool to represent the functional relationships.

Consolidate the learning by having students examine the recursive relationships of a variety of linear patterns to determine if the patterns increase by 1, 2, 3 and so on.

For example, provide students with the following problem.

Odd Number Problem 3
Find the 100th term in the pattern 7, 9, 11, 13, ….

Guided Solution

Suggest that the pattern be represented in a table of values showing the step number and the number of elements in each step.

Step Number	1	2	3	4	…	100
Number of Elements in the Step	7	9	11	13	…	?

Through discussion, have students verbalize the following explanation.

Since the pattern in the bottom row of the chart increases by two each time, then the pattern rule that relates the two rows in the table (functional relationship) will include doubling the step number. If you double step number 1, you get 2. In order to get the required number of elements (7) for this first step, you must add five. Similarly, if you double step number 2, you get 4. In order to get the required number of elements (9) for this second step, you must add five. Therefore, the pattern rule, using mathematical language, is: double the step number and add five to get the number of elements for that step.

The 100th step would have $2 \times 100 + 5 = 205$ elements.

To conclude, review that the recursive relationships for patterns are needed to determine their functional relationships. It is the functional relationships, written as pattern rules, which show the power of algebra, providing a general rule that can be used to find the number of elements in any step when given the step number.

Have students discuss the similarities and differences between recursive and functional relationships, recognizing that both are included in patterns but each of them has a different role:

- a recursive relationship describes the pattern between successive numbers in one of the rows in a table of values
- a functional relationship is a general rule to describe the relationship between two rows of numbers in a table of values.

Sample Activity 3: Input–Output

Read the story *Two of Everything* by Lily T. Hong. The story is based on a traditional Chinese folktale. Mr. Haktak digs up a brass pot that is a magic pot. Everything that is placed inside the magic pot is doubled. Use this story as a context for the following Input–Output activity.

Tell students to work in pairs and follow these instructions:

- Partner A thinks of a pattern rule (functional relationship) and keeps it a secret from his or her partner.
- Partner B places a number in the input row of a table of values.
- Partner A uses the pattern rule he or she created and writes the corresponding output in the table of values.
- Partner B continues to provide numbers in the input row until he or she can discover the pattern rule used to make the output numbers.
- Plot the pattern on a graph, using coordinate points to represent the input–output pairs.

Look For …

Do students:

- ☐ record the input–output data in a chart?
- ☐ use successive numbers as input so that the pattern is more easily seen?
- ☐ apply their understanding of recursive relationships (how a pattern changes from step-to-step) to find the functional relationship (the relationship between two rows of numbers in a chart)?
- ☐ translate the pattern efficiently from the chart to a graph?
- ☐ show improvement as the input–output game progresses?

Example:

Partner A creates the pattern rule: double the input number and add one to get the output number.

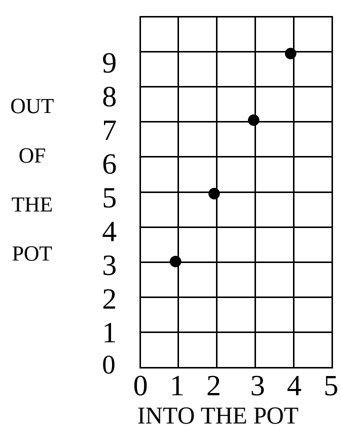
Partner B places the number 1 in the input row. Partner A uses the pattern rule, $2 \times 1 + 1 = 3$ and writes 3 in the output row.

Similarly complete for the remaining numbers.

Input Number	1	2	3	4
Output Number	3	5	7	9

Partner B discovers the pattern rule and both students plot the graph.

INPUT–OUTPUT GRAPH



Have students reverse the roles and do the activity again. Remind students that the pattern rules should include only linear patterns; i.e., patterns that increase or decrease by the same amount each time producing a linear graph with discrete elements (*Patterns and Pre-Algebra, Grades 4–6*, Alberta Education 2007, pp. 133–134).

Sample Activity 4: Problem Solving with Patterns

Provide students with problems using everyday contexts in which they can apply their understanding of functional relationships.

