Planning Guide**:** *Grade 7 Probability*

Strand: Statistics and Probability (Chance and Uncertainty)
Outcome: 6

Curriculum Highlights

This sample targets the following changes in the curriculum:

- The General Outcome focuses on using experimental or theoretical probabilities to represent and solve problems involving uncertainty, whereas the previous mathematics curriculum focused on creating and solving problems, using probability.
- The Specific Outcome focuses on conducting probability experiments with two independent events and comparing the experimental results to the theoretical probability that is found by using tables or tree diagrams, whereas the previous mathematics curriculum focused on using a table to find the outcomes for two independent events and then finding probability by dividing the favourable outcomes by the possible outcomes.

Step 1: Identify Outcomes to Address

Guiding Questions

- What do I want my students to learn?
- What can my students currently understand and do?
- What do I want my students to understand and be able to do, based on the Big Ideas and specific outcomes in the program of studies?

Big Ideas

Probability is a measure of how likely an event is to occur. It involves predicting the likelihood of an event occurring over a period of time rather than predicting the likelihood of occurrence for a specific event for a given time.

Theoretical probability of an event is the ratio of the number of outcomes in an event to the total number of possible outcomes, when all possible outcomes are equally likely.

Experimental probability or relative frequency of an event is the ratio of the number of observed occurrences of the event to the total number of trials (Van de Walle and Lovin 2006, p. 334). The greater the number of trials, the closer the experimental probability approaches the theoretical probability.

The **sample space** of a probability experiment is the set of all possible outcomes for that experiment.

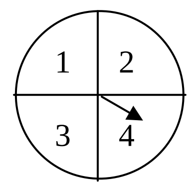
Equally likely outcomes have the same probability or the same chance happening in a given probability experiment (Cathcart 1997).

An **outcome** is the result of a single trial of an experiment, whereas an **event** is one or more outcomes (a set of outcomes) of an experiment. Both an outcome and an event form a subset of the sample space.

A **single-stage** probability experiment is a probability experiment that involves only one action, such as tossing one coin, to determine an outcome. A **two-stage** probability experiment is a probability experiment that involves two actions, such as tossing two coins, to determine an outcome.
Adapted from Van de Walle, John A., LouAnn H. Lovin, *Teaching Student-Centered Mathematics: Grades 5–8* (p. 341). Published by Allyn and Bacon, Boston, MA. Copyright ©2006 by Pearson Education. Adapted by permission of the publisher.

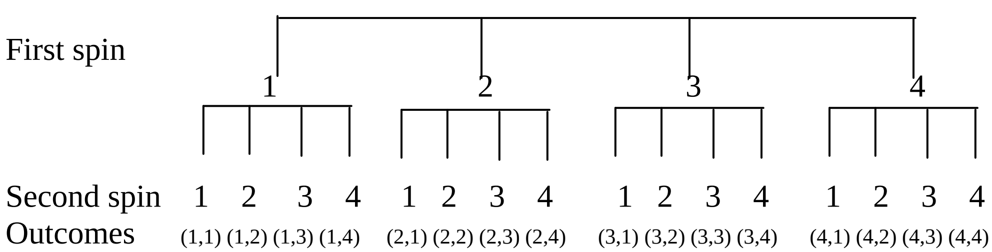
Two events are **independent** if the fact that one event occurs does not affect the probability of the second event occurring.

For example, the possible outcomes or sample space for one spin of the spinner shown at the left is 1, 2, 3, 4. All the outcomes are equally likely; i.e., the probability of each outcome is the same—1/4. One event from this sample space is spinning a prime number and the probability of this event is 2/4 or 1/2.



Spinning the spinner twice results in two independent events because the result of the first spin does not affect the result of the second spin. The equally likely possible outcomes for two spins of the spinner can be represented in a **tree diagram** or **table** as follows.

Tree Diagram



Table

Second Spin / First Spin	1	2	3	4
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)

One event of spinning the spinner twice is obtaining a sum that is a multiple of 3; i.e., (1, 2), (2, 1), (2, 4), (3, 3), (4, 2). The theoretical probability of this event is 5/16.

$$\text{Theoretical probability of event X} = \frac{\text{Number of ways that event X can occur}}{\text{Total number of possible outcomes}}$$

If an experiment is conducted by spinning the spinner twice, then a **trial** is the result of spinning the spinner twice and the experimental probability or relative frequency of a specific event is calculated as follows.

$$\text{Experimental probability of event X} = \frac{\text{Number of observed occurrences of event X}}{\text{Total number of trials in the experiment}}$$

Sequence of Outcomes from the Program of Studies

See (Web address) for the complete program of studies.

