Planning Guide: *Grade 7 Preservation of Equality*

Strand: Patterns and Relations (Variables and Equations)
Outcome: 3

Curriculum Highlights

This sample targets the following changes in the curriculum:

- The General Outcome is exactly the same as in the previous mathematics curriculum—represent algebraic expression in multiple ways.
- The Specific Outcome focuses on understanding the preservation of equality and solving equations concretely, pictorially and symbolically, whereas the previous mathematics curriculum included solving equations but not specific reference to the preservation of equality.

Step 1: Identify Outcomes to Address

Guiding Questions

- What do I want my students to learn?
- What can my students currently understand and do?
- What do I want my students to understand and be able to do, based on the Big Ideas and specific outcomes in the program of studies?

Big Ideas

Equality and inequality express relationships between quantities. When the quantities balance, there is equality. The equal sign is a symbol that indicates the quantity on the left side of the sign is the same as the quantity on the right. When there is an imbalance, there is inequality. The symbols on either side of the equality or inequality represent a quantity; e.g., $2 + 3$ and $2n + 4$ are both expressions for numbers.

The National Council of Teachers of Mathematics states, "In understanding equality, one of the first things students must realize is that equality is a relationship, not an operation" (2000–2007). Students often think of the equal sign as a symbol that tells them to do something or find the answer, but "they should come to view the equals sign as a symbol of equivalence and balance" (NCTM 2000, p. 39). For example, $5 + 2 = ?$ means that adding 2 to 5 results in 7. When students have this "operator view of equality," they have difficulty making sense out of a number sentence like $8 + 4 = ? + 5$.

Liping Ma views the equal sign as "the soul of mathematical operations. In fact, changing one or both sides of an equal sign for certain purposes while preserving the 'equals' relationship is the 'secret' of mathematical operations" (1999, p. 111).

Patterns and Pre-algebra, Grades 4–6 discusses the various relationships that can be shown using equality.

Equality and inequality between quantities can be considered as:
- whole to whole relationships (five red chips = five blue chips or 5 = 5)
- part-part to whole relationships (3 + 5 = 8)
- whole to part-part relationships (8 = 3 + 5)
- part-part to part-part relationships (4 + 4 = 3 + 5).

Reproduced from Alberta Education, *Patterns and Pre-Algebra, Grades 4–6* (unpublished workshop handout) (Edmonton, AB: Alberta Education, 2007), p. 54.

Compensation is used in deciding if the expressions on both sides of the equal sign represent the same quantity; e.g., 38 + 72 = 40 + 70 is a true statement because the 2 that is added to 38 to make 40 is compensated by subtracting 2 from 72 to make 70. Similarly, 85 – 28 = 87 – 30 is a true statement because the difference is constant; i.e., 2 is added to each number on the left side of the equal sign to make the expression on the right side so the difference between the two numbers remains the same.

Solving equations requires that the balance of the equation is maintained so that the expressions on either side of the equal sign represent the same quantity. Relations between the expressions on either side of the equal sign are examined and used to simplify the process; e.g., if a quantity is added to one side of the equation then, to maintain equality, the same quantity must be added to the other side of the equation. Similarly, the equality or balance must be maintained when a number is subtracted, multiplied or divided.

Sequence of Outcomes from the Program of Studies

Grade 6	Grade 7	Grade 8
Specific Outcomes 5. Demonstrate and explain the meaning of preservation of equality, concretely and pictorially.	Specific Outcomes 3. Demonstrate an understanding of preservation of equality by: • modelling preservation of equality, concretely, pictorially and symbolically • applying preservation of equality to solve equations.	Specific Outcomes 2. Model and solve problems concretely, pictorially and symbolically, using linear equations of the form: • $ax = b$ • $x/a = b$, $a \neq 0$ • $ax + b = c$ • $x/a + b = c$, $a \neq 0$ • $a(x + b) = c$ where a, b and c are integers.

Step 2: Determine Evidence of Student Learning

Guiding Questions

- What evidence will I look for to know that learning has occurred?
- What should students demonstrate to show their understanding of the mathematical concepts, skills and Big Ideas?

As you begin planning lessons and learning activities, keep in mind ongoing ways to monitor and assess student learning. One starting point for this planning is to consider the achievement indicators listed in *The Alberta K–9 Mathematics Program of Studies with Achievement Indicators* (Alberta Education 2007). You may also generate your own indicators and use these to guide your observation of students.

The following achievement indicators may be used to determine whether students have met this specific outcome.

- Model the preservation of equality for each of the four operations, using concrete materials, or, using pictorial representations, explain the process orally and record it symbolically.
- Write equivalent forms of a given equation by applying the preservation of equality and verify, using concrete materials; e.g., $3b = 12$ is the same as $3b + 5 = 12 + 5$ or $2r = 7$ is the same as $3(2r) = 3(7)$.
- Solve a given problem by applying preservation of equality.

