

Planning Guide

Grade 8 *Percents*

Number

Specific Outcome 3

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Planning Guide: Grade 8 Percents

Strand: Number

Specific Outcome: 3

This *Planning Guide* addresses the following outcomes from the Program of Studies:

Strand: Number

Specific Outcome: 3. Demonstrate an understanding of percents greater than or equal to 0%, including greater than 100%.

Curriculum Focus

The changes to the curriculum targeted by this sample include:

- The general outcome focuses on developing number sense; whereas the previous mathematics curriculum focused on illustrating the use of rate, ratio, percentages and decimals in solving problems.
- The specific outcome focuses on understanding percents greater than or equal to 0%, including greater than 100%; whereas the previous mathematics curriculum focused on representing and applying fractional percents, and percents greater than 100%, in fraction or decimal form and vice versa.

What Is a Planning Guide?

Planning Guides are a tool for teachers to use in designing instruction and assessment that focuses on developing and deepening students' understanding of mathematical concepts. This tool is based on the process outlined in *Understanding by Design* by Grant Wiggins and Jay McTighe.

Planning Steps

The following steps will help you through the Planning Guide:

- **Step 1: Identify Outcomes to Address** (p. 3)
- **Step 2: Determine Evidence of Student Learning** (p. 5)
- **Step 3: Plan for Instruction** (p. 6)
- **Step 4: Assess Student Learning** (p. 14)
- **Step 5: Follow-up on Assessment** (p. 21)

Step 1: Identify Outcomes to Address

Guiding Questions

- What do I want my students to learn?
- What can my students currently understand and do?
- What do I want my students to understand and be able to do based on the Big Ideas and specific outcomes in the program of studies?

Big Ideas

Percent is:

- a different notation and terminology for the known concept *hundredths*
- "when the decimal identifies the hundredths position as the units, the word *percent* can be specified as a synonym for hundredths. Thus, 0.659 (of some whole or 1) is 65.9 hundredths or 65.9 percent of the same whole ... the notion of placing the decimal point to identify the percent position is conceptually more meaningful than the apparently arbitrary rule: 'To change a decimal to a percent, move the decimal two places to the right.' A better idea is to equate hundredths with percent both orally and in notation" (Van de Walle and Lovin 2006, pp. 119–120).
- a way to write a fraction with a denominator of 100
- strongly connected to fraction and decimal concepts
- best linked to fractions and decimals by the use of models such as base 10 blocks, number lines, fraction strips, 10×10 grids and area models
- "particularly useful when comparing fractional parts of sets or numbers of unequal size," including percents less than 1% and greater than 100% (NCTM 2000, p. 217)
- frequently used in problem-solving situations encountered in every day life.

The exploration of relationships amongst fractions, decimals and percents develops number sense. Number sense means "having intuition about or a flexible understanding of numbers" (Van de Walle and Lovin 2006, p. 113).

Sequence of Outcomes from the Program of Studies

See <http://education.alberta.ca/teachers/core/math/programs.aspx> for the complete program of studies.

Grade 7	Grade 8	Grade 9
Specific Outcomes	Specific Outcomes	Specific Outcomes
3. Solve problems involving percents from 1% to 100%.	3. Demonstrate an understanding of percents greater than or equal to 0%, including greater than 100%.	There are no directly related specific outcomes in Grade 9.

Step 2: Determine Evidence of Student Learning

Guiding Questions

- What evidence will I look for to know that learning has occurred?
- What should students demonstrate to show their understanding of the mathematical concepts, skills and Big Ideas?

Using Achievement Indicators

As you begin planning lessons and learning activities, keep in mind ongoing ways to monitor and assess student learning. One starting point for this planning is to consider the achievement indicators listed in the *Mathematics Kindergarten to Grade 9 Program of Studies with Achievement Indicators*. You may also generate your own indicators and use them to guide your observation of the students.

The following indicators may be used to determine whether or not students have met this specific outcome. Can students:

- provide a context where a percent may be more than 100% or between 0% and 1%?
- represent a given fractional percent using grid paper?
- represent a given percent greater than 100% using grid paper?
- determine the percent represented by a given shaded region on a grid and record it in decimal, fraction and percent form?
- express a given percent in decimal or fraction form?
- express a given decimal in percent or fraction form?
- express a given fraction in decimal or percent form?
- solve a given problem involving percents?
- solve a given problem involving combined percents; e.g., addition of percents, such as GST + PST?
- solve a given problem that involves finding the percent of a percent; e.g., “A population increased by 10% one year and by 15% the next year. Explain why there was not a 25% increase in population over the two years”?

Sample behaviours to look for related to these indicators are suggested for some of the activities listed in **Step 3, Section C: Choosing Learning Activities** (p. 9).

Step 3: Plan for Instruction

Guiding Questions

- What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should I use?
- How will I meet the diverse learning needs of my students?

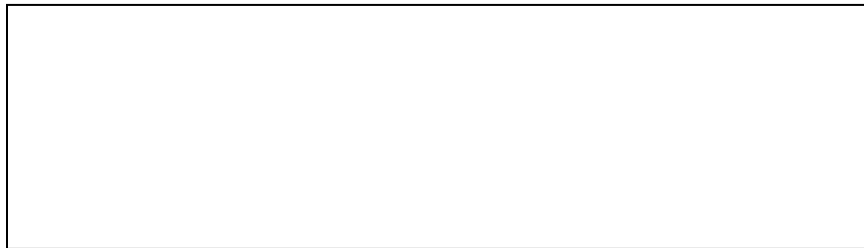
A. Assessing Prior Knowledge and Skills

Before introducing new material, consider ways to assess and build on students' knowledge and skills related to percents. For example:

Activity: Have the students solve the following science fair problem.

Three middle schools are going to have a science fair. The science fair will be in an auditorium. The amount of space given to each school is based on the number of students. Three Streams Middle School has about 1000 students, Boss Middle School has about 600 students and Jackson Middle School has about 400 students.

1. The rectangle below represents the auditorium. Divide the rectangle to show the amount of space each school should get on the basis of number of students. Label the sections TS for Three Streams, B for Boss and J for Jackson.

