Mathematics

Planning Guide

Grade 8 *Independent Events*

Statistics and Probability (Chance and Uncertainty)

Specific Outcome 2

This Planning Guide can be accessed online at:
http://www.learnalberta.ca/content/mepg8/html/pg8_independentevents/index.html

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Planning Guide: *Grade 8 Independent Events*

Strand: Statistics and Probability (Chance and Uncertainty)
Specific Outcome: 2

This *Planning Guide* addresses the following outcomes from the Program of Studies:

Strand: Statistics and Probability (Chance and Uncertainty)	
Specific Outcome:	2. Solve problems involving the probability of independent events.

Curriculum Focus

The changes to the curriculum targeted by this sample include:

- The general outcome focuses on using experimental or theoretical probabilities to represent and solve problem involving uncertainty; whereas the previous curriculum focused on comparing the theoretical and experimental probability of independent events.
- The specific outcome focuses on solving problems involving the probability of independent events; whereas the previous curriculum focused on determining the probability of two independent events.

What Is a Planning Guide?

Planning Guides are a tool for teachers to use in designing instruction and assessment that focuses on developing and deepening students' understanding of mathematical concepts. This tool is based on the process outlined in *Understanding by Design* by Grant Wiggins and Jay McTighe.

Planning Steps

The following steps will help you through the Planning Guide:

Step 1: Identify Outcomes to Address

Guiding Questions

- What do I want my students to learn?
- What can my students currently understand and do?
- What do I want my students to understand and be able to do based on the Big Ideas and specific outcomes in the program of studies?

Big Ideas

- The probability of an event is a number between 0 (impossible) and 1 (certain). It is a measure of the chance that a given event will occur. A probability of $\frac{1}{2}$ indicates an even chance of the event occurring.
- Probability is a way to measure uncertainty. It involves predicting the likelihood of an event occurring over a period of time rather than predicting the likelihood of occurrence of an event for a specific time. For example, when rolling a die, we expect a two to occur about one sixth of the time when we conduct a large number of rolls; however, we cannot predict with much certainty what the next roll of the die will be.
- Theoretical probability of an event is the ratio of the number of outcomes in an event to the total number of possible outcomes, when all possible outcomes are equally likely.
- Experimental probability or relative frequency of an event is the ratio of the number of observed occurrences of the event to the total number of trials. The greater the number of trials, the closer the experimental probability approaches the theoretical probability.
- Two events are independent if the fact that one event occurs does not affect the probability of the second event occurring.

Sequence of Outcomes from the Program of Studies

See http://education.alberta.ca/teachers/core/math/programs.aspx for the complete program of studies.

Grade 7	Grade 8	Grade 9
Specific Outcomes	**Specific Outcomes**	**Specific Outcomes**
5. Identify the sample space (where the combined sample space has 36 or fewer elements) for a probability experiment involving two independent events. 6. Conduct a probability experiment to compare the theoretical probability (determined using a tree diagram, table or other graphic organizer) and experimental probability of two independent events.	2. Solve problems involving the probability of independent events.	4. Demonstrate an understanding of the role of probability in society.

Step 2: Determine Evidence of Student Learning

Guiding Questions

- What evidence will I look for to know that learning has occurred?
- What should students demonstrate to show their understanding of the mathematical concepts, skills and Big Ideas?

Using Achievement Indicators

As you begin planning lessons and learning activities, keep in mind ongoing ways to monitor and assess student learning. One starting point for this planning is to consider the achievement indicators listed in the *Mathematics Kindergarten to Grade 9 Program of Studies with Achievement Indicators*. You may also generate your own indicators and use them to guide your observation of the students.

The following indicators may be used to determine whether or not students have met this specific outcome. Can students:

- determine the probability of two given independent events and verify the probability using a different strategy?
- generalize and apply a rule for determining the probability of independent events?
- solve a given problem that involves determining the probability of independent events?

Sample behaviours to look for related to these indicators are suggested for some of the activities listed in **Step 3, Section C: Choosing Learning Activities** (p. 8).

Step 3: Plan for Instruction

Guiding Questions

- What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should I use?
- How will I meet the diverse learning needs of my students?

A. Assessing Prior Knowledge and Skills

Before introducing new material, consider ways to assess and build on students' knowledge and skills related to probability, such as conducting an experiment and using graphic organizers such as a tree diagram or a table to find the equally likely possibilities.

