Planning Guide: *Grade 7 Linear Relations*
Strand: Patterns and Relations (Patterns)
Outcomes: 1, 2

Curriculum Highlights

This sample targets the following changes in the curriculum:
- The General Outcome focuses on using patterns to describe the world and solve problems, as did the previous mathematics curriculum.
- The Specific Outcomes focus on understanding the connections among patterns, their equivalent linear relations, tables of values and graphs to solve problems, whereas the previous mathematics curriculum focused on understanding the connections among patterns, their equivalent oral and written expressions and graphs to solve problems.

Step 1: Identify Outcomes to Address

Guiding Questions

- What do I want my students to learn?
- What can my students currently understand and do?
- What do I want my students to understand and be able to do, based on the Big Ideas and specific outcomes in the program of studies?

Big Ideas

Mathematics is often referred to as the science of patterns. Patterns permeate every mathematical concept and are found in everyday contexts. The various representations of patterns, including symbols and variables, provide valuable tools in making generalizations of mathematical relationships. Some characteristics of patterns include the following.
- There are different types of patterns that can be modelled in a variety of ways.
- Patterns include repetitive patterns and growth patterns.
- Patterns using concrete and pictorial representations can be translated into patterns using numbers to represent the quantity in each step of the pattern. The steps in a pattern are often translated as the sequence of items in the pattern.
- Growth patterns are evident in a wide variety of contexts, including arithmetic and geometric situations. Arithmetic patterns are formed by adding or subtracting the same number each time. Geometric patterns are formed by multiplying or dividing by the same number each time.
- Patterns are used to generalize relationships: recursive and functional.
- The description that represents how a pattern changes from one step to another step is a recursive relationship and describes the evolution of the pattern; i.e., an expression that explains what you do to the previous number in the pattern to get the next one (Van de Walle and Lovin 2006).

- The description that determines the number of elements in a step by using the number of the step is a functional relationship and is known as a pattern rule; i.e., an expression that explains what you do to the step number to get the value of the pattern for that step (Van de Walle and Lovin 2006).
- Variables are used to describe generalized relationships in the form of an expression or an equation (formula) (Van de Walle and Lovin 2006).

Algebraic reasoning is directly related to patterns because this reasoning focuses on making generalizations based on mathematical experiences and recording these generalizations by using symbols or variables (Van de Walle and Lovin 2006).
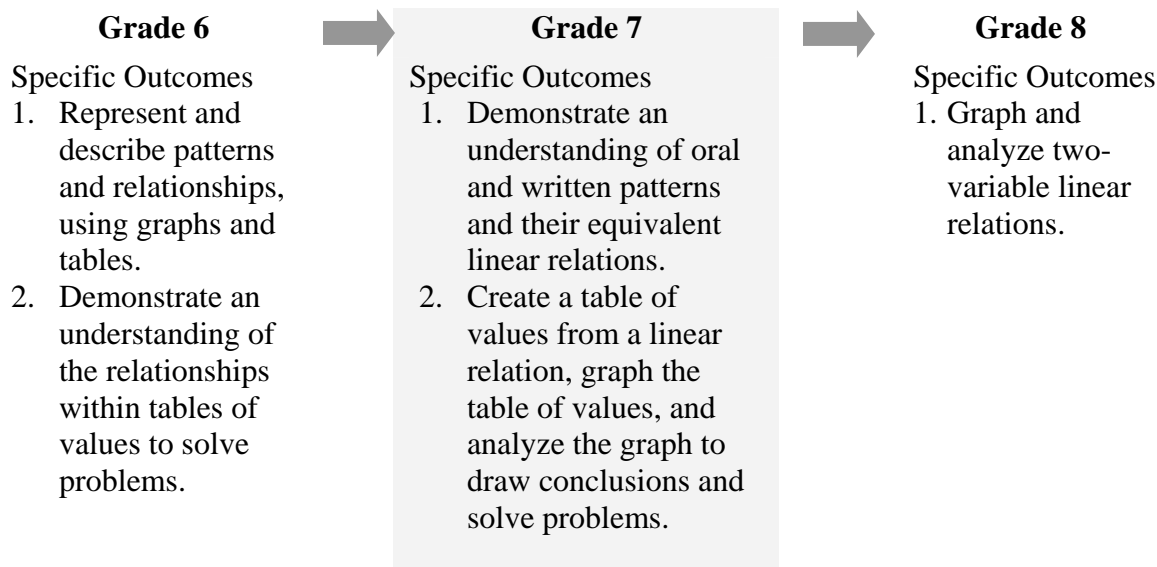
"A variable is a symbol that can stand for any one of a set of numbers or other objects" (Van de Walle and Lovin 2006, p. 274). Variables are used in different ways as mathematical literacy is developed. They can be used:
- in equations as unknown numbers; e.g., $4 + x = 6$
- to describe mathematical properties; e.g., $a + b + b + a$
- to describe functions; e.g., input (n): 1, 2, 3, 4 . . . output ($3n$): 3, 6, 9, 12 …
- in formulas to show relationships; e.g., $C = \pi d$.

Adapted from W. George Cathcart, Yvonne M. Pothier and James H. Vance, *Learning Mathematics in Elementary and Middle Schools* (2nd ed.) (Scarborough, ON: Prentice-Hall Canada, 1997), pp. 352–353. Adapted with permission from Pearson Education Canada.

Sequence of Outcomes from the Program of Studies
See (Web address) for the complete program of studies.

Grade 6	Grade 7	Grade 8
Specific Outcomes	Specific Outcomes	Specific Outcomes
1. Represent and describe patterns and relationships, using graphs and tables.	1. Demonstrate an understanding of oral and written patterns and their equivalent linear relations.	1. Graph and analyze two-variable linear relations.
2. Demonstrate an understanding of the relationships within tables of values to solve problems.	2. Create a table of values from a linear relation, graph the table of values, and analyze the graph to draw conclusions and solve problems.	

Step 2: Determine Evidence of Student Learning

Guiding Questions

- What evidence will I look for to know that learning has occurred?
- What should students demonstrate to show their understanding of the mathematical concepts, skills and Big Ideas?

As you begin planning lessons and learning activities, keep in mind ongoing ways to monitor and assess student learning. One starting point for this planning is to consider the achievement indicators listed in *The Alberta K–9 Mathematics Program of Studies with Achievement Indicators* (Alberta Education 2007). You may also generate your own indicators and use these to guide your observation of students.