Grade 6	Grade 7	Grade 8
Specific Outcomes 4. Demonstrate an understanding of probability by: • identifying all possible outcomes of a probability experiment • differentiating between experimental and theoretical probability • determining the theoretical probability of outcomes in a probability experiment • determining the experimental probability of outcomes in a probability experiment • comparing experimental results with the theoretical probability for an experiment.	Specific Outcomes 6. Conduct a probability experiment to compare the theoretical probability (determined using a tree diagram, table or other graphic organizer) and experimental probability of two independent events.	Specific Outcomes 2. Solve problems involving the probability of independent events.

Step 2: Determine Evidence of Student Learning

Guiding Questions

- What evidence will I look for to know that learning has occurred?
- What should students demonstrate to show their understanding of the mathematical concepts, skills and Big Ideas?

As you begin planning lessons and learning activities, keep in mind ongoing ways to monitor and assess student learning. One starting point for this planning is to consider the achievement indicators listed in *The Alberta K–9 Mathematics Program of Studies with Achievement Indicators* (Alberta Education 2007). You may also generate your own indicators and use these to guide your observation of students.

The following achievement indicators may be used to determine whether students have met this specific outcome.

- Distinguish between an outcome and an event.
- Distinguish between equally likely and unequally likely outcomes.
- Determine the theoretical probability of a given event involving two independent events with each event having equally likely outcomes.
- Determine the theoretical probability of a given event involving two independent events with at least one event having unequally likely outcomes.
- Use tree diagrams, charts and the area model to determine the theoretical probability of an event.
- Conduct a probability experiment for an event involving two independent events (with equally likely outcomes or unequally likely outcomes), with and without technology, to compare the experimental probability to the theoretical probability.
- Solve a given probability problem involving two independent events, using a graphic organizer to determine the probability rather than multiplying the probabilities of the two independent events.

Some sample behaviours to look for in relation to these indicators are suggested for many of the instructional activities in Step 3, Section C, Choosing Learning Activities.

Step 3: Plan for Instruction

Guiding Questions

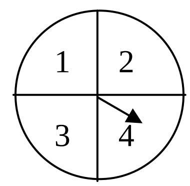
- What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should I use?
- How will I meet the diverse learning needs of my students?

A. Assessing Prior Knowledge and Skills

Before introducing new material, consider ways to assess and build on students' knowledge and skills related to counting. For example:

- A probability experiment consists of rolling a 12-sided die. One face has the number 1, two faces have the number 2, three faces have the number 3, four faces have the number 4 and two faces have the number 5. Use this information to answer the questions below.
  a) List all the possible outcomes for this probability experiment.
  b) Are the outcomes equally likely or unequally likely? Explain your thinking.
  c) Predict the probability of rolling a number 4. Explain your thinking.
  d) Describe how you would carry out an experiment to determine the experimental probability of rolling a number 4.
  e) Will the theoretical and experimental probability for rolling a number 4 be the same? Explain why or why not.

- A probability experiment consists of spinning the spinner shown at the right once. The outcomes of the probability experiment are shown in the tally chart below.



Outcomes	Tallies
1	~~////~~ ~~////~~ ~~////~~ //
2	~~////~~ ~~////~~ ////
3	~~////~~ ~~////~~ ~~////~~ ////
4	~~////~~ ~~////~~

  a) How many trials were completed in this experiment? How do you know?
  b) What is the experimental probability of spinning a 4? Explain your thinking.
  c) What is the theoretical probability of spinning a 4? Explain your thinking.
  d) If there is a difference between your answers in parts (b) and (c), what could be done to bring the experimental probability closer to the theoretical probability?
  e) Are the outcomes in this experiment equally likely or unequally likely? Explain your thinking.

- Is it more likely to draw a red counter from Bag A, containing 3 red counters and 2 blue counters, or from Bag B, containing 4 red counters and 6 blue counters? Explain your thinking.

If a student appears to have difficulty with these tasks, consider further individual assessment, such as a structured interview, to determine the student's level of skill and understanding. See <u>Sample Structured Interview: Assessing Prior Knowledge and Skills</u>.

B. Choosing Instructional Strategies

Consider the following strategies when planning lessons.

- Build on students' understanding of experimental and theoretical probability focusing on a single action (e.g., tossing a coin once) from the previous grade and extend it to include two independent events (e.g., tossing a coin twice).

- Clarify terminology used in probability; e.g., theoretical probability, experimental probability, sample space, outcomes, events, equally likely outcomes, unequally likely outcomes and trials.
- Provide examples and nonexamples of independent events to deepen students' understanding of independent events.
- Provide various strategies for creating the sample space and then calculating the theoretical probability of independent events without using multiplication. These strategies may include using tree diagrams, tables and area models.
- Integrate technology after students have done hands-on work in carrying out experiments with independent events.
- Provide a variety of manipulatives in illustrating independent events; e.g., coins, dice, spinners and drawing cards from a deck or objects from a bag with replacement.
- Have students predict the results of any experiment with independent events by using theoretical probability.

