Planning Guide: *Grade 7 Area*

Strand: Shape and Space (Measurement)
Outcome: 2

Curriculum Highlights

This sample targets the following changes in the curriculum:

- The General Outcome focuses on using direct or indirect measurement to solve problems, whereas the previous mathematics curriculum focused on generalizing measurement patterns and procedures and solving problems involving area.
- The Specific Outcomes for finding the areas of triangles, parallelograms and circles were included in Grade 8 in the previous mathematics curriculum and were not included at all in Grade 7.

Step 1: Identify Outcomes to Address

Guiding Questions

- What do I want my students to learn?
- What can my students currently understand and do?
- What do I want my students to understand and be able to do, based on the Big Ideas and specific outcomes in the program of studies?

Big Ideas

Van de Walle and Lovin define area as "a measure of the space inside a region or how much it takes to cover a region" (2006, p. 240).

In using any type of measurement, such as length, area or volume, it is important to discuss the similarities in developing understanding of the different measures: first identify the attribute to be measured, then choose an appropriate unit and finally compare that unit to the object being measured (NCTM 2000, p. 171). As with other attributes, it is important to understand the attribute of area before measuring.

Key ideas in understanding the attribute of area include:

- conservation—an object retains its size when the orientation is changed or when it is rearranged by subdividing it in any way
- iteration—the repetitive use of identical nonstandard or standard units of area to cover the surface of the region entirely
- tiling—the units used to measure the area of a region must not overlap and must completely cover the region, leaving no gaps
- additivity—add the measures of the area for each part of a region to obtain the measure of the entire region

- proportionality—there is an inverse relationship between the size of the unit used to measure area and the number of units needed to measure the area of a given region
- i.e., the smaller the unit, the more units are needed to measure the area of a given region
- congruence—comparison of the area of two regions can be done by superimposing one region on the other region, subdividing and rearranging as necessary
- transitivity—when direct comparison of two areas is not possible, a third item is used that allows comparison; e.g., to compare the area of two windows, find the area of one window using nonstandard or standard units and compare that measure with the area of the other window; i.e., if $A = B$ and $B = C$, then $A = C$, and similarly for inequalities
- standardization—using standard units for measuring area, such as cm^2 and m^2, facilitates communication of measures globally
- unit/unit–attribute relations—units used for measuring area must relate to area; e.g., cm^2 must be used to measure area and not cm or mL.

Adapted from Alberta Education, *Measurement: Activities to Develop Understanding* (Research section) (unpublished workshop handout) (Edmonton, AB: Alberta Education, 2005), pp. 2–4.

Formulas for finding areas of 2-D shapes provide a method of measuring area by using only measures of length (Van de Walle and Lovin 2006, p. 230). The areas of rectangles, parallelograms, triangles and circles are related, with the area of rectangles forming the foundation for the areas of the other 2-D shapes.

Sequence of Outcomes from the Program of Studies

See [Web address] for the complete program of studies.

Grade 6	Grade 7	Grade 8
Specific Outcomes 3. Develop and apply a formula for determining the: • perimeter of polygons • area of rectangles • volume of right rectangular prisms.	Specific Outcomes 2. Develop and apply a formula for determining the area of: • triangles • parallelograms • circles.	Specific Outcomes 3. Determine the surface area of: • right rectangular prisms • right triangular prisms • right cylinders to solve problems. 4. Develop and apply formulas for determining the volume of right rectangular prisms, right triangular prisms and right cylinders.

Step 2: Determine Evidence of Student Learning

Guiding Questions

- What evidence will I look for to know that learning has occurred?
- What should students demonstrate to show their understanding of the mathematical concepts, skills and Big Ideas?

As you begin planning lessons and learning activities, keep in mind ongoing ways to monitor and assess student learning. One starting point for this planning is to consider the achievement indicators listed in *The Alberta K–9 Mathematics Program of Studies with Achievement Indicators* (Alberta Education 2007). You may also generate your own indicators and use these to guide your observation of students.

The following achievement indicators may be used to determine whether students have met this specific outcome.

- Explain how to estimate the area of a circle.
- Illustrate and explain how the area of a rectangle can be used to determine the area of a triangle.
- Generalize a rule to create a formula for determining the area of a triangle.
- Apply a formula for determining the area of a given triangle.
- Explain how to estimate the area of a parallelogram.
- Illustrate and explain how the area of a rectangle can be used to determine the area of a parallelogram.
- Generalize a rule to create a formula for determining the area of a parallelogram.
- Apply a formula for determining the area of a given parallelogram.
- Explain how to estimate the area of a circle.
- Illustrate and explain how the area of a parallelogram or a rectangle can be used to determine the area of a circle.
- Generalize a rule to create a formula for determining the area of circles.
- Apply a formula for determining the area of a given circle.
- Solve a given problem involving the area of triangles, parallelograms and/or circles.

Some sample behaviours to look for in relation to these indicators are suggested for many of the instructional activities in Step 3, Section C, Choosing Learning Activities.

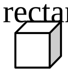
Step 3: Plan for Instruction

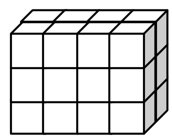
Guiding Questions

- What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should I use?
- How will I meet the diverse learning needs of my students?