> **Look For …**
> Do students:
> - ☐ transfer the information in the problem to another model such as a chart or a diagram?
> - ☐ apply their understanding of recursive relationships (how a pattern changes from step-to-step) to find a functional relationship (the relationship between two rows of numbers in chart)?
> - ☐ use the pattern rule for the functional relationship to solve the problem using larger numbers?

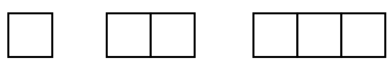
Pizza Problem
Pete's Pizza Parlour has square tables that each seat four people. If you push two tables together, six people can be seated. If you push three tables together, eight people can be seated.

a. Write a pattern rule that can be used to calculate the number of people that can be seated given any number of tables put end-to-end.
b. Use your pattern rule to find how many people can be seated if 50 tables are put end-to-end.

Guided Solution

Build on students' knowledge of creating charts for patterns and have them suggest how the information in the problem can be represented in a chart. Encourage students to draw diagrams to represent the pattern and place the data in a chart.

For example:
Number of Tables: 1 2 3



Number of People: 4 6 8

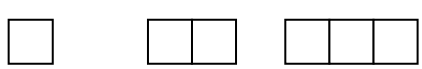
Number of Tables	1	2	3	4	…	50
Number of People	4	6	8	?	…	?

Have students describe the recursive relationship of the pattern of numbers in the bottom row of the chart; i.e., each succeeding number increases by two.

Build on student's understanding of patterns in writing functional relationships that connect the step number with the number of elements in each step (see Sample Activity 3, p. 29).

Provide scaffolding for students, if necessary, by having them examine the diagrams in the pattern and notice what changes and what stays the same.

Number of Tables: 1 2 3



$2 \times \boxed{1} + 2$ $2 \times \boxed{2} + 2$ $2 \times \boxed{3} + 2$

Discuss that the constant, 2, is added in each expression because two people sit at the ends in each diagram.

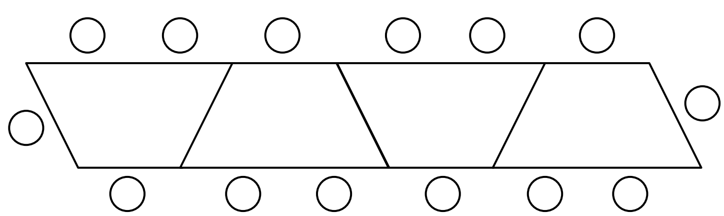
Discuss that the constant, 2, multiplies each step number because for each table there are two people seated on the sides that are not the ends; i.e., for one table, two people can be seated at the sides that are not the ends; for two tables, $2 \times 2 = 4$ people can be seated at the sides that are not the ends; and for three tables, $2 \times 3 = 6$ people can be seated at the sides that are not the ends, and so on. See the expressions written below the diagrams.

Instruct students to write a pattern rule, using mathematical language, which can be used to find the number of people seated at a given number of tables; e.g., double the number of tables and add two to get the number of people seated for that step.

Have students use their pattern rule to find the number of people that can be seated with 50 tables placed end-to-end; e.g., $2 \times 50 + 2 = 102$. Then have students write a sentence to answer the question asked in part (b) of the problem; e.g., when 50 tables are placed end-to-end, 102 people can be seated.

Provide other real-world problems for students to write pattern rules (functional relationships) and use them to solve the problems. Remind them to use diagrams and charts to represent the problems so that they are better able to write the pattern rules.

Example: Trapezoid Problem (Blanton and Kaput 2003, pp. 75–76)
Find how many people can be seated at 20 trapezoidal tables, modelled by the following diagram. Explain your thinking.
How many people can be seated at 100 trapezoidal tables? Explain your thinking.



Step 4: Assess Student Learning

Guiding Questions

- Look back at what you determined as acceptable evidence in Step 2.
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

In addition to ongoing assessment throughout the lessons, consider the following sample activities to evaluate students' learning at key milestones. Suggestions are given for assessing all students as a class or in groups, individual students in need of further evaluation, and individual or groups of students in a variety of contexts.

