Mathematics

Planning Guide

Grade 8 *Fractions*

Number

Specific Outcome 6

This Planning Guide can be accessed online at:
http://www.learnalberta.ca/content/mepg8/html/pg8_fractions/index.html

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Planning Guide: *Grade 8 Fractions*

Strand: Number
Specific Outcome: 6

This *Planning Guide* addresses the following outcomes from the Program of Studies:

Strand: Number	
Specific Outcome:	6. Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially and symbolically.

Curriculum Focus

The changes to the curriculum targeted by this sample include:

- The general outcome focuses on developing number sense; whereas the previous mathematics curriculum focused on applying arithmetic operations on rational numbers to solve problems.
- The specific outcome focuses on understanding multiplication and division of positive fractions and mixed numbers concretely, pictorially and symbolically; whereas the previous mathematics curriculum included addition, subtraction, multiplication and division of fractions, concretely, pictorially and symbolically.

What Is a Planning Guide?

Planning Guides are a tool for teachers to use in designing instruction and assessment that focuses on developing and deepening students' understanding of mathematical concepts. This tool is based on the process outlined in *Understanding by Design* by Grant Wiggins and Jay McTighe.

Planning Steps

The following steps will help you through the Planning Guide:

- **Step 1: Identify Outcomes to Address** (p. 3)
- **Step 2: Determine Evidence of Student Learning** (p. 4)
- **Step 3: Plan for Instruction** (p. 5)
- **Step 4: Assess Student Learning** (p. 38)
- **Step 5: Follow-up on Assessment** (p. 45)

Step 1: Identify Outcomes to Address

Guiding Questions

- What do I want my students to learn?
- What can my students currently understand and do?
- What do I want my students to understand and be able to do based on the Big Ideas and specific outcomes in the program of studies?

Big Ideas

- Multiplication and division of fractions is similar to multiplication and division of whole numbers.
- Estimation of multiplication and division helps fraction computations make sense.
- For multiplication by a fraction, it is useful to recall that the denominator is a divisor (Van de Walle and Lovin 2006, p. 66).
- For division by a fraction, the two ways of thinking about the operation—partitive model (equal-sharing) and measurement model (equal grouping)—are very important (Van de Walle and Lovin 2006, p. 66).

Sequence of Outcomes from the Program of Studies

See http://education.alberta.ca/teachers/core/math/programs.aspx for the complete program of studies.

Grade 7	Grade 8	Grade 9
Specific Outcomes	**Specific Outcomes**	**Specific Outcomes**
5. Demonstrate an understanding of adding and subtracting positive fractions and mixed numbers, with like and unlike denominators, concretely, pictorially and symbolically (limited to positive sums and differences).	6. Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially and symbolically.	There are no directly related specific outcomes in Grade 9.

Step 2: Determine Evidence of Student Learning

Guiding Questions

- What evidence will I look for to know that learning has occurred?
- What should students demonstrate to show their understanding of the mathematical concepts, skills and Big Ideas?

Using Achievement Indicators

As you begin planning lessons and learning activities, keep in mind ongoing ways to monitor and assess student learning. One starting point for this planning is to consider the achievement indicators listed in the *Mathematics Kindergarten to Grade 9 Program of Studies with Achievement Indicators*. You may also generate your own indicators and use them to guide your observation of the students.

The following indicators may be used to determine whether or not students have met this specific outcome. Can students:

- identify the operation required to solve a given problem involving positive fractions?
- provide a context that requires the multiplying of two given positive fractions?
- provide a context that requires the dividing of two given positive fractions?
- estimate the product of two given positive proper fractions to determine if the product will be closer to 0, $\frac{1}{2}$ or 1?
- estimate the quotient of two given positive fractions and compare the estimate to whole number benchmarks?
- express a given positive mixed number as an improper fraction and a given positive improper fraction as a mixed number?
- model multiplication of a positive fraction by a whole number concretely or pictorially and record the process?
- model multiplication of a positive fraction by a positive fraction concretely or pictorially using an area model, and record the process?
- model division of a positive proper fraction by a whole number concretely or pictorially and record the process?
- model division of a whole number by a positive proper fraction concretely or pictorially using an area model, and record the process?
- model division of a positive proper fraction by a positive proper fraction pictorially and record the process?
- generalize and apply rules for multiplying and dividing positive fractions, including mixed numbers?
- solve a given problem involving positive fractions, taking into consideration order of operations (limited to problems with positive solutions)?
- apply a personal strategy to solve, symbolically, a given division problem involving improper fractions?
- refine personal strategies to increase their efficiency?

Sample behaviours to look for related to these indicators are suggested for some of the activities listed in **Step 3, Section C: Choosing Learning Activities** (p. 9).

Step 3: Plan for Instruction

Guiding Questions

- What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should I use?
- How will I meet the diverse learning needs of my students?

A. Assessing Prior Knowledge and Skills

Before introducing new material, consider ways to assess and build on students' knowledge and skills related to fractions. For example:

Activity 1: Give the students the following problem: $\frac{1}{2} + \frac{1}{3}$. Ask them to show you what the problem means with any concrete material, or a drawing, or by relating it to some real-life situation.

Adapted from Burns, Marilyn. *About Teaching Mathematics: A K–8 Resource, Second Edition*, p. 224. Copyright © 2000 by Math Solutions Publications. Adapted by permission. All rights reserved. (Note: This title is now in its third edition, copyright © 2007.)

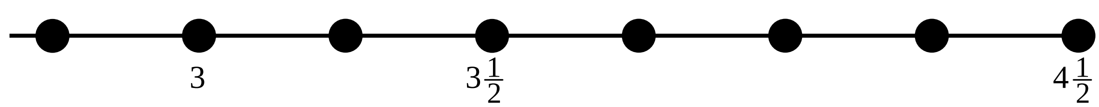
Activity 2:

Which of the following sums is more than 1? Explain how you know.

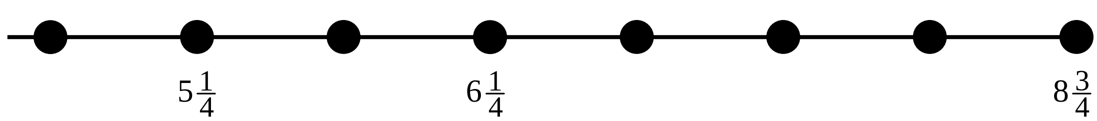
$\frac{7}{15} + \frac{5}{12}$ $\frac{1}{2} + \frac{4}{9}$ $\frac{5}{9} + \frac{8}{15}$

Activity 3: Have the students find the patterns in the following number lines.

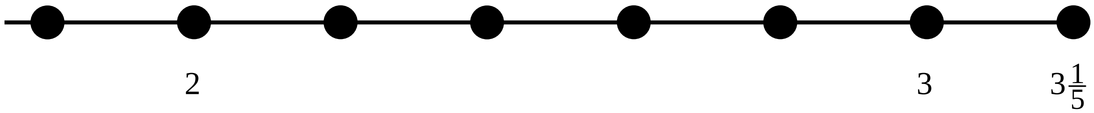
a.



b.



c.



This activity reproduced with permission from Grayson H. Wheatley and George E. Abshire, *Developing Mathematical Fluency: Activities for Grades 5–8* (Tallahassee, FL: Mathematics Learning, 2002), p. 113. www.mathematicslearning.org

Activity 4: Add the following. Express the answer in lowest terms.

$$3\frac{5}{6} + 1\frac{1}{4} =$$

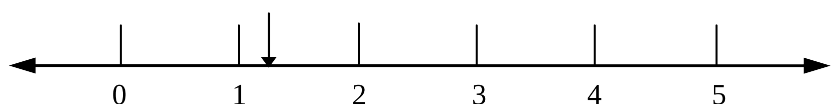
Activity 5: Subtract the following. Express the answer in lowest terms.

$$4\frac{2}{5} - 3\frac{1}{2} =$$

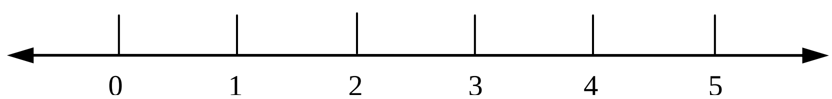
Activity 6: Have the students estimate the answers and indicate them with an arrow on the number line, as illustrated in the example.

Example:

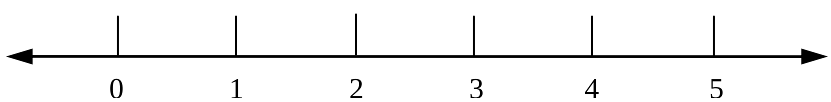
$\frac{1}{2} + \frac{3}{4}$



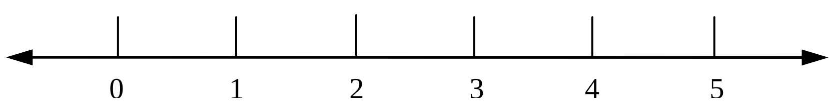
$\frac{7}{8} + \frac{2}{5}$



$1\frac{5}{9} + \frac{2}{3}$



$2\frac{3}{5} + 1\frac{1}{2}$



This activity adapted from *Developing Number Sense in the Middle Grades* (p. 33) by Barbara J. Reys, copyright 1992 by the National Council of Teachers of Mathematics. Source: *Mathematics Resource Project: Number Sense and Arithmetic Skills* by Alan R. Hoffer (ed.) (Palo Alto, CA: Creative Publications, 1978).

If a student appears to have difficulty with these tasks, consider further individual assessment, such as a structured interview, to determine the student's level of skill and understanding. See **Sample Structured Interview: Assessing Prior Knowledge and Skills** (p. 7).

Sample Structured Interview: Assessing Prior Knowledge and Skills

Directions	Date:	
	Not Quite There	**Ready to Apply**
1. Give the student the following problem: $\frac{1}{2}+\frac{1}{3}$. Ask them to show you what the problem means with any concrete material, a drawing or by relating it to some real-life situation. Adapted from Burns, Marilyn. *About Teaching Mathematics: A K–8 Resource, Second Edition*, p. 224. Copyright © 2000 by Math Solutions Publications. Adapted by permission. All rights reserved. (Note: This title is now in its third edition, copyright © 2007.)	1. Student responds with an answer of $\frac{2}{5}$ or student responds with an answer of $\frac{5}{6}$ but is unable to explain what it means using concrete materials, a drawing or relating it to some real-life situation.	1. Student responds with an answer of $\frac{5}{6}$ and is able to explain the meaning using concrete materials, a drawing or some real-life situation.
2. Which of the following sums is more than one? Explain how you know. a. $\frac{7}{15}+\frac{5}{12}$ b. $\frac{1}{2}+\frac{4}{9}$ c. $\frac{5}{9}+\frac{8}{15}$	2. Student calculates the answers for each, and is unable to determine which one is greater than one without doing so.	2. Student responds with the answer that the sum $\frac{5}{9}+\frac{8}{15}$ is more than one because both of fractions are more than $\frac{1}{2}$.