A. Assessing Prior Knowledge and Skills

Before introducing new material, consider ways to assess and build on students' knowledge and skills related to perimeter, area and volume. For example:

- Write a formula for finding the perimeter of any regular polygon and use this formula to find the perimeter of a hexagonal plate that is 15 cm on each side. Show all your work.
- Write the formula for finding the area of a rectangle. Explain how the formula for the area of a rectangle refers to length and width, which are linear units, but area is measured in square units. Use a diagram to explain the process.
- Write the formula for finding the volume of a right rectangular prism and use the formula to find the volume of the following prism. Each cube, , is 1 cm by 1 cm by 1 cm. Explain your thinking.



- A runway for a dog is in the shape of a rectangle that is 50 m long and 300 cm wide.
  a) Find the length of fencing needed to enclose the runway. Explain your thinking.
  b) Find the area of the runway. Explain your thinking.
- The area of a rectangular garden plot is 36 m^2. If the length is 9 m, what is the width of the plot? Explain your thinking.

If a student appears to have difficulty with these tasks, consider further individual assessment, such as a structured interview, to determine the student's level of skill and understanding. See Sample Structured Interview: Assessing Prior Knowledge and Skills.

B. Choosing Instructional Strategies

Consider the following strategies when planning lessons.

- Build on students' prior knowledge of area.
- Have students construct the formulas by applying their knowledge of the area of rectangles and rearranging the shapes of triangles, parallelograms and circles.
- Have students demonstrate conservation of area so that they know the area remains the same when shapes are rearranged.
- Provide students with many types of triangles and parallelograms when constructing meaning for the areas of these 2-D shapes.
- Connect the area of a parallelogram to motion geometry. Convert a parallelogram into a rectangle by sliding a triangle.
- Have students build meaning for the area of a parallelogram based on their understanding of the area of a rectangle. Then have them construct meaning for the area of a triangle by connecting it to the area of a parallelogram. Finally, have students construct meaning for the area of a circle by rearranging it into a parallelogram or a rectangle. Emphasize the connections among the formulas.
- Promote discussions that guide students to discover that pi (π) is used for finding the circumference and the area of circles; i.e., $\pi = C/d$ and $\pi = A/r^2$.

- Have students estimate before calculating the areas of parallelograms, triangles and circles.
- Emphasize that the area of any 2-D shape has a numerical value and a unit. The unit for area is always the square unit even though units of length are used in the formula. Have students communicate the connection between the linear units of length and the square units for area by using diagrams.
- Have students communicate why the linear units used in any formula must always be the same; i.e., if the base of a parallelogram is measured in centimetres, then the height must also be measured in centimetres.

C. Choosing Learning Activities

The following learning activities are examples of activities that could be used to develop student understanding of the concepts identified in Step 1.

Look For …

Do students:

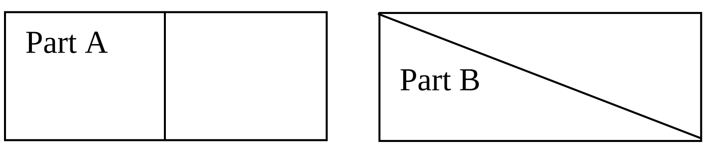
- ☐ apply and use the term congruency in solving the problem?
- ☐ explain that "equal parts" means the same area but not necessarily the same shape?
- ☐ communicate clearly why Part A and Part B have the same area?

Sample Activities for Teaching the Formulas for the Areas of Parallelograms, Triangles and Circles

1. Conservation of Area

   Provide students with the following problem.

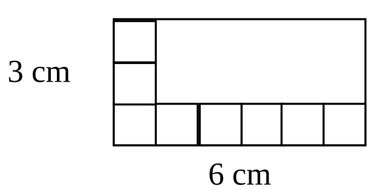
   > You are given two rectangular fields with the same area as shown below. Each field is divided into two equal parts. Does Part A have the same area as Part B? Why or why not?



   Have students share their answers and explanations. Guide the discussion so students conclude that halves of equal areas must be equal to each other. The area is conserved even though the shapes of the two parts are different (NCTM 2000, p. 190).

2. Area of Parallelograms Using Rectangles

   Review the area of rectangles and the formula $A = LW$. Use diagrams to show why the area of a rectangle is recorded in square units but the length and width in the formula are recorded in linear units.



Look For …

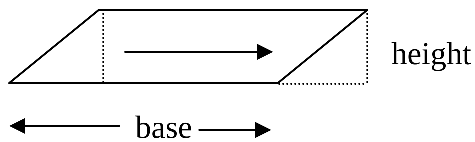
Do students:

- ☐ draw diagrams to show the area of rectangles, using a given number of square units in each row and a given number of rows?
- ☐ apply their understanding of slides (or translations) to rearrange a parallelogram into a rectangle of equal area?
- ☐ draw the perpendicular height for a given parallelogram?
- ☐ apply the formula for parallelograms in any orientation?

The area of the rectangle is found by taking three groups of 6 cm^2 to make 18 cm^2. Therefore, the area of a rectangle is found by multiplying 3 cm by 6 cm, both of which are the dimensions of the rectangle—3 cm x 6 cm = 18 cm^2.

Provide students with parallelograms that are not rectangles. Have them estimate the area of each parallelogram, using a referent such as a small fingernail representing 1 cm^2. Have students work in groups or independently to find the area of the parallelograms and generalize a formula. Provide students with scissors so that they may cut out the parallelograms and rearrange them if desired.