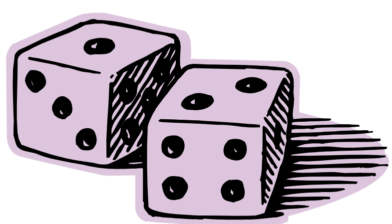
Activity: Give the students a copy of the following activity.

Fair Game

You need: a partner
a pair of dice

1. Take turns rolling the two dice. Player A scores a point if the sum is even. Player B scores a point if the sum is odd. Is the game fair? If not, how could you make the game fair? Explain your reasoning.

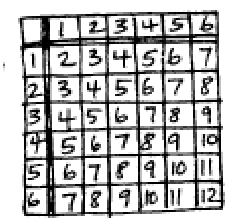
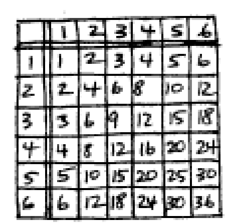
2. Play the game again, this time determining the product. Player A scores a point if the product is even. Player B scores a point if the product is odd. Is the game fair? If not, how could you make it fair? Explain your reasoning.



Adapted from Burns, Marilyn. *About Teaching Mathematics: A K–8 Resource, Second Edition*, p. 74. Copyright © 2000 by Math Solutions Publications. Adapted by permission. All rights reserved. (Note: This title is now in its third edition, copyright © 2007.)

If a student appears to have difficulty with these tasks, consider further individual assessment, such as a structured interview, to determine the student's level of skill and understanding. See **Sample Structured Interview: Assessing Prior Knowledge and Skills** (p. 7).

Sample Structured Interview: Assessing Prior Knowledge and Skills

Directions	Date:	
	Not Quite There	**Ready to Apply**
1. Take turns rolling the two dice. Player A scores a point if the sum is even. Player B scores a point if the sum is odd. Is the game fair? If not, how could you make the game fair? Explain your reasoning.	1. Student responds that the possible sums when rolling two dice are 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12. Student also responds that the game is fair but can give no explanation.	1. Student responds with the 11 possible sums when rolling two dice. Student also responds with the 36 combinations of possible sums when rolling two dice in a form such as  and can correctly calculate the probabilities of the sums: P(2) = $\frac{1}{36}$, P(3) = $\frac{2}{36}$ = $\frac{1}{18}$, P(4) = $\frac{3}{36}$ = $\frac{1}{12}$. From this information, or by counting in the chart, the P(even) = $\frac{18}{36}$ = $\frac{1}{2}$ and so is the P(odd). The student responds that this is a fair game.
2. Play the game again, this time determining the product. Player A scores a point if the product is even. Player B scores a point if the product is odd. Is the game fair? If not, how could you make it fair? Explain your reasoning.	2. Student responds that the possible products when rolling two dice are: 1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 16, 18, 20, 24, 25, 30, 36. Student also responds that the game is not fair but is unable to provide an explanation.	2. Student responds with the 18 possible products. Student also responds with a chart such as  and is able to calculate the probabilities P(even) = $\frac{27}{36}$ = $\frac{3}{4}$ and P(odd) = $\frac{9}{36}$ = $\frac{1}{4}$. Student responds that this game is not fair.

Directions and graphics adapted from Burns, Marilyn. *About Teaching Mathematics: A K–8 Resource, Second Edition*, pp. 267, 271. (Note: This title is now in its third edition, copyright © 2007.)

B. Choosing Instructional Strategies

Consider the following general strategies for teaching when planning lessons:

- Build on the students' understanding of experimental and theoretical probability, focusing on two independent events (e.g., tossing a coin twice).
- Provide a variety of hands-on activities using manipulatives such as coins, dice, spinners, playing cards or objects in a bag.
- Provide various strategies for calculating probability (e.g., the use of tree diagrams, tables and area models).
- Provide opportunities for the students to develop critical thinking skills by choosing activities that stem from real world situations. These skills are necessary so that students will not be misled or blinded by the statistical society that we live in.

C. Choosing Learning Activities

The following learning activities are examples of activities that could be used to develop student understanding of the concepts identified in Step 1.

Sample Activities:

1. **The Game of Sticks and Stones** (p. 9)
2. **Sticks and Stones Revisited** (p. 12)

Sample Activity 1: The Game of Sticks and Stones

This activity adapted from National Council of Teachers of Mathematics, "Sticks and Stones," *NCTM Illuminations*, 2000–2008, http://illuminations.nctm.org/LessonDetail.aspx?ID=L585 (Accessed April 2008). Used with permission.