A. Whole Class/Group Assessment

Translations for Growing Patterns

In this assessment task, students will solve problems to demonstrate their understanding of patterns by using concrete materials, as needed, diagrams and symbols. In the first question, they will create at least two pictorial representations of a pattern described in a table of values. Then they will translate the pattern to a graph (a linear graph with discrete elements).

In the second question, the students will create a table of values to represent a pictorial pattern of rectangles, made up of grey and white tiles. They will write a rule, using mathematical language, that can be used to find the number of grey tiles when given the number of white tiles in a rectangle. Finally, they will use their rule to find the number of grey tiles in a rectangle with 100 white tiles.

Materials required: square tiles, pattern blocks, centimetre grid paper.

Question 1
In creating a pictorial representation of the relationship shown in a table of values, students could use the square tiles or pattern blocks to create concrete patterns and then translate the patterns into pictorial representations. The patterns created along with the written explanations should clearly show what stays the same and what changes from step-to-step. In other words, the patterns must show "some repetition or regularity" (Small 2009, p. 3).

Students should draw and label a graph to represent the pattern shown in the table of values. The graph consists of discrete elements with each point representing an ordered pair from the table of values.

Question 2

Students should create a table of values to represent the pictorial pattern of growing rectangles, label it appropriately and justify why the table represents the pattern. For example:

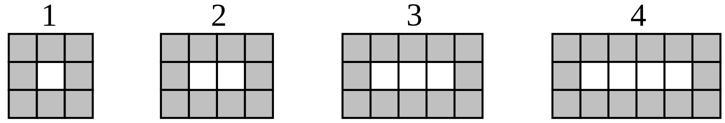
Number of White Tiles	1	2	3	4
Number of Grey Tiles	8	10	12	14

The number of white tiles starts at 1 and increases by one with each step. The number of grey tiles starts at 8 and increases by two with each step.

Students write a pattern rule, using mathematical language, which describes the relationship between the number of white tiles and the number of grey tiles in the growing rectangles. This rule may include variables but it is not required. Students should explain clearly how the rule was derived by stating what changes and what stays the same in the pattern.

Sample Explanation and Pattern Rule

Number of White Tiles: 1 2 3 4



Number of Grey Tiles: 8 10 12 14

$2 \times \boxed{1} + 6$ $2 \times \boxed{2} + 6$ $2 \times \boxed{3} + 6$ $2 \times \boxed{4} + 6$

The three grey tiles on each end of each rectangle stay the same, making six grey tiles that are constant for any rectangle. Also, there are always two equal groups of grey tiles on each rectangle that do not form the ends of the rectangle. The number of white tiles changes and matches the number of each diagram (or the step number); i.e., the first diagram has one white tile, the second diagram has two white tiles and so on.

The pattern rule is: double the number of white tiles (or the step number) and add six to get the number of grey tiles in any step of the pattern. Using variables, the rule could be written as $2W + 6 = G$, where W = the number of white tiles and G = the number of grey tiles in any step in the pattern.

The students use the rule to find the number of grey tiles if given 100 white tiles and write a statement; e.g., $2 \times 100 + 6 = 206$. There are 206 grey tiles in the step that has 100 white tiles.

Early finishers can:

- use the information from Question 2 to find the number of white tiles in a rectangle that has 50 grey tiles
- use the information from Question 2 to find the total number of grey and white tiles in the 100^{th} diagram
- find the pattern rule for the pattern 5, 7, 9, 11, … and use it to find the 300^{th} term.

Task-specific Criteria

Each student will:

- create a concrete or pictorial representation of the relationship in a given table of values
- translate a pattern from a table of values to a graph (limited to linear graphs with discrete elements)
- state, using mathematical language, the relationship in a given table of values
- formulate a rule to describe the relationship between two rows of numbers in a table of values
- generate values in one row of a table of values, given values in the other row and a pattern rule
- create a table of values to record and reveal a pattern to solve a given problem
- draw linear graphs with discrete elements to represent a pattern
- create and apply a pattern rule to solve problems.