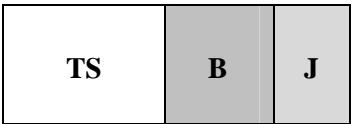


2. What fraction of the space should each school get on the basis of the number of students?
3. If the schools share the cost of the science fair on the basis of number of students, what percent of the cost should each school pay?
4. If the cost of the science fair is \$300, how much should each school pay on the basis of number of students? Justify your answers.

Adapted with permission from *Mathematics Assessment: A Practical Handbook for Grades 6–8* (p. 120) by Pam Beck et al., copyright 2000 by the National Council of Teachers of Mathematics.

If a student appears to have difficulty with these tasks, consider further individual assessment, such as a structured interview, to determine the student's level of skill and understanding. See **Sample Structured Interview: Assessing Prior Knowledge and Skills** (p. 7).

Sample Structured Interview: Assessing Prior Knowledge and Skills

Directions	Date:	
	Not Quite There	Ready to Apply
1. The rectangle shown in Step 3, Part A, Activity 1 represents the auditorium. Divide the rectangle to show the amount of space each school should get on the basis of number of students. Label the sections TS for Three Streams, B for Boss and J for Jackson.	1. Student does not recognize the rectangle as the whole or the size of each space is not reflective of the amounts presented in the question.	1. Student draws an accurate diagram, such as: 
2. What fraction of the space should each school get on the basis of number of students?	2. Student does not recognize that 2000 is the whole.	2. Student responds with the correct fractions of $\frac{1}{2}$, $\frac{3}{10}$ and $\frac{2}{5}$. Student shows correct calculations and provides an explanation, indicating that the whole is 2000, 1000 is $\frac{1}{2}$ of the whole, 600 is $\frac{3}{10}$ of the whole and 400 is $\frac{2}{5}$ of the whole.
3. If the schools share the cost of the science fair on the basis of the number of students, what percent of the cost should each school pay?	3. Student is unable to convert the fractions to percent.	3. Student responds with the correct percentages of 50%, 30% and 20%. Student has calculations and explanations to justify his or her answers.
4. If the cost of the science fair is \$300, how much should each school pay on the basis of number of students? Justify your answers.	4. Student is unable to determine cost based on the percentages.	4. Student provides correct calculations and answers of \$150, \$90 and \$60 respectively.

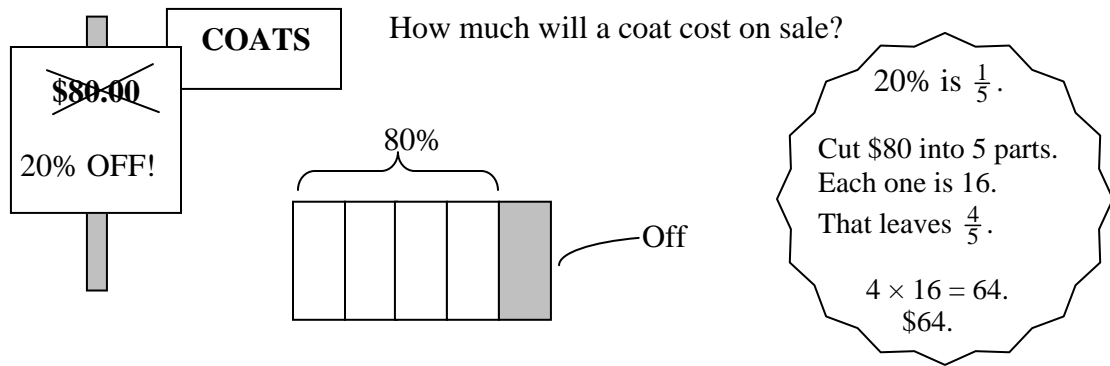
B. Choosing Instructional Strategies

Consider the following strategies when planning lessons:

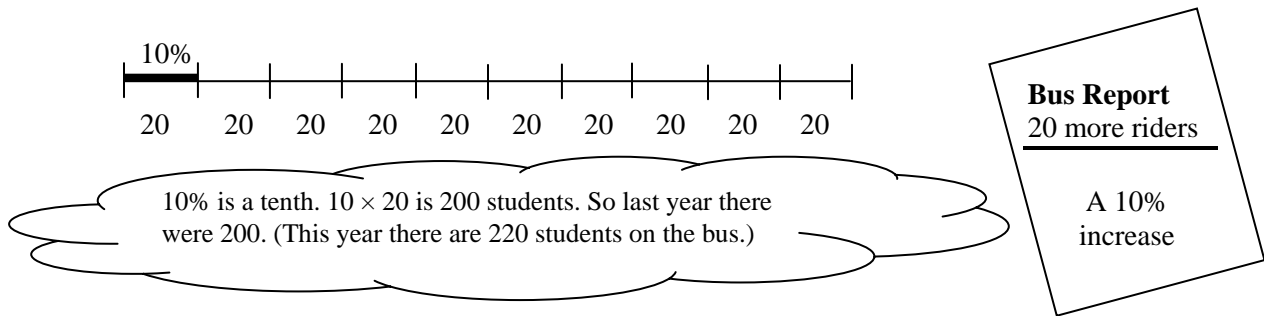
- Build on students' understanding of percents between 1% and 100% from the previous grade.
- Problems involving percent should be presented in realistic and relevant contexts.
- For percents less than 1%, use familiar fractions (halves, thirds, fourths, fifths and eighths) or easy percents ($\frac{1}{10}$, $\frac{1}{100}$), and use numbers compatible with these fractions. The focus of these exercises is the relationships involved, not complex computational skills (see examples below).
- Allow the students to develop their own strategies when solving problems.
- Use the terms part, whole and percent (or fraction). Fraction and percent are interchangeable. Help the students connect percentage exercises to the same types of exercises they did with simple fractions.
- Encourage the students to use models or drawings to explain their solutions. Remember that the purpose is the exploration of relationships, not computation skill.
- Encourage mental calculation in the solution of percentages.