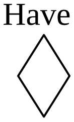
Have students share their ideas with one another. Provide guidance as necessary, suggesting that the parallelogram may be rearranged to form a rectangle by sliding a triangular section. Have students slide the triangle in the parallelogram to make a rectangle as shown below.



Have students generalize that any parallelogram can be rearranged to form a rectangle. Therefore, the area can be found by multiplying the base of the parallelogram by the height of the parallelogram, using the formula for the area of a rectangle. Emphasize that the height of the parallelogram is always the **perpendicular height** because a rectangle always has a base and height perpendicular to each other.

Ask students to measure the base and the height of the parallelogram, apply the formula ($A = bh$) and calculate the area of the parallelogram. Students could also place centimetre grid transparencies over the parallelogram and count the number of square centimetres inside the parallelogram. Have students compare the calculated area to the estimated area of the parallelogram.

Have students draw other parallelograms, estimate their areas, measure the base and perpendicular height, and calculate the areas. Have them include examples of parallelograms in various orientations, such as the following.



Encourage students to change the orientation of a given figure to whatever orientation suits them best as they apply the formula (NCTM 2000, p. 244).

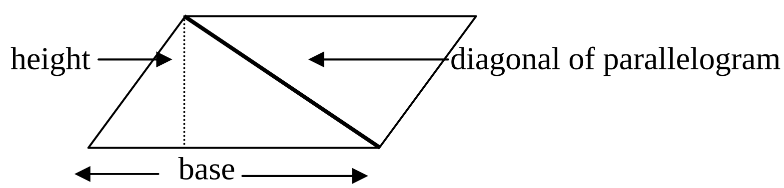
3. Area of Triangles Using Parallelograms
Provide students with copies of parallelograms, including parallelograms that are rectangles. Present them with the following problem:

> Use what you know about the area of a parallelogram ($A = bh$) to help you discover the formula for the area of any triangle.

Provide prompts if necessary, such as suggesting that they cut
out a parallelogram and cut along its diagonal to make two triangles. Ask them how these two triangles are related. If necessary, suggest that they turn one triangle 180° to superimpose on the other triangle, showing congruency.

Look For …
Do students:
- ☐ apply their knowledge of congruency to the two triangles that make up the parallelogram?
- ☐ apply the formula for the area of triangles to triangles of all different shapes and orientations?

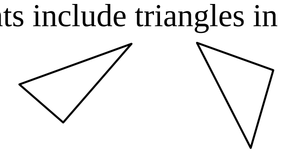
Ask how this information can be used to find a formula for the area of a triangle. Have them draw and label diagrams to show this relationship. Example:



After students discover that the triangle has the same base and perpendicular height as the related parallelogram but has only half the area of the parallelogram, have them write the formula, using appropriate letters, such as $A = bh/2$, and explain the meaning of each letter used.

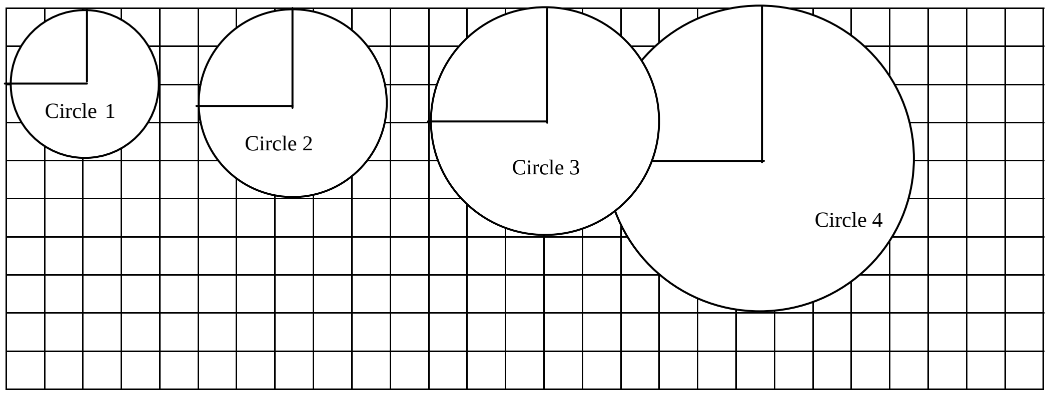
Have students draw many different triangles, estimate their areas, measure the base and the perpendicular height, calculate the areas and compare their calculated answers to their estimates. Encourage students to draw the parallelogram that is double the area of the given triangle.

Have students include triangles in different orientations, such as the following.



Adapted from Van de Walle, John A., LouAnn H. Lovin, *Teaching Student-Centered Mathematics, Grades 5–8* (pp. 255–256). Published by Allyn and Bacon, Boston, MA. Copyright © 2006 by Pearson Education. Adapted by permission of the publisher.

4. Estimating the Area of a Circle Using the Radius Square as a Referent
Provide students with circles drawn on centimetre grid paper with the radius of each circle shown as illustrated.



Suggest that students estimate the area of each circle. Have them share their strategies. Guide the discussion to include the strategy of using the square with two sides as the radii of the related circle. For example, in Circle 1, the square formed by using two radii for two of the sides has an area of 4 cm^2. The area of the circle would be less than four of these squares so the area of the circle might be about 4 x 4 – 4 cm^2 or about 3 x 4 = 12 cm^2. Have students find the area of the circle by drawing the grid lines and counting the squares. They should compare the area found by counting the squares to their estimated area.