C. Choosing Learning Activities

The following learning activities are examples of activities that could be used to develop student understanding of the concepts identified in Step 1.

Sample Activities for Teaching Theoretical and Experimental Probability of Two Independent Events

1. Independent Events with Equally Likely Outcomes

   Build on students' understanding of equally likely outcomes, using a single-stage experiment such as rolling a single standard die. Review that each of the outcomes (1, 2, 3, 4, 5, 6) has the same theoretical probability (ratio of the number of favorable outcomes to the total possible outcomes)—1/6. Therefore each outcome is equally likely.

   Review that an event is a subset of the sample space (set of outcomes) and the events may or may not be equally likely. Have students provide examples of events for rolling a standard die once; e.g., rolling an even number, odd number, prime number or a number that is a multiple of three. Through discussion, guide students to verbalize that the probabilities of these different events are different and are therefore not equally likely.

   Explain that instead of using a single-stage experiment such as rolling one die, students will use two-stage experiments such as rolling two dice. Ask students whether the outcome on the first die affects the outcome on the

Look For …

Do students:

- ☐ use appropriate terminology for single-stage probability experiments?
- ☐ describe the similarities and differences between a single-stage and a two-stage probability experiment?
- ☐ explain the characteristics of independent events?
- ☐ create the sample space for probability experiments with two independent events in more than one way; e.g., tree diagrams, tables?
- ☐ compare theoretical and experimental probability for two independent events?
- ☐ demonstrate flexibility in using a variety of manipulatives to solve problems involving probability?

second die. Have them provide other examples in which the outcome in the first stage of the experiment does not affect the outcome in the second stage of the experiment; e.g., tossing two coins and spinning a spinner twice. Explain that these are examples of independent events.

Have students provide examples of events that are not independent, such as drawing two cards from a deck of cards or drawing two marbles from a bag of marbles without replacement. Explain that the focus of the two-stage experiments will be on independent events and each of the outcomes in the experiment will be equally likely.

Present students with the following problem:

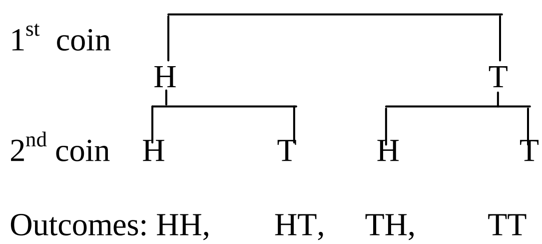
> You conduct an experiment of tossing two fair coins.
> About how many times in an experiment with 64 trials would you expect to get two heads? Explain your thinking.

Have students predict the answer to the problem by using theoretical probability with a sample space developed by using a table or a tree diagram as shown below.

Table

2nd coin / 1st coin	H	T
H	HH	HT
T	TH	TT

Tree Diagram



Theoretical probability: $P(TT) = \dfrac{\text{number of ways the event (TT) occurs}}{\text{total number of outcomes}}$

$$= \frac{1}{4}$$

64/4 = 16 Therefore, you would expect to get two heads about 16 times in 64 trials.

Have students work in pairs to carry out the experiment with each group doing 10 or 20 trials. Collate the results to obtain 64 trials and then add more trials as needed to show that experimental probability approaches theoretical probability as the number of trials increases.

Suggest that students tally the results of their experiment in a chart as shown below.

Outcomes	Tallies	Number
HH		
HT		
TH		
TT		
	Total	

Then have them calculate the experimental probability of getting two heads when two coins are tossed.

$$\text{Experimental probability} = \frac{\text{number of observed occurrences of two heads}}{\text{total number of trials}}$$

Have students compare the experimental probability to the theoretical probability, using ratios, fractions, decimals and/or percents.

Extend the problem as follows:

> You conduct an experiment of tossing three fair coins.
> About how many times in an experiment with 64 trials would you expect to get two heads? Explain your thinking.

Have students use a similar procedure as was used in the previous problem, predicting the answer first by using theoretical probability and then conducting the experiment with a collated total of at least 64 trials. Encourage students to compare, always, the theoretical and the experimental probabilities.

Provide other manipulatives for students to use in conducting experiments with two independent events, such as rolling two standard dice, spinning a spinner twice in which each of the parts of the spinner are congruent, tossing one coin and one standard die, and spinning a spinner with two congruent parts and tossing one coin.