Some points adapted from John A. Van de Walle, LouAnn H. Lovin, *Teaching Student-Centered Mathematics: Grades 5–8*, 1e (p. 121). Published by Allyn and Bacon, Boston, MA. Copyright © 2006 by Pearson Education. Reprinted by permission of the publisher.

Note: These examples also illustrate how simple drawings can help with reasoning.



This year, 20 more students rode the bus than last year.
If that is a 10% increase, how many rode the bus last year?



Examples adapted from John A. Van de Walle, LouAnn H. Lovin, *Teaching Student-Centered Mathematics: Grades 5–8*, 1e (p. 122) Person Education. Reprinted by permission of the publisher.

C. Choosing Learning Activities

The following learning activities are examples of activities that could be used to develop student understanding of the concepts identified in Step 1.

Sample Activities:

1. **Representing Percents** (p. 10)
2. **Benchmarks** (p. 13)

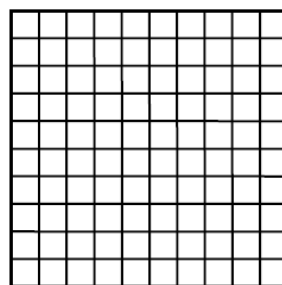
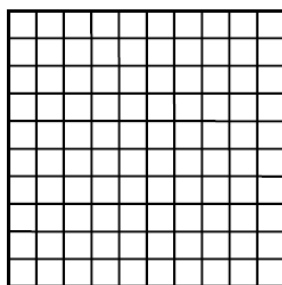
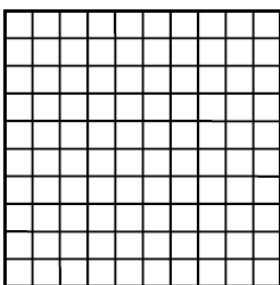
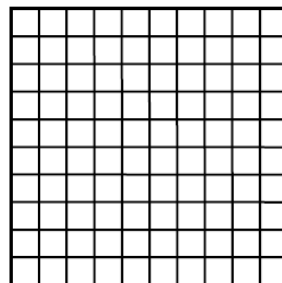
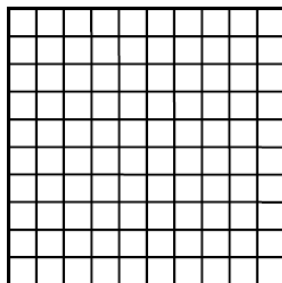
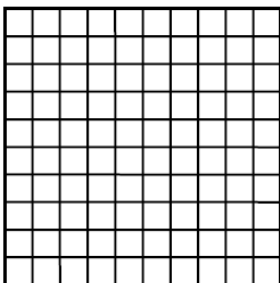
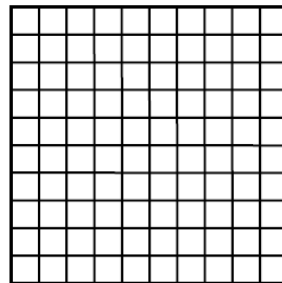
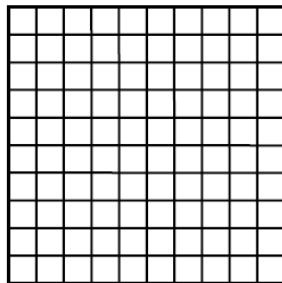
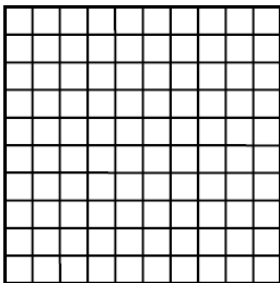
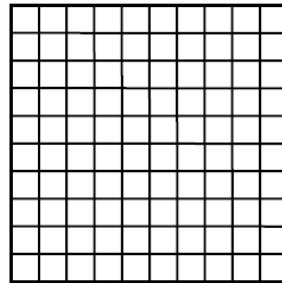
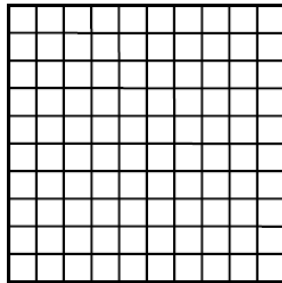
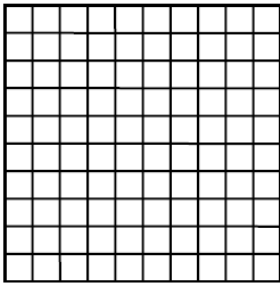
Sample Activity 1: Representing Percents

This activity adapted from National Council of Teachers of Mathematics, "Grid and Percent It," *NCTM Illuminations*, 2000–2008, <http://illuminations.nctm.org/LessonDetail.aspx?ID=L249> (Accessed April 2008). Used with permission.

- Distribute a grid worksheet to all students (blackline master provided on the following page).

10 × 10 Grids

NAME _____

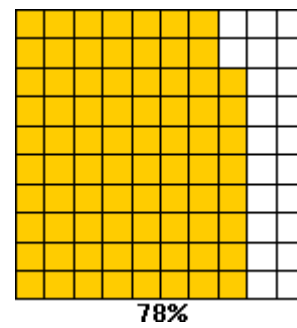
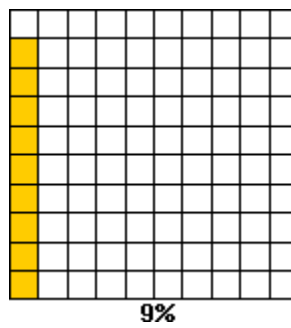
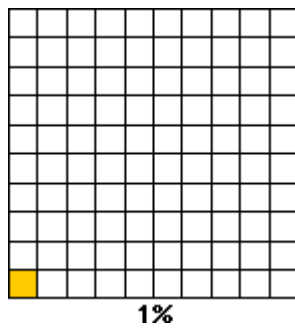


- Ask the students to represent various percents on these grids—be sure to include whole number percents such as 10%, 39% and 84% as well as decimals and fractions such as 28.5% and $76\frac{1}{2}\%$.
- Display some shaded grids on the overhead, and have the students determine the percent for a shaded amount (see the examples below). Again, be sure to include decimals and fractions.