3. Have the students find the patterns in the following number lines. a. _, 3, _, $3\frac{1}{2}$, _, _, _, $4\frac{1}{2}$ b. _, $5\frac{1}{4}$, _, $6\frac{1}{4}$, _, _, _, $8\frac{1}{4}$ c. _, 2, _, _, _, _, 3, $3\frac{1}{5}$	3. Student is unable to complete the patterns.	3. Student responds with answers: a. $2\frac{3}{4}$, $3\frac{1}{4}$, $3\frac{3}{4}$, 4, $4\frac{1}{4}$, indicating a change of $\frac{1}{4}$. b. $4\frac{3}{4}$, $5\frac{3}{4}$, $6\frac{3}{4}$, $7\frac{1}{4}$, $7\frac{3}{4}$ indicating a change of $\frac{2}{4}$ or $\frac{1}{2}$. c. $1\frac{4}{5}$, $2\frac{1}{5}$, $2\frac{2}{5}$, $2\frac{3}{5}$, $2\frac{4}{5}$ indicating a change of $\frac{1}{5}$.
4. Add the following and leave the answer in lowest terms. $3\frac{5}{6} + 1\frac{1}{4} =$	4. Student responds with an answer of $4\frac{6}{10}$ or $4\frac{3}{5}$.	4. Student responds with an answer of $5\frac{1}{12}$.
5. Subtract the following and leave the answer in lowest terms. $4\frac{2}{5} - 3\frac{1}{2} =$	5. Student responds with an answer of $1\frac{2}{3}$.	5. Student responds with an answer of $\frac{9}{10}$.
6. Have the students estimate the answer to each by indicating the answer with an arrow on the number line, as illustrated in the example in Step 3, Part A, Activity 6.	6. Student is unable to respond or can respond using the whole numbers only and is unable to work with the fractions.	6. Answers may vary as students describe their thinking. One student might explain his or her thinking for the last problem in this way: "First, I'd find $2\frac{3}{5}$ and then I'd move 1 whole unit to the right of it, and then I'd move another $\frac{1}{2}$ unit right, ending here." (Which would be a little more than 4.)

B. Choosing Instructional Strategies

Consider the following general strategies for teaching multiplication and division of fractions:

- Connect the meaning of fraction computation with whole number computation.
- Let estimation and informal methods play a large role in the development of strategies. Estimation keeps the focus on the meanings of the numbers and the operations, encourages reflective thinking and helps build number sense with fractions.
- Explore each of the operations using models (Van de Walle 2001, p. 229).
- Use graphic organizers, such as a concept definition map, a modified Frayer model or a generalization/principle diagram.
- Use classroom strategies, such as an anticipation/reaction guide and think–pair–share discussion.
- Use problem solving as the principal instructional strategy to facilitate the learning of these concepts.

Caution: Research indicates that the teaching of fractions by memorizing rules has significant dangers; the rules do not help students think in any way about the meanings of the operations or why they work and the mastery observed in the short term is often quickly lost (Van de Walle 2001, p. 228).

C. Choosing Learning Activities

The following learning activities are examples of activities that could be used to develop student understanding of the concepts identified in Step 1.

Sample Activities:

1. **Anticipation/Reaction Guide** (p. 10)
2. **Multiplication of Fractions** (p. 11)
3. **Using Patterns to Understand the Multiplication of Fractions** (p. 21)
4. **Division of Fractions** (p. 22)
5. **Fraction Two-ways** (p. 36)

Sample Activity 1: Anticipation/Reaction Guide

- Using the think–pair–share strategy, have the students complete an anticipation guide such as the one provided.
- After the lesson has been completed, have the students revisit the anticipation/reaction guide to see if they are still in agreement with their original decisions.

Note: The Anticipation/Reaction Guide is a strategy consisting of a set of carefully selected statements that serve as a pre/post inventory for a topic to be learned or a reading selection. This strategy accesses prior knowledge, sets a purpose for learning the topic or reading the selection and requires justification of the truth or the mistruth of the statements.

When using the think–pair–share strategy:

- Allow the students to work independently to answer the questions (think).
- After a reasonable amount of time, ask the students to express their thoughts to another student (pair).
- And then, facilitate a class discussion (share).

Anticipation/Reaction Guide

Directions: In the column labelled *before*, place a T if you believe the statement to be true or place an F if you believe it is not true. After learning more about the topic, place either a T or F in the *after* column.

OPERATIONS WITH FRACTIONS

Before	Statements	After
	1. Fractions can represent part of a whole region or part of a whole set.	
	2. In a fraction, the numerator counts and the denominator shows what is counted.	
	3. In adding fractions, add the numerators to get the numerator of the sum and add the denominators to get the denominator of the sum.	
	4. The product of two fractions is always larger than either of the fractions being multiplied.	
	5. The quotient of a mixed number and a fraction is always greater than the mixed number.	

Sample Activity 2: Multiplication of Fractions

a. Multiplying a fraction and a whole number

This activity adapted from John A. Van de Walle, LouAnn H. Lovin, *Teaching Student-Centered Mathematics: Grades 5–8*, 1e (p. 94). Published by Allyn and Bacon, Boston, MA. Copyright © 2006 by Pearson Education. Reprinted by permission of the publisher. AND from John A. Van de Walle, *Elementary and Middle School Mathematics: Teaching Developmentally* (p. 233), 4/e. Published by Allyn and Bacon, Boston, MA. Copyright © 2001 by Pearson Education. Reprinted by permission of the publisher.

Have the students solve simple story problems using carefully chosen numbers (fractions with denominators less than 12). For example:

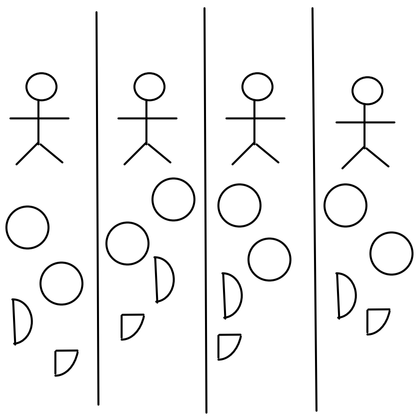
> There are 15 cars in Michael's toy car collection. Two thirds of the cars are red. How many red cars does Michael have?
>
> Suzanne has 11 cookies. She wants to share them with her three friends. How many cookies will Suzanne and each of her friends get?
>
> Wayne filled 5 glasses with $\frac{2}{3}$ of a litre of soda in each glass. How much soda did Wayne use?

Have the students explain their solutions.

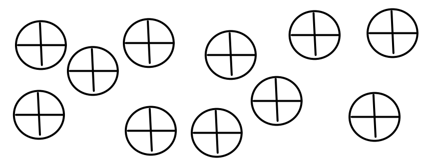
Instructional suggestions:

- Have the students make an estimate of each answer.
- Allow the students to solve the problems in their own way, using models and vocabulary of their own choosing.
- The first two problems involve finding the fractional part of a whole number.
  - In Michael's car problem, think of the fifteen cars as the whole and you are asked to find $\frac{2}{3}$ of the whole. First, find thirds by dividing 15 by 3. Multiplying by thirds, regardless of how many thirds, involves dividing by 3. The denominator is a divisor. Two of these groups of thirds is 10 red cars.

– Suzanne's cookie problem is a sharing problem. Dividing by 4 is the same as multiplication by one quarter. Solutions to the cookie problem could look like:



Pass out whole cookies.
Cut two cookies in half.
Cut last cookie into fourths.
Each girl gets $2\frac{3}{4}$ cookies.



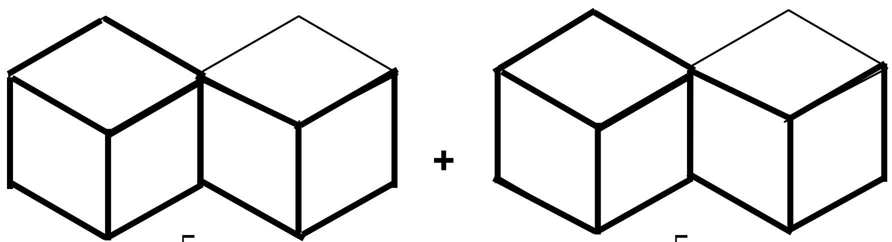
Cut all eleven cookies in fourths.
Give a fourth of each cookie to each girl.
Each girl will get $\frac{11}{4}$ or $2\frac{3}{4}$ cookies.

- Wayne's problem may be solved in different ways. Some students will put the thirds together as they go. Others will count all of the thirds and then find out how many whole litres are in ten thirds.

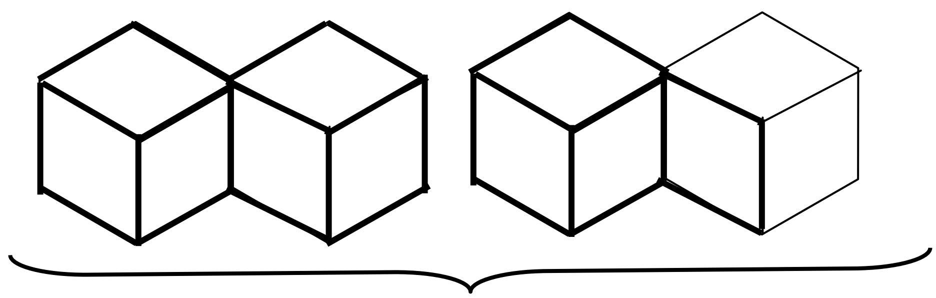
b. Multiplying a fraction and a whole number using fraction blocks

Lauren practises for $\frac{5}{6}$ of an hour each day for 2 days. What is the total time that she practised on these 2 days?

We use repeated addition to solve this multiplication problem.



$$\frac{5}{6} + \frac{5}{6}$$



$$\frac{10}{6} = \frac{5}{3} = 1\frac{2}{3}$$

Lauren practised $1\frac{2}{3}$ hours on the 2 days.

c. Multiplication of a positive fraction by a positive fraction using fraction strips

There is $\frac{4}{5}$ of a cake left. Mandy eats $\frac{3}{4}$ of this leftover cake.

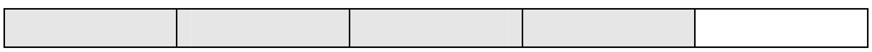
a. Estimate what fraction of the entire cake Mandy ate.
b. What fraction of the entire cake did Mandy eat?

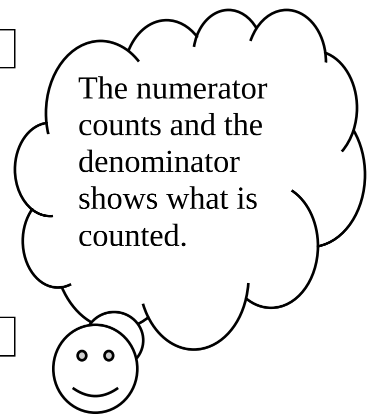
Include a diagram and a number sentence.

Answers:

a. If Mandy ate $\frac{1}{2}$ of the leftover cake, then she will have eaten $\frac{2}{5}$ of the entire cake. Since she ate $\frac{3}{4}$ of the leftover cake, then she has eaten more cake – probably a little more than $\frac{1}{2}$ of the entire cake.

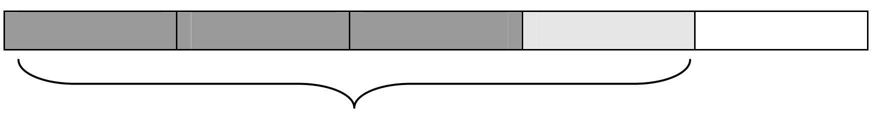
b. The shaded part in the diagram below shows $\frac{4}{5}$ of a cake left.





The $\frac{4}{5}$ is now the whole region and we will take $\frac{3}{4}$ of it.
The denominator tells us to divide the shaded region into 4 equal parts.