Some sample behaviours to look for in relation to these indicators are suggested for many of the instructional activities in Step 3, Section C, Choosing Learning Activities.

Step 3: Plan for Instruction

Guiding Questions

- What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should I use?
- How will I meet the diverse learning needs of my students?

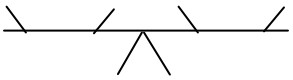
A. Assessing Prior Knowledge and Skills

Before introducing new material, consider ways to assess and build on students' knowledge and skills related to preservation of equality. For example:

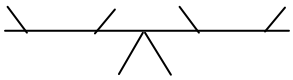
- To balance the second balance scale, how many blocks would have to be placed in the empty pan? Assume that each cylinder weighs the same and that each block weighs the same. Explain your thinking.

- Draw a diagram of a balance scale to illustrate whether or not the following statement is true: $68 + 82 = 70 + 80$.

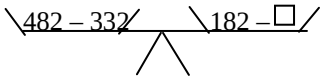
- Use the balance scale below to illustrate and solve the equation $5 + 7 = \square + 5$.



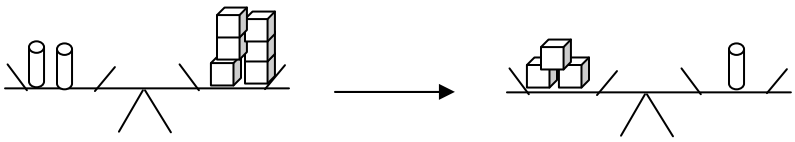
- Draw a diagram on the balance scale to show that $2 \times 3 = 3 \times 2$.



- To maintain equality, what number must be placed in the square on the balance scale below? Explain your thinking.



a) Explain the process that was used to connect the two balance scales shown below.



b) Using the information provided on the two scales, give one example of:
   - what the weight of the cylinder might be
   - what the corresponding weight of the cube would be.

If a student appears to have difficulty with these tasks, consider further individual assessment, such as a structured interview, to determine the student's level of skill and understanding. See Sample Structured Interview: Assessing Prior Knowledge and Skills.

B. Choosing Instructional Strategies

Consider the following strategies when planning lessons.

- Build on understanding of equality from previous grades, using the four operations of addition, subtraction, multiplication and division.
- Have students use balance scales to illustrate equality and then connect the concrete to the pictorial and symbolic representations.
- Use balance scales and alge-tiles together with an unknown weight, such as a film container filled with pinto beans, to illustrate equivalent forms of a given equation. Have students draw the diagram and write the symbolic representation.
- Use balance scales and alge-tiles together with an unknown weight, such as a film container filled with pinto beans, to illustrate the solution of equations in which the answer is a whole number. Have students draw the diagram and write the symbolic representation.
- Use the pictorial representation of alge-tiles on balance scales to solve equations in which the solution is a negative integer.
- Have students solve problems by writing the appropriate equation, illustrating the solution concretely or pictorially using balance scales and recording the solution symbolically. This addresses the mathematical process of problem solving included in the Specific Outcome.

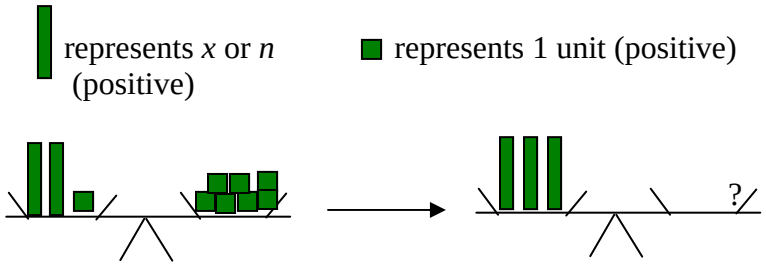
C. Choosing Learning Activities

The following learning activities are examples of activities that could be used to develop student understanding of the concepts identified in Step 1.

Sample Activities for Teaching an Understanding of Preservation of Equality

1. Modelling Equality Using the Four Operations

   Provide students with alge-tiles to model equality on a pictorial balance scale. Use the tiles that represent a positive *x* and a positive unit. Have students draw diagrams and write number sentences to represent the action pictorially and symbolically. Include examples in which students must add, subtract, multiply and divide to solve the problem presented on the pictorial balance scales; e.g., present the following problem to students.



   Have students share their answers of what alge-tiles to place on the second balance scale to balance the scale. Students might subtract a unit from each side of the first scale and divide

Look For …

Do students:

- ☐ apply prior knowledge of using the four operations to preserve equality?
- ☐ apply prior knowledge of equality in using alge-tiles?
- ☐ demonstrate flexibility in finding more than one way to balance the scale?
- ☐ translate easily among the concrete, pictorial and symbolic modes?

both sides by two so that one x balances three units. Then on the second balance scale, they have three x's on one side so they must place three groups of three units, or nine units, to balance the scale.