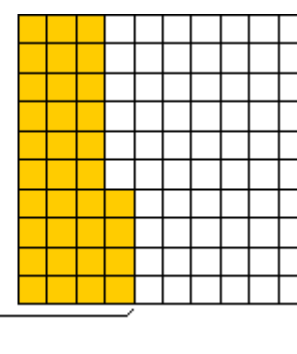
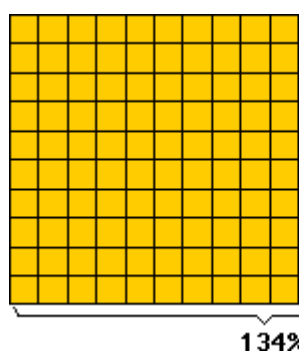
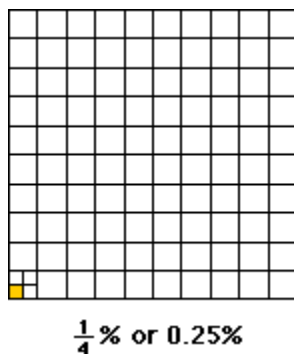
Look For ...

Do students:

- recognize that 100% is represented by one whole square (10×10 grid) and that one percent is represented by one small square, or one hundredth of a unit square?
- recognize that a percent between 0 and 1 is represented within one small square?
- recognize that a percent greater than 100 is represented by more than one whole square?



- Have the students represent percents less than 1 percent and greater than 100 percent. The figures below show that when $\frac{1}{4}$ of a square is shaded, this represents $\frac{1}{4}\%$, and when one entire unit square plus another 34 small squares are shaded, this represents 134%.



- Have the students represent $\frac{1}{2}\%$, $\frac{3}{4}\%$, 116%, 232% and others on the grids and provide explanations of what each representation means.

Sample Activity 2: Benchmarks

Research indicates that it is helpful for students to be able to calculate percents without first having to convert the number to a decimal or a fraction. The use of benchmarks, such as 50% and 10%, allows students to see other percentages of a number such as 23%, 25%, 30% and so on. Have the students solve the following problem using mental calculations.

There are 2000 students in our high school. How many students make up 1% of the student body? How many students make up 10% of the student body? If 30% of the student body are freshmen, how many freshmen are there? If the senior class makes up 22% of the population, how many seniors are there? If $\frac{1}{2}$ % of the student body are red heads, how many red heads are there?

Have the students share how they came up with the answers. Some possible responses follow:

- 1% of the student body is 20.
- 10% of the student body is 200 (10 times the 1% amount)
- 30% of the freshmen is 600 (2 times the 10% amount or 30 times the 1% amount)
- 22% of the senior class is 440 (2 times the 10% amount + 2 times the 1% amount or the 30% amount – 8 times the 1% amount).
- $\frac{1}{2}$ % is 10 ($\frac{1}{2}$ the 1% amount).

Have the students work in groups to solve the following problem and share their answers.

Mark is the manager of The Athlete Shop and is in charge of pricing sale items. In his job, he often needs to mentally compute percentages of a specific dollar amount. A jacket is currently priced at \$80.00 (Wheatley and Abshire 2002, p. 220).

- a. How could Mark use benchmarks to calculate 20%, 25%, 33% and 75% of the price of the jacket?
- b. Would the use of benchmarks be a useful tool for Mark to use?
- c. If the mark-up on clothing is 200%, what is the cost of the jacket to the owner of the store?

Step 4: Assess Student Learning

Guiding Questions

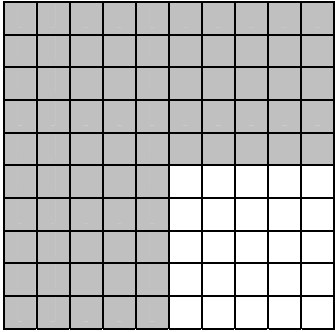
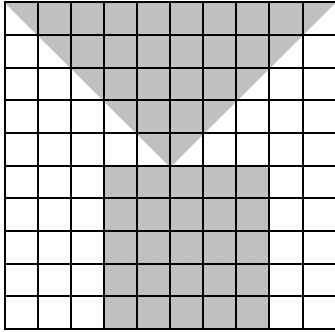
- Look back at what you determined as acceptable evidence in Step 2.
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

In addition to ongoing assessment throughout the lessons, consider the following sample activities to evaluate students' learning at key milestones. Suggestions are given for assessing all students as a class or in groups, individual students in need of further evaluation, and individual or groups of students in a variety of contexts.

A. Whole Class/Group Assessment

Activity 1: In groups, have the students write what they have learned about percent. Encourage the use of a Frayer model, a modified Frayer model or a concept definition map. An example and blackline masters have been provided.

Modified Frayer Model for Percents

Definition	Examples	Non-examples
<ul style="list-style-type: none"> • a way to write a fraction with a denominator of 100 • another name for hundredths 	<p>3%</p> <p>1.5%</p> <p>100%</p> <p>$\frac{1}{4}$ %</p> <p>250%</p>	<p>0.3</p> <p>$\frac{3}{4}$</p> <p>6.25</p> <p>14</p> <p>275</p>
Visual and Numeric Representation	Word Problem	
<div style="text-align: center;">  </div> <p>The diagram represents 75%.</p> <div style="text-align: center;">  </div> <p>The diagram represents 50%.</p>	<p>A package of cupcakes contains the following varieties: 6 chocolate, 2 coconut, 1 lemon-lime, 1 peppermint and 2 peanut butter. What percentage of the cupcakes are peanut butter?</p>	