The numerator tells us to take 3 of these 4 equal parts.



The leftover cake is already divided into 4 equal parts.
The dark shading shows the 3 out of 4 equal parts, or $\frac{3}{4}$ of $\frac{4}{5}$, which is $\frac{3}{5}$ of the entire cake.

We can write the number sentence $\frac{3}{4} \times \frac{4}{5} = \frac{3 \times 4}{4 \times 5} = \frac{4 \times 3}{4 \times 5} = 1 \times \frac{3}{5} = \frac{3}{5}$

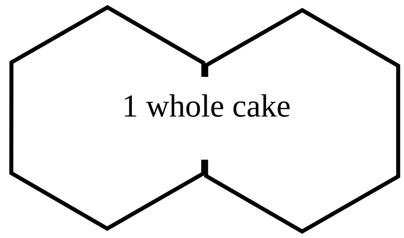
1

Another method: $\frac{3}{4} \times \frac{4}{5} \longrightarrow \frac{3}{4}$ of 4 is 3, $3 \times \frac{1}{5} = \frac{3}{5}$

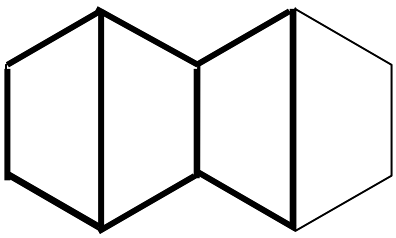
Mandy eats $\frac{3}{5}$ of the entire cake.

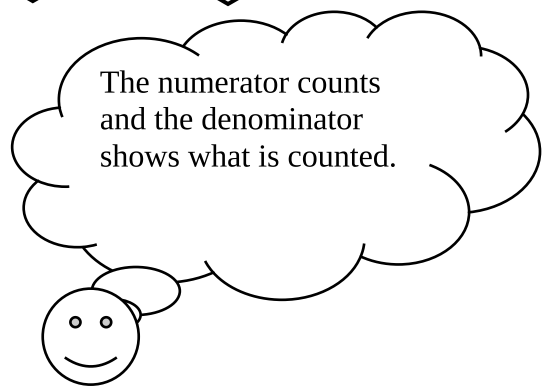
d. Multiplication of a positive fraction by a positive fraction using fraction blocks

There is $\frac{3}{4}$ of a cake left. Mark eats $\frac{2}{3}$ of this leftover cake. What fraction of the entire cake does Mark eat?



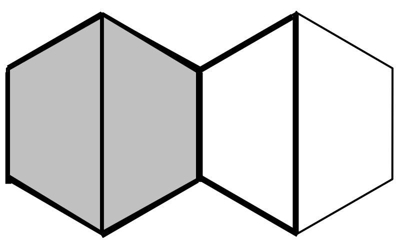
Answer:
The part drawn with dark lines shows $\frac{3}{4}$ of a cake.





We will take $\frac{2}{3}$ of the $\frac{3}{4}$.

The numerator tells us to take 2 of the 3 equal parts in $\frac{3}{4}$ of the cake.
The shaded parts show the 2 equal parts.



Number sentence: $\frac{2}{3}\times\frac{3}{4} = \frac{\cancelto{1}{2}\times\cancel{3}}{\cancelto{1}{3}\times\cancel{2}\times 2} = \frac{1}{2}$

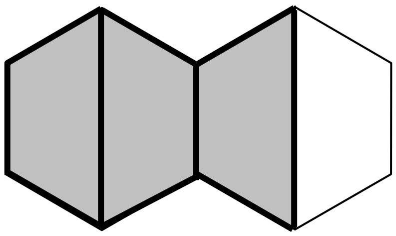
Mark eats $\frac{1}{2}$ of the entire cake.

Another example of multiplication of a positive fraction by a positive fraction using fraction blocks.

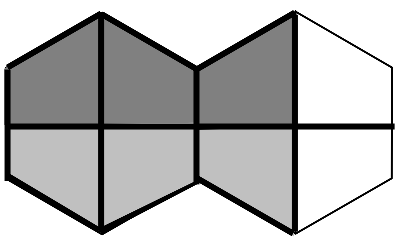
Carpet covers $\frac{3}{4}$ of the floor. $\frac{1}{2}$ of the carpeted area is covered with furniture. What fraction of the entire floor is covered by furniture?

Answer:

The part drawn with dark lines shows $\frac{3}{4}$ of a floor.



We draw a line segment across the diagram to show $\frac{1}{2}$.



The darker shaded parts show $\frac{1}{2}$ of $\frac{3}{4}$, which is $\frac{3}{8}$.

Number sentence: $\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$

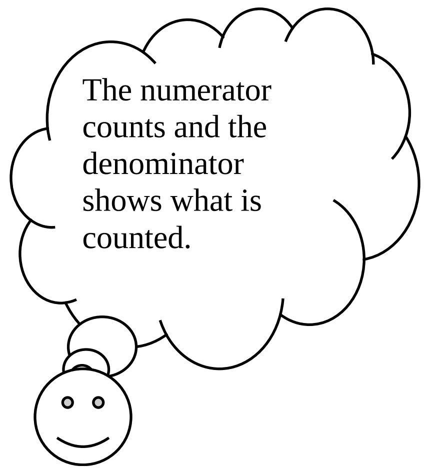
$\frac{3}{8}$ of the entire floor is covered by furniture.

e. Multiplication of a positive fraction by a postive fraction using arrays and factoring

Carpet covers $\frac{9}{10}$ of a floor. $\frac{2}{3}$ of the carpeted area is covered with furniture.

i. Use mental calculation to find the fraction of the carpet that is covered by furniture.

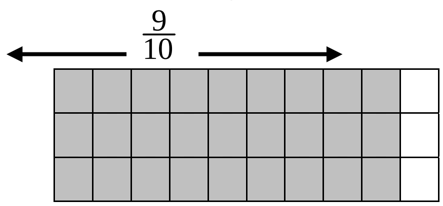
ii. What fraction of the entire floor is covered by furniture?
Include a diagram and a number sentence.



Answer:

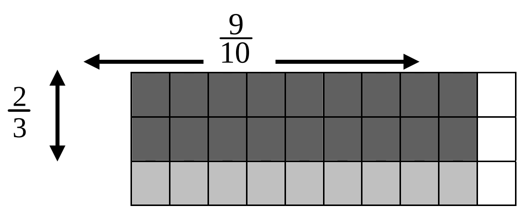
i. We can divide the $\frac{9}{10}$ into 3 equal parts and then count 2 of these parts. Each of the 3 equal parts are $\frac{3}{10}$, so 2 of them are $\frac{6}{10}$.

ii. We use arrays to multiply whole numbers and this strategy can also be used to multiply fractions.



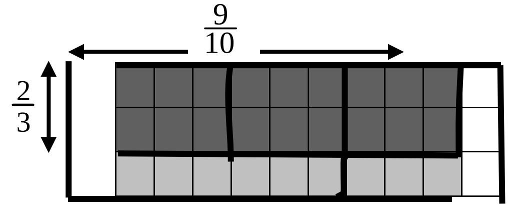
$\frac{9}{10}$ of the figure becomes the part we work with.

To show $\frac{2}{3}$ of $\frac{9}{10}$, focus on the fraction $\frac{2}{3}$. The denominator tells us to divide the region shown by $\frac{9}{10}$ into 3 equal parts. The numerator tells us to count 2 of these 3 equal parts.



$$\frac{2}{3} \times \frac{9}{10} = \frac{18}{30}$$

To simplify the fraction, we can divide the entire floor into groups of 6, because 6 is a common factor of 18 and 30. There are 5 groups of 6, as shown in the diagram below.



3 out of the 5 equal groups (or $\frac{3}{5}$ of the floor) have dark shading and represent $\frac{2}{3}$ of $\frac{9}{10}$.

Another way to express the product in its simplest form is to use factorization. $9 = 3 \times 3$ and $10 = 2 \times 5$.

Therefore,

$$\frac{2}{3} \times \frac{9}{10} = \frac{2 \times 3 \times 3}{3 \times 2 \times 5} = \frac{3 \times 2 \times 3}{3 \times 2 \times 5} = \frac{3}{5}$$

1 1

$\frac{3}{5}$ of the entire floor is covered by furniture.

f. Making sense of the algorithm

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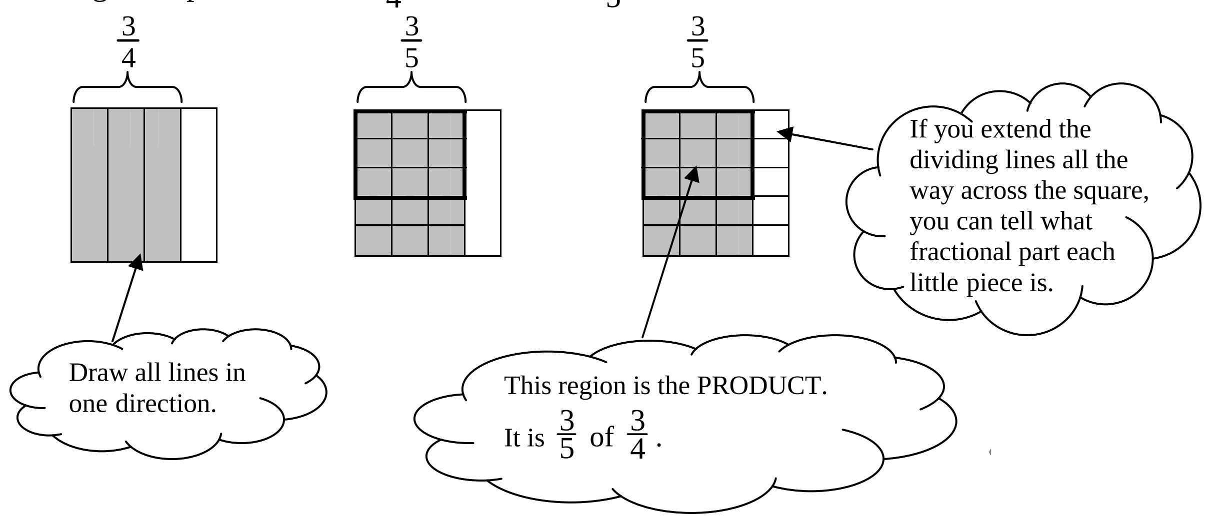
How much is $\frac{3}{5}$ of $\frac{3}{4}$?

- Another way to represent this is $\frac{3}{5} \times \frac{3}{4}$, which means "$\frac{3}{5}$ of a set of $\frac{3}{4}$."
- To get the product, make $\frac{3}{4}$ and then take $\frac{3}{5}$ of it, as illustrated below:



There are three rows and three columns in the product, or 3 × 3 rows.