2. Independent Events with Unequally Likely Outcomes

   Build on students' understanding of one-stage experiments in which the outcomes were unequally likely, such as rolling a six-sided die that has on each face one of the following numbers: 1, 2, 2, 3, 3, 3. Compare the probability of rolling a 2 with the probability of rolling a 3. Have students suggest other experiments in which the outcomes would be unequally likely, such as spinning a spinner with parts that are not congruent.

Look For …

Do students:

- ☐ explain the similarities and differences between equally likely and unequally likely outcomes in probability experiments?
- ☐ apply their knowledge of equally likely outcomes for two independent events to unequally likely outcomes?

Present students with the following problem:

> You conduct an experiment of spinning the spinner at the right twice and finding the sum of the numbers from the two spins. Predict which sum will appear most often. Explain your thinking.

Have students explain their predictions based on previous knowledge but do not provide guidance in using theoretical probability until after the experiment is carried out. The results of the experiment will be a surprise to most students.

Have students work in pairs to carry out the experiment with each group doing 10 or 20 trials. Collate the results to obtain at least 100 trials.

Suggest that students tally the results of their experiment in a chart as shown below.

Events: Sums	Tallies	Number
2		
3		
4		
5		
6		
	Total	

Have students compare the experimental results to their prediction and explain why there may be differences.

Have students calculate the experimental probability of each event.

$$\text{Experimental probability} = \frac{\text{number of observed occurrences of the event}}{\text{total number of trials}}$$

After sharing ideas about why the experimental results show that 4 is the sum that appears most often, you may wish to share the following strategy if it was not already suggested by students.

Since the outcomes of obtaining a 1, 2 or 3 on the spinner are unequally likely with the likelihood of 3 occurring twice as often as either 1 or 2, then the outcomes can be presented in a chart as follows:

1st Spin \ 2nd Spin	1	2	3	3
1	2	3	4	4
2	3	4	5	5
3	4	5	6	6
3	4	5	6	6

This area model shows that outcomes containing 3 as one of the addends has twice as much as the area for outcomes containing 1 as an addend and also twice as much as the area for the outcomes containing 2 as an addend.

$$\text{Theoretical probability: P(sum of 4)} = \frac{\text{number of ways the event (sum of 4) occurs}}{\text{total number of outcomes}}$$

$$= \frac{5}{16}$$

Have students calculate the theoretical and experimental probabilities of all the possible sums and compare the results.

Have students predict the results and then carry out other two-stage experiments with independent events in which the outcomes for each stage of the experiment are unequally likely. Examples might include tossing two six-sided dice that have on each face one of the following numbers: 1, 2, 2, 3, 3, 3. Have students predict which sum will appear most often when the two dice are tossed and the numbers are added. Then have students conduct the experiment and compare the experimental results to their predictions that should be based on some logical reasoning using theoretical probability.

3. Using Technology to Provide Trials for Probability Experiments with Two Independent Events

   The Maths Online Web site—http://www.mathsonline.co.uk/nonmembers/resource/prob/spinners.html—quickly tabulates the experimental results for two independent events, such as tossing coins or spinning spinners. The student enters the number of coins or the type of spinners and also the number of trials. The computer does the experiment and graphs the results. For example, tossing three coins provides a bar graph drawn to represent the experimental results including no heads, one head, two heads and three heads.

4. Critiquing the Fairness of Games

   Provide students with various games for two players in which two dice are rolled and rules are given that relate the two numbers rolled. Have students predict whether the games are fair. Encourage students to use theoretical probability in justifying their predictions. Have students play the games with at least 30 trials to compare the theoretical probability with the experimental probability.

Look For …

Do students:

- ☐ use the large number of trials generated by the computer to verify that the experimental probability approaches the theoretical probability as the number of trials increases?
- ☐ explore various probability experiments by predicting the results and comparing their predictions to the experimental results generated by the computer?

Examples:
For each of the following games, have students answer these questions:

- Did the same player win each time?
- Do both players have the same chance of winning? Explain.

Look For …
Do students:
- ☐ share strategies for determining whether or not a probability game is fair?
- ☐ apply their knowledge of creating sample spaces using tree diagrams and tables in determining the theoretical probability of winning a game?
- ☐ create other probability games and critique them for fairness?

Game A: Fractions Between Zero and One
Roll two standard dice and make a fraction less than or equal to 1 with the numbers showing on the dice. Player A scores one point if the fraction is in simplest form, otherwise Player B scores one point.

Game B: Fractions
Roll two standard dice, one red indicating the numerator and one blue indicating the denominator. Player A scores one point if the fraction is greater than 1. Player B scores one point if the fraction is less than 1. Both players score one point if the fraction is equal to 1.

Adapted from W. George Cathcart, Yvonne M. Pothier and James H. Vance, *Learning Mathematics in Elementary and Middle Schools* (2nd ed.) (Scarborough, ON: Prentice-Hall Canada, 1997), p. 259. Adapted with permission from Pearson Education Canada.