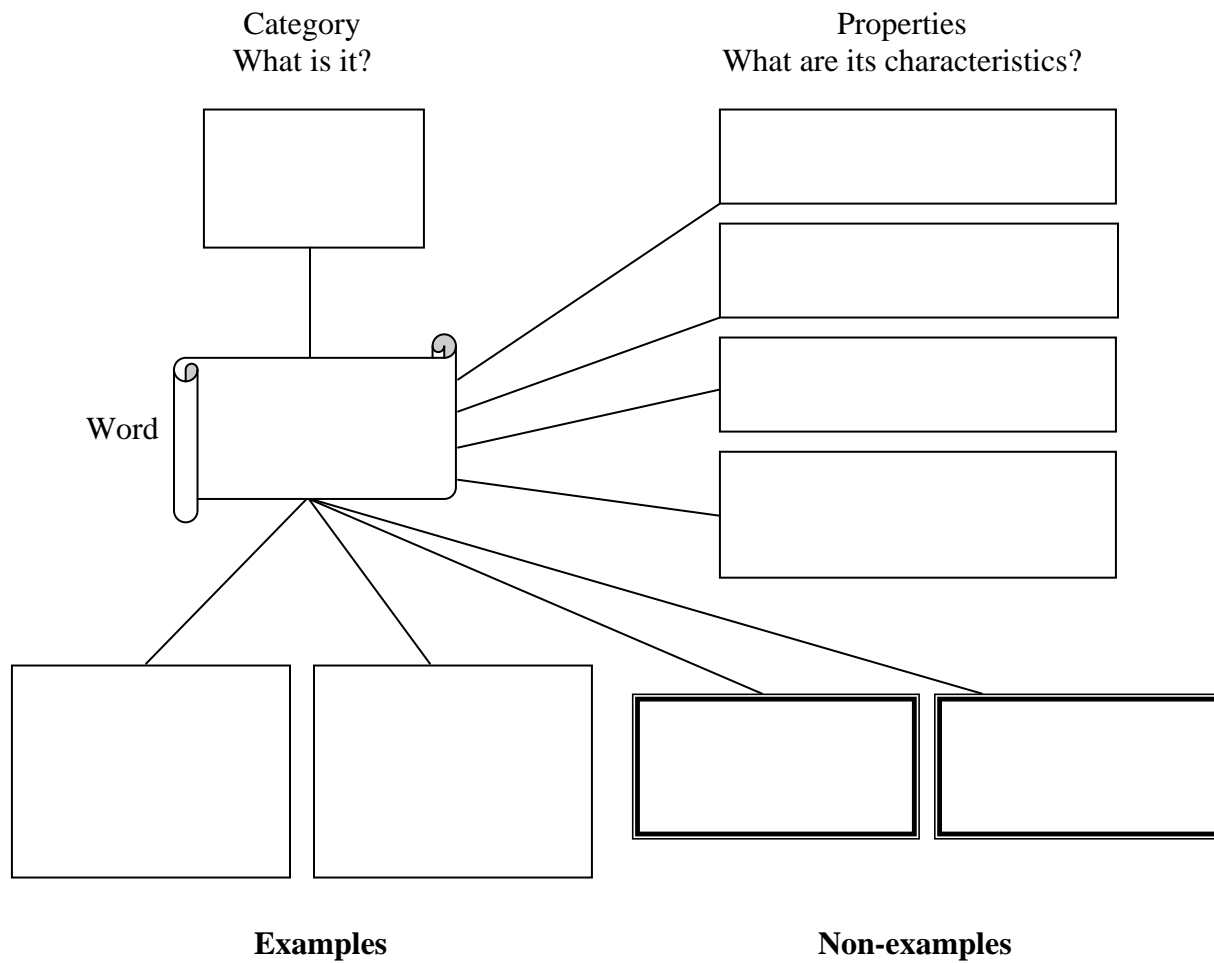
Format adapted from D. A. Frayer, W. C. Frederick and H. J. Klausmeier, *A Schema for Testing the Level of Concept Mastery* (Working Paper/Technical Report No. 16) (Madison, WI: Research and Development Center for Cognitive Learning, University of Wisconsin, 1969). Adapted with permission from the Wisconsin Center for Education Research, University of Wisconsin-Madison.

Modified Frayer Model for _____

<p>Definition</p>	<p>Examples</p> <p>Non-examples</p>
<p>Visual and Numeric Representation</p>	<p>Word Problem</p>

Format adapted from D. A. Frayer, W. C. Frederick and H. J. Klausmeier, *A Schema for Testing the Level of Concept Mastery* (Working Paper/Technical Report No. 16) (Madison, WI: Research and Development Center for Cognitive Learning, University of Wisconsin, 1969). Adapted with permission from the Wisconsin Center for Education Research, University of Wisconsin-Madison.

Concept Definition Map



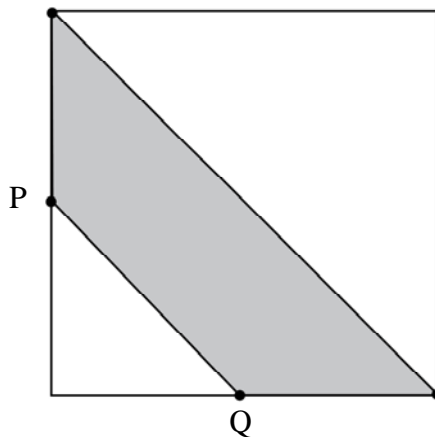
Adapted from Robert M. Schwartz, "Learning to Learn Vocabulary in Content Area Textbooks," *Journal of Reading* 32, 2 (November 1988), p. 110, Example 1. Adapted with permission from International Reading Association.

Activity 2: Find 75% of \$20. Show how to get your answer three different ways.

Activity 3: In groups, have the students write problems involving the everyday use of percents showing how percent benchmarks could be useful. Encourage the students to try to find examples using percents between 0% and 1% and greater than 100%. Have the students exchange papers and solve each other's problems.

B. One-on-one Assessment

Activity 1: What fraction of the square shown below is shaded? Record your answer in decimal, fraction and percent forms and provide an explanation. Points P and Q are midway along their sides.



This activity adapted with permission from Grayson H. Wheatley and George E. Abshire, *Developing Mathematical Fluency: Activities for Grades 5–8* (Tallahassee, FL: Mathematics Learning, 2002), p. 53.
www.mathematicslearning.org

Activity 2: Solve the following problems.

- A recent article in the *Calgary Herald* stated that Peter Munk, an 80-year-old billionaire, made rather a lot of money in 2006 when he sold one of his companies for \$4.8 billion, tripling his investment.
 - a. Explain the meaning of "tripling" his investment.
 - b. Find the value of his original investment.