The WHOLE is now five rows and four columns, so there are 5 x 4 parts in the whole.

$$\text{PRODUCT} = \frac{3}{5} \times \frac{3}{4}$$

$$= \frac{\textbf{Number} \text{ of parts in the product}}{\textbf{Kind} \text{ of parts}}$$

$$= \frac{3 \times 3}{5 \times 4} = \frac{9}{20}$$

Note: Encourage the students to count each small part in the drawings rather than have them notice that the number of rows and columns is actually the numerators and the denominators multiplied together. Encourage the understanding of the algorithm rather than applying by rote procedures, even when drawing the diagrams.

g. Making sense of the algorithm using mixed numbers

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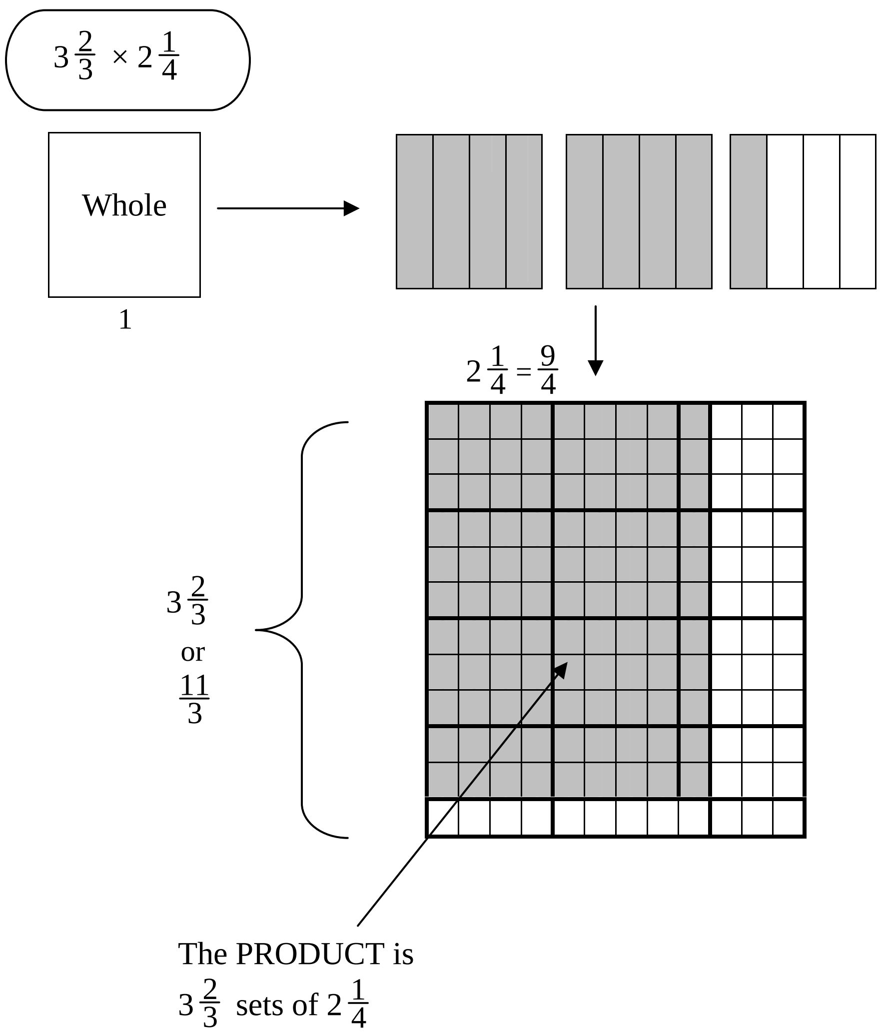
How much is $3\frac{2}{3} \times 2\frac{1}{4}$?



There are 11 rows and 9 columns, or 11 × 9 parts, in the Product.

The WHOLE now has three rows and four columns, or 3 × 4 parts.

$$\text{PRODUCT} = \frac{\text{Number of parts}}{\text{Kind of parts}}$$

$$3\frac{2}{3} \times 2\frac{1}{4} = \frac{11}{3} \times \frac{9}{4} = \frac{11 \times 9}{3 \times 4} = \frac{99}{12} = 8\frac{1}{4}$$

Sample Activity 3: Using Patterns to Understand the Multiplication of Fractions

When multiplying a whole number by a fraction, we could use the following pattern:

$8 \times 4 = 32$

$8 \times 2 = 16$

$8 \times 1 = 8$

$8 \times \frac{1}{2} = 4 \longrightarrow \frac{8}{1} \times \frac{1}{2} = \frac{8}{2} = 4$

$8 \times \frac{1}{4} = 2 \longrightarrow \frac{8}{1} \times \frac{1}{4} = \frac{8}{2} = 2$

$8 \times \frac{1}{8} = 1 \longrightarrow \frac{8}{1} \times \frac{1}{8} = \frac{8}{8} = 1$

This pattern can be extended to include other fraction products and ultimately to a generalization about multiplying fractions.

Instructional suggestion: Have the students use another set of numbers to see if a pattern develops and then have them develop a rule for multiplying fractions using their own words.

Sample Activity 4: Division of Fractions

It is important to recall here that there are two meanings of division: **partitive model** (equal sharing) and **measurement model** (equal grouping). In problems using the partitive model, the question being asked is: "How much for one?" or "How much for the whole?" In problems using the measurement model, the question being asked is: "How many equal groups can be made?"

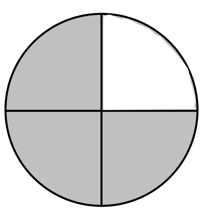
> **Equal Sharing (Partitive Model) Example:**
> If 36 cookies are shared equally among 9 children, how many cookies does each child receive?
> **Equal Grouping (Measurement Model) Example:**
> If 36 cookies are put into bags with 4 cookies in each bag, how many bags can be filled?

Suggested beginning lesson:

- Ask the students how they would solve the following story problem if they did not know their multiplication facts:
  Oliver had 12 cupcakes to divide among his 4 friends. How many cupcakes did each friend get?
- Have the students draw a picture or think about how they would act out the story to determine the answer. Listen to students' ideas. Capitalize on ideas that emphasize the sharing action of the problem.
- Activities a through e illustrate division of fractions using the partition or equal sharing concept.
- Activities f, g, h and j illustrate division of fractions using the measurement or equal grouping concept.

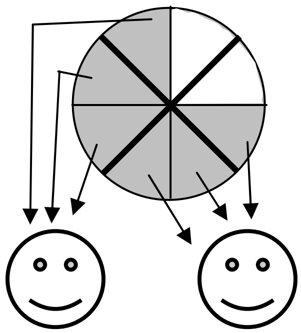
a. An example of division of a fraction by a whole number, illustrating the partition concept

You have $\frac{3}{4}$ of a pizza to divide evenly between 2 people. How much pizza would each person receive?



The diagram shows $\frac{3}{4}$ of a pizza.

To divide it evenly among 2 people,



cut each piece in half and share the 6 pieces equally.

Each person would receive $\frac{3}{8}$ of a pizza.

When you divide a number by 2, it is the same as taking $\frac{1}{2}$ of that number. We show how division and multiplication are related in the following number sentence:

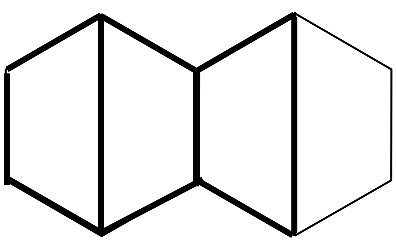
$$\frac{3}{4} \div 2 = \frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$$

Each person would receive $\frac{3}{8}$ of a pizza.

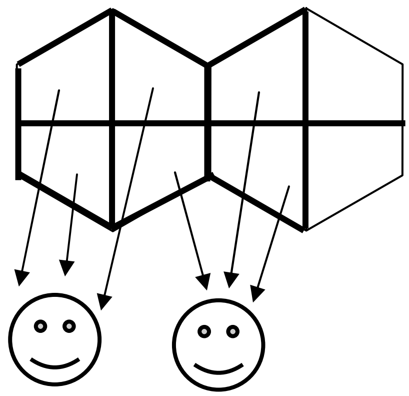
b. An example of division of fraction by a whole number, illustrating the partition concept and using fraction blocks

You have $\frac{3}{4}$ of a pizza to divide equally between 2 people. How much pizza would each person receive?

The part drawn with dark edging shows $\frac{3}{4}$ of a pizza.



Since 3 equal pieces cannot be divided equally between 2 people, we draw a line segment across the diagram to cut each piece in half. Then we share the 6 pieces equally.



Each person would receive $\frac{3}{8}$ of a pizza.

When you divide a number by 2, it is the same as taking $\frac{1}{2}$ of that number. We show how division and multiplication are related in the following number sentence:

$$\frac{3}{4} \div 2 = \frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$$

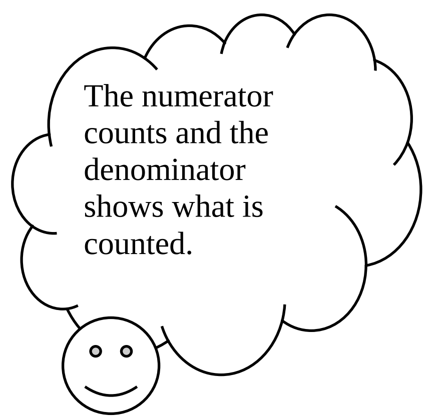
Each person would receive $\frac{3}{8}$ of a pizza.

c. An example of division of whole number by a fraction, illustrating the partition concept

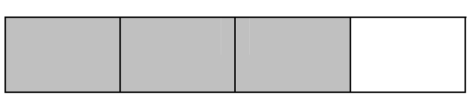
You pay \$3 for $\frac{3}{4}$ kg of nuts.

i. Estimate the cost of 1 kg.
ii. Calculate the cost of 1 kg. Include a diagram and a number sentence.

i. 1 kg is a little more than $\frac{3}{4}$ kg. Therefore, the price should be a little more than \$3 or about \$3.80.
ii. Since the numerator counts, we know that there are 3 quarters in all.

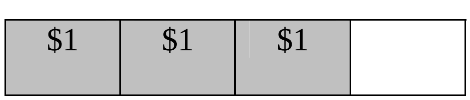


The shaded part shows $\frac{3}{4}$ kg.



The shaded part ($\frac{3}{4}$ kg) costs \$3.

So each of the shaded parts (or each quarter) costs \$1.



It follows that 4 quarters or 1 whole kilogram would cost \$4.

\$1	\$1	\$1	\$1

$$3 \div \frac{3}{4} = \frac{\cancel{3}}{1} \times \frac{4}{\cancel{3}} = 4$$

1

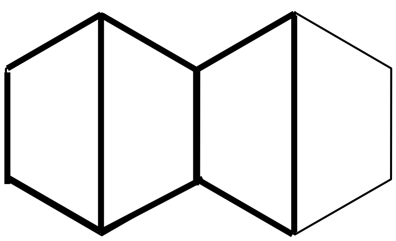
The cost of 1 kg of nuts is \$4.

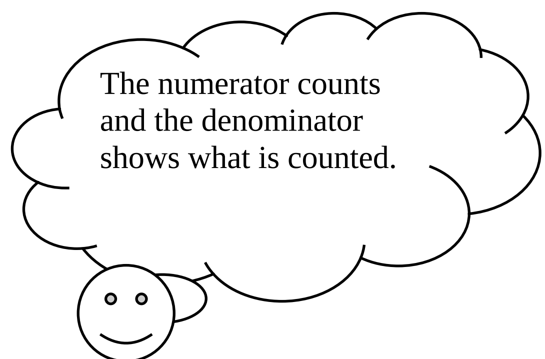
d. Dividing a whole number by a fraction, illustrating the partition concept and using fraction blocks

Look For …
Do students:
- ☐ apply rules for multiplying and dividing fractions without understanding?

You pay \$3 for $\frac{3}{4}$ kg of nuts. How much would 1 kg of these nuts cost?