Symbolic representation could be as follows:
$2x + 1 = 7$. Subtract one from both sides and the result is $2x = 6$. Divide both sides by two and the result is $x = 3$. Multiply both sides by three and the result is $3x = 3(3)$.

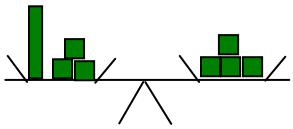
Another option is to place two x's and three units in the right pan of the second balance scale to balance the scale.

Symbolic representation could be as follows:
$2x + 1 = 7$. Subtract one from both sides and the result is $2x = 6$. Divide both sides by two and the result is $x = 3$. Add $2x$ to both sides and the result is $3x = 2x + 3$.

Have students create other balance scale problems to solve by using the four operations to preserve equality.

2. Equivalent Forms of Equations—Preservation of Equality

Provide students with alge-tiles as described in the previous activity. Have students draw a balance scale and place the alge-tiles on the balance scale as in the example below.



Instruct students to draw a diagram and write the equation represented by the alge-tiles. Then have them use the alge-tiles to create an equivalent equation on the balance scale, draw the corresponding diagram and write the equation symbolically. Have students share their equivalent equations and explain how their equations were derived from the original equation by using one or more of the four operations.

Look For …

Do students:

- ☐ apply prior knowledge of using the four operations to preserve equality?
- ☐ apply prior knowledge of equality in using alge-tiles?
- ☐ demonstrate flexibility in finding more than one way to balance the scale?
- ☐ translate easily among the concrete, pictorial and symbolic modes?

Reverse the procedure and provide students with two equivalent symbolic representations of equations. Have them use alge-tiles and draw corresponding diagrams to explain why these two equations are equivalent; e.g.,

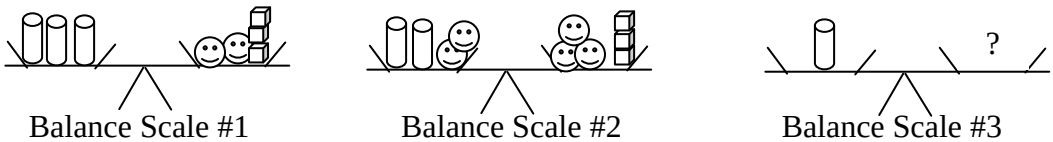
Present the following linear relations to students and have them show, by modelling and drawing diagrams, how equality is preserved; i.e., by multiplying both sides of the original equation by 2 and then adding 1 to both sides. Have students create other equivalent forms for the given equation.

Given equation:	Equivalent equation:
$2n = 5$	$2(2n) + 1 = 2(5) + 1$

3. Solving Problems by Applying Preservation of Equality

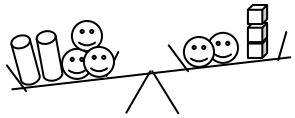
Provide students with pictorial balance scale problems. Have them solve the problems and explain their thinking. Encourage them to find as many solutions as possible for each problem and then create some problems that they could share with the class.

Example 1:
Use the information from the first two balance scales to find which objects are needed to balance the third scale. Explain your thinking.



Balance Scale #1 Balance Scale #2 Balance Scale #3

Extension of Example 1:
Suppose one of the happy faces from Balance Scale #2 moved from the pan on the right to the pan on the left as shown below. Make a list of all the combinations of objects that will balance the scale below. Explain your thinking.



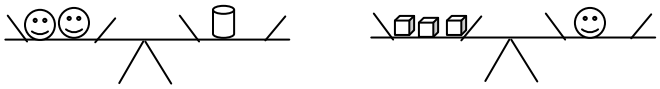
Adapted from Robert Mann, "Responses to the 'A Walk in the Park' Problem," *Teaching Children Mathematics* 10, 3 (September 2003), pp. 54, 55. Adapted with permission of the National Council of Teachers of Mathematics.

Look For …

Do students:

- ☐ apply prior knowledge of using the four operations to preserve equality and simplify the number of objects on the scales?
- ☐ make connections between the balance scales to solve the problem?
- ☐ demonstrate flexibility in finding more than one way to balance the scale?
- ☐ communicate clearly the thinking done to solve the problem?

Example 2:



Which shape weighs the most? Explain your thinking.
Which shape weighs the least? Explain your thinking.

Example 3:

What will balance two happy faces? Explain your thinking.