- This game is based on the Apache game "Throw Sticks," which was played at celebrations.
- Students will collect data and investigate the various probabilities of throwing the sticks.
- To play the game, students throw three sticks, each decorated on one side only; the other side should be blank. Students will use these sticks to determine how far they move.
- To create the game board, arrange 40 round markers in a circle, preferably divided into four groups of 10. As an alternative you can use a game board, which consists of 10 squares on each side of the board.

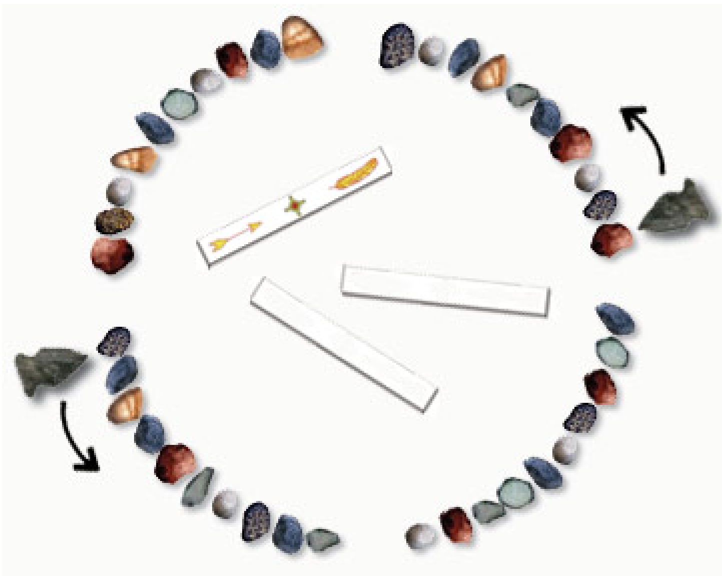
Look For …

Do students:

☐ use correct vocabulary such as more *likely*, *less likely*, *probable*, *outcomes and tree diagram*?

☐ calculate the possible outcomes of independent events in more than one way; e.g., tree diagrams, tables?

☐ demonstrate flexibility in using a variety of manipulatives to solve problems involving



Rules of the Game:

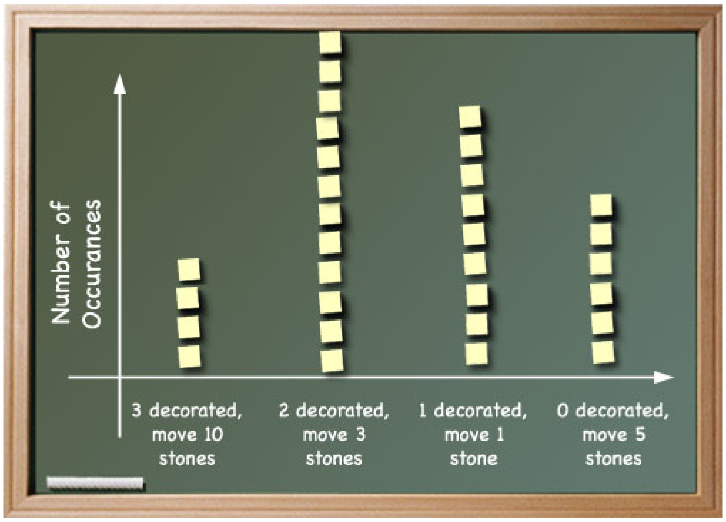
- **Object of the Game**: Be the first player to move your piece around the board past your starting point.
- **Set-Up**: Each student should place a marker on opposite sides of the circle (game board). The area inside the circle (the middle of the board) is used for throwing the sticks.
- **Play:** Determine which player will go first. Player 1 throws the three sticks into the centre of the circle and moves his or her piece according to the results (shown in the chart below). Player 2 then throws the sticks and moves accordingly. Play continues with players alternating turns.
- **Special Rule:** If one player's marker lands on or passes another player's, the player passed over must move his or her piece back to the starting point.