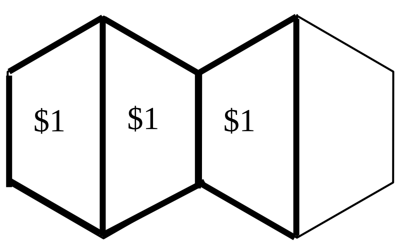
The part drawn with dark edging shows $\frac{3}{4}$ kg.



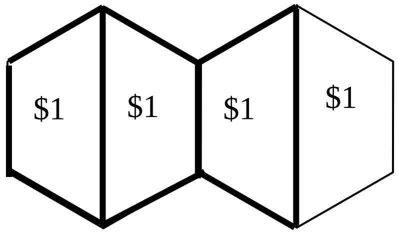


The part drawn with dark edging ($\frac{3}{4}$ kg) costs \$3.

So each of the 3 equal parts in (or each quarter) would cost \$1.



It follows that 4 quarters or 1 whole kilogram would cost $4.



$$3 \div \frac{3}{4} = \frac{\cancel{3}}{1} \times \frac{4}{\cancel{3}} = 4$$

1

The cost of 1 kg of nuts is $4.

e. An example of division of a fraction by a fraction illustrating the partition concept

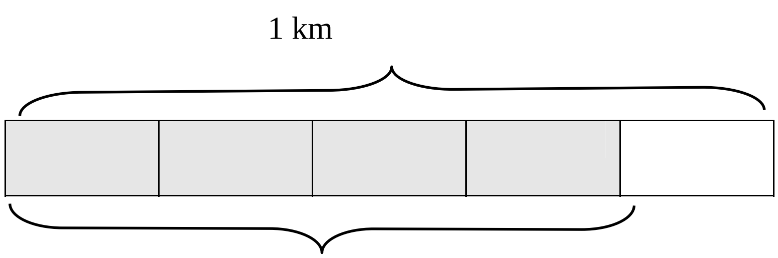
Dylano walks $\frac{4}{5}$ km in $\frac{2}{3}$ of an hour. Assuming a constant speed,

i. estimate how far he walks in 1 hour.
ii. calculate how far he walks in 1 hour. Include a diagram and a number sentence.

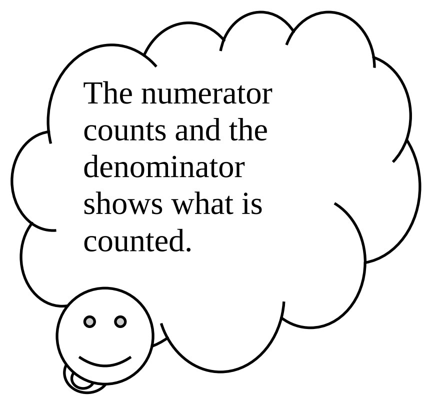
Answer:

i. Since $\frac{2}{3}$ of an hour is a little less than 1 hour, then Dylano will walk a little more than $\frac{4}{5}$ km. Dylano will walk a little more than 1 km in an hour.

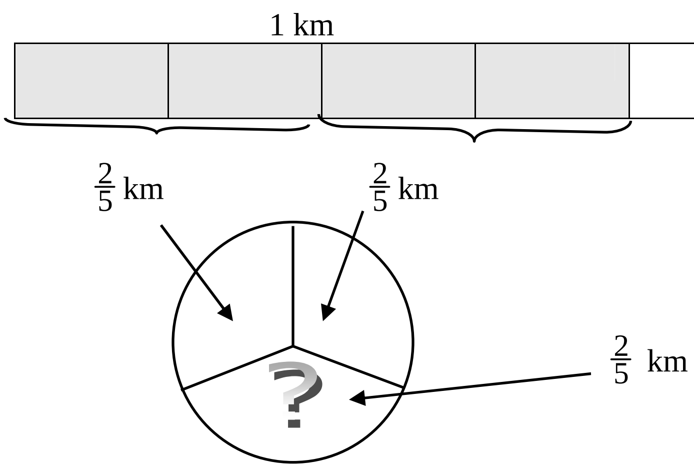
ii. The shaded part of the following diagram shows $\frac{4}{5}$ km.



Distance walked in $\frac{2}{3}$ of an hour.



The numerator of $\frac{2}{3}$ counts; therefore, the $\frac{4}{5}$ km is shared equally between the two equal groups of $\frac{1}{3}$ hour.



For each $\frac{1}{3}$ hour, Dylano runs $\frac{2}{5}$ km.

Therefore, he would also run $\frac{2}{5}$ km in the final $\frac{1}{3}$ hour.

$$\frac{2}{5} + \frac{2}{5} + \frac{2}{5} = \frac{6}{5} = 1\frac{1}{5}\text{ km}$$

Dylano runs $1\frac{1}{5}$ km in 1 hour, assuming a constant speed.

This process can also be shown by the following number sentence.

$$\frac{4}{5} \div \frac{2}{3} = \frac{4}{5} \times \frac{3}{2} = \frac{2 \times \cancel{2}}{5} \times \frac{3}{\cancel{2}} = \frac{6}{5} = 1\frac{1}{5}$$

1

Dylano runs $1\frac{1}{5}$ km in 1 hour, assuming a constant speed.

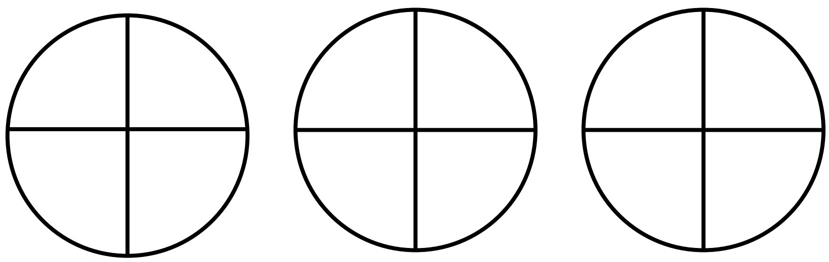
f. An example of division of a whole number by a fraction illustrating the measurement concept

You have 3 pizzas. Each person eats $\frac{3}{4}$ of a pizza and all the pizzas are completely eaten.

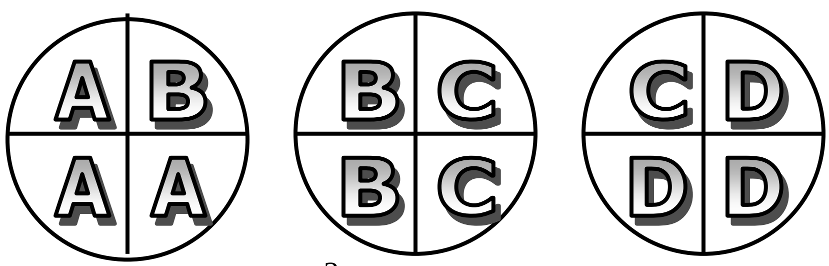
i. About how many people eat the pizzas?
ii. Calculate how many people eat the pizzas. Include a diagram and number sentence.

Answer:

i. $\frac{3}{4}$ of a pizza is a little less than 1 whole pizza. Since there are 3 whole pizzas, then slightly more than 3 people could eat the pizzas. About 4 people eat the 3 pizzas.
ii. In taking out groups of $\frac{3}{4}$ from 3 whole pizzas, first divide the pizzas into quarters to make 12 quarters.



Then take out equal groups of $\frac{3}{4}$. Label each group with a different letter.



Person A forms 1 group $\rightarrow \frac{3}{4}$ of a pizza. The same goes for B, C and D.

The 3 pizzas can be divided into 4 equal groups of $\frac{3}{4}$.

This process is shown by the following number sentence.

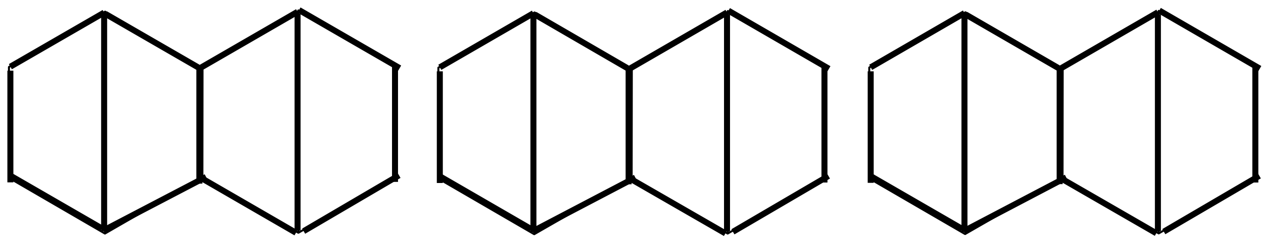
$$3 \div \frac{3}{4} = \frac{3}{1} \times \frac{4}{3} = \frac{12}{3} = 4$$

Four people eat the 3 pizzas, with each person eating $\frac{3}{4}$ of a pizza.

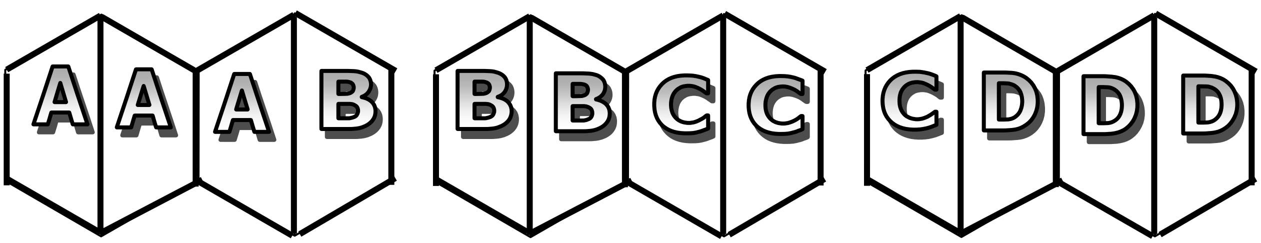
g. An example of division of a whole number by a fraction, illustrating the measurement concept and using fraction blocks

You have 3 pizzas. Each person eats $\frac{3}{4}$ of a pizza and all the pizzas are completely eaten. How many people eat the pizzas?

Answer:
In taking out groups of $\frac{3}{4}$ from 3 whole pizzas, first divide the pizzas into quarters to make 12 quarters.



Then take out equal groups of $\frac{3}{4}$. Label each group with a different letter.



Person A forms 1 group $\longrightarrow \frac{3}{4}$ of a pizza. The same goes for B, C and D.

$$3 \div \frac{3}{4} = \frac{3}{1} \times \frac{4}{3} = \frac{12}{3} = 4$$

Four people eat the 3 pizzas, with each person eating $\frac{3}{4}$ of a pizza.

h. An example of division of a fraction by a fraction illustrating the measurement concept using fraction strips

You have $\frac{5}{6}$ of a litre of ice cream.

i. About how many $\frac{1}{2}$ litre cartons could you fill with this ice cream?

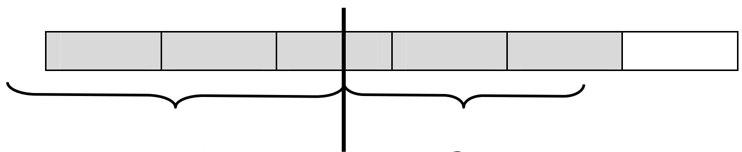
ii. Calculate how many $\frac{1}{2}$ litre cartons you could fill with this ice cream. Include a diagram and a number sentence.