The following achievement indicators may be used to determine whether students have met this specific outcome.
- Formulate a linear relation to represent the relationship in a given oral or written pattern.
- Provide a context for a given linear relation that represents a pattern.
- Represent a pattern in the environment, using a linear relation.
- Create a table of values for a given linear relation by substituting values for the variable.
- Create a table of values, using a linear relation, and graph the table of values (limited to discrete elements).
- Sketch the graph from a table of values created for a given linear relation and describe the patterns found in the graph to draw conclusions; e.g., graph the relationship between n and $2n + 3$.
- Describe the relationship shown on a graph, using everyday language in spoken or written form, to solve problems.
- Match a given set of linear relations to a given set of graphs.
- Match a given set of graphs to a given set of linear relations.

Some sample behaviours to look for in relation to these indicators are suggested for many of the instructional activities in Step 3, Section C, Choosing Learning Activities.

## Step 3: Plan for Instruction

Guiding Questions

- What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should I use?
- How will I meet the diverse learning needs of my students?

A. Assessing Prior Knowledge and Skills

Before introducing new material, consider ways to assess and build on students' knowledge and skills related to patterns. For example:

Provide students with graph paper or centimetre grid paper.

- Use the following pattern to answer the following three parts to this question.

    Diagram:
    Step Number: 1   2   3   4
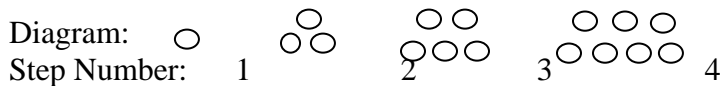
    a) Represent the pattern by completing the table of values:

Step Number	1	2	3	?	?	?
Number of Elements in the Step	1	?	?	?	?	?

    b) Draw a graph, using the data included in the table of values.
    c) Describe the relationship shown on the graph, using everyday language.

- Draw a pictorial representation of the relationship shown in the following table of values.

Step Number	1	2	3	4	…
Number of Elements in the Step	2	4	6	8	…

- a) Complete the following table of values. Explain your thinking.

Step Number	1	2	3	4	…	8	…	?
Number of Elements in the Step	5	8	11	?	…	?	…	38

    b) Describe the pattern of the numbers in the second row: 5, 8, 11 …
    c) Find the number of elements in the 50th step in the pattern. Explain your thinking.
    d) Formulate a rule to describe the relationship between the two rows of numbers in the table of values. Justify your rule.

- Create a table of values that represents the following graph.

Money in My Pocket over Time
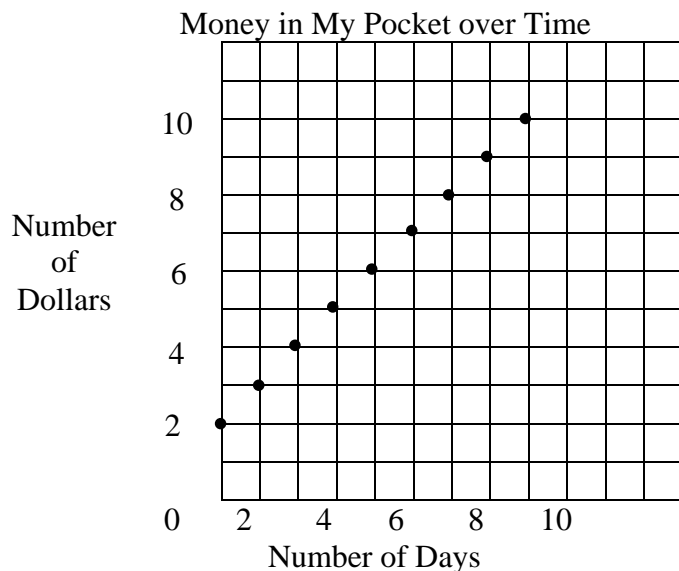
Number of Days	0	1	2	3	4	5	6	7
Number of Dollars	?	?	?	?	?	?	?	?

- Complete the following table of values, using the pattern rule multiply the step number by 5 and add 1 to get the number of elements in the step (the output is $5n + 1$ when the input is n).

Step Number (input is n)	1	2	3	4	5	?
Number of Elements in the Step (output is $5n + 1$)	?	?	?	?	?	?

If a student appears to have difficulty with these tasks, consider further individual assessment, such as a structured interview, to determine the student's level of skill and understanding. See Sample Structured Interview: Assessing Prior Knowledge and Skills.

B. Choosing Instructional Strategies

Consider the following strategies when planning lessons.
- Build on understanding of patterns from Grade 6—connecting the concrete, pictorial and symbolic representations of patterns and developing rules for patterns.
- Provide experiences with various models for patterns and the translations among the models; i.e., various diagrams for the same pattern, tables of values, graphs.
- Encourage students to describe patterns and rules orally and in writing before using algebraic symbols.
- Provide opportunity to connect the concrete and pictorial representations to symbolic representations as well as connecting the symbolic representations to pictorial and concrete representations.

- Provide examples of growth patterns that are arithmetic (the same number is added or subtracted each time) and geometric (the same number is multiplied or divided each time).
- In creating tables of values and graphs for linear relations, use real-world contexts.
- In creating a table of values for a given linear relation by substituting values for the variable, encourage students to draw diagrams to illustrate the linear relation in which the step number replaces the variable to obtain the number of elements in each step.
- In formulating linear relations representing oral or written patterns, encourage students to draw diagrams and create tables of values to assist them in visualizing the relationship.
- Provide examples in which students formulate linear relations that are recursive (describing how a pattern changes from one step to another) and functional (describing the relationship between two columns or rows of numbers in a table of values).
- In creating linear relations that are functional for a given pattern, have students represent and extend the pattern in a table of values, describe the patterns shown in the table and use these patterns to write a functional relationship or a formula in terms of the step number. Have students use the created formula or the functional linear relation to calculate the 20th or 50th entry in the table (Van de Walle and Lovin 2006).
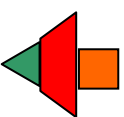
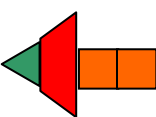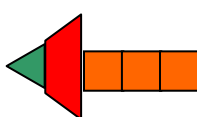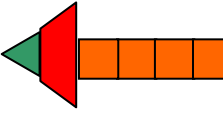## C. Choosing Learning Activities

The following learning activities are examples of activities that could be used to develop student understanding of the concepts identified in Step 1.

Sample Activities for Teaching Understanding of Patterns and Linear Relations

1. Variables (What Changes) and Constants (What Stays the Same)

   The purpose of this activity is to develop understanding of functional relationships by examining pictorial patterns and determining what changes and what remains the same in each successive step of the pattern.