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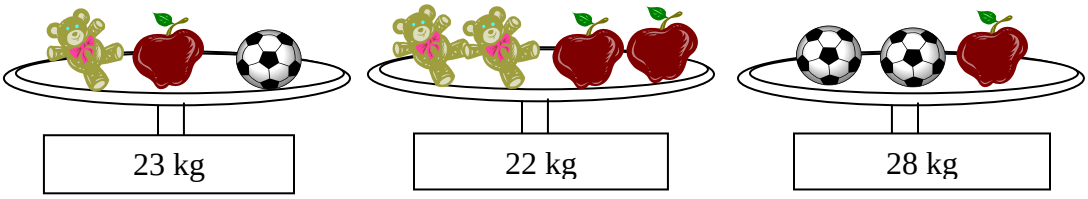
Example 4:



What is the weight of each item? Explain your thinking.

Adapted from the National Council of Teachers of Mathematics, "Responses to the 'Guess the Weight' Problem," *Teaching Children Mathematics* 6, 1 (September 1999), p. 31. Adapted with permission of the National Council of Teachers of Mathematics.

4. Balance Scales and Solving Equations

Provide students with balance scales, weights and film containers filled with pinto beans. The film containers with pinto beans should all weigh the same and the weight depends on the equations that students are to solve by using the balance scales. For example, if students are to solve the equation $x + 5 = 15$, then x represents the weight of the film container and pinto beans, which is 10 g.

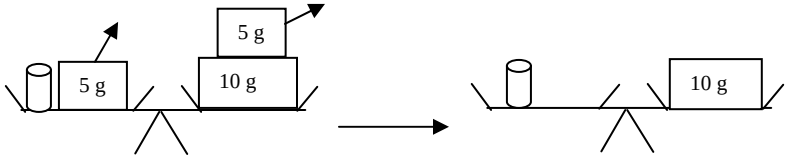
Look For …

Do students:

- ☐ apply prior knowledge in using balance scales?
- ☐ remove equal weights from both sides of the balance scale to preserve equality?
- ☐ translate easily among the concrete, pictorial and symbolic modes?

Present students with a problem; e.g.,
If you add 5 g to the weight of an object (in this case the film container filled with pinto beans), you get a total weight of 15 g. What is the weight of the object? Use the balance scales to solve the problem, draw a diagram and write equations to show your work.

Sample of student work:



$x + 5 = 15$
$-5 \quad -5$
$x = 10$

The weight of the object is 10 g.

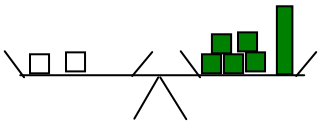
These basic equations provide the foundation for the work with integers using negative numbers in the next activity.

5. Alge-tiles, Pictorial Balance Scales and Solving Equations

Provide students with alge-tiles and review the concept of zero, using the tiles. Present students with equations and have them solve the equations by placing the alge-tiles on pictorial balance scales. Then have students draw the corresponding diagrams and record the process symbolically by writing appropriate equations.

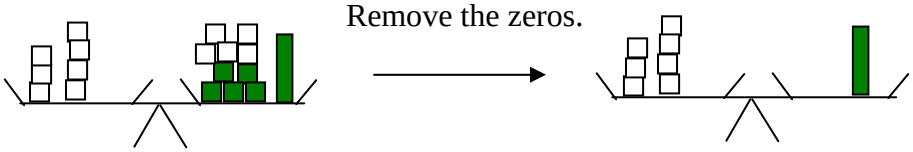
Example: Given the equation $-2 = 5 + x$, solve for x.

☐ White (–) ■ Green (+) ▮ unknown or x



Add –5 to both sides of the balance scale to solve for x.

$-2 = x + 5$
$-5 = \quad -5$



$-2 = x + 5$
$\underline{-5 = \quad -5}$
$-7 = x$

Provide opportunities for students to interpret diagrams drawn by others and write the corresponding equations to represent the process.

Look For …

Do students:

- ☐ apply prior knowledge about integers and using integer tiles to using alge-tiles?
- ☐ use a systematic approach to determine the solution?
- ☐ have a clear sense that = means that the expressions on either side of it are equivalent amounts?
- ☐ translate easily among the concrete, pictorial and symbolic modes?

Step 4: Assess Student Learning

Guiding Questions

- Look back at what you determined as acceptable evidence in Step 2.
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

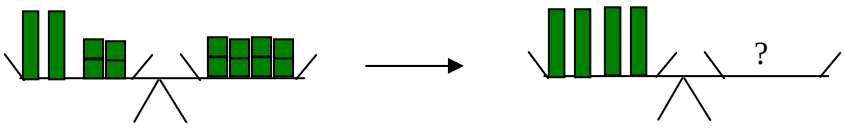
In addition to ongoing assessment throughout the lessons, consider the following sample activities to evaluate students' learning at key milestones. Suggestions are given for assessing all students as a class or in groups, individual students in need of further evaluation and individual or groups of students in a variety of contexts.