Have students apply their strategies for estimating the area of a circle to two other circles and share their results with one another.

Encourage students to look for patterns by tabulating their findings in a chart, such as the following:

Look For …
Do students:
- ☐ apply the formula for the area of a square in finding the radius of a square?
- ☐ apply their understanding of central angles circle—four right angles or four radius squares?
- ☐ use estimation skills to explain that the area of the circle is about 3 radius squares?

Circle	1	2	3	4
Radius (cm)	2	2.5	3	4
Estimated Area (cm^2)	12	18	27	48
Area by Counting the Squares (cm^2)	12.5	19.5	28	50

The exploratory work done by students in estimating the areas of circles, using the square of the radius as a referent, provides a foundation for developing the formula for the area of a circle as described in the next activity.

Adapted from Alberta Education, *Measurement: Activities to Develop Understanding* (unpublished workshop handout) (Edmonton, AB: Alberta Education, 2005), pp. 59–66.

5. Area of a Circle Using Patterns
   Have students build on the work done in the previous activity to explore what the formula for the area of any circle might be. Extend the previous activity by having students complete the following chart for the circles studied before. Encourage the use of the calculator in finding the values for the last row in the chart.

Circle	1	2	3	4
Radius (cm)	2	2.5	3	4
Area of the Radius Square (cm^2)	4	6.25	9	16
Area of Circle by Counting the Number of Centimetre Squares (cm^2)	12.5+	19.5+	28+	50+
Area of Circle/Radius Square or A/r^2	12.5/4 = 3.125	19.5/6.25 = 3.12	28/9 = 3.11	50/16 = 3.125

Through discussion, guide students to verbalize that the area of the circle divided by the square of the radius of that circle is always a little more than 3. Relate this pattern to the pattern that students discovered in dividing the circumference of a circle by the diameter of that circle. Have students generalize that in both patterns the constant is a little more than 3 and is known as π .

Have students apply their understanding of related number sentences for multiplication and division to manipulate the division sentence, $A/r^2 = \pi$, into the formula using a multiplication sentence for the area of a circle, namely, $A = \pi r^2$.

Review the relationship between the radius and the diameter of a circle. Provide students with the opportunity to apply the formula for the area of a circle in solving problems where the area, radius or diameter of a circle is provided and the other two are unknown.

Look For …
Do students:
- ☐ use the calculator to find A/r^2?
- ☐ connect previous learning about C/d to A/r^2?
- ☐ manipulate the formula $A/r^2 = \pi$ to $A = \pi r^2$?

Adapted from Alberta Education, *Measurement: Activities to Develop Understanding* (unpublished workshop handout) (Edmonton, AB: Alberta Education, 2005), pp. 59–66.

6. Area of a Circle Using a Parallelogram
Present the following story to students and have them cut out a circle, make the sectors and rearrange them as described in the story. You may wish to have a model already made out of cardboard so that after students do their explorations, they may explore your model and see how the different sectors of the circle fit together to make a parallelogram, recognizing that if the sectors were cut small enough, a rectangle could be made out of the interlocking sectors.

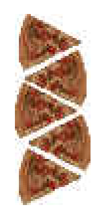
Look For …
Do students:
- ☐ cut the sectors of a circle and rearrange them into a parallelogram?
- ☐ transfer the measures of the circle (radius and half the circumference) to the measures of the parallelogram?
- ☐ apply the formula for the area of a parallelogram to create the formula for the area of a circle?

The Real Story behind the Area of a Circle

"One sunny afternoon, Dominic Candalara asks Archimedes if he is interested in walking down to a very popular pizza shop in downtown Syracuse (not New York, but Sicily) for lunch. Even though Archimedes is extremely busy, he does not want to upset his friend so he accepts his luncheon invitation.

They place an order for a large, 14-inch round pizza, but just as it is served they hear the sounds of several rounds of machine gun fire. Dominic immediately jumps from his seat hoping to catch a glimpse of what is taking place outside, leaving Archimedes alone with the pizza.

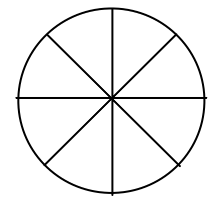
Not interested in the activities on the street and not really hungry, Archimedes calls to the chef, 'Hey, Rosseti, bringa me a big, sharp knife!' Not understanding why Archimedes wants the knife, the chef obliges, and watches in horror while Archimedes begins to cut the round pizza into very thin slices. Each slice of pizza is uniform in size and the slices are even in number.

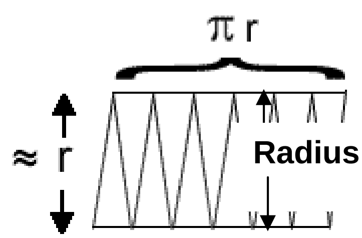


For every slice, there is a 'top' and a 'bottom.' Placing a top and bottom together, side by side, Archimedes discovers that the circular pizza can be rearranged into a different shape."

Reproduced with permission from the Mathematics Department of Edison Community College, "The real story behind the 'Area of a Circle'," *Edison Community College*, 2007, www.edison.cc.oh.us/Math/AreaCircle.htm (Accessed October 2007).