Translations for Growing Patterns—Student Assessment Task

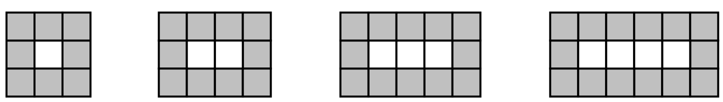
1. A growing pattern is shown in the table of values below:

Step Number	1	2	3	4
Number of Elements in the Step	3	6	9	12

   a. Draw and label at least two different pictorial representations of the pattern shown in the table of values. Explain your thinking.

   b. Draw and label a graph on centimetre grid paper that represents the pattern shown in the table of values.

2. A pattern of growing rectangles using grey and white tiles is shown below:



   a. Create a table of values to represent the pattern of white and grey tiles in each rectangle. Explain your thinking.

   b. Write a rule that could be used to find the number of grey tiles in any rectangle if you are given the number of white tiles. Explain your thinking.

   c. Use your rule to find the number of grey tiles in a rectangle that has 100 white tiles. Explain your thinking.

Student: ________________________________

SCORING GUIDE:
Translations for Growing Patterns

Level / Criteria	4 Excellent	3 Proficient	2 Adequate	1 Limited *	Insufficient / Blank *
Translates a pattern from a table of values to a pictorial representation. Question 1 (a)	The student draws and labels very accurate diagrams in three or more different ways to represent the pattern.	The student draws and labels accurate diagrams in two ways to represent the pattern.	The student draws diagrams in only one way, including most labels, with some inaccuracies to represent the pattern.	The student draws inaccurate or incorrect diagrams in only one way to represent the pattern with few, if any, labels.	No score is awarded because there is insufficient evidence of student performance based on the requirements of the assessment task.
Translates a pattern from a table of values to a graph. Question 1 (b)	The student draws and labels a very accurate graph to represent the pattern shown in the table of values.	The student draws and labels an accurate graph to represent the pattern shown in the table of values.	The student draws the graph, including most labels, to represent the pattern shown in the table of values with slight inaccuracies.	The student draws an inaccurate graph of the pattern shown in the table of values with few, if any, labels.	No score is awarded because there is insufficient evidence of student performance based on the requirements of the assessment task
Creates a table of values to represent a pictorial pattern. Question 2 (a)	The student creates a table of values to represent the pattern of white and grey tiles in growing rectangles, and explains it clearly, using precise mathematical language.	The student creates a table of values to represent the pattern of white and grey tiles in growing rectangles, and provides a clear explanation.	The student creates a table of values to represent the pattern of white and grey tiles in growing rectangles with limited or no explanation.	The student attempts to create a table of values to represent the pattern of white and grey tiles in growing rectangles, but has difficulty with the labels and/or the pattern of numbers and provides no explanation.	No score is awarded because there is insufficient evidence of student performance based on the requirements of the assessment task.
Formulates and applies a pattern rule that connects two rows of a table of values to solve a problem. Question 2 (b) and (c)	The student describes clearly, using precise mathematical language, a pattern rule that is used to find how many grey tiles there are in a rectangle with 100 white tiles.	The student describes clearly a pattern rule that is used to find how many grey tiles there are in a rectangle with 100 white tiles.	The student creates and uses a pattern rule with a limited explanation to find how many grey tiles there are in a rectangle with 100 white tiles.	The student is unable to create and use a pattern rule to find how many grey tiles there are in a rectangle with 100 white tiles, but the student may extend the pattern to show that the number of grey tiles increases by two with each step.	No score is awarded because there is insufficient evidence of student performance based on the requirements of the assessment task.

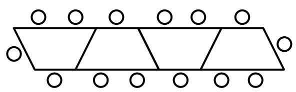
* When work is judged to be limited or insufficient, the teacher makes decisions about appropriate interventions to help the student improve.

B. One-on-one Assessment

Provide the students with graph paper, pattern blocks and square tiles.

Pattern Rules and Translations

1. Present the following problem to the student:
   The following diagram shows four trapezoidal tables with 14 people seated. Following this pattern, find how many people can be seated at 10 trapezoidal tables. Explain your thinking.