Moves	
3 Decorated Sides	10 spaces
3 Blank Sides	5 spaces
2 Decorated, 1 Blank	3 spaces
2 Blank, 1 Decorated	1 space

Instruction suggestions:

- Students will need markers, a Monopoly game board (or 40 round markers to simulate the original game with stones), and popsicle sticks.
- Have students decorate three popsicle sticks on one side only, using different colours for each stick.
- Pair students together, and let them play the game once, for fun.
- Before playing a second time, have students make a list of all throws that are possible. The possible outcomes could be organized as follows:
  – 3 Decorated, 0 Blank
  – 2 Decorated, 1 Blank
  – 1 Decorated, 2 Blank
  – 0 Decorated, 3 Blank
- During the second game, have students keep track of how often each throw occurs.

- After tallying the total of each type of throw, have students draw a graph that shows the possible throws on the horizontal axis and the number of occurrences on the vertical axis. Gather data from 4–6 pairs of students to give a larger sample size (this should yield experimental results closer to the theoretical possibilities). A completed graph could look something like this.



Facilitate a discussion about what the graph tells us. You may want to ask: Which type of throw is most likely to occur? Least likely? Is this what you would expect to happen?

- Determine the probability of each outcome using the table.

Sample Activity 2: Sticks and Stones Revisited

Revisit the Sticks and Stones activity to develop an understanding of theoretical probability.

- Using only 2 sticks, e.g., one decorated green and one decorated red, have students identify all possible outcomes.

  Students should identify the following possibilities in some organized manner; e.g., tree diagram, chart:

Outcome Number	Green Decorated Stick	Red Decorated Stick
1	Decorated	Blank
2	Decorated	Decorated
3	Blank	Decorated
4	Blank	Blank

- Have students determine the probability of each of the outcomes using this information; e.g.: P(green decorated, red blank) = $\frac{1}{4}$, P(2 blank) = $\frac{1}{4}$, P(1 decorated, 1 blank) = $\frac{1}{2}$
- Instead of drawing a chart or tree diagram each time, we can use the probability of each individual event (e.g., tossing a green decorated stick and tossing a red decorated stick) in order to determine the probabilities as shown above. Discuss with students the meaning of independent events and how, for example, the green stick being tossed decorated side up has no bearing on the red stick being tossed decorated side up. Explain to students that this is useful only when you are looking at one event happening. It cannot be used to determine P(1 decorated, 1 blank) because there are really two different events that could meet this outcome – P(green decorated, red blank) or P(green blank, red decorated).

  Have students determine the probability of each event.
  P(decorated side up) = $\frac{1}{2}$ = P(blank side up)

  The P(2 blank) is the same as P(blank of green and blank of red). Since we want both of these to happen at the same time, we can multiply the probabilities of each individual event.
  P(2 blank) = P(blank of green and blank of red)
  = P(blank of green) × P(blank of red)
  = $\frac{1}{2} \times \frac{1}{2}$
  = $\frac{1}{4}$

- Return to the original Sticks and Stones game and have students draw a chart or tree diagram showing all possible outcomes listed by colour of stick. From this chart, have students determine the theoretical probability of:
  - 3 Decorated, 0 Blank
  - 2 Decorated, 1 Blank
  - 1 Decorated, 2 Blank
  - 0 Decorated, 3 Blank

  Do these probabilities match the probabilities they determined when the game was played? Why or why not? Why do you think the moves in the game are made the way they are?

Based on National Council of Teachers of Mathematics, "Sticks and Stones," *NCTM Illuminations*, 2000–2008, http://illuminations.nctm.org/LessonDetail.aspx?ID=L585 (Accessed April 25, 2008).

Step 4: Assess Student Learning

Guiding Questions

- Look back at what you determined as acceptable evidence in Step 2.
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

In addition to ongoing assessment throughout the lessons, consider the following sample activities to evaluate students' learning at key milestones. Suggestions are given for assessing all students as a class or in groups, individual students in need of further evaluation, and individual or groups of students in a variety of contexts.

A. Whole Class/Group Assessment

Ask the students to use the sticks to create a new game. They should provide complete rules for the game, including how players move and win. Then they should explain the possible outcomes when a player throws the sticks, and the probability of each.

Adapted from National Council of Teachers of Mathematics, "Sticks and Stones," *NCTM Illuminations*, 2000–2008, http://illuminations.nctm.org/LessonDetail.aspx?ID=L585 (Accessed April 25, 2008). Used with permission.

B. One-on-one Assessment

Activity 1: A group of three students is playing the game rock, paper, scissors. The group decides to analyze whether the game is fair or not fair. One way to do this is to account for all of the equally likely possibilities. The group makes the following list of possibilities in which the three columns A, B and C, represent the three players and *r*, *p* and *s* and represent rock, paper, scissors.