Have students model the action in the story and cut the sectors of their circles, rearranging them into a different shape. Guide students' thinking to use the information that they know about the areas of rectangles and parallelograms. (Note: it is easier to rearrange the sectors of the circle if the circle is first cut in half and then the sectors are cut to the edge of the circle without cutting the circumference. This way, all the sectors for each half of the circle stay intact and can be opened to look like teeth in a comb. The two halves can then be interlocked to form the shape shown below.) Have students explain why the base of the parallelogram can be represented by πr. If necessary, review the formula for the circumference of a circle; i.e., $C = 2\pi r$; therefore $C/2 = \pi r$.





Reproduced with permission from the Mathematics Department of Edison Community College, "The real story behind the 'Area of a Circle'," *Edison Community College*, 2007, www.edison.cc.oh.us/Math/AreaCircle.htm (Accessed October 2007).

Have students develop the formula for the area of the circle by finding the area of the parallelogram created by the rearranged circle; i.e., $A = \pi r^2$.

7. Solving Problems by Applying the Formulas for Area
   Have students solve problems with everyday contexts, using a single formula such as $A = bh$ or $A = bh/2$ or $A = \pi r^2$. Encourage them to focus on the needed information to solve the problem, recognize what the question is asking for, draw and label diagrams if necessary, estimate the answer, write the appropriate formula and then substitute the numbers into the formula to solve the problem. Emphasize that students must include both a numerical value and the correct unit in all their answers.

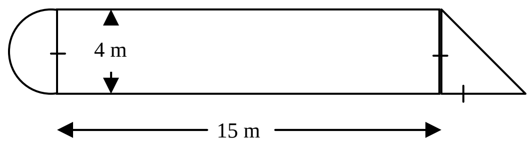
   After students understand and can apply the formulas to simpler problems, present them with more complex problems, such as the following, in which they apply one or more of the formulas created.

Look For …

Do students:

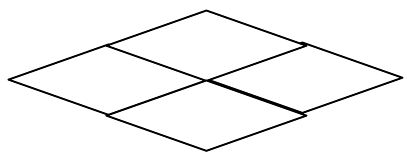
- ☐ estimate the answer, using appropriate strategies?
- ☐ analyze the diagrams to determine which formulas to use in finding the area?
- ☐ substitute the correct numbers into the formulas?
- ☐ find the total area of all the parts of the composite diagrams?
- ☐ use a numerical value and a square unit for area when writing the answer to the problem?

- A garden plot was made in the following shape.



a) Estimate the area of the garden plot. Explain your thinking.
b) Find the area of the garden plot. Explain your thinking.
c) If the width (4 m) of the plot is doubled, is the area of the plot doubled? Explain.

- A stained glass window is made up of four rhombi as shown in the diagram below. The length of one side of each rhombus is 10 cm. The perpendicular height of each rhombus is 8 cm. Find the area of the window. Show all your work.



8. Frayer Model for Area
   Provide students with a template for the Frayer model and have them fill in the sections, individually or as a group, to consolidate their understanding of area. A sample of a Frayer model is provided below.

Look For …

Do students:
- ☐ describe the essential characteristics of area?
- ☐ create real-world problems, applying their knowledge of area of various shapes?
- ☐ draw and label appropriate diagrams to illustrate area?
- ☐ provide examples and nonexamples of area?

Frayer Model for Area

Definition

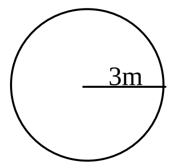
Area is the measure of the space inside a region.

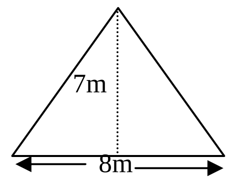
Characteristics

- the area of a region remains the same when the region is rearranged
- area can be measured in nonstandard or standard units of measure
- the smaller the unit of measure, the greater the number of units needed to measure a given area
- when comparing areas, the same units must be used
- standard units for area include cm^2 and m^2
- for a given area, there are many different shapes

Real-life Problem and Visual Representation

Cindy has a circular flowerbed that has a radius of 3 m. Marty has a triangular flowerbed with a base of 8 m and a perpendicular height of 7 m. Which flowerbed has the greater area?





Area

Examples

Area is used in the following:

- tiling floors
- seeding lawns
- painting walls
- buying windows

Area formulas include:

$A = LW$ $\quad A = bh/2$

$A = bh$ $\quad A = \pi r^2$

Nonexamples

Area is not used in the following:

- fencing around a garden
- lace around a tablecloth
- liquid in a glass

Area is not used in these formulas:

$P = 2L + 2W$

$C = 2\pi r$

Format adapted from D. A. Frayer, W. C. Frederick and H. J. Klausmeier, *A Schema for Testing the Level of Concept Mastery* (Working Paper No. 16/Technical Report No. 16) (Madison, WI: Research and Development Center for Cognitive Learning, University of Wisconsin, 1969). Adapted with permission from the Wisconsin Center for Education Research, University of Wisconsin-Madison.

Step 4: Assess Student Learning

Guiding Questions

- Look back at what you determined as acceptable evidence in Step 2.
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

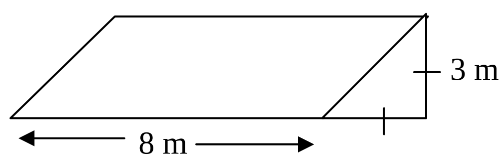
In addition to ongoing assessment throughout the lessons, consider the following sample activities to evaluate students' learning at key milestones. Suggestions are given for assessing all students as a class or in groups, individual students in need of further evaluation and individual or groups of students in a variety of contexts.