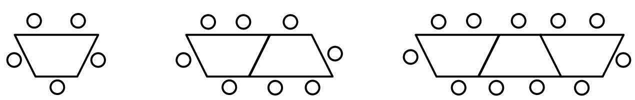
   

   (Blanton and Kaput 2003, pp. 75–76)

   If the student has difficulty solving the problem, suggest that he or she use the trapezoidal pattern blocks to model the problem for fewer tables; e.g., one table, two tables, three tables, four tables. Ask the student how many more people can be seated with the addition of each new table. The student may use 10 trapezoids to show concretely the solution to the problem.

   The goal is to have the student use pattern rules that describe functional relationships to solve problems. Suggest that the student translate the data to another pictorial representation showing step numbers and then to a table of values or a chart so that the pattern is more easily seen. Provide assistance, as necessary, in connecting the concrete and pictorial representations, as well as creating the headings for the chart. Remind the student that the step number refers to the number of the diagram in the pattern, or, in this case, the number of trapezoidal tables in the diagram. Help the student model the first step using a trapezoid, then the second step using two trapezoids and so on.

   Step Number: 1 2 3

   Number of People Seated: 5 8 11

Step Number or Number of Tables	1	2	3	4
Number of People Seated	5	8	11	?

   The student may continue the pattern in the chart to include 10 tables to answer to the problem or verify the work done with concrete materials. Then use larger numbers in the problem to motivate the student to apply a pattern rule describing a functional relationship.

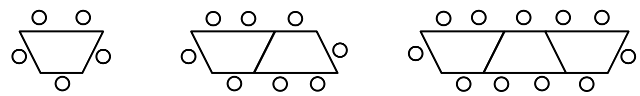
   Ask the student, "How many people can be seated at 100 trapezoidal tables?"

   If the student has difficulty answering the problem, suggest that the student find a pattern rule that describes the relationship between the two rows in the table of values or chart. Remind the student to look at the pictorial representation and decide what changes and what stays the same for each step in the pattern.

The student should see that the number of tables changes with each step as well as the number of people seated. What stays the same is the fact that three more people are added each time and there are always two people seated at each end.

Suggest that the student write an expression under each step to show what changes and what stays the same for the number of people seated. The variable, the step number, is placed in a box in each expression.

Step Number: 1 2 3



Number of People Seated: $3 \times \boxed{1} + 2$ $3 \times \boxed{2} + 2$ $3 \times \boxed{3} + 2$

Have students state, in words, the functional relationship that between the step number and the number of people seated; i.e., triple the step number (or the number of trapezoidal tables) and add two to get the number of people seated for that step.

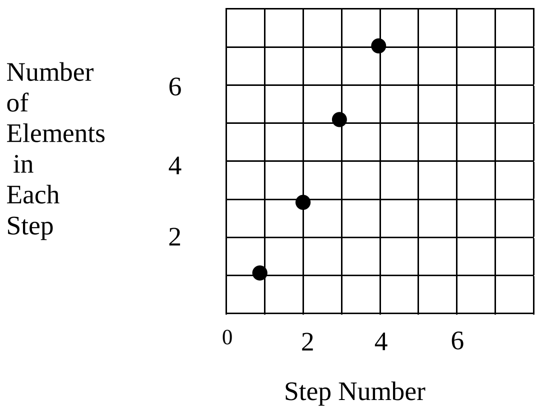
Now, the student should be able to apply this pattern rule to find the number of people seated if there are 100 tables; i.e., $3 \times 100 + 2 = 302$.
There can be 302 people seated at 100 trapezoidal tables.

2. Ask the student to graph the pattern from the previous problem.

   If the student has difficulty drawing a graph, point to the headings from the table of values and say that these are the labels of the axes. Have the student use the step number as the label for the horizontal axis. Remind the student that the numbers placed along the vertical and horizontal axis must be evenly spaced.

   Take one pair of numbers from the table of values and ask the student where each number is represented on the graph, remembering that the step number is the first member of the ordered pair. Have the student check the accuracy of his or her graph by using a ruler to determine if the points placed on the graph line up, as is always the case with linear relations such as this one.