1. Represent all possible outcomes in at least two ways.
2. Calculate the probability of each outcome.
3. Which player would you like to be? Why?
4. This game is not fair. Why? How could you make the game more fair?

Note to Teachers: The selection of rock, paper or scissors is equally likely for each individual playing the game. Also, order does not matter, therefore pps, psp and spp are all equally likely possibilities.

Adapted from Burns, Marilyn. *About Teaching Mathematics: A K–8 Resource, Second Edition*, pp. 74, 270, 271. Copyright © 2000 by Math Solutions Publications. Used with permission. All rights reserved. (Note: This title is now in its third edition, copyright © 2007.)

Activity 2: Lucky Draw? Fund Raiser

At the Winston School's Fall Fair, the Charity for Children Club is planning to run a money-raising booth. One of the members in the club proposed the following game.

Lucky Draw

There are equal numbers of red and blue balls buried in sawdust in each barrel.

10¢ Per Turn

One turn allows you to make
One Lucky Draw from each barrel.

Win $1.00

If you draw three balls of the same colour on one turn, you win $1.00.

Chris, the chairperson of the festival, likes the idea of the game, but she isn't sure whether this is a good moneymaker. You have been asked to prepare a report to the festival committee on this issue.

Your job is to recommend keeping the game, or to show how to modify it to make it a moneymaker. Support your conclusion with data and explanations.

Adapted with permission from *Mathematics Assessment: A Practical Handbook for Grades 6–8* (p. 127) by Pam Beck et al., copyright 2000 by the National Council of Teachers of Mathematics.

C. Applied Learning

Provide opportunities for the students to use their understanding of probability in a practical situation and notice whether or not the understanding transfers.

For example, have the students analyze a game of their own choice to predict the probability of winning. This could be a game from home or one that is available in the classroom.

Step 5: Follow-up on Assessment

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction?

A. Addressing Gaps in Learning

- Provide hands-on experiences that would allow the students to become more familiar with representing equally likely possibilities in tree diagrams or tables, calculating theoretical and experimental probability, and conducting experiments involving two given independent events.
- Provide the students with time to reflect on their learning by creating graphic organizers such as Frayer models for terms such as *independent events*, *tree diagrams* and *probability*.

B. Reinforcing and Extending Learning

Students who have achieved or exceeded the outcomes will benefit from ongoing opportunities to apply and extend their learning. These activities should support students in developing a deeper understanding of the concept and should not progress to the outcomes in subsequent grades.

Consider strategies such as:

Activity 1: The Game of SKUNK

Every day we all make choices such as: My mathematics teacher might collect homework today; should I do it? The game of Skunk, which can be found at http://illuminations.nctm.org/LessonDetail.aspx?id=L248, presents students with an experience that clearly involves both choice and chance. Skunk is a variation on a dice game also known as "pig" or "hold'em." The object of Skunk is to accumulate points by rolling the dice. Points are accumulated by making several "good" rolls in a row and choosing to stop before a "bad" roll comes and wipes out all your points. Skunk can be played by groups, by the whole class at once or by individuals.

(This activity adapted from National Council of Teachers of Mathematics, "The Game of Skunk," *NCTM Illuminations*, 2000–2008, http://illuminations.nctm.org/LessonDetail.aspx?id=L248 (Accessed April 25, 2008). Used with permission.)

Activity 2: **The Popcorn Problem**

The Popcorn Problem

You need: 10 each of six colours of cubes
a paper sack

A popcorn company found that sales improved when prizes were put in the popcorn boxes. They decided to include a felt-tip pen in every box and to use pens in six different colours. The company bought equal numbers of pens in each colour and were careful when shipping popcorn to stores to send boxes with the same number of each colour pen. When you buy a box, you have an equal chance to find any one of the six colours of pens inside.

About how many boxes of popcorn do you need to buy to have a good chance to get a complete set of six coloured pens? Explain your reasoning.

Try the following experiment to simulate the situation: Put six cubes in a sack, one each of six colours to represent the pens. Reach into the sack and, without looking, draw a cube. Note its colour and replace it. Continue until you have drawn out one of each colour, keeping track of the number of draws you make. How many draws do you think will be needed?

Repeat the experiment. Then compile results from other class members.

Discuss the popcorn problem in light of the information gathered from the simulations.

Extension: Repeat the simulation, this time putting 60 cubes in the sack, 10 each of six colours. Compare the results with the first simulation.

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