Have students share their strategies for determining the fairness of each game. An example of a strategy would be to describe the sample space for the probability experiment in a chart as shown below.

Blue / Red	1	2	3	4	5	6
1	**(1, 1)**	**(1, 2)**	**(1, 3)**	**(1, 4)**	**(1, 5)**	**(1, 6)**
2	***(2, 1)**	(2, 2)	**(2, 3)**	(2, 4)	**(2, 5)**	(2, 6)
3	***(3, 1)**	***(3, 2)**	(3, 3)	**(3, 4)**	**(3, 5)**	(3, 6)
4	***(4, 1)**	*(4, 2)	***(4, 3)**	(4, 4)	**(4, 5)**	(4, 6)
5	***(5, 1)**	***(5, 2)**	***(5, 3)**	***(5, 4)**	(5, 5)	**(5, 6)**
6	***(6, 1)**	*(6, 2)	*(6, 3)	*(6, 4)	***(6, 5)**	(6, 6)

Game A: The ordered pairs in bold show the outcomes for which Player A would win. Therefore, the probability of Player A winning is 23/36. Since 23/36 is greater than 1/2, then Player A will win more often and the game is not fair.

Game B: The starred ordered pairs show the outcomes for which Player A would win. Player A would also score for the 6 outcomes in the diagonal that produce a fraction equal to 1. Therefore, the probability of Player A winning is (15 + 6)/36 or 21/36. The probability of Player B winning is also (15 + 6)/36. Since the probability of winning is the same for each player, the game is fair.

Encouraging students to create other games and explain the fairness of each game.

5. Frayer Model for Independent Events

Provide students with a template for the Frayer Model and have them fill in the sections, individually or as a group, to consolidate their understanding of independent events. Other Frayer Models could be drawn for theoretical probability or experimental probability. A sample of a Frayer Model is provided below.

Look For …

Do students:
- ☐ write the definition and characteristics of independent events in their own words?
- ☐ create a real world problem, using two independent events?
- ☐ provide examples and nonexamples of independent events?

Frayer Model for Independent Events

Definition

Two events are **independent** if the fact that one event occurs does not affect the probability of the second event occurring.

Characteristics

- in experiments with two independent events, the experimental probability approaches the theoretical probability for a given event as the number of trials are increased
- the outcomes of each independent event may be equally likely or unequally likely
- the sample space (set of all possible outcomes) can be created by using a table or a tree diagram
- the sample space consists of ordered pairs of outcomes—the first member represents the first event and the second member represents the second event

Real-life Problem and Visual Representation

Find the probability of obtaining a sum of 7 when two fair standard dice are tossed.

2nd die (columns); 1st die (rows)

+	**1**	**2**	**3**	**4**	**5**	**6**
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$$\text{Probability} = \frac{\text{number of ways the event (sum of 7) occurs}}{\text{total number of outcomes}} = \frac{6}{36} = \frac{1}{6}$$

The probability of obtaining a sum of 7 is 1/6.

Independent Events

Examples

- tossing two or more coins
- tossing two or more dice
- tossing one coin and one die
- spinning two or more spinners
- spinning a three-section spinner and a four-sided die
- tossing a coin and spinning a four-section spinner

Nonexamples

- drawing two cards from a deck of cards without replacement
- drawing two marbles from a bag of marbles without replacement
- choosing two friends to go skating
- picking two pencils from a box of pencils without replacement

Format adapted from D. A. Frayer, W. C. Frederick and H. J. Klausmeier, *A Schema for Testing the Level of Concept Mastery* (Working Paper No. 16/Technical Report No. 16) (Madison, WI: Research and Development Center for Cognitive Learning, University of Wisconsin, 1969). Adapted with permission from the Wisconsin Center for Education Research, University of Wisconsin-Madison.

Step 4: Assess Student Learning

Guiding Questions

- Look back at what you determined as acceptable evidence in Step 2.
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

In addition to ongoing assessment throughout the lessons, consider the following sample activities to evaluate students' learning at key milestones. Suggestions are given for assessing all students as a class or in groups, individual students in need of further evaluation and individual or groups of students in a variety of contexts.

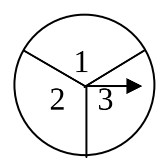
A. Whole Class/Group Assessment

Note: Performance-based assessment tasks are under development.

Provide students with calculators, compasses and protractors.

1. A probability experiment consists of tossing two four-sided fair dice (tetrahedra). Use this information to answer the questions below.
   a) Does this experiment describe two independent events? Explain.
   b) Draw a tree diagram or create a table to show all the possible outcomes for this experiment.
   c) Find the theoretical probability of obtaining a sum of 5 on the two dice in this experiment. Show all your work.
   d) Describe how you could conduct this experiment by using two spinners instead of two four-sided dice.