3. Present the following graph to the student and ask the student to:
   - create a table of values for the pattern
   - create a pictorial representation for the pattern.



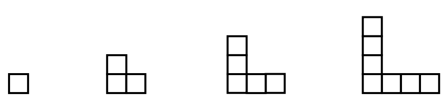
If the student has difficulty creating a table of values, suggest that the headings for the table of values must match the labels on the axes of the graph. Ask, "How many elements are there in the first step?" Show the student the location of the point, (1, 1), on the graph, if necessary, and explain the meaning of each number in the ordered pair. Then continue with the other steps as the data is recorded in the table of values.

If the student has difficulty creating a pictorial representation for the pattern, provide the student with concrete materials such as square tiles and have him or her show a pattern by using one item for the first step, three items for the second step, five items for the third step and so on. Remind the student that the pattern must show "some repetition or regularity" (Small 2009, p. 3). Have the student show you what part repeats in each succeeding step.

Then ask students to draw the diagrams to match the concrete representation. Have students describe the pattern made by explaining what changes and what stays the same with each successive step.

Sample Pictorial Representation of the Pattern:

Step Number:	1	2	3	4



Number of Elements: (or area in square units)	1	3	5	7

See Step 3, Section C: Choosing Learning Activities, Sample Activities for Teaching Pattern Rules (Recursive and Functional Relationships) p. 21 for a detailed description of activities to promote understanding of what changes and what stays the same in patterns.

C. Applied Learning

Provide opportunities for students to use patterns in a practical situation and notice whether or not the understanding transfers. For example, have the student predict and then calculate how much money he or she will save on the 30th day if $3 are saved on the first day, $5 on the second day, $7 on the third day and so on. Does the student:

- communicate the pattern that is shown in the problem?
- predict correctly and justify his or her prediction?
- explain the functional relationship between the number of the day and the money saved on that day?
- calculate the total amount saved in a given number of days such as 10 days?
- predict the number of the day on which a given amount of money is saved?

Step 5: Follow-up on Assessment

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction?

A. Addressing Gaps in Learning

If students are having difficulty in solving the basic facts using strategies, consider the following.

- Build on students' understanding of patterns, area and perimeter from previous grades.
- Use patterns with everyday contexts so students understand the purpose of the patterns and are motivated to complete the tasks.
- Encourage students to use manipulatives to represent various patterns and then draw corresponding diagrams along with charts or tables of values.
- Have students explain their thinking and provide scaffolding to overcome any misconceptions or misunderstandings.
- Use friendly numbers in the patterns initially so students can easily draw diagrams to represent the patterns.
- Explain that there are two types of pattern rules or relationships: a rule that describes how a pattern changes from one step to another step (recursive relationship), and a rule that explains what you do to the step number to get the value of the pattern for that step (functional relationship). Emphasize that the rule connecting the step number to the number of elements in that step is used to find how many elements are in any given step number, such as the 100^{th} step. Provide examples of each type of rule for a given pattern.
- In analyzing patterns to establish pattern rules, remind students to focus on what changes with each step in the pattern and what stays the same.
- In creating functional relationships for a given pattern, have students explain the recursive relationship among the numbers in the individual rows in the table of values and use this relationship in establishing the functional relationship that relates the two rows or columns of numbers in a table of values.
- Emphasize the importance of creating a table of values when connecting pattern rules with their corresponding graphs. Provide scaffolding in labelling the table of values, if necessary.
- Reinforce information from a problem that can be translated into a table of values and then represented on a graph. The graph provides a visual tool to aid in solving the problem.
- Reinforce understanding of patterns by integrating patterns in every strand and emphasize the power of patterns in mathematics.

B. Reinforcing and Extending Learning

Students who have achieved or exceeded the outcomes will benefit from ongoing opportunities to apply and extend their learning. Consider strategies such as the following to support students in developing a deeper understanding of the concept.