A. Whole Class/Group Assessment

Note: Performance-based assessment tasks are under development.

Provide students with alge-tiles to use as needed.

1. a) Solve the following problem by drawing alge-tiles in the right pan of the second balance scale.



   b) Write an equation to represent what is shown on each balance scale.

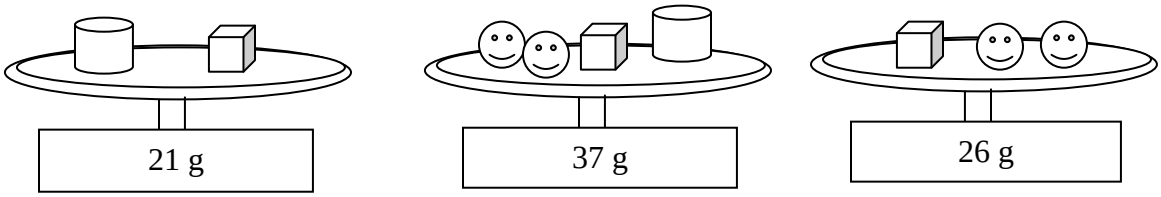
2. Draw diagrams on the balance scales below to show that $2n = 5$ is the same as $2(2n) = 2(5)$.

3. Write an equation that is equivalent to $3n + 1 = 5$ and draw diagrams on the balance scales below to represent the process.

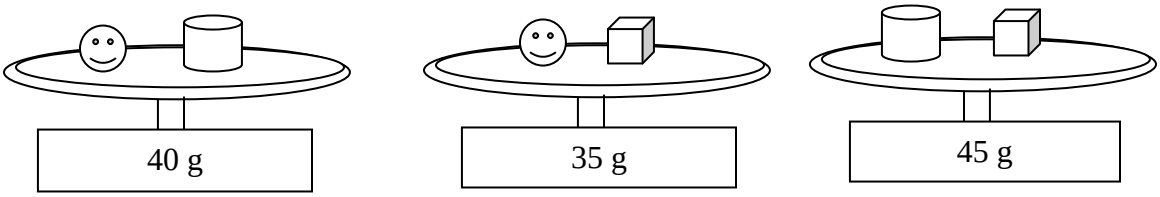
4. Solve the following equation and draw diagrams of alge-tiles on the balance scales below to represent the process.

$$-5 = -3 + n$$

5. Solve the following problems. Explain your thinking.
   a)



b)



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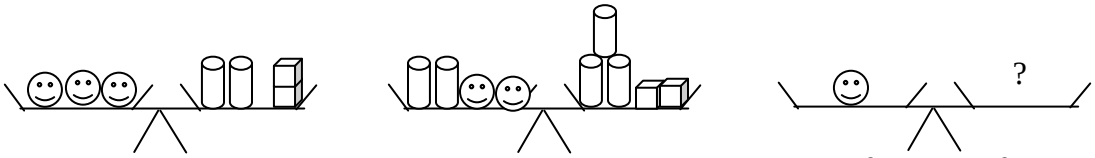
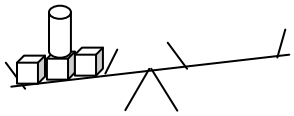
c) (i) Find all the possible ways to balance the happy face on the third balance scale, using the information from the first two scales. Explain your thinking.



(ii) Find all the possible ways to balance the scale below, using the information from part (i) of this problem. Explain your thinking.



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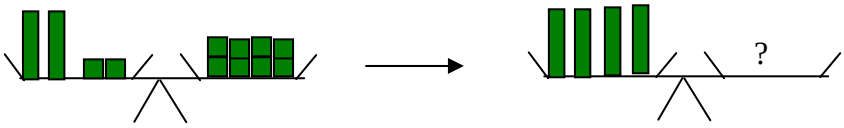
B. One-on-One Assessment

Assessment activities can be used with individual students, especially students who may be having difficulty with the outcome.

Provide the student with alge-tiles to use as needed.

1. Present the following problem to the student.
   a) Solve the following problem by drawing alge-tiles in the right pan of the second balance scale.



   b) Write an equation to represent what is shown on each balance scale.

   If the student has difficulty, have him or her use the alge-tiles and place them on a pictorial balance scale to represent the relations shown on the first balance scale. Have the student look at the second balance scale to determine what the question is asking for and how it is related to the first balance scale. Guide the student to discover that there are twice as many long tiles or *x*'s on the second scale as there are on the first scale. Ask, "How could you

remove the two unit tiles from the left pan of the first scale and preserve the equality?" Remind the student that equality or balance is preserved by removing the same tiles from each side of the scale. After the student removes two unit tiles from each side of the scale to find what balances the two *x* tiles, have him or her use that information by doubling it to find what balances the four *x* tiles on the second balance scale.