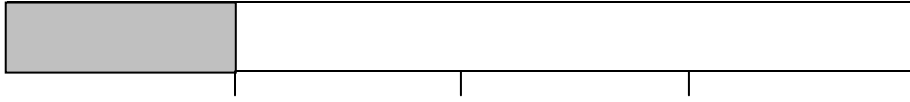
Justify your solutions with a drawing and an explanation.

- The same article stated that when Mr. Munk first got into the gold business 25 years ago, his business was worth \$40 million and the price of gold was \$220 an ounce. Now the price of gold is around \$950 an ounce and his business has a value of more than \$40 billion. Calculate the percent increase of:
 - a. the price of gold, and
 - b. the net worth of Mr. Munk's company.

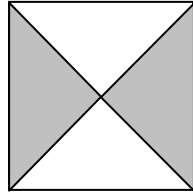
Be sure to justify your calculations with diagrams and an explanation.

Activity 3: Parts of the following figures are shaded. Express these shaded parts as fractions, decimals and percents.

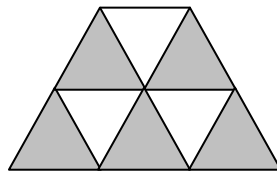
a.



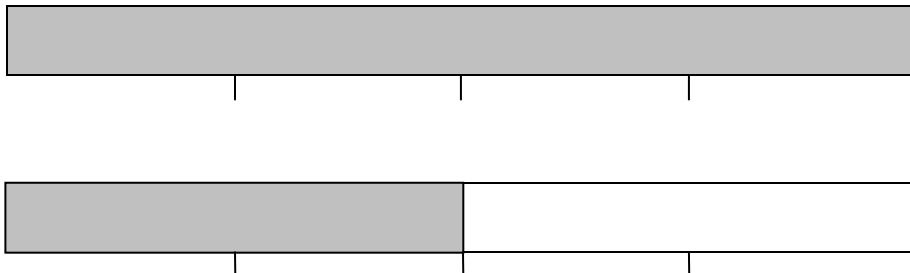
b.



c.



d.



Activity 4: You are travelling in British Columbia with your family. The family stops at a restaurant for lunch and orders pizza. They ordered two large pizzas: one house special and one pepperoni and mushroom. Mom and Dad order coffee. You and your brother order a pop. The waitress brought your drinks, including a glass of water for each person, right away. The pizzas were ready soon afterward, and she brought them right over, with a stack of extra napkins. The bill was \$34.80 before tax. Answer the following questions:

- What percent tip would you give and what amount would it be?
- In British Columbia, you must pay 7% PST and 5% GST. What was the final total of the bill for your family's lunch? (PST is calculated first and then the GST).

This activity adapted from Burns, Marilyn. *About Teaching Mathematics: A K–8 Resource, Second Edition*, p. 250. Copyright © 2000 by Math Solutions Publications. Adapted by permission. All rights reserved. (Note: This title is now in its third edition, copyright © 2007.)

Activity 5: Have the students solve the following problems and provide explanations and diagrams with their solutions.

- Boss Middle School has an enrollment of 575 students. This represents 320 families who attend the school. At last night's PTA meeting, 70 percent of the families were represented. How many families were represented at the meeting?
- Margaret has read 40 out of the 320 pages from her latest book club read. What percentage of the book has she read?
- In Hazard County, 38 percent or 57 of the schools have a pupil-teacher ratio that meets or exceeds the requirements set out by the Department of Education in that area. How many schools are in Hazard County?

These problems adapted from John A. Van de Walle, LouAnn H. Lovin, *Teaching Student-Centered Mathematics: Grades 5–8*, 1e (p. 121). Published by Allyn and Bacon, Boston, MA. Copyright © 2006 by Pearson Education. Reprinted by permission of the publisher. AND from National Council of Teachers of Mathematics, "Grid and Percent It," *NCTM Illuminations*, 2000–2008, <http://illuminations.nctm.org/LessonDetail.aspx?ID=L249> (Accessed April 2008). Used with permission.

C. Applied Learning

Provide opportunities for the students to use what they have learned about percents from 0% to 1% and greater than 100% in a practical situation and assess whether or not knowledge transfers.

Activity 1: Have the students look for articles (not advertisements) in which percents are used. Choose one to present to the class. In your presentation, explain the meaning of percents.

Activity 2: Provide the students with six copies of a cartoon or drawing, including one full size, three reductions and two enlargements. Have the students work in groups to figure out what percent was used for each reduction and enlargement. The groups should record their solutions and be prepared to share an explanation of the procedures they used with the class.

Activities 1 and 2 adapted from Burns, Marilyn. *About Teaching Mathematics: A K–8 Resource, Second Edition*, p. 250. Copyright © 2000 by Math Solutions Publications. Adapted by permission. All rights reserved. (Note: This title is now in its third edition, copyright © 2007.)

Step 5: Follow-up on Assessment

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction?

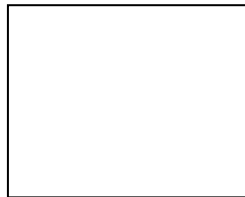
A. Addressing Gaps in Learning

Activity: Have the students who are having difficulties review some basic fraction concepts and then relate them to decimals and percents. For example:

- a. If this strip represents one whole, find one half, one quarter, three fifths and five quarters. What percents do these fractions represent?



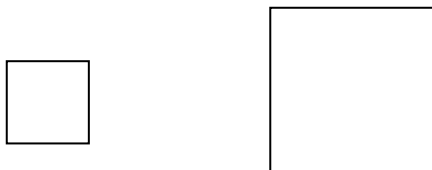
- b. If this rectangle is one quarter, what would the whole look like? What percent is the original rectangle of the whole?