- Provide tips for parents on writing generalizing patterns at home or in the community.
  - Predict the salary for a given month if the starting monthly salary is known and if on each successive month the salary increases by a constant number such as $2.
  - Predict how many pages are read in 10 days if five pages are read on the first day and each successive day, the number of pages read increases by a constant number such as four.
  - Generalize common quantities such as the following:
    - the number of wheels on any number of bicycles
    - the number of legs for any number of dogs
    - the number of minutes in any number of hours
    - the number of eggs in any number of egg cartons.

- Have students play the game Guess My Rule. One student operates the input–output machine that can be represented by a chart such as the following:

Input				
Output				

  The operator has a pattern rule describing a functional relationship in mind. The other students try to guess the rule by suggesting input numbers while the operator records the output number. The list of input–output numbers should be recorded in the chart on the board or overhead. Students who think they know the rule provide the output number using the rule and the operator decides if the output follows the pattern or not. The game continues until most students have guessed the rule. Students could play this game in small groups (Van de Walle and Lovin 2006, p. 272; Cathcart, Pothier and Vance 1994, p. 375).

- Have students create a concrete or pictorial representation of the relationship shown in the following tables of values.

  a.

Step Number	1	2	3	4	5	6
Number of Elements in Each Step	2	5	8	11	14	17

  Have students graph the data shown in the table of values.
  Then have students find the pattern rule that shows the relationship between the step number and the number of elements in each step. Have them use the pattern rule to find the number of elements in the 50^{th} step.

b.

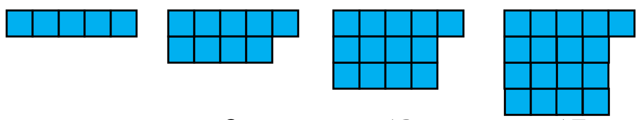
Step Number	1	2	3	4	5	6
Number of Elements in Each Step	1	3	6	10	15	21

Challenge students to find the pattern rule that shows the relationship between the step number and the number of elements in each step. Have them use the pattern rule to find the number of elements in the 50th step.

- Provide students with pictorial patterns. Ask students to:
  – create a table of values from the given pattern
  – draw a graph to represent the given pattern and the table of values
  – write a pattern rule to describe the relationship between the step number and the number of elements in each step
  – use their pattern rule to find the number of elements in the 100th step.

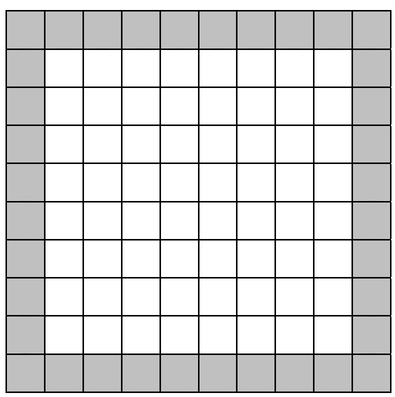
Sample Pictorial Pattern (Small 2009, p. 5)

Step Number: 1 2 3 4



Area (square units): 5 9 13 17

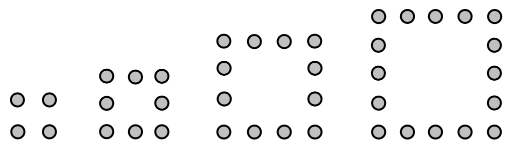
- Provide students with a 10-by-10 grid to solve the following border problem.



  – How many squares are there in the border of the 10-by-10 grid? Explain how you know.
  – Describe a different way to get your answer.
  – Write a rule that could be used to find the number of squares in the border of a square grid of any size. Explain your thinking.
  (Burns 1989, p. 27; National Council of Teachers of Mathematics 2000, p.185)

- Provide students with the following post problem.

  The dots represent the posts needed to enclose square fields of various sizes.



  – Draw the 5th square. Explain your thinking.
  – How many posts would be needed for the 10th square field? Explain your thinking.
  – Write a rule that could be used to find the number of posts needed if you are given the number of posts on one side. Explain your thinking.
  – Use your rule to find the number of posts needed if you have 100 posts on one side of the square field. Explain your thinking.

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