2. Present the following problem to the student.
   Draw diagrams on the balance scales below to show that $3n = 6$ is the same as $2(3n) = 2(6)$.

   If the student has difficulty, have him or her use the alge-tiles to represent the problem. Prompt the student to recall that $3n$ means 3 groups of n. Have him or her place the tiles on a pictorial balance scale to show that $3n = 6$. Similarly, prompt the student that $2(3n)$ means 2 groups of $3n$ and have him or her represent the second equation on the second balance scale. Ask, "How do you know the second scale balances?" If necessary, prompt the student to say that multiplying both sides of an equation by the same number preserves the equality.

3. Present the following instructions to the student.
   Write an equation that is equivalent to $2n + 3 = 5$ and draw diagrams on the balance scales below to represent the process.

   If the student has difficulty, have him or her use the alge-tiles to represent the equation and draw these tiles on the first balance scale. Remind the student that he or she can add a number to both sides of the equation and preserve the equality. This also applies to subtraction, multiplication and division. After the student uses one of the operations to preserve equality, have him or her draw the tiles on the second balance scale.

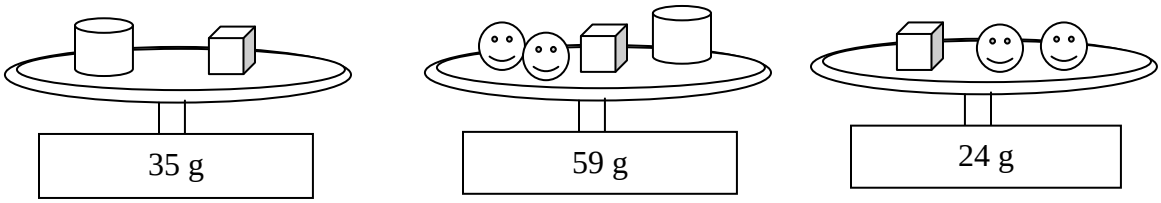
4. Present the following instructions to the student.
   Solve the following equation and draw diagrams of alge-tiles on the balance scales below to represent the process.

$$-1 = -4 + n$$

   If the student has difficulty, have him or her use the alge-tiles to represent the equation and draw these tiles on the first balance scale. If necessary, remind the student which tiles are used to represent positive and negative integers. Ask, "How would you use the alge-tiles to solve for *n*, in other words, how would you remove –4?" Prompt the student to add +4 to both sides of the equation and use zeros to solve for *n*. Remind the student to draw the tiles to

represent +4 added to both sides on the first balance scale. Have the student place the solution on the second balance scale.

5. Present the following problems, one at a time, to the student.
   a) Solve the following problems. Explain your thinking.



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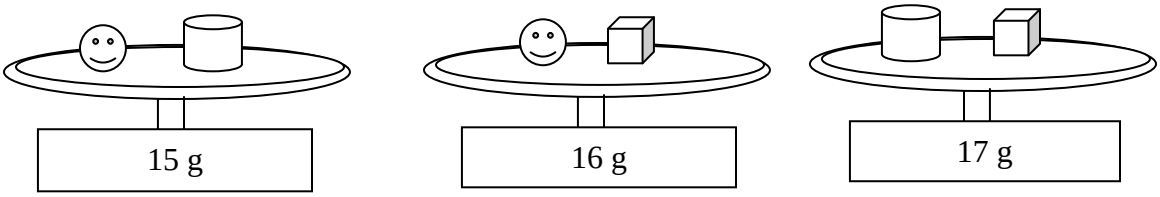
If the student has difficulty, ask how he or she might use the information from either the first or third balance scale in the second balance scale. If necessary, point to two objects on the second balance scale, such as the cube and the cylinder, and then point to these objects and their total weight shown on the first balance scale. Suggest that these objects on the second balance scale be replaced with their combined weight; i.e., 35 g.

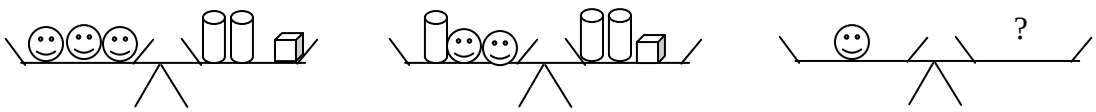
   b)



Adapted from Van de Walle, John A., LouAnn H. Lovin, *Teaching Student-Centered Mathematics, Grades 5–8* (p. 280). Published by Allyn and Bacon, Boston, MA. Copyright © 2006 by Pearson Education. Adapted by permission of the publisher.