Answer:

i. Since $\frac{5}{6}$ is a little more than $\frac{1}{2}$, we could fill a little more than one of the $\frac{1}{2}$ litre cartons.

ii. The diagram shows $\frac{5}{6}$ of a litre shaded.

We will divide the diagram to show how many $\frac{1}{2}$ litre cartons can be made from the $\frac{5}{6}$ of a litre that is given.



$\frac{1}{2}$ of a litre $\qquad$ $\frac{2}{3}$ of $\frac{1}{2}$ of a litre

I could fill $1\frac{2}{3}$ of the $\frac{1}{2}$ litre cartons with the ice cream.

This process can also be shown by the following number sentence.

$$\frac{5}{6} \div \frac{1}{2} = \frac{5}{6} \times \frac{2}{1} = \frac{5}{3 \times \cancel{2}} \times \frac{\cancel{2}}{1} = \frac{5}{3} = 1\frac{2}{3}$$

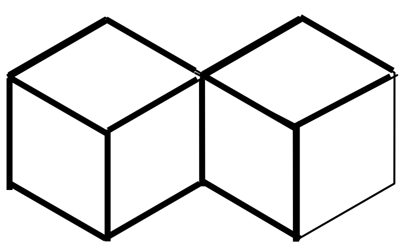
1

i. An example of division of a fraction by a fraction, illustrating the measurement concept and using fraction blocks

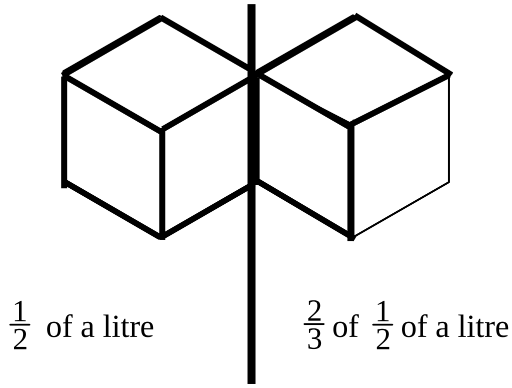
You have $\frac{5}{6}$ of a litre of ice cream. How many $\frac{1}{2}$ litre cartons could you fill with this ice cream?

Answer:

The part drawn with dark edging shows $\frac{5}{6}$ of a litre.



We will divide the diagram to show how many $\frac{1}{2}$ litre cartons can be made from the $\frac{5}{6}$ litre that is given.



I could fill $1\frac{2}{3}$ of the $\frac{1}{2}$ litre cartons with the ice cream.

This process can also be shown by the following number sentence.

$$\frac{5}{6} \div \frac{1}{2} = \frac{5}{6} \times \frac{2}{1} = \frac{5}{3 \times 2} \times \frac{2}{1} = \frac{5}{3} = 1\frac{2}{3}$$

j. Developing the algorithm

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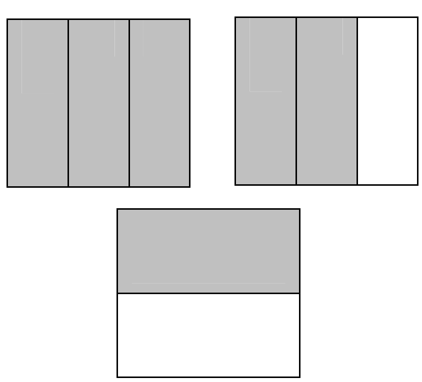
The **common denominator algorithm** relies on the measurement concept, as illustrated by the example below.

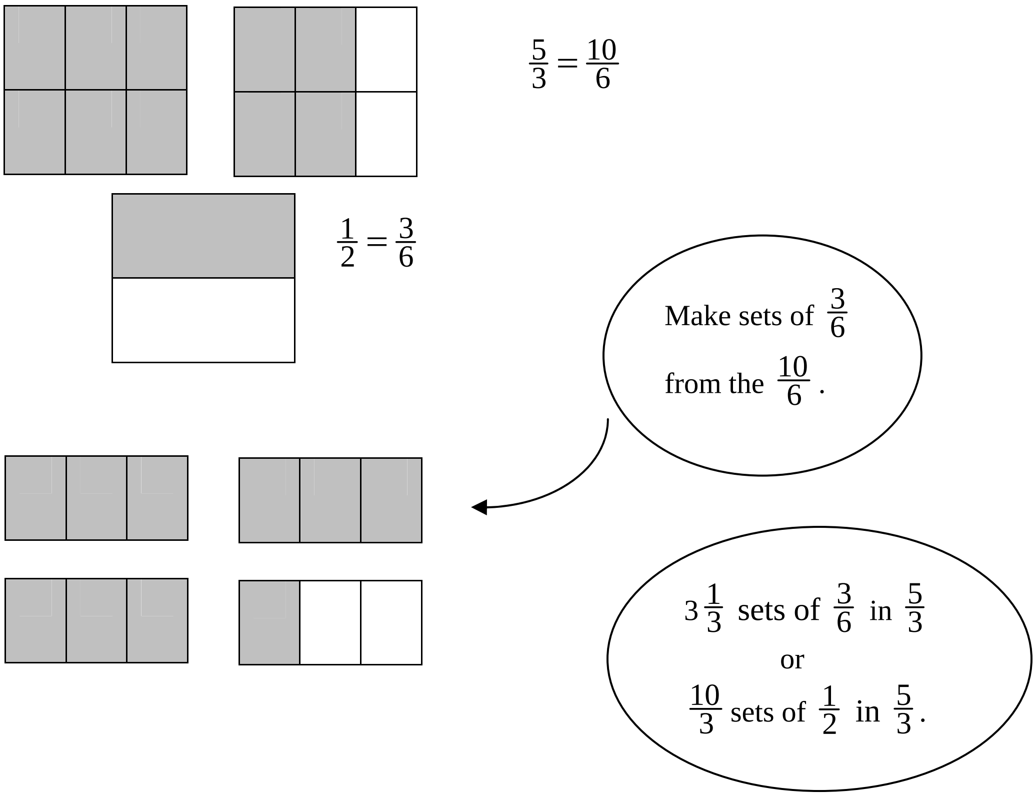
$\frac{5}{3} \div \frac{1}{2}$

Means "How many sets of $\frac{1}{2}$ are in $\frac{5}{3}$?"



Restate the problem with common denominators.

"How many sets of $\frac{3}{6}$ are in $\frac{10}{6}$?"



$$\frac{5}{3} \div \frac{1}{2} = \frac{10}{6} \div \frac{3}{6} = 10 \div 3 \text{ or } \frac{10}{3}$$

Note: Activities f and g could be written as:

$$3 \div \frac{3}{4} = \frac{12}{4} \div \frac{3}{4} = 12 \div 3 = 4$$

Activities h and i could be written as:

$$\frac{5}{6} \div \frac{1}{2} = \frac{5}{6} \div \frac{3}{6} = 5 \div 3 = \frac{5}{3}$$

The **invert and multiply algorithm** is illustrated in the examples using the partition concept, illustrated by the following example.

A water bottle can be filled to $\frac{7}{8}$ full using $\frac{2}{3}$ of a litre of water. How much will the bottle hold if it is completely filled?

Answer:
We are looking for the whole. The given amount is $\frac{7}{8}$ of a bottle. A full bottle is $\frac{8}{8}$. Because the water in the bottle is seven of the eight parts needed to fill the bottle, dividing the water by 7 and multiplying that amount by 8 solves the problem. Therefore, take the $\frac{2}{3}$, divide it by 7 and multiply by 8.

The denominator of a fraction divides the whole into parts—it is a divisor. The numerator tell us the number of those parts—it is a multiplier. In the problem we divided the $\frac{2}{3}$ by 7 and multiplied by 8. Therefore, we multiplied $\frac{2}{3}$ by $\frac{8}{7}$. Therefore, we inverted and multiplied.

Here is another example.

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$\frac{3}{4} \div \frac{5}{6} = \square$

Write the equation in an equivalent form as a product with a missing factor.

$\frac{3}{4} = \square \times \frac{5}{6}$

Multiply both sides by $\frac{6}{5}$.

($\frac{6}{5}$ is the inverse of $\frac{5}{6}$).

$\frac{3}{4} \times \frac{6}{5} = \square \times \left(\frac{5}{6} \times \frac{6}{5}\right)$

$\frac{3}{4} \times \frac{6}{5} = \square \times 1$

$\frac{3}{4} \times \frac{6}{5} = \square$

But $\frac{3}{4} \div \frac{5}{6} = \square$ also.

Therefore, $\frac{3}{4} \div \frac{5}{6} = \frac{3}{4} \times \frac{6}{5} = \square$

In general, $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$

k. Using patterns is another method to understand the division of fractions

When dividing a whole number by a fraction, one could use the following pattern:

$8 \div 4 = 2$

$8 \div 2 = 4$

$8 \div 1 = 8$

$8 \div \frac{1}{2} = 16 \longrightarrow \frac{8}{1} \div \frac{1}{2} = \frac{8}{1} \times \frac{2}{1} = 16$

$8 \div \frac{1}{4} = 32 \longrightarrow \frac{8}{1} \div \frac{1}{4} = \frac{8}{1} \times \frac{4}{1} = 32$

$8 \div \frac{1}{8} = 64 \longrightarrow \frac{8}{1} \times \frac{1}{8} = \frac{8}{1} \times \frac{8}{1} = 64$

This pattern can be extended to include other fraction quotients and ultimately to a generalization about dividing fractions.

l. Applying the algorithm to division of mixed numbers

You have $1\frac{2}{3}$ litres of ice cream.

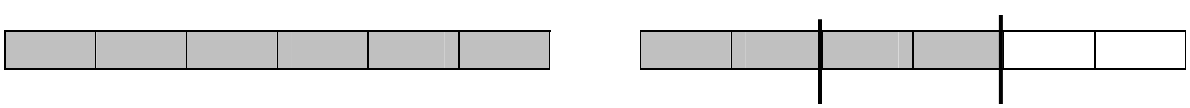
i. About how many $\frac{1}{2}$ litre cartons could you fill with this ice cream?

ii. Calculate how many $\frac{1}{2}$ litre cartons you could fill with this ice cream. Include a diagram and a number sentence.

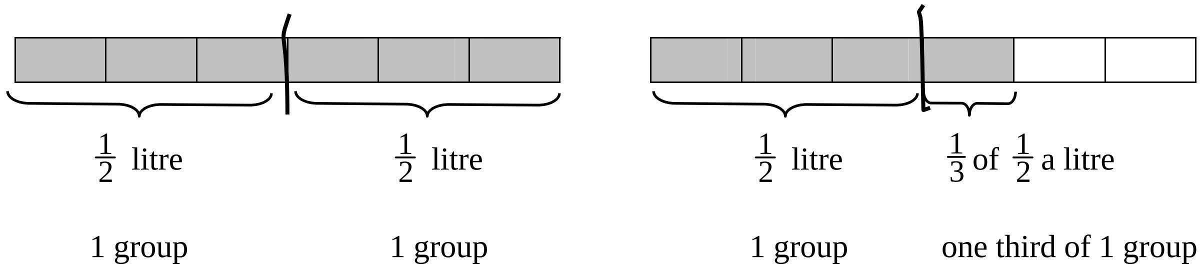
Answer:

i. Since $1\frac{2}{3}$ is quite a bit more than $\frac{1}{2}$, we could fill about 3 of the $\frac{1}{2}$ litre containers.

ii. The diagram shows $1\frac{2}{3}$ or $1\frac{4}{6}$ litres of ice cream.