A. Whole Class/Group Assessment

Note: Performance-based assessment tasks are under development.

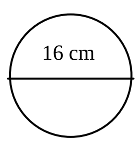
Provide students with calculators.

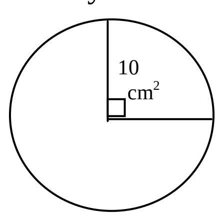
1. A triangle and a parallelogram have the same base and the same height. Explain how their areas compare. Include diagrams in your explanation.
2. Explain how the formulas for the area of rectangles, parallelograms and triangles are the same. Explain how they are different.
3. A garden plot was made in the following shape:



   a) Will the total area of the garden plot be greater than 24 m^2? Explain your thinking.
   b) Calculate the total area of the garden plot. Write the formulas that you use. Show all your work.

4. Find the area of the following circles. Show all your work.

a) 

b) 

5. A circular plate has a radius of 10 cm.
   a) Estimate the area of the plate. Explain your thinking.

   b) Write the formula for the area of a circle.

   c) Calculate the area of the plate. Show all your work.

B. One-on-One Assessment

Assessment activities can be used with individual students, especially students who may be having difficulty with the outcome.

Provide the student with a calculator.

1. Present the following problem to the student.

   A triangle and a parallelogram have the same base and the same height. Explain how their areas compare. Include diagrams in your explanation.

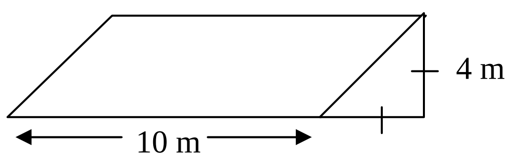
   If the student has difficulty drawing the diagram, suggest that he or she draw a rectangle and then slide the base over a couple of centimetres. Review the terms base and height if necessary, explaining the meaning of perpendicular height. Suggest the student draw a diagonal in the parallelogram to create two triangles. If necessary, have the student cut out the triangles and superimpose them to show congruency.

2. Ask the student to explain how the formulas for the areas of rectangles, parallelograms and triangles are the same. Then have him or her explain how they are different.

   If the student has difficulty, draw a diagram of a rectangle, a parallelogram, a triangle that is half the parallelogram and another triangle that is half the rectangle. Have the student cut out the shapes and rearrange them as necessary; e.g., fold the parallelogram to make a triangle that can be cut and slid to make a rectangle.

3. Present the following problem to the student.

   A garden plot was made in the following shape:

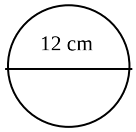
   

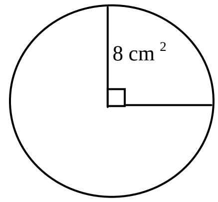
   a) Will the total area of the garden plot be greater than 40 m^2? Explain your thinking.
   b) Calculate the total area of the garden plot. Write the formulas that you use. Show all your work.

   If the student has difficulty with part (a), remind the student that the garden plot is made up of two shapes—a parallelogram and a triangle. Point out that the height of the parallelogram is 4 m. Then ask what the area of the parallelogram would be.

   If the student has difficulty with part (b), indicate that the base and the height of the triangle are the same—4 m each. Remind him or her that the formulas for both the parallelogram and the triangle require that the base is multiplied by the height. Ask what other operation is included in the formula for the area of a triangle. Guide the student to find both areas, add them and write the appropriate units.

4. Ask the student to find the areas of the following circles, showing all his or her work.

a) 

b) 

If the student has difficulty with part (a), ask him or her what the formula is for the area of a circle. If the student knows the formula, ask him or her what each variable in the formula represents. If the student does not know the formula, provide it for him or her. Remind the student that the formula requires the measure for the radius. Prompt the student to discuss that the radius is half the diameter and is therefore 6 cm.

If the student has difficulty with part (b), remind him or her that the box in the corner of the angle means it is a right angle. Have the student draw in the other three sectors, all of which are congruent.

5. Present the following problem to the student.

   A circular plate has a radius of 20 cm.

   a) Estimate the area of the plate. Explain your thinking.

   b) Write the formula for the area of a circle.

   c) Calculate the area of the plate. Show all your work.

   If the student has difficulty with part (a), remind the student that the formula requires that the radius is multiplied by itself. Ask the student for the approximate value of pi (about 3) and use that number to multiply the product of the radius by itself to get an estimate.

   If the student has difficulty with part (b), provide the formula for him or her.

   If the student has difficulty with part (c), suggest that he or she draw and label an appropriate diagram. Provide guidance in substituting the radius into the formula. Suggest that the student use a calculator to find the area.

C. Applied Learning

Provide opportunities for students to use their area formulas in a practical situation and notice whether or not the understanding transfers. For example, have students find how many square metres of canvas are needed to repair the triangular face of a tent with the base of the triangle as 2.8 m and the perpendicular height as 1.2 m. Does the student:

- estimate about how many square metres of canvas would be needed?
- apply the formula for the area of a triangle and explain the process?
- explain more than one way to apply the formula, such as finding the product of the base and height and dividing by two or dividing the height by two and multiplying the quotient by the base?

- readily adjust the area if you double the base of the triangle or the height of the triangle or both and explain the process?
- find the base of the triangle if provided with the area and the height and explain the process?

Step 5: Follow-up on Assessment

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction?