   Begin with patterns that increase by one; e.g.,

   | Step: | 1 | 2 | 3 | 4 |

   Number of blocks: $2 + \boxed{1}$ $2 + \boxed{2}$ $2 + \boxed{3}$ $2 + \boxed{4}$

   Have students create the pattern by using pattern blocks and explain what changes and what remains the same in each step. Have them draw the pictorial representation as well. Guide discussion as follows:

Since the one triangle and one trapezoid appear in each step, then these shapes remain the same and the number 2 can be written under the diagram in each step as shown above. This number is called the constant because it doesn't change with each succeeding step. Since one more square is added to each diagram for each step, then 1 can be added to 2 to show the total number of blocks for the first step, 2 added to 2 for the second step, 3 added to 2 for the third step and 4 added to 2 for the fourth step as shown above.

Label the numbers that change in each step in some way; e.g., put a box around them as shown above. Review the meaning of a variable—a symbol that can stand for any one of a set of numbers or objects. Have students suggest a symbol to use for the variable in the pattern above and rewrite the addition expressions under each diagram, using the constant 2 and the variable they choose; e.g., $2 + n$.
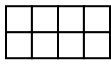Pose the following question:
How can you use the expression $2 + n$ to find the total number of blocks in the 50th diagram? Explain your thinking.

Guide the discussion to conclude that when you add 2 to the step number, you obtain the number of blocks for that step. Students may wish to write the linear relation as an equation; e.g., $b = 2 + n$, where b = the number of blocks in a diagram and n = the step number.

Have students apply their understanding of linear relations in which the coefficient of n is 1 to another pattern where the coefficient of n is another counting number.

Use another pattern with a different recursive relationship, such as the following, and discuss what changes and what stays the same in the different diagrams for the different steps.

The pattern 2, 4, 6, 8 … can be represented as follows:

Step: 1   2   3   4

Number of squares: 2 x 1   2 x 2   2 x 3   2 x 4
(or area of rectangle)

In deciding what stays the same and what changes, guide the discussion to include the dimensions of the composite rectangle in each step; i.e., the width stays the same at 2 units but the length increases by 1 unit with each new step in the pattern.

Have students write a linear relation to represent the pattern; e.g., $A = 2n$, where A equals the number of squares in each step and n represents the length of the rectangle or the number of squares in the bottom row of each step.

Have students draw diagrams to show other similar patterns with the coefficient of the variable being 3 or 4; e.g.,
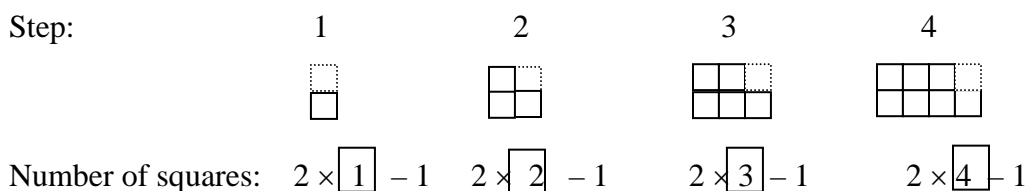3, 6, 9, 12, … $3n$
4, 8, 12, 16, … $4n$

To guide students in writing linear relations with the coefficient of the variable greater than 1 and a constant added or subtracted, adjust the pattern with the rectangles as shown below.

The pattern 3, 5, 7, 9, … can be represented as follows:
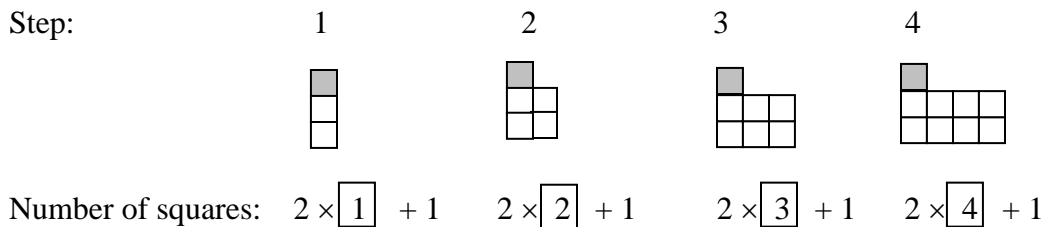
Step: 1   2   3   4

Number of squares: $2 \times \boxed{1} + 1$ $2 \times \boxed{2} + 1$ $2 \times \boxed{3} + 1$ $2 \times \boxed{4} + 1$

Encourage students to use variables to write the linear relation; e.g., $B = 2n + 1$, where $B = $ the number of blocks in a given step and $n = $ the step number. Have students use the linear relation to find the number of blocks in the 50th step.

Similarly, the pattern 1, 3, 5, 7, … can be represented as follows:

Step: 1   2   3   4

Number of squares: $2 \times \boxed{1} - 1$ $2 \times \boxed{2} - 1$ $2 \times \boxed{3} - 1$ $2 \times \boxed{4} - 1$

Encourage students to use variables to write the linear relation; e.g., $B = 2n - 1$, where $B = $ the number of blocks in a given step and $n = $ the step number. Have students use the linear relation to find the number of blocks in the 50th step.

Consolidate the learning by having students examine the recursive relationships of a variety of patterns for linear relations to determine if the patterns increase by 1, 2, 3 and so on. Their explorations should lead them to conclude that a pattern that increases by 1 has the variable in the linear relation (functional relationship) with a coefficient of 1, a pattern that increases by 2 has the variable in the linear relation with a coefficient of 2 and so on. Once the coefficient of the variable is determined, then by substituting the step number for the variable it logically follows whether a constant must be added or subtracted to obtain the required number of elements for that step; e.g.,

Step Number	1	2	3	4	…	50
Number of Elements in the Step	7	9	11	13	…	?

Guide the exploration and discussion as follows:
Since the pattern (recursive relationship) in the bottom row of the chart increases by 2 each time, then the linear relation (functional relationship) for this pattern will have $2n$ included in it. Substituting 1 for n in the first step, you get $2 \times 1 = 2$. To get the required number of elements (7) for this first step, you must add 5. Therefore, the linear relation can be written as $E = 2n + 5$, where E is the number of elements in each step and n is the step number.

The 50th step would have $2 \times 50 + 5 = 105$ elements.

To conclude, review that the recursive relationships for patterns are needed to determine their functional relationships. It is the functional relationships written as linear relations that show the power of algebra, providing a general rule that can be used to find the number of elements in any step when given the step number.

Have students discuss the similarities and differences between recursive and functional relationships, recognizing that both are included in patterns but each of them has a different role:
- A recursive relationship describes the pattern between successive numbers in one of the columns of a table of values.
- A functional relationship is a general rule to describe the relationship between two columns of numbers in a table of values.