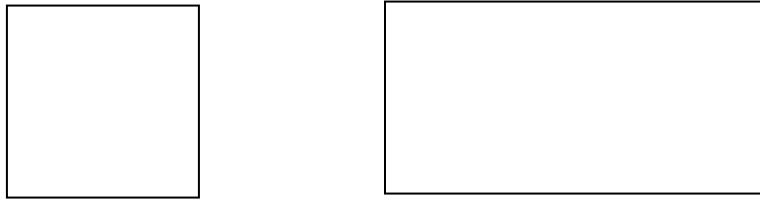
- c. If this rectangle represents three fourths of the whole, draw a shape that could be the whole. What percentage is the original rectangle of the whole?



- d. What fraction of the big square does the small square represent? What percent is this?



e. What fraction of the big square does the small square represent below? What percent is this?



This activity adapted from John A. Van de Walle, LouAnn H. Lovin, *Teaching Student-Centered Mathematics: Grades 5–8*, 1e (pp. 71, 72). Published by Allyn and Bacon, Boston, MA. Copyright © 2006 by Pearson Education. Reprinted by permission of the publisher.

B. Reinforcing and Extending Learning

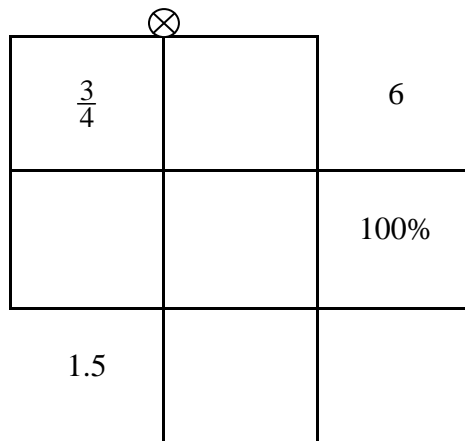
Students who have achieved or exceeded the outcomes will benefit from ongoing opportunities to apply and extend their learning. These activities should support students in developing a deeper understanding of the concept and should not progress to the outcomes in subsequent grades.

Consider strategies such as:

Activity: Fraction–Decimal–Percent Two Ways

The three numbers in any row or column must form a correct multiplication sentence. The self-checking nature of these tasks is an asset in providing immediate feedback. Have the students share their strategies for solving the two-ways. Encourage the students to develop their own two-ways and have other students in the class complete them.

1.



2.

⊗		
2	10.5	
$\frac{1}{6}$	200%	

3.

⊗		
$\frac{1}{4}$	40%	
		10
.5		

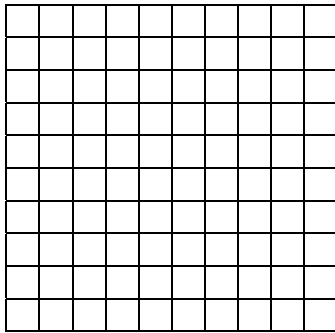
4.

⊗		
25%	40%	
	.1	
$1\frac{1}{4}$		

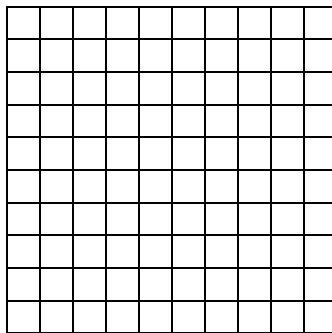
These two-ways reproduced with permission from Grayson H. Wheatley and George E. Abshire, *Developing Mathematical Fluency: Activities for Grades 5–8* (Tallahassee, FL: Mathematics Learning, 2002), p. 247.
www.mathematicslearning.org

Percents Units

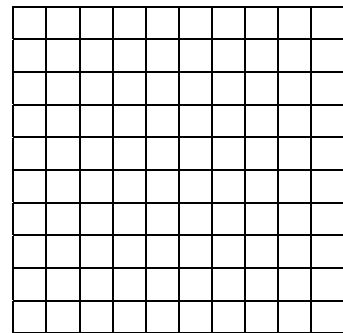
Shade the given percent of the region.