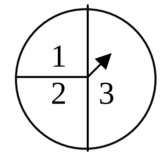
2. A probability experiment consists of tossing a fair coin and spinning the spinner at the right. The outcomes of the probability experiment are shown in the tally chart below.



Outcomes	Tallies
H1	~~////~~ ~~////~~ ~~////~~ //
H2	~~////~~ ~~////~~ ////
H3	~~////~~ ~~////~~ ~~////~~ ////
T1	~~////~~ ~~////~~ ~~////~~
T2	~~////~~ ~~////~~ ~~////~~ ///
T3	~~////~~ ~~////~~ ~~////~~ //

   a) How many trials were in the experiment? Explain.
   b) What is the experimental probability of tossing a head and spinning an odd number? Explain.
   c) What is the theoretical probability of tossing a head and spinning an odd number? Explain.

d) Compare the answers in parts b and c. Explain any discrepancy. What would be the theoretical probability of tossing a head and spinning an odd number if the spinner showed unequally likely outcomes as illustrated to the right? Show all your work.



B. One-on-One Assessment

Assessment activities can be used with individual students, especially students who may be having difficulty with the outcome.

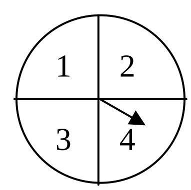
Provide the student with a calculator, a compass and a protractor.

1. Present the following problem to the student.
   A probability experiment consists of tossing two six-sided fair dice. Use this information to answer the questions below.
   a) Does this experiment describe two independent events? Explain.
   b) Draw a tree diagram or create a table to show all the possible outcomes for this experiment.
   c) Find the theoretical probability of obtaining a sum of 5 on the two dice in this experiment. Show all your work.
   d) Describe how you could conduct this experiment by using two spinners instead of two six-sided dice.

   Terminology might be a problem so prompt the student by giving examples of independent events, tree diagrams and theoretical probability.

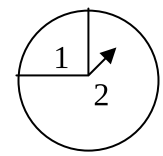
   Provide the student with two six-sided fair dice so that the experiment can be carried out and the various outcomes generated. If the student has difficulty drawing the spinner for part d, remind the student that a circle has 360^o so each sector of the spinner must be $360^o/6$ or 60^o.

2. Present the following problem to the student.
   A probability experiment consists of tossing a fair coin and spinning the spinner at the right. The outcomes of the probability experiment are shown in the tally chart below.



Outcomes	Tallies
H1	~~////~~ ~~////~~ ///
H2	~~////~~ ~~////~~ ///
H3	~~////~~ ~~////~~ ~~////~~
H4	~~////~~ ~~////~~ /
T1	~~////~~ ~~////~~ /
T2	~~////~~ ~~////~~ ~~////~~
T3	~~////~~ ~~////~~
T4	~~////~~ ~~////~~ //

a) How many trials were in the experiment? Explain.
b) What is the experimental probability of tossing a head and spinning an odd number? Explain.
c) What is the theoretical probability of tossing a head and spinning an odd number? Explain.
d) Compare the answers in parts b and c. Explain any discrepancy.
e) What would be the theoretical probability of tossing a head and spinning an even number if the spinner showed unequally likely outcomes as illustrated to the right? Show all your work.



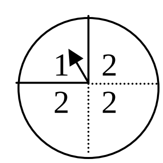
If the student has difficulty with part a, review the meaning of a trial and count the tallies for one outcome. Then encourage the student to count the rest of the tallies to determine the total number of trials in the experiment.

If the student has difficulty with part b, review the meaning of experimental probability and provide the formula if necessary. Also, provide one example of an outcome that would be part of the event described so that the student will then be able to find other outcomes for that event.

If the student has difficulty with part c, review the meaning of theoretical probability and provide the formula if necessary. Also, remind the student that he or she should draw a tree diagram or create a chart to display the sample space (all the possible outcomes) for the experiment. Start the tree diagram or the chart for the student if necessary.

If the student has difficulty with part d, remind the student that a very large number of trials is necessary in conducting an experiment before the experimental and theoretical probabilities approach the same number.

If the student has difficulty with part e, mark the sectors on the spinners to show that spinning a 2 is three times as likely as spinning a 1. Prompt the student to create a table to display the unequally likely outcomes. If necessary, help the student to begin the table:



Spinner / Coin	1	2	2	2
H				
T				

C. Applied Learning

Provide opportunities for students to use their understanding of probability in a practical situation and notice whether or not the understanding transfers. For example, have students analyze a probability game for the mathematics fair and decide which option is best to choose because it has the greatest probability of winning.