Look For …

Do students:
- ☐ transfer the information in the problem to another model, such as a chart or diagram?
- ☐ apply their understanding of recursive relationships (how a pattern changes from step to step) to find a rule or a functional relationship (the relationship between two rows of numbers in a chart)?
- ☐ use the general rule (linear relation) to solve the problem, using larger numbers?

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2. Problem Solving with Patterns

   Provide students with problems, using everyday contexts in which they can apply their understanding of functional relationships; e.g.,
   > Patsy's Pizza Parlor has square tables that each seat 4 people. If you push 2 tables together, 6 people can be seated. If you push 3 tables together, 8 people can be seated.
   > a) Write a general rule (linear relation) that can be used to calculate the number of people that can be seated given any number of tables put end-to-end.
   > b) Use your general rule to find how many people can be seated if 50 tables are put end-to-end.

   Build on students' knowledge of creating charts for patterns and have them suggest how the information in the problem can be represented in a chart. Encourage students to draw diagrams to represent the pattern and place the data in a chart; e.g.,

   Number of Tables: 1   2   3

   Number of People: 4   6   8
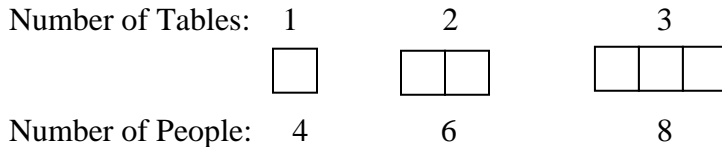
Number of Tables	1	2	3	4	…	50
Number of People	4	6	8	?	…	?

   Have students describe the recursive relationship of the pattern of numbers in the bottom row of the chart; i.e., each succeeding number increases by 2.

   Build on students' understanding of patterns in writing functional relationships that connect the step number with the number of elements in each step (see previous activity). Provide scaffolding for students if necessary by having them examine the diagrams in the pattern and notice what changes and what stays the same.

   Number of Tables: 1   2   3

   $2 \times \boxed{1} + 2$ $2 \times \boxed{2} + 2$ $2 \times \boxed{3} + 2$
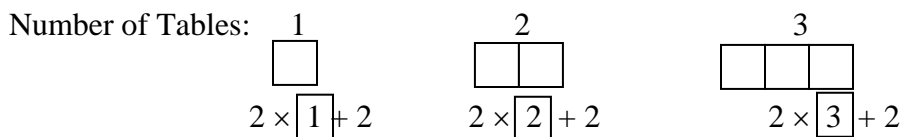
   Explain that the constant is 2 because 2 people sit at the ends in each diagram.

   Have students verbalize that the number of people seated on the sides that are not the ends of the table varies with each new step or table, i.e., for 1 table, 2 people can be seated at the sides that are not then ends; for 2 tables, $2 \times 2 = 4$ people can be seated at the sides that are not the ends; and for 3 tables, $2 \times 3 = 6$ people can be seated at the sides that are not the ends. See the expressions written below the diagrams.

Instruct students to write a linear relation, using a variable to represent the numbers in the boxes; e.g., $2n + 2 = P$, where n is the number of tables placed end-to-end and P is the number of people.

Have students use their linear relation to find the number of people that can be seated with 50 tables placed end-to-end by substituting 50 for n; e.g., $2 \times 50 + 2 = 102$. Have students write a sentence: When 50 tables are placed end-to-end, 102 people can be seated.

Provide other real-world problems for students to write linear relations and use them to solve the problems. Remind them to use diagrams and charts to represent the problems so that they are better able to write the linear relations.

3. Contexts for Linear Relations

    Provide students with a linear relation and have them create a context to represent that relation; e.g.,
    Given the linear relation, $N \rightarrow 3N + 2$ or $D = 3N + 2$, create a problem that could be solved using this linear relation.

Use the think-aloud strategy to create a problem using this linear relation. Explain that translating the linear relation into a pattern in a chart helps to understand the meaning of the linear relation.

Start the following chart by explaining that the step number or N is the label for the first row and the number of elements in the step or $3N + 2$ is the label for the second row. This connects previous learning about creating charts to the present learning. Have students make suggestions as to what the numbers in the second row of the chart will be and justify their answers.

Step Number (N)	1	2	3	4
Number of Elements in the Step (3N + 2)	?	?	?	?

> **Look For …**
> Do students:
> □ apply previous learning with patterns, using diagrams to create contexts for linear relations?
> □ create everyday problem-solving contexts for linear relations?
> □ check that the context created satisfies the linear relation provided?

After students create the pattern in the second row, i.e., 5, 8, 11, 14 …, continue the think-aloud strategy. Explain that you will now have to think of an everyday context in which numbers increase by 3 each time, starting with 5. Pick a context that students can relate to; e.g., money. Suggest that money earned could increase so a problem could be created with money that would represent this linear relation.

Example: You earn \$5 the first day, \$8 dollars the second day, \$11 dollars the third day and so on. If this pattern continues, how much would you earn on the 20th day?

Have students suggest other everyday contexts that could be used to represent the given linear relation.

Provide other linear relations and have students work in groups to find the pattern represented by the linear relations and create everyday contexts in problem form for them.

Sample Activities for Teaching Connections among Linear Relations, Tables of Values and Graphs

1. Translating a Linear Relation into a Table of Values and a Graph

   Build on students' prior knowledge about working with charts and linear relations (see previous activities). Explain that the charts used previously to display patterns using the step number and the number of elements in the step are really tables of values for the given linear relations. Provide students with experience in using horizontal and vertical tables of values (or charts).

   Direct students to create a vertical and a horizontal table of values (chart) for a given linear relation, such as the following. Explain that the labels for the chart must include the variables in the linear relation.
   Example: $N \longrightarrow 2N - 3$

N	$2N - 3$

N				
$2N - 3$				

   Have students suggest what number might be used for the variable in completing the charts or tables of values. If students suggest random numbers, suggest that they place the numbers in the chart in order, starting with zero, so that the pattern is more evident and easier to graph.

   Have students complete the tables of values, explaining their thinking for each entry in the table. Encourage them to use the pattern created as a check for their work.

   Look For …
   Do students:
   ☐ transfer their understanding of charts to represent patterns by using *N* as the step number and the linear expression as the number of elements in the step?
   ☐ apply their understanding of operations with integers to complete the tables of values or charts?
   ☐ translate a pattern from a table of values to a graph, including negative numbers?

N	0	1	2	3
2N – 3	–3	–1	+1	+3

N	2N – 3
0	–3
1	–1
2	+1
3	+3

Provide students with graph paper or centimetre grid paper to draw a graph of the points included on the tables of values. Explain that the values for *N* are placed along the horizontal axis of the graph and the values for 2*N* – 3 are placed along the vertical axis. Build on graphing experiences from previous grades and extend this knowledge by explaining that negative numbers must be placed on the vertical axis because they are in the tables of values. Build on students' understanding of integers as the numbers are placed on the graph, using the horizontal and vertical axes. Have students place the points on the graph as they are placed on a grid on the overhead or on the board.