We will divide the diagram to show how many $\frac{1}{2}$ litre cartons can be made from the $1\frac{2}{3}$ litres that are given.



I could fill $3\frac{1}{3}$ containers that each hold $\frac{1}{2}$ litre of ice cream.

This process can also be shown by the following number sentence, illustrating the invert and multiply algorithm.

$$1\frac{2}{3} \div \frac{1}{2} = \frac{5}{3} \div \frac{1}{2} = \frac{10}{6} \times \frac{2}{1} = \frac{10}{3\times\cancel{2}} \times \frac{\cancel{2}}{1} = \frac{10}{3} = 3\frac{1}{3}$$

1

It can also be shown by this number sentence, illustrating the common denominator algorithm.

$$1\frac{2}{3} \div \frac{1}{2} = \frac{5}{3} \div \frac{1}{2}$$
$$= \frac{10}{6} \div \frac{3}{6}$$
$$= 10 \div 3$$
$$= 3\frac{1}{3}$$

Sample Activity 5: Fraction Two-ways

This activity reproduced with permission from Grayson H. Wheatley and George E. Abshire, *Developing Mathematical Fluency: Activities for Grades 5–8* (Tallahassee, FL: Mathematics Learning, 2002), pp. 140, 141. www.mathematicslearning.org

In two-ways, the numbers in a row must multiply to make a number at the right and the numbers in each column must multiply to a number at the bottom. Then the two numbers at the right must multiply to make the number in the bottom right corner and the numbers along the bottom must also multiply to make the same number. Thus, there is a self-check built into the design of the activity.
For example:

⊗

$\frac{4}{5}$		4
20		16

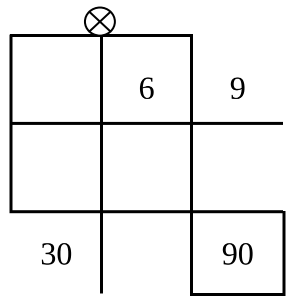
On the right vertically, $4 \times 4 = 16$. That was easy. Across the top, I needed five $\frac{4}{5}$ to make 4 so the answer was 5. Along the bottom, I thought that $\frac{1}{5}$ of 20 is 4 and
$4 \times 4 = 16$ so the answer is $\frac{4}{5}$ (multiplying by 4 and dividing by 5). On the left, I saw that $5 \times \frac{4}{5}$ is 4 and multiplying by 5 again gives 20, so the number is 25. In the middle vertically, "What can I multiply 5 by to get $\frac{4}{5}$?" $\frac{1}{5}$ of 5 is 1 and $\frac{1}{5}$ of that is $\frac{1}{5}$. I need 4 of those to make $\frac{4}{5}$ so the answer is $\frac{4}{25}$. Now to check, I looked across the middle.
$25 \times$ ____ = 4. 4 divided by 25 is $\frac{4}{25}$. Since I got $\frac{4}{25}$ both ways I know all my answers are correct.

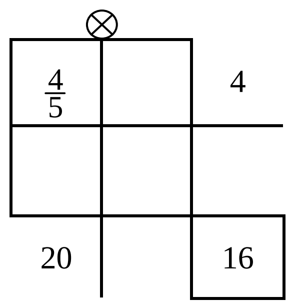
⊗

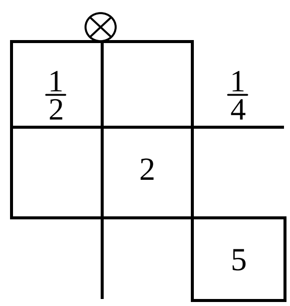
$\frac{4}{5}$	5	4
25	$\frac{4}{25}$	*4*
20	$\frac{4}{5}$	16

Note: Solving multiplication two-ways will provide the students with rich computational experiences and opportunities to relate the mathematical operations of multiplication and division.

Examples to try:







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Step 4: Assess Student Learning

Guiding Questions

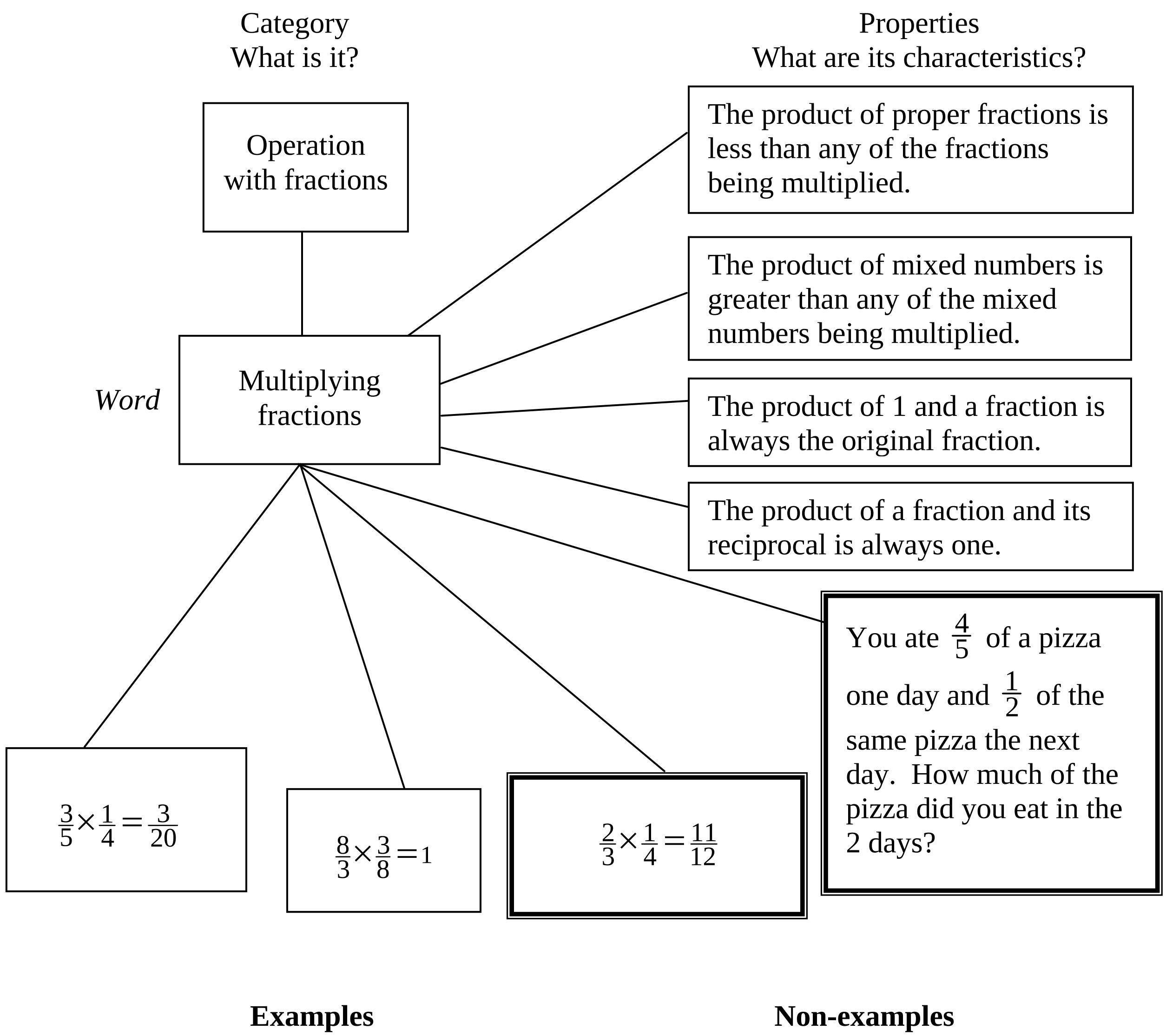
- Look back at what you determined as acceptable evidence in Step 2.
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

In addition to ongoing assessment throughout the lessons, consider the following sample activities to evaluate students' learning at key milestones. Suggestions are given for assessing all students as a class or in groups, individual students in need of further evaluation, and individual or groups of students in a variety of contexts.

A. Whole Class/Group Assessment

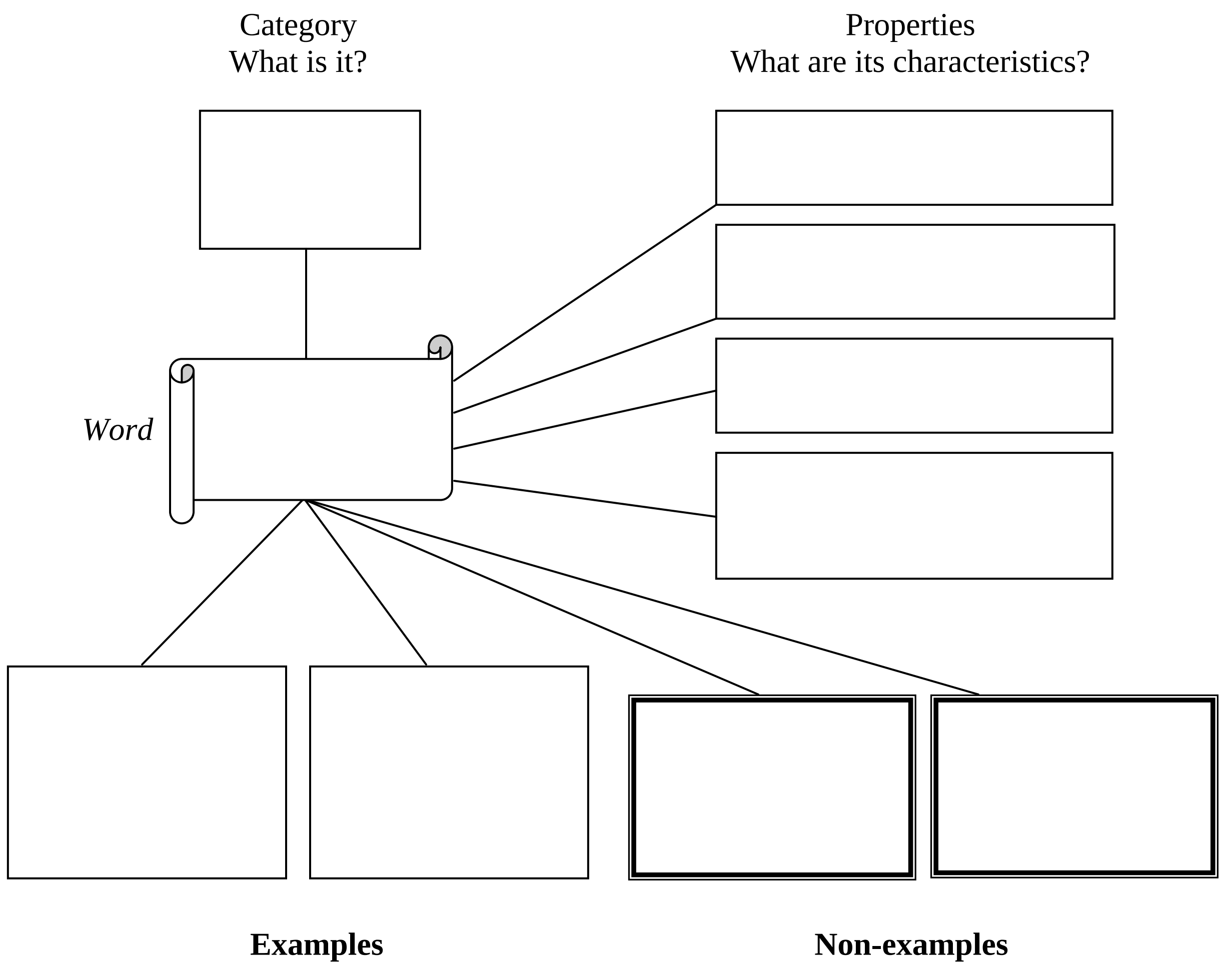
Activity 1: Have the students work in groups to complete a concept definition map or a modified Frayer model for the concepts of multiplication and division of fractions. See the following sample.