Example of a probability game:

> Place a marker on one of the following numbers: 0, 1, 2, 3, 4, 5. Roll two dice. Subtract the smaller number from the larger number. If your marker is on the number that shows this difference, you are a winner.

Does the student:

- use logical reasoning in determining the sample space for the game?
- use theoretical probability in determining the likelihood each number has of winning?
- compare the theoretical probabilities of all the possible outcomes in explaining which number would be best to choose to have the greatest likelihood of winning?
- create a similar probability game, e.g., adding the two numbers shown on four-sided fair dice, and explain the sum that would have the greatest likelihood of winning?

Step 5: Follow-up on Assessment

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction?

A. Addressing Gaps in Learning

- Review single-stage probability experiments with equally likely and unequally likely outcomes before beginning the two-stage probability experiments with two independent events.
- Provide hands-on experiences by having students conduct the probability experiments to verify predictions made using theoretical probability.
- Have students develop flexibility in using tree diagrams or tables as strategies for displaying the sample space for a given probability experiment.
- Provide time for students to consolidate their thinking by creating graphic organizers, such as Frayer models or Venn diagrams, for terms such as probability, theoretical probability, experimental probability, sample space, independent events, equally likely outcomes, events, outcomes and tree diagrams.
- Encourage students to use calculators in comparing theoretical and experimental probabilities.

B. Reinforcing and Extending Learning

Students who have achieved or exceeded the outcomes will benefit from ongoing opportunities to apply and extend their learning. These activities should support students in developing a deeper understanding of the concept and should not progress to the outcomes in subsequent grades.

Consider strategies, such as the following.

- Provide tips for parents on working with probability at home or in the community.
  - Roll two dice. If the sum of the two numbers is 5, your child gets a double treat; e.g., allowance or free-reading time. If the sum of the two numbers is 10, he or she gets half the intended treat. Have the child decide if this would be fair and explain why or why not.
  - Play games with two dice, two spinners with numbers or one die and one spinner. Have the child create rules to determine a winner; e.g., add the two numbers and the winner has an even sum. Encourage the child to explain why these rules would be fair or unfair.
  - When playing games, such as *Sorry!* or *Frustration!*, have the child explain the likelihood of getting two 1s in a row.
  - Encourage the child to create probability games and explain whether or not they are fair.

- At the Carnival
  At a carnival, you will receive a prize if you toss two dice and the sum is a prime number. Which of the following options provides the greatest likelihood of winning a prize? Explain your thinking.
  - tossing two six-sided dice
  - tossing one six-sided die and one four-sided die

- Most Likely Sum
  Find the most likely sum when you toss two four-sided dice with numbers on each face as follows.
  - Die A: 2, 4, 6, 8
  - Die B: 1, 3, 5, 7

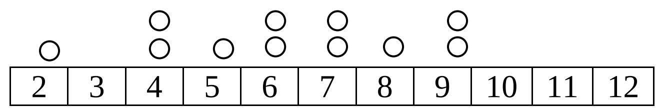
  Explain your thinking.

- Unequally Likely Sectors of a Spinner
  The sectors of a spinner are divided as follows:
  - Sector A is 60^o
  - Sector B is 120^o
  - Sector C is 180^o

  a) What is the probability of the spinner stopping on Sector A after one spin? Explain.
  b) What is the probability of the spinner stopping on Sector B on two consecutive spins? Use a tree diagram or a table to explain.
  c) If Sector A has a value of 1, Sector B has a value of 2 and Sector C has a value of 3, what is the most likely sum of the two numbers on two consecutive spins of the spinner? Explain.

- Two-dice Sum Game
  This game is played with partners. Each player has 11 counters and a pair of dice. Each player draws a number line from 2 to 12 and places his or her counters on the number line in any arrangement. There may be more than one counter placed on some numbers and no counters on other numbers. Players take turns rolling the dice. On each roll, each player removes one counter that is on the number that matches the sum on the dice. The winner is the first player to remove all 11 counters. The players should decide on the best winning arrangement of counters on the number line and explain their thinking.

  Example:



- Two-coloured Counters
  Place 20 two-coloured counters (red and white) in a bag. Predict how many of each colour will appear if the bag is shaken and the counters are emptied onto a flat surface. Explain your thinking. Empty the counters to check your prediction. Try it again with more counters. Will your prediction change? Why or why not?

- Three-dice Problem
  Lucky Lucy has three dice:
  The blue die has 2, 2, 4, 4, 9, 9 on its faces.
  The red die has 3, 3, 5, 5, 7, 7 on its faces.
  The green die has 1, 1, 6, 6, 8, 8 on its faces.