Example of the graph:
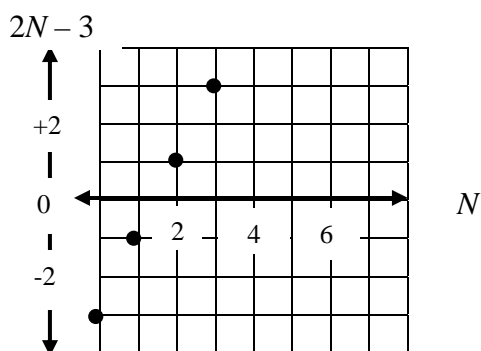
Have students check their work by using a ruler to determine if the points on the graph are in a straight line. Emphasize that graphs of linear relations include points that form a straight line.

Provide other examples of linear relations and have students create tables of values for them and graph them.

2. Solving Problems Using Data from a Graph

Provide students with a graph that includes discrete points representing a linear relation. Explain that the points on the graph represent various places on the rock wall that Keri is climbing. Through discussion, establish the meaning of the two axes in relation to the context and then have them solve and create problems, using data from the graph.

Keri's Climb on the Rock Wall
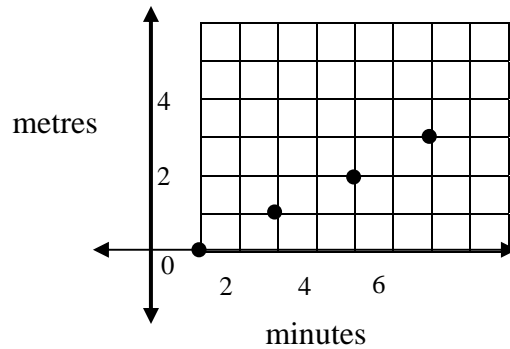
Examples of problems:
- How far did Keri climb in 3 minutes? Explain your thinking.
- Predict how far Keri will climb in 8 minutes. Explain your thinking. What, if any, assumptions are you making in determining your answer?
- Predict how far Keri will climb in 20 minutes. Explain your thinking.

After students interpret the graph using the data from Keri's climb, have them draw on the same axes another pattern to represent another climber. Then provide questions that compare the two graphs for students to discuss and answer.

Provide other similar graphs and have students solve and create problems, using the data from the graphs.

3. Frayer Model for Linear Relations

   Provide students with a template for the Frayer model and have them fill in the sections, individually or as a group, to consolidate their understanding of linear relations. A sample Frayer model is provided below.

Look For …

Do students:
- ☐ interpret the data provided in the graph?
- ☐ make predictions by extrapolating the data in the graph?
- ☐ compare the data of two graphs on the same axes?
- ☐ create problems that can be answered, using the data from the graph?

Frayer Model for Linear Relations

Definition

A linear relation is a relationship that maps a first number onto a second number so that the pattern formed by the second number is constant growth; i.e., the same number is added to the previous number each time.

Characteristics

- linear relation can be represented in a variety of ways, such as diagrams, tables of values, graphs or symbols
- the symbols used to represent a linear function include variables (what changes) and/or constants (what stays the same)
- linear relation is a rule that describes the relationship between two columns in a table of values
- the graph is linear

Real–life Problem

Sarah's Spaghetti Surprise has pentagonal tables that each seat 5 people. If you push 2 tables together, 8 people can be seated. If you push 3 tables together, 11 people can be seated.

a) Write a general rule (linear relation) that can be used to calculate the number of people that can be seated given any number of tables put end-to-end.

b) Use your general rule to find how many people can be seated if 50 tables are put end-to-end.
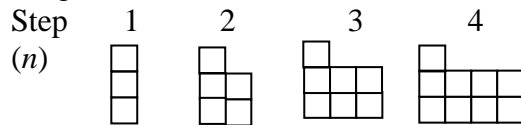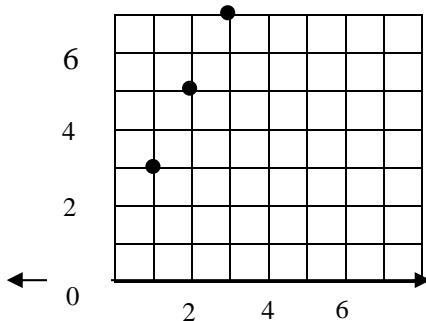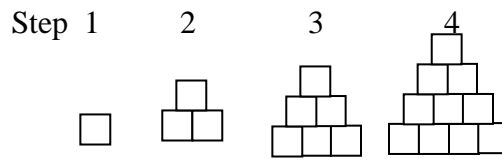
Linear Relation

Examples

Diagrams:

Step (n)	1	2	3	4

Table of Values

n	1	2	3
$2n + 1$	3	5	7

Symbolic Representation: $n \rightarrow 2n + 1$

Nonexamples

Step 1 2 3 4

n	1	2	3	4
$(n \times n + n)/2$	1	3	6	10

$n \rightarrow (n \times n + n)/2$
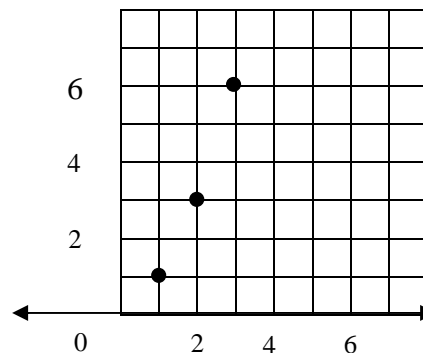
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Step 4: Assess Student Learning

Guiding Questions

- Look back at what you determined as acceptable evidence in Step 2.
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

In addition to ongoing assessment throughout the lessons, consider the following sample activities to evaluate students' learning at key milestones. Suggestions are given for assessing all students as a class or in groups, individual students in need of further evaluation and individual or groups of students in a variety of contexts.

A. Whole Class/Group Assessment

Note: Performance-based assessment tasks are under development.

Provide students with graph paper.

1. a) Create a table of values for the linear relation $n \rightarrow 3n - 4$.
    b) Draw a graph for the table of values that you created.

2. Create a problem or draw diagrams to represent each of the following linear relations:
    a) $N \rightarrow 3N$
    b) $N \rightarrow 3N + 1$

3. Sam's Steak House has hexagonal tables that each seat 6 people. If you push 2 tables together, 10 people can be seated. If you push 3 tables together, 14 people can be seated.
    a) Write a general rule (linear relation) that can be used to calculate the number of people that can be seated given any number of tables put end-to-end. Explain your thinking.
    b) Use your general rule to find how many people can be seated if 30 tables are put end-to-end. Show your work.