If the student has difficulty, prompt him or her to place all the objects on one balance scale and determine their total weight. If necessary, point out that the combined weight of all the objects is double the combined weight on one cylinder, one cube and one happy face. Ask, "How can you use this information to find the weight of each item?" If necessary, have the student focus on the first balance scale and draw a cube on it to include all three items. Have him or her write the total weight of all three items in the box below the scale, crossing out the 15 g but still keeping it in view. Ask, "How can you now find the weight of the cube?"

   c) (i) Find all the possible ways to balance the happy face on the third balance scale, using the information from the first two balance scales. Explain your thinking.



If the student has difficulty, have him or her focus on the second balance scale, remove a cylinder from each side and determine what balances two happy faces. Ask, "How can this information be used on the first balance scale?" If necessary, point out that two of the happy faces on the first balance scale can be replaced by a cylinder and a cube. After the student makes this substitution, have him or her remove the same objects from each side of the first balance scale and determine what one happy face is equal to. Encourage the student to find more than one answer to what object could balance the happy face on the third balance scale.

(ii) Find all the possible ways to balance the scale below, using the information from part i of this problem. Explain your thinking.

Adapted from Robert Mann, "Responses to the 'A Walk in the Park' Problem," *Teaching Children Mathematics* 10, 3 (September 2003), p. 55. Adapted with permission of the National Council of Teachers of Mathematics.

If the student has difficulty, remind him or her of the information obtained from part (i) of this problem. Suggest that the student make an organized list to obtain all the possible combinations that would balance the scale. For example, since one happy face is equivalent to one cylinder or one cube, then the following options occur:

# of happy faces	3	2	2	1	1	1	0	0	0	0
# of cylinders	0	1	0	1	0	2	3	2	1	0
# of cubes	0	0	1	1	2	0	0	1	2	3

If necessary, make the problem simpler by including only one cylinder and one cube in the pan on the left side of the balance scale.

C. Applied Learning

Provide opportunities for students to use their preservation of equality strategies in a practical situation and notice whether or not the strategies transfer. For example, ask the student how many times greater the circumference of circular plot would be if the diameter is doubled. Does the student:

- use equality in describing the formula for the circumference of a circle?
- explain that multiplying both sides of an equality by 2 maintains the equality?
- explain that if the diameter is doubled then the circumference must be doubled as well so that the equality of the formula is maintained?
- transfer this understanding of equality by responding correctly to other similar questions, such as tripling the length of one side of a square and finding the effect on the perimeter of the square?

Step 5: Follow-up on Assessment

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction?

A. Addressing Gaps in Learning

- Build on students' understanding of preservation of equality from previous grades.
- Encourage students to use balance scales to represent various equalities, draw corresponding diagrams and record the process symbolically.
- Have students explain their thinking and provide scaffolding to overcome any misconceptions or misunderstandings.
- Use examples of equality with everyday contexts so that students can relate to the situation and understand it better.
- Use friendly numbers in modelling preservation of equality, initially, so that students can easily draw diagrams to represent the situation.
- Include examples with positive and negative integers, using alge-tiles and a balance scale, and reviewing the concept of zero to illustrate equality on both sides of the scale.
- Solve a variety of everyday problems by applying the preservation of equality.
- Reinforce the role of variables and constants in the preservation of equality.
- Integrate the strands in mathematics by applying the preservation of equality to number operations, measurement and statistics and probability.
- Encourage students to create problems that can be solved by applying the preservation of equality.

B. Reinforcing and Extending Learning

Students who have achieved or exceeded the outcomes will benefit from ongoing opportunities to apply and extend their learning. These activities should support students in developing a deeper understanding of the concept and should not progress to the outcomes in subsequent grades.

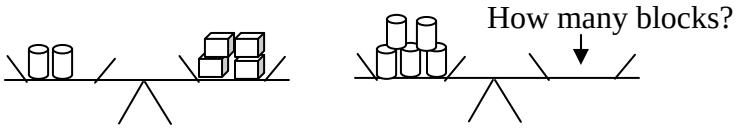
Consider strategies, such as the following.

- Provide tips for parents on applying preservation of equality at home or in the community. For example:
  - If two jars of nuts fill three cans, then 10 jars of nuts will fill how many cans?
  - You read 12 pages in an hour. At that rate, how many pages would you read in 15 minutes?
  - A carton of 12 boxes weighs the same as a carton of eight cans. How many boxes would weigh the same as six cans?

– You are putting two types of nuts into each bag: 356 grams of peanuts and 124 grams of cashews. You want the total weight for each bag to remain the same but you only have 120 grams of cashews for the last bag. How many grams of peanuts will you need to put into the bag?

- Use an electronic slide show presentation, such as the one created by Enzo Timoteo, in solving equations ($a + x = b$) pictorially with balance scales and integer tiles.