A. Addressing Gaps in Learning

- Have students provide real-world examples of where area is used. Have them compare area to perimeter by explaining how the two concepts are the same and how they are different. They could record their ideas in a Venn diagram.
- Review prior knowledge about the formula for the area of a rectangle. This forms the basis for the formulas for the area of parallelograms, triangles and circles. Also review circumference as this measure is used in one method of finding the area of a circle.
- Provide students with geoboards and geopaper. Have them construct rectangles, parallelograms and triangles on the geoboards, find the areas by counting the nonstandard units on the geoboard, draw the corresponding diagrams on geopaper and write the areas of the figures. Have them explore what happens to the area when the height of the figure is cut in half or doubled.
- Emphasize the connections among the formulas for the area of rectangles, parallelograms, triangles and circles.
- Encourage students to use calculators as they apply the formulas in solving real-world problems.
- In solving problems, have students read the problem orally, identify the needed information, draw and label diagrams if necessary, estimate the answer, choose the correct formula, substitute the numbers into the formula and write their answers, using both a numerical value and the unit.
- Remind students that the units used in the formulas must be the same; e.g., if the base of the parallelogram is measured in metres, then the height must also be measured in metres.

B. Reinforcing and Extending Learning

Students who have achieved or exceeded the outcomes will benefit from ongoing opportunities to apply and extend their learning. These activities should support students in developing a deeper understanding of the concept and should not progress to the outcomes in subsequent grades.

Consider strategies, such as the following.

- Provide tips for parents on working with area at home or in the community. For example:
  - Find the area of a lawn to determine how much sod is needed. Include shapes that are triangular and circular.
  - Find the area of a circular tabletop that will be covered with glass.
  - Find the total area of a quilt that is being made. Each congruent piece of material in the quilt will be shaped like parallelograms or squished rectangles.

- Use the Geometer's Sketchpad to draw circles and explore the relationships among radius, diameter and the quotient of the area and the radius squared. Explore the effect on the area of the circle when the radius is doubled or tripled. Compare it to the effect on the circumference of the circle (Bennett 1999).

- Use the Geometer's Sketchpad to draw parallelograms and triangles and explore the relationship between the base and the height as the area changes, or explore the relationship between the base and the area as the height changes.

- Use the Geometer's Sketchpad to construct meaning for the formulas describing the area of parallelograms and triangles. Students can maintain the same base and height for a parallelogram and convert it into a rectangle. They may also draw a triangle, copy it, turn it 180° and attach the two triangles together to make a parallelogram.

- Read the book *Sir Cumference and the Isle of Immeter* (Neuschwander 2007) to students and have them predict the answers to the riddles that appear in the story. Encourage students to act out the story by cutting the sectors in a circle to create a rectangle as described in the story. Provide time for students to relate the circumference of a circle to its area by using diagrams and number sentences. The number sentences include the following:
  $C = 2\pi r$, $\frac{1}{2} C = \pi r$, $A = \pi r \times r$, $A = \pi r^2$.

- Have students explore what shape would provide the maximum area for a given perimeter. Present students with the following problem:

  > You have 30 m of fencing to make the largest possible pen for your pet. What shape would you make the pen? Explain.

  Provide students with string so that they can act out the problem. They could use 30 cm of string to represent the 30 m of fencing and place the various created shapes on centimetre grid paper to determine which shape would provide the greatest area.

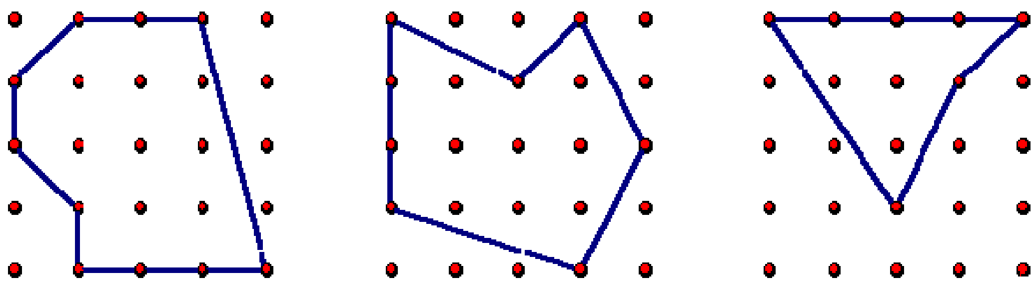
- Provide students with a variety of problems in which two of the following three measures are provided and the student must find the third measure:
  - radius, diameter, area
  - base, height, area.

- Have students create problems about everyday contexts for areas of parallelograms, triangles and circles.

- Challenge students to compare the graph for the circumference of circles ($C = 3.14\ d$) with the graph for the area of circles ($A = \pi r^2$). Encourage them to discuss the similarities and differences between the two graphs.

- Provide students with small circular objects to trace (e.g., jar lids, cups), string, centimetre grid paper and tape. Have students trace around one circular object, use centimetre grid paper to measure the area of the circle and record the area inside the circle. Have them use string to measure the circumference of the circle. Instruct them to cut another piece of string that is double the length of the circumference and form another circle, taping the string in place. Have students predict if the area of the new circle is twice the area of the original circle and then measure the area using the centimetre grid paper. Discuss the relationship between the circumferences and the areas of the two circles, and have students write a sentence that describes this relationship.