4. Which of the following linear relations best describes the graph below? Explain your thinking.

?
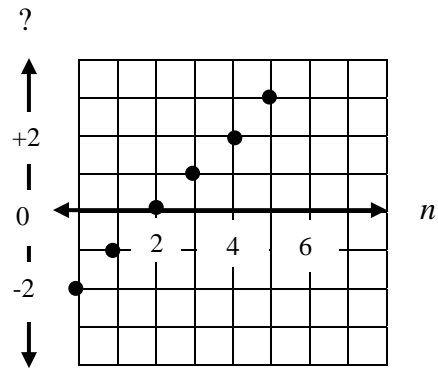
A. $n \rightarrow n - 2$ B. $n \rightarrow 2n - 2$
C. $n \rightarrow 2 - n$ D. $n \rightarrow 2 - 2n$

5. Use the following graph to answer the questions. Assume that Peter and Flopsy start the race at the same place.
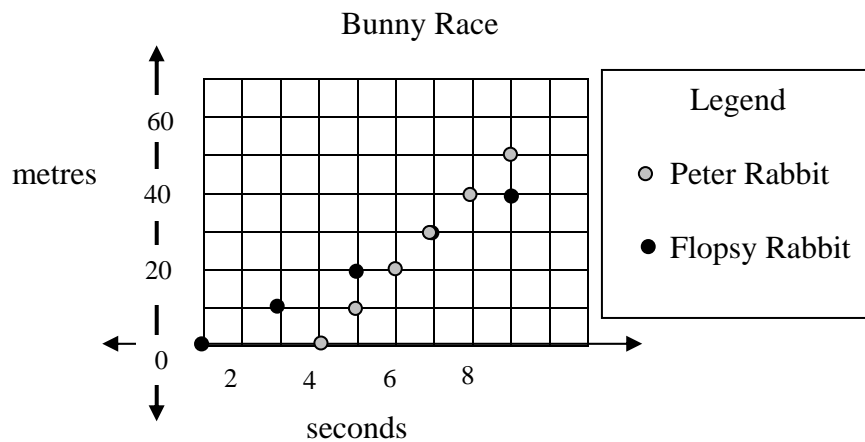
Bunny Race

Legend

○ Peter Rabbit

● Flopsy Rabbit

a) If the distance for the race is 60 m, who wins the race and by how much?
b) How far from the start did Peter overtake Flopsy?
c) How many seconds from the start did Peter overtake Flopsy?
d) Who was ahead after 8 seconds? By how much?
e) Provide a reason why Peter had a delayed start to the race.
f) Create another question that can be answered from this graph.

B. One-on-One Assessment

Assessment activities can be used with individual students, especially students who may be having difficulty with the outcome.

Provide students with graph paper.

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1. Present the following linear relation to the student: $n \rightarrow 2n - 4$.
   a) Ask the student to create a table of values for the linear relation.
   b) Ask the student to draw a graph for the table of values that he or she created.

   If the student has difficulty setting up a table of values, provide the chart for the student with the labels "n" and "$2n - 4$" included in the chart. Remind the student to use values such as 0, 1, 2 and so on for n.

   If the student has difficulty drawing a graph, remind the student that the numbers placed along the vertical and horizontal axis must be evenly spaced. Take one pair of numbers from the table of values and ask the student where each number is represented on the graph, remembering that the values for n are along the horizontal axis. Have the student check the accuracy of his or her graph by using a ruler to determine if the points placed on the graph line up as is always the case with linear relations.

2. Ask the student to create a problem or draw diagrams to represent each of the following linear relations:
   a) $N \rightarrow 2N$
   b) $N \rightarrow 2N + 1$
   If the student has difficulty, suggest that he or she create a table of values for each linear relation and examine the pattern carefully. Remind the student that the first linear relation as shown in the table of values created by him or her is a doubling relation. It might be necessary to start the pattern by drawing the first diagram and then the student can draw the next diagram by drawing the number of elements that are double the step number or the value of N. Suggest some topics for everyday problems if necessary, such as money or time jogging.

3. Present the following problem to the student:
   Harry's Hamburger Haven has triangular tables that each seat 3 people. If you push 2 tables together, 4 people can be seated. If you push 3 tables together, 5 people can be seated.
   a) Write a general rule (linear relation) that can be used to calculate the number of people that can be seated given any number of tables put end-to-end. Explain your thinking.
   b) Use your general rule to find how many people can be seated if 30 tables are put end-to-end. Show your work.

   If the student has difficulty, have him or her read the problem orally and represent the problem in a drawing, using 1 table with 3 people, 2 tables with 4 people and so on. Then have the student put the data into a chart with the labels "number of tables" and "number of people."

   Remind the student to examine the pattern of the numbers in the chart. Each number increases by 1 so that should be a hint for the student, knowing that the variable will have a coefficient of 1. Ask the student what number must be added to 1 to get 3. Then ask what number must be added to 2 to get 4 and so on. In this way the student must focus on describing the relationship between two rows of numbers in the chart and therefore find the general rule (linear relation).

If the student wants to extend the pattern all the way to 30, suggest that he or she use the linear relation created by replacing the variable with 30 to get the number of people that can be seated.

4. Present the following graph to the student along with the linear relations shown below. Ask the student to choose the linear relation that best describes the graph and to explain his or her thinking.
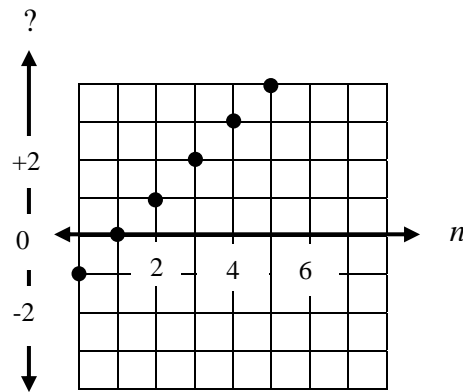
A. $n \rightarrow n - 1$

B. $n \rightarrow 2n - 1$

C. $n \rightarrow 1 - n$

D. $n \rightarrow 1 - 2n$

If the student has difficulty, suggest that he or she create a table of values for the data shown in the graph. The labels for the table of values can be "n" and "expression." After the table of values is done, have the student take one value for n and substitute it into the different linear relations provided to determine which one is correct. If difficulty arises, review operations with integers.