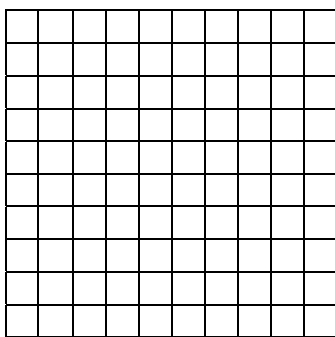
50%



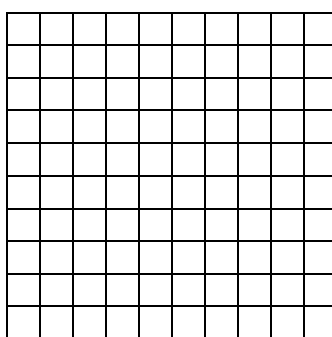
25%



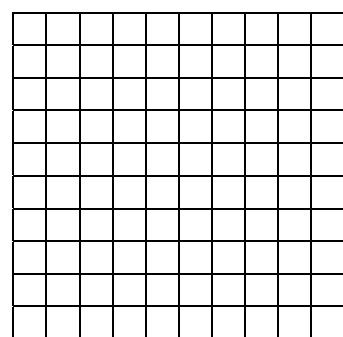
75%



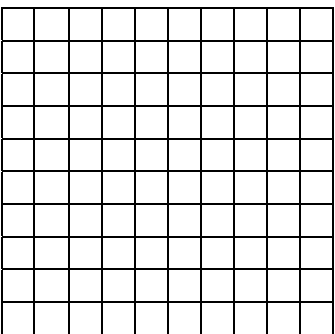
10%



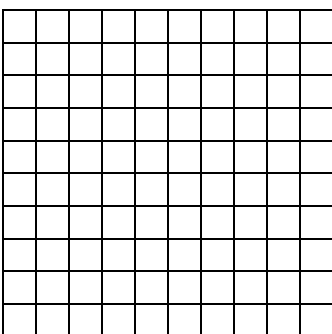
20%



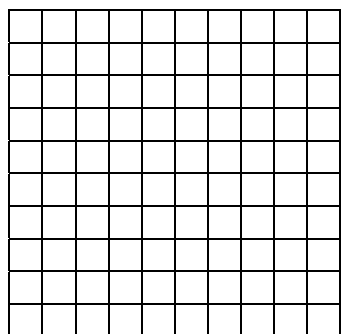
40%



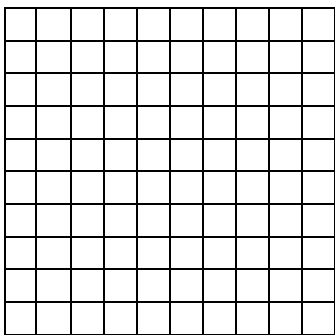
30%



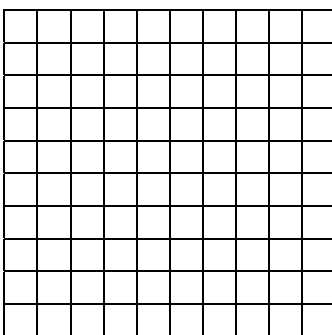
60%



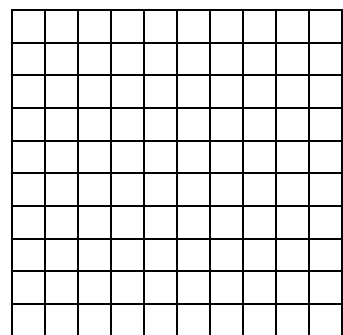
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28%



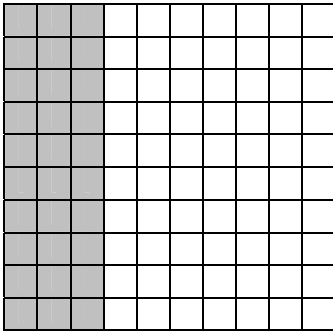
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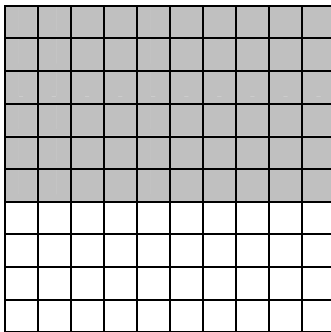


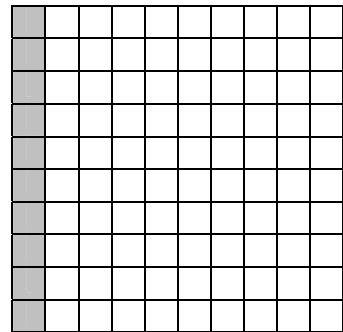
1%

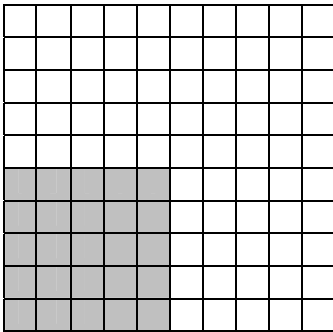
Reproduced with permission from Grayson H. Wheatley and George E. Abshire, *Developing Mathematical Fluency: Activities for Grades 5–8* (Tallahassee, FL: Mathematics Learning, 2002), p. 216. www.mathematicslearning.org

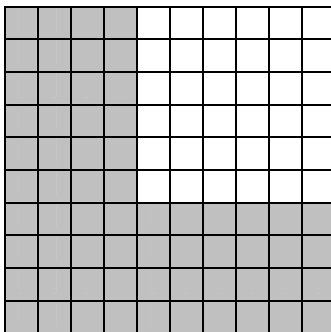
Write the percent of each region that is shaded.

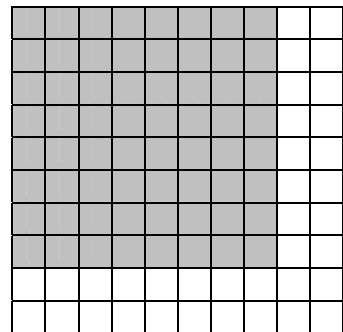


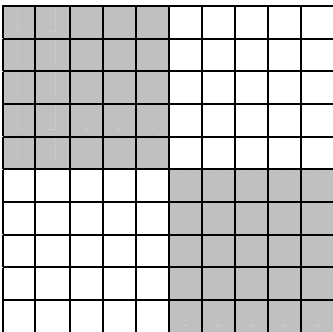


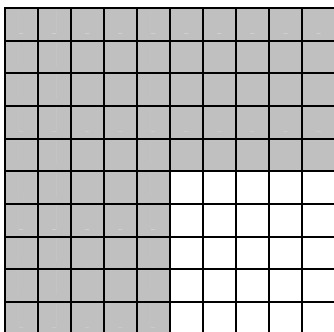


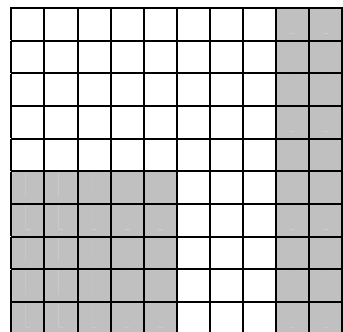


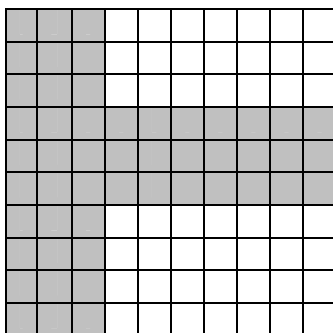


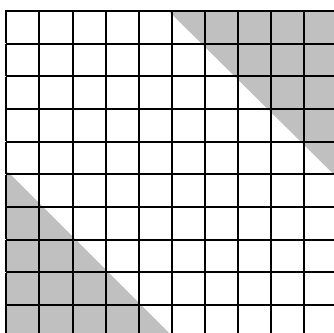


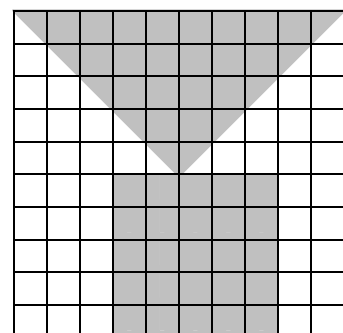












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