  Lucky Lucy is confident that if her opponent chooses one die first, she can select a die that would give her a better chance of beating her opponent. Explain which die Lucky Lucy would choose for each of the following:
  a) opponent chooses the blue die
  b) opponent chooses the red die
  c) opponent chooses the green die.
  Explain your thinking.

- Is This Game Fair?
  Roll two standard fair dice. If the sum of the two numbers is a prime number, you score a point. If the sum of the two numbers is not a prime number, your partner scores a point. Is this a fair game? Explain. Play the game with at least 30 trials to test your prediction.

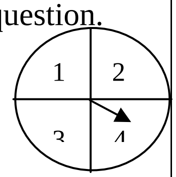
- Create a Game
  Design games using coins, dice or spinners with two independent events in which the likelihood of you winning is:
  a) 50%
  b) 75%.

- Sample Structured Interview: Assessing Prior Knowledge and Skills

Directions	Date:	
	Not Quite There	**Ready to Apply**
Present the following probability experiment to the student. Have the student respond to each question. "A probability experiment consists of rolling a 12-sided die. One face has the number 1, two faces have the number 2, three faces have the number 3, four faces have the number 4 and two faces have the number 5. Use this information to answer the questions below. a) List all the possible outcomes for this probability experiment. b) Are the outcomes equally likely or unequally likely? Explain your thinking. c) Predict the probability of rolling a number 4. Explain your thinking. d) Describe how you would carry out an experiment to determine the experimental probability of rolling a number 4. e) Will the theoretical and experimental probability for rolling a number 4 be the same? Explain why or why not."	a) Lists 12 numbers or neglects to include at least one of the numbers. b) Responds incorrectly or responds that the outcomes are unequally likely but is unable to explain why. c) Predicts the probability by using a random guess rather than using theoretical probability. d) Provides a partial description of a probability experiment, omitting critical information. e) Says that the experimental and theoretical probabilities are not the same but is unable to explain why.	a) Lists the four possible outcomes: 1, 2, 3, 4 and 5. b) Responds that the outcomes are unequally likely because there are a different number of faces on the die for each number. c) Predicts the probability by using theoretical probability, obtaining the answer 4/12 or 1/3. d) Provides a detailed description of how to carry out the probability experiment. e) Says that the experimental and theoretical probabilities are not the same because the number of trials in an experiment is limited and therefore provides a probability that is close to but not exactly the same as the theoretical probability.

Present the following probability experiment to the student. Have the student respond to each question.

"A probability experiment consists of spinning the spinner shown at the right once. The outcomes of the probability experiment are shown in the tally chart below.



Outcomes	Tallies
1	~~////~~ ~~////~~ //// //
2	~~////~~ ~~////~~ ////
3	~~////~~ ~~////~~ //// ////
4	~~////~~ ~~////~~

a) How many trials were completed in this experiment? How do you know?
b) What is the experimental probability of spinning a 4? Explain your thinking.
c) What is the theoretical probability of spinning a 4? Explain your thinking.
d) If there is a difference between your answers in parts b and c, what could be done to bring the experimental probability closer to the theoretical probability?
e) Are the outcomes in this experiment equally likely or unequally likely? Explain your thinking."

a) Guesses the number of trials and may not understand what a trial is.
b) Randomly chooses an experimental probability, misinterprets the tally chart in producing an answer or provides the correct experimental probability but is unable to explain his or her answer.
c) Does not provide the theoretical probability or provides the correct answer but is unable to explain why.
d) Says that there is a difference between the two probabilities but is unable to explain how to bring the experimental probability closer to the theoretical probability.
e) Does not understand the question or says the outcomes are equally likely but is unable to explain why.

a) Counts the tallies and responds that the number of trials for the experiment is 60.
b) Refers to the tally chart, counts the tallies for spinning a 4 and writes the probability as a ratio: 4/60 or 1/15.
c) Refers to the spinner, stating that each outcome is equally likely because the sectors of the circle are congruent and therefore the theoretical probability of spinning a 4 is the ratio of the favourable outcome (1) to the total number of outcomes (4), or 1/4.
d) Explains that there is a difference between the answers in parts b and c. Says that the experimental probability will approach the theoretical probability as the number of trials increases.
e) Reiterates the explanation provided in part c, saying that each outcome is equally likely because the sectors of the circle are congruent—have the same size and shape.

Present the following problem to the student. "Is it more likely to draw a red counter from Bag A, containing 3 red counters and 2 blue counters, or from Bag B, containing 4 red counters and 6 blue counters? Explain your thinking."	• Chooses the correct bag randomly and is not able to explain the process.	• Chooses Bag A and explains that the probability of drawing a red counter from Bag A is 3/5 whereas the probability of drawing a red counter from Bag B is 4/10 or 2/5.

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Strand: Statistics and Probability (Chance and Uncertainty)
Outcome: 6

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