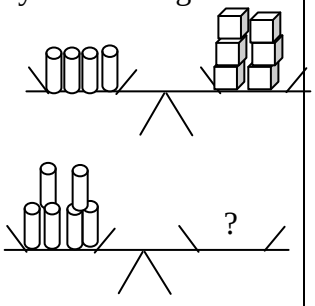
- Have students solve problems, using illustrations with balance scales, such as the following:

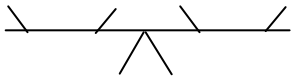
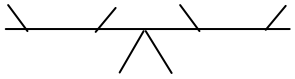


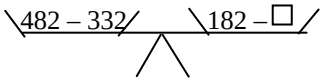
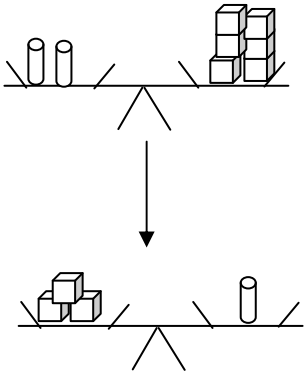
If a block weighs 5 g, how much does the cylinder weigh?

- Use the *Illuminations* Web site (http://illuminations.nctm.org/ActivityDetail.aspx?ID=33), created by the National Council of Teachers of Mathematics, to work with virtual balance scales, using shapes and also numbers.

- Sample Structured Interview: Assessing Prior Knowledge and Skills

Directions	Date:	
	Not Quite There	**Ready to Apply**
Place the two balance scales shown below before the student and present the following problem: "To balance the second balance scale, how many blocks would have to be placed in the empty pan? Assume that each cylinder weighs the same and that each block weighs the same. Explain your thinking." 	• Does not indicate the correct number of blocks that must be placed in the second balance scale to maintain balance. • Says that nine blocks will balance the second scale but is unable to explain his or her thinking.	• Says that nine blocks are needed to balance the second scale and explains his or her thinking; e.g., dividing both sides of the first balance scale by two gives the information that two cylinders balance three blocks, and multiplying both of these groups by three to maintain balance, you have six cylinders balancing nine blocks.

Write the equation $68 + 82 = 70 + 80$ before the student. Then say, "Draw a diagram of a balance scale to illustrate whether or not the following statement is true."	• Calculates both sides to determine equality but does not use the balance scale. • Places the quantity of $68 + 82$ on one side of a balance scale and the quantity of $70 + 80$ on the other side of the balance scale but makes no attempt to use compensation.	• Draws a balance scale and places $68 + 82$ on one side of the balance scale. Adds 2 to 68 and subtracts 2 from 82 and then writes $70 + 80$ on the other side of the balance scale, illustrating equality.
Write the equation $5 + 7 = \square + 5$ before the student and present the balance scale.  Then say, "Use the balance scale to illustrate and solve this equation."	• Places $5 + 7$ on one side of the balance scale and $\square + 5$ on the other side of the scale but is unable to solve the equation correctly.	• Places $5 + 7$ on one side of the balance scale and $\square + 5$ on the other side of the scale and explains that the unknown number must be 7 so that both sides of the equation represent 12 and balance each other.
Present the balance scale and the equation $2 \times 3 = 3 \times 2$ to the student. Say, "Draw a diagram on the balance scale to show that $2 \times 3 = 3 \times 2$." 	• Places 2×3 on one side of the balance scale and 3×2 on the other side but is unable to draw appropriate diagrams to illustrate the multiplication.	• Draws appropriate diagrams to illustrate the multiplication sentence; e.g., draws two groups of three cylinders on one side of the balance scale and three groups of two cylinders on the other side of the scale.

Present the balance scale with numbers as shown below. Say, "To maintain equality, what number must be placed in the square on the balance scale below? Explain your thinking."  	• Does not find the correct number to put in the square. • Finds the difference on the left side of the scale and uses that information to find an equal difference on the right side of the scale.	• Uses compensation to find the number 32 to put in the square and explains that 300 is subtracted from each number on the left side to create the numbers on the right side so the difference is constant on both sides of the scale. 332 – 300 = 32.
Present the student with the two balance scales shown below. Say, "Explain the process that was used to connect the two balance scales shown below."  After the student finishes the first part, say, "Using the information provided on the two scales, give one example of: • what the weight of the cylinder might be • what the corresponding weight of the cube would be."	• Says that there are fewer blocks and cylinders on the second balance scale than on the first balance scale but does not explain why equality is maintained on the second scale. • Randomly chooses weights for the cylinder and the cube but does not illustrate that the weight of the cylinder is three times the weight of one cube.	• Explains that half the cylinders were removed from one side of the balance scale and half the cubes were removed from the other side of the balance scale; therefore equality is maintained because both sides were divided by two. • Chooses a weight for a cube, say 8 grams, and multiplies it by three to obtain the weight of the cylinder—24 grams.

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Strand: Patterns and Relations (Variables and Equations)
Outcome: 3

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Step 5 References

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