- Have students discover Pick's Rule ($A = 1/2\ b + i - 1$) by constructing various polygons on geoboards and geopaper and examining the patterns formed. Have them record in a chart the number of dots on the boundary of the polygon (b), the number of dots inside the polygon (i) and the number of squares for the area (A) of the polygon. For example:

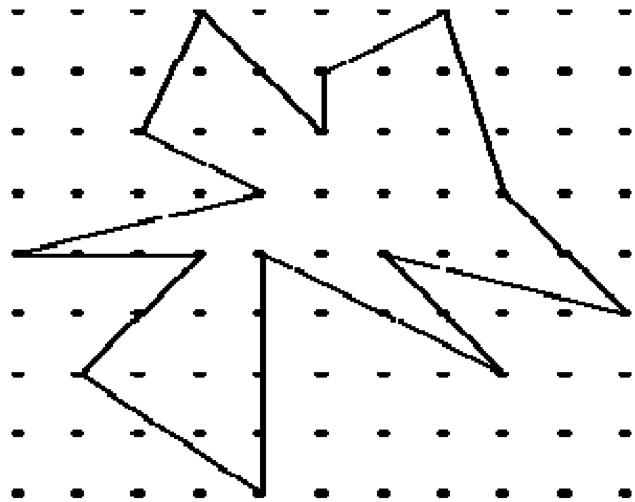


Polygon #1 Polygon #2 Polygon #3

Polygon	1	2	3
Number of dots on the boundary of the polygon	10	8	7
Number of dots inside the polygon	8	8	3
Area of the polygon (square units)	12	11	5.5

Have students examine the patterns in the table to develop a formula that can be used to find the area of any polygon, given the number of dots on the boundary and the number of dots inside the polygon.

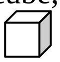
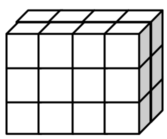
Encourage students to create other polygons and apply their formula to finding the areas of these polygons; e.g., Find the area of the following polygon, using Pick's Rule.



Adapted from Alberta Education, *Measurement: Activities to Develop Understanding* (unpublished workshop handout) (Edmonton, AB: Alberta Education, 2005), pp. 79–85.

- Sample Structured Interview: Assessing Prior Knowledge and Skills

Directions	Date:	
	Not Quite There	**Ready to Apply**
Instruct the student as follows: "Write a formula for finding the perimeter of any regular polygon and use this formula to find the perimeter of a hexagonal plate that is 15 cm on each side. Show all your work."	• Does not write the formula for the perimeter of any regular polygon but may write the formula for the perimeter of a square or a regular hexagon. • Multiplies 15 by 6 but makes computational errors, has difficulty relating the process to the formula and may not include the correct units in the answer.	• Writes the formula for the perimeter of any regular polygon; i.e., $P = ns$, where P is the perimeter, n is the number of sides on the polygon and s is the length of each side. • Uses the formula correctly to find the perimeter of the plate with correct units.
Say to the student, "Write the formula for finding the area of a rectangle. Explain how the formula for the area of a rectangle refers to length and width, which are linear units, but area is measured in square units. Use a diagram to explain the process."	• Does not write the formula for finding the area of a rectangle. • Draws a diagram of a rectangle but does not explain how the formula relates to the area of the rectangle and connects to measures of length.	• Writes the formula for the area of a rectangle; i.e., $A = LW$, where A is the area of the rectangle, L is the lengths and W is the width. • Draws a diagram of a rectangle and explains that the number used for the length is also the number of square units along the length of the rectangle (the units are different but the number is the same) and the width relates to the number of rows of square units to complete the area of the rectangle.

Say to the student, "Write the formula for finding the volume of a right rectangular prism and use the formula to find the volume of the following prism. Each cube,  , is 1 cm by 1 cm by 1 cm. Explain your thinking." 	• Does not write a formula for the volume of a right rectangular prism. • Counts the cubes in the diagram or constructs the prism and counts the cubes rather than using the formula to find the volume. • May use incorrect units to record the volume.	• Writes the formula for the volume of a right rectangular prism; i.e., $V = LWH$, where V is the volume of the prism, L is the length, W is the width and H is the height. • Uses the formula to find the volume of the prism in the diagram and writes the answer with the correct units.
Present the following problem to the student, first part (a) and then part (b): "A runway for a dog is in the shape of a rectangle that is 50 m long and 300 cm wide. a) Find the length of fencing needed to enclose the runway. Explain your thinking. b) Find the area of the runway. Explain your thinking."	• Neglects to convert the 300 cm to metres. • Makes an error in finding the perimeter, such as adding only the length and width but not doubling the sum. • Makes an error in finding the area by calculating incorrectly. • Does not use the correct units for perimeter and/or area.	• Converts the 300 cm into 3 m and correctly calculates the perimeter and area of the runway, using the correct units and explaining his or her thinking.

Say to the student, "The area of a rectangular garden plot is 36 m^2. If the length is 9 m, what is the width of the plot? Explain your thinking."	• Multiplies the given numbers together to find the area. • Does not use the correct units for the width. • Does not explain his or her thinking clearly.	• Finds the width by dividing the area by the length and uses the correct units for the length. • Explains the process by using a diagram or referring to the formula for the area of a rectangle and substituting the given numbers into the formula.

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Strand: Shape and Space (Measurement)
Outcome: 2

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Step 2 References

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Step 4 References

No references.

Step 5 References

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