5. Present the following graph to the student and state the assumption that Peter and Flopsy start the race at the same place. Ask the questions shown below. Have the student explain his or her thinking for each question.
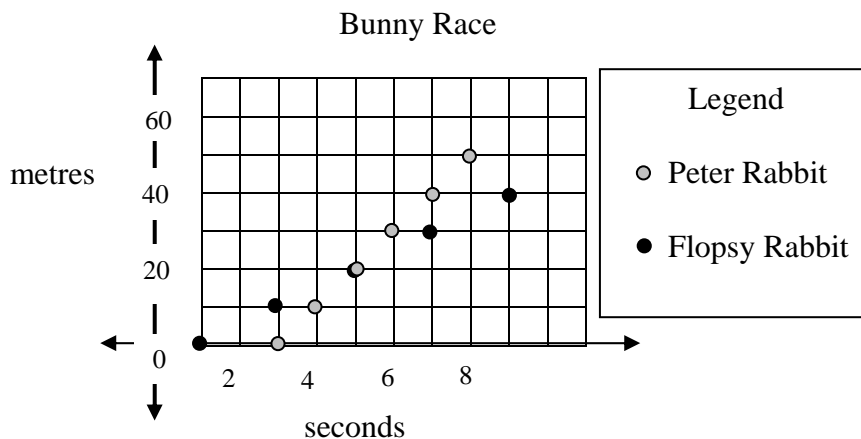
Bunny Race

a) If the distance for the race is 60 m, who wins the race and by how much?
b) How far from the start did Peter overtake Flopsy?

c) How many seconds from the start did Peter overtake Flopsy?
d) Who was ahead after 8 seconds? By how much?
e) Provide a reason why Peter had a delayed start to the race.
f) Create another question that can be answered from this graph.

If the student has difficulty with any of the questions, have the student describe what is happening on the graph prior to answering any of the questions. It might be useful to connect the points on the graph so that it is easier for the student to compare the two sets of data. For each question, have the student point to the part of the graph that the question is referring to. If the student is unable to do so, prompt the student by explaining the meaning of the question, reviewing the meanings of the labels on the axes and pointing to the correct part of the graph if necessary.

C. Applied Learning

Provide opportunities for students to use linear relations in a practical situation and notice whether or not the understanding transfers; e.g., have the student predict how many push-ups will be done on the 30th day if 3 push-ups are done on the first day, 4 push-ups on the second day, 5 push-ups on the third day and so on. Does the student:
- communicate the pattern that is shown in the problem?
- predict correctly and justify his or her prediction?
- explain the functional relationship between the number of the day and the number of push-ups done on that day?
- calculate the total number of push-ups done in a given number of days, such as 10 days?
- predict the number of the day on which a given number of push-ups is done?
- apply his or her understanding of patterns to similar problems and explain the thinking?

Step 5: Follow-up on Assessment

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction?

A. Addressing Gaps in Learning

- Build on students' understanding of patterns from previous grades.
- Encourage students to use manipulatives to represent various patterns and then draw corresponding diagrams along with charts.
- Have students explain their thinking and provide scaffolding to overcome any misconceptions or misunderstandings.
- Use patterns with everyday contexts so that students understand the purpose of the patterns and are motivated to complete the tasks.

- Use friendly numbers in the patterns initially so that students can easily draw diagrams to represent the patterns.
- In creating linear relations for a given pattern, have students explain the recursive relationship among the numbers in the individual rows in the table of values and use this relationship in establishing a rule (functional relationship) that relates the two rows or columns of numbers in a table of values.
- Emphasize the importance of creating a table of values when connecting linear relations with their corresponding graphs.
  – Reinforce that variables stand for what changes in a pattern while constants stand for what remains the same in the pattern.
  – Reinforce understanding of patterns by integrating patterns in every strand and emphasize the power of patterns in mathematics.
  – Encourage students to bring examples of graphs from newspapers and magazines to school for analysis.

B. Reinforcing and Extending Learning

Students who have achieved or exceeded the outcomes will benefit from ongoing opportunities to apply and extend their learning. These activities should support students in developing a deeper understanding of the concept and should not progress to the outcomes in subsequent grades.

Consider strategies, such as the following.
- Provide tips for parents on writing generalizing patterns at home or in the community.
  – Predict the salary for a given day if the starting daily salary is known and on each successive day the salary increases by a constant number, such as $2.
  – Predict how many days are worked in a given month if the initial number of days worked in a month is known and on each successive month the number of days worked increases by a constant number, such as 4 days.
  – Generalize common quantities, such as the following:
    - the number of tires needed for any number of cars
    - the pieces of cutlery needed for any number of place settings
    - the number of chicken legs for any number of chickens
    - the number of minutes in any number of hours
    - the number of eggs in any number of egg cartons.
  – Discuss and interpret different graphs that are in newspapers, magazines and on various Web sites.

- Reinforce the functional relationship of linear relations by using the Function Machine. To make a Function Machine, cut three holes (circle, square and triangle) in a vertical alignment from a blank piece of paper. The circle is the input, the triangle is the output and the square is the rule or linear relation. Cut a vertical slit on each side of the shapes. Cut transparent slips that are wide enough to fit through the slits. On two of the strips, write numbers to represent the input and the output. On the third strip write the rule or the linear relation. Place the template with the three shapes on the overhead. Place two strips so that they are exposed through two shapes and have students find the appropriate response for the third shape; e.g., if the input and output are provided, students must find the function rule, and if the input and the function rule are provided, students must find the output for each input.

Transparent strip
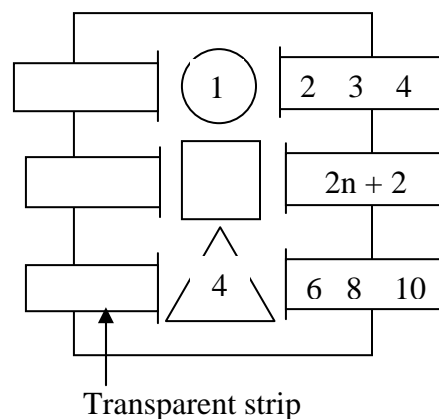
Adapted from Elizabeth Phillips, *Patterns and Functions: Curriculum and Evaluation Standards for School Mathematics Addenda Series, Grades 5–8* (Reston, VA: The National Council of Teachers of Mathematics, 1991), pp. 67, 68. Adapted with permission of the National Council of Teachers of Mathematics.

- Have students play the game Guess My Rule. One student operates the input–output machine that can be represented by a chart; e.g.,
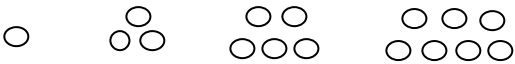
Input				
Output				

The operator has a rule or linear relation in mind. The other students try to guess the rule by suggesting input numbers while the operator records the output numbers. The list of input and output numbers should be recorded in the chart on the board or overhead. Students who think they know the rule provide the output number using the rule and the operator decides if the output follows the pattern or not. The game continues until most students have guessed the rule. Students could play this game in small groups.