Concept Definition Map



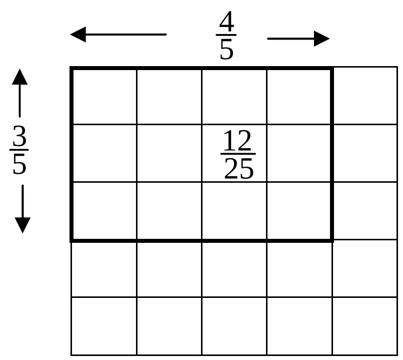
Format adapted from Robert M. Schwartz, "Learning to Learn Vocabulary in Content Area Textbooks," *Journal of Reading* 32, 2 (November 1988), p. 110, Example 1. Adapted with permission from International Reading Association.

Concept Definition Map



Format adapted from Robert M. Schwartz, "Learning to Learn Vocabulary in Content Area Textbooks," *Journal of Reading* 32, 2 (November 1988), p. 110, Example 1. Adapted with permission from International Reading Association.

Modified Frayer Model for Product of Fractions

Definition The product of fractions is the result of multiplying the fractions together.	**Examples** $\frac{2}{3}\times\frac{3}{4}=\frac{6}{12}$ $\frac{5}{6}\times\frac{6}{5}=1$ **Non-examples** $\frac{3}{8}-\frac{1}{4}=\frac{1}{8}$ $\frac{1}{3}+\frac{1}{6}=\frac{1}{2}$
Visual and Numeric Representation  $\frac{3}{4}\times\frac{4}{5}=\frac{12}{25}$	**Word Problem** Mary bought $\frac{3}{4}$ m of material. She used $\frac{1}{2}$ of the material. How many metres of the material did she use?

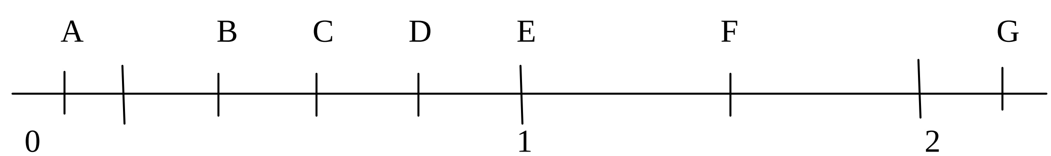
Format adapted from D. A. Frayer, W. C. Frederick and H. J. Klausmeier, *A Schema for Testing the Level of Concept Mastery* (Working Paper/Technical Report No. 16) (Madison, WI: Research and Development Center for Cognitive Learning, University of Wisconsin, 1969). Adapted with permission from the Wisconsin Center for Education Research, University of Wisconsin-Madison.

Modified Frayer Model for ____________

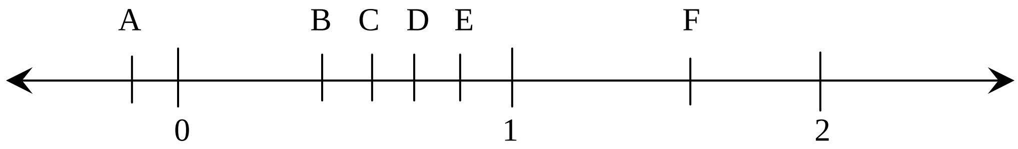
Definition	Examples Non-examples
Visual and Numeric Representation	Word Problem

Format adapted from D. A. Frayer, W. C. Frederick and H. J. Klausmeier, *A Schema for Testing the Level of Concept Mastery* (Working Paper/Technical Report No. 16) (Madison, WI: Research and Development Center for Cognitive Learning, University of Wisconsin, 1969). Adapted with permission from the Wisconsin Center for Education Research, University of Wisconsin-Madison.

Activity 2:



Activity 3: Display the following number line and ask the students questions such as those indicated here and other that may come to mind. Encourage the students to justify their answers by explaining their reasoning.



a. If the fractions represented by the points D and E are multiplied, what point on the number line best represents the product?
b. If the fractions represented by the points C and D are multiplied, what point on the number line best represents the product?
c. If the fractions represented by the points B and F are multiplied, what point on the number line best represents the product?
d. Suppose 20 is multiplied by the number represented by E on the number line. Estimate the product.
e. Suppose 20 is divided by the number represented by E on the number line. Estimate the quotient.

Activity 4: Have the students complete the following exercise. Encourage them to make generalizations about operations involving fractions of various sizes and to justify their solutions.

Suppose that a > 1, 0 < b < 1, and 0 < c < 2. Ask students to complete each sentence by filling in the blank with <, =, >, or CT (for "can't tell").

1. $a \quad b \ \square \ a$	5. $a \div c \ \square \ a$
2. $b \quad c \ \square \ b$	6. $b \div c \ \square \ b$
3. $a \quad b \quad c \ \square \ b$	7. $\frac{c}{c} \ \square \ c$
4. $a \div b \ \square \ a$	8. $b^2 \ \square \ b$

Solutions: 1. <; 2. CT; 3. CT; 4. >; 5. CT; 6. <; 7. CT; 8. <. Students should use more than one example to justify the "can't tell" answers.

B. One-on-one Assessment

Activity 1: Solve the following problems. Solutions should contain diagrams and written explanations.

- Cassie has $5\frac{1}{4}$ metres of ribbon to make three bows for birthday packages. How much ribbon should she use for each bow if she wants to use the same length of ribbon for each?
- Karen drinks 3 bottles of water, each holding $1\frac{2}{3}$ litres. How much water does Karen drink?
- Anna watches TV for $2\frac{1}{3}$ hours. Maria watches TV $1\frac{1}{2}$ times as long as Anna. How long does Maria watch TV?
- How many containers, each with a capacity of $\frac{4}{5}$ litre, can be filled with $2\frac{2}{5}$ litres of paint?
- Michael walked $10\frac{1}{2}$ km to help raise money for a charitable cause. Ariel walked $\frac{1}{3}$ the distance that Michael walked. Shannon walked twice as far as Ariel. How far did Shannon walk (Warrington and Kamii 1998, p. 341)?
- Tobin went apple picking and later baked 4 apple pies. He ate $\frac{1}{5}$ of one of the pies and put the remaining pies on his back porch to cool. His sister and her friends came along and ate $\frac{1}{2}$ of the pies on the porch. How much pie was left on the porch (Warrington and Kamii 1998, p. 342)?

Activity 2:

Given: $2\frac{2}{3} \div \frac{1}{6} = 16$

Using this information, find the answers to these questions.
Be sure to explain your thinking.

a. $2\frac{2}{3} \div \frac{1}{3} =$ b. $2\frac{2}{3} \div \frac{1}{12} =$

C. Applied Learning

Provide opportunities for the students to use the knowledge they have gained about multiplying and dividing fractions and notice whether or not this knowledge transfers.

Activity 1: Have the students get a favourite family recipe and half, double and triple the ingredients.

Activity 2: Have the students find five examples of fractions from the real world.

Step 5: Follow-up on Assessment

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction?

A. Addressing Gaps in Learning

- Review the concepts of multiplication and division of fractions by having the students work with models such as fraction strips or fractions blocks.
- Provide time for the students to reflect on their learning by the use of the graphic organizers such as the Frayer models or concept definition map.
- Make sure that the students are solving problems by drawing pictures and providing explanations to justify their thinking.
- If students are experiencing difficulty with identifying the whole, they may need more time to explore part and whole tasks, such as:

If this rectangle is one third, what could the whole look like (Van de Walle and Lovin 2006, p. 71)?

B. Reinforcing and Extending Learning

Students who have achieved or exceeded the outcomes will benefit from ongoing opportunities to apply and extend their learning. These activities should support students in developing a deeper understanding of the concept and should not progress to the outcomes in subsequent grades.

Consider strategies such as:

Activity 1: Write a real world problem for the following operations using fractions:

a. 3 divided by $\frac{1}{4}$

b. \$2.50 multiplied by $\frac{3}{5}$

c. $1\frac{2}{3}$ divided by $\frac{1}{6}$

d. $1\frac{3}{4}$ multiplied by $\frac{1}{2}$.

Activity 2: Fraction Bingo

Players: 2 or more

Materials: a bingo card for each player, two dice, with one fraction on each face; e.g.,
$\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{2}{3}$, $\frac{3}{4}$ first die

$\frac{3}{8}$, $\frac{2}{4}$, $\frac{4}{4}$, $\frac{3}{2}$, $\frac{4}{3}$, $\frac{2}{3}$ second die

Description: One player (or the teacher) rolls the two dice. Each player multiplies or divides the two numbers to obtain a number on his or her bingo card, if possible. The player uses a marker to cover the product or quotient on his or her card.

Goal: The first player to complete a row, a column or a diagonal wins.

Sample Bingo Card

$\frac{1}{4}$	$\frac{9}{8}$	$\frac{8}{3}$	$\frac{2}{3}$	$\frac{3}{2}$
$\frac{3}{8}$	$\frac{3}{4}$	$1\frac{1}{2}$	3	$4\frac{1}{2}$
4	1	Free Space	2	$\frac{8}{9}$
$\frac{9}{4}$	$\frac{4}{3}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{6}$
$\frac{1}{2}$	$\frac{1}{8}$	$\frac{3}{16}$	$\frac{5}{8}$	$\frac{4}{9}$

Variations (The bingo card may need to be changed depending on the variation chosen.)

- Roll three dice instead of two.
- Use addition and subtraction.
- Use three or four operations.
- Change the fractions on the dice.

Activity 3: Fraction Fun

Players: individual, small group or whole class
Materials: the problems described below
Description: Students work individually or in teams to solve the problems.
Goal: Find as many ways as possible to solve the problems and create similar problems.

Problems

The Market Gardener

In your job as a gardener, you must decide how to use your garden as you plant. You mark $\frac{1}{2}$ of the garden for potatoes. You use $\frac{1}{4}$ of the remaining area for corn. Then you plant cucumbers in $\frac{1}{3}$ of what is left. The remainder of your garden is used for carrots. What fraction of your garden is used for carrots?

Muffin Mania

(This problem adapted with permission from "Muffin Mania" (p. 292) by Robert Mann, in Volume 10, Issue 6 of *Teaching Children Mathematics*, copyright 2004 by the National Council of Teachers of Mathematics.)

In a role reversal story based on “Goldilocks and the Three Bears,” Goldilocks puts some muffins on the porch to cool and the three bears come along and eat some of them. The following problem was created from this story.
Papa Bear took $\frac{1}{4}$ of the original muffins, Mama Bear took $\frac{1}{3}$ of what he left, Baby Bear took $\frac{1}{2}$ of what remained and only 3 muffins are left in the basket.

a. How many muffins were in the basket when Goldilocks first put them on the porch to cool?
b. How many muffins did each bear eat?
c. Suppose the problem remains the same, except that the number of muffins left is 4 instead of 3. How would this affect the solution? Explain.
d. Suppose the bears ate the muffins in reverse order. How many muffins did each bear eat?

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