Adapted from Van de Walle, John A., LouAnn H. Lovin, *Teaching Student-Centered Mathematics, Grades 5–8* (p. 272). Published by Allyn and Bacon, Boston, MA. Copyright © 2006 by Pearson Education. Adapted by permission of the publisher.
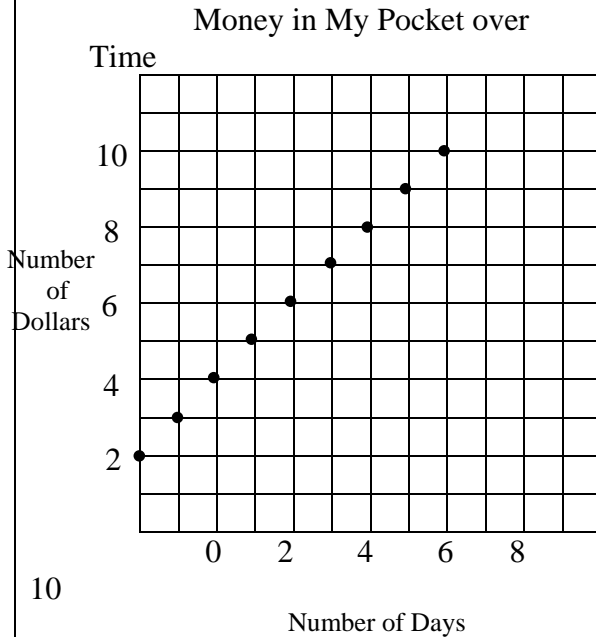
- Have students write expressions to generalize special numbers; e.g., even numbers ($2n$), odd numbers ($2n + 1$), multiples of 5 ($5n$), consecutive whole numbers (n and $n + 1$) and consecutive even numbers ($2n$ and $2n + 2$). Also have students write equations used to relate centimetres and metres (e.g., $100\,M = C$, where M is the number of metres and C is the number of centimetres), grams and kilograms, hours in a day, months in a year, perimeter of a regular hexagon, perimeter of a square (Van de Walle and Lovin 2006).

- Have students write expressions to translate real-world situations with the unknowns represented by variables:
  − one more than triple the number of pencils in each box
  − five less than twice the number of cans in each carton
  − three subtracted from double the number of sticks of gum in each package.

- Challenge students to draw graphs for two linear relations on the same coordinate axes and discuss the similarities and differences; e.g., $n \rightarrow n + 2$ and $n \rightarrow 2n$. Have students apply their learning to predict how graphs for other similar linear relations will be drawn, e.g., $n \rightarrow n + 3$ and $n \rightarrow 3n$. Encourage students to draw the graphs to verify their predictions. Also, have them suggest everyday contexts for the various graphs drawn.

- Sample Structured Interview: Assessing Prior Knowledge and Skills

Directions	Date:	
Provide the student with graph paper or centimetre grid paper.	**Not Quite There**	**Ready to Apply**
Present the following problem to the student, one part at a time. "Use the following pattern to answer the following three parts to this question. ○ ○ ○○ ○○ ○ ○○ ○○○ ○○○○ a) Represent the pattern by completing the table of values: b) Draw a graph, using the data included in the table of values. c) Describe the relationship shown on the graph, using everyday language."	- Does not connect the step with the number of elements in each step to complete the table of values. - Completes the table of values but is unable to draw the graph. - Completes the table of values and draws the graph but is unable to describe the relationship shown by the graph.	- Completes the table of values, draws the graph and describes the relationship shown by the graph as doubling the step number minus one to obtain the number of elements in each step, written as $2n - 1$.

Step Number	1	2	3	?	?	?
Number of Elements in the Step	1	?	?	?	?	?

Place the following table of values before the student and say, "Draw a pictorial representation of the relationship shown in the following table of values."	• Draws diagrams but shows no patterning in keeping something constant in each step. • Draws a series of related steps but does not consistently show an increase of 2 elements for each succeeding step.	• Draws a pictorial representation that shows a pattern that consistently grows by adding two more elements to each succeeding step.

Step Number	1	2	3	4	…
Number of Elements in the Step	2	4	6	8	…

Present the following problem to the student, one part at a time. a) Complete the following table of values. Explain your thinking.	• Completes the first part of the table of values but has difficulty extending the pattern to the 8th step. • Has difficulty explaining his or her thinking. • Continues the pattern to find the 50th step rather than using a rule that connects the numbers in the two rows of the table. • Cannot formulate a rule to describe the relationship between the two rows of numbers in the table of values.	• Completes the table of values, describes the pattern, calculates the number of elements in the 50th step by using the rule 3n + 2, which is represented algebraically or in words (triple the number of the step and add 2 to get the number of elements in the step). • Justifies the rule by using it, given step numbers, to get the number of elements in those steps.

Step Number	1	2	3	4	…	8	…	?
Number of Elements in the Step	5	8	11	?	…	?	…	38

b) Describe the pattern of the numbers in the second row: 5, 8, 11 …
c) Find the number of elements in the 50th step in the pattern. Explain your thinking.
d) Formulate a rule to describe the relationship between the two rows of numbers in the table of values. Justify your rule."

Present the following graph and incomplete table of values to the student and say, "Complete a table of values that represents the graph."	• Places numbers in the table of values that are not represented in the graph. • Makes one or two errors in the table of values and does not correct the errors by following a pattern.	• Completes the table of values correctly and uses the pattern to verify that the numbers make sense.

Money in My Pocket over Time

Number of Dollars (vertical axis: 2, 4, 6, 8, 10)

Number of Days (horizontal axis: 0, 2, 4, 6, 8, 10)

Number of Days	0	1	2	3	4	5	6	7
Number of Dollars	?	?	?	?	?	?	?	?

• Place the following table of values before the student and say, "Complete the following table of values, using the pattern rule multiply the step number by 5 and add 1 to get the number of elements in the step (the output is $5n + 1$ when the input is n)."	• Does not know how to apply the rule in finding the number of elements in each step. • Makes errors in calculating the number of elements in some of the steps and does not use the pattern to correct the errors.	• Correctly calculates the number of elements for each step, using the rule, and verifies his or her work by describing the pattern created in the table of values.

Step Number (input is n)	1	2	3	4	5	?
Number of Elements in the Step (output is $5n + 1$)	?	?	?	?	?	?

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Outcomes: 1, 2

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