Mathematics

Planning Guide

Grade 8
Perfect Squares and Square Roots

Number
Specific Outcomes 1 and 2

This Planning Guide can be accessed online at:
http://www.learnalberta.ca/content/mepg8/html/pg8_perfectsquaresandsquareroots/index.html

Table of Contents

Planning Guide: *Grade 8 Perfect Squares and Square Roots*

Strand: Number
Specific Outcomes: 1 and 2

This *Planning Guide* addresses the following outcomes from the program of studies:

Strand: Number	
Specific Outcomes:	1. Demonstrate an understanding of perfect squares and square roots, concretely, pictorially and symbolically (limited to whole numbers). 2. Determine the approximate square root of numbers that are not perfect squares (limited to whole numbers).

Curriculum Focus

This sample targets the following changes to the curriculum:

- The general outcome focus is to develop number sense; this has not changed from the previous curriculum.
- The specific outcomes focus on understanding perfect squares and square roots and determining approximate square roots of whole numbers that are not perfect squares.
- A specific outcome in the previous program of studies required students to distinguish between a square root and its decimal approximation on a calculator. In the current program of studies, students use technology to approximate the square root of a given number that is not a perfect square and explain why it is an approximation. These achievement indicators may be used to determine whether students have met Specific Outcome 2.

What Is a Planning Guide?

Planning Guides are a tool for teachers to use in designing instruction and assessment that focuses on developing and deepening students' understanding of mathematical concepts. This tool is based on the process outlined in *Understanding by Design* by Grant Wiggins and Jay McTighe.

Planning Steps

The following steps will help you through the Planning Guide:

Step 1: Identify Outcomes to Address

Guiding Questions

- What do I want my students to learn?
- What can my students currently understand and do?
- What do I want my students to understand and be able to do based on the Big Ideas and specific outcomes in the program of studies?

Big Ideas

- Perfect squares can be represented by a square region. The side length of the square region is the square root of the perfect square.
- The product of two identical factors is a perfect square.
- When you take the square root of a perfect square, the result is one of the two identical factors.
- Squaring a number and taking the square root of the resulting product are inverse operations.
- The square root of a non-perfect square can be approximated as a decimal.

Sequence of Outcomes from the Program of Studies

See http://education.alberta.ca/teachers/program/math/educator/progstudy.aspx for the complete program of studies.

Grade 7	Grade 8	Grade 9
Specific Outcomes	**Specific Outcomes**	**Specific Outcomes**
	1. Demonstrate an understanding of perfect squares and square roots, concretely, pictorially and symbolically (limited to whole numbers). 2. Determine the approximate square root of numbers that are not perfect squares (limited to whole numbers).	5. Determine the square root of positive rational numbers that are perfect squares. 6. Determine an approximate square root of positive rational numbers that are non-perfect squares.

Step 2: Determine Evidence of Student Learning

Guiding Questions

- What evidence will I look for to know that learning has occurred?
- What should students demonstrate to show their understanding of the mathematical concepts, skills and Big Ideas?

Using Achievement Indicators

As you begin planning lessons and learning activities, keep in mind ongoing ways to monitor and assess student learning. One starting point for this planning is to consider the achievement indicators listed in *The Alberta K–9 Mathematics Program of Studies with Achievement Indicators*. You may also generate your own indicators and use them to guide your observation of the students.

The following indicators may be used to determine whether or not students have met the specific outcome for demonstrating and understanding perfect squares and square roots. Can students:

- represent a given perfect square as a square region, using materials, such as grid paper or square shapes?
- determine the factors of a given perfect square and explain why one of the factors is the square root and the others are not?
- determine whether or not a given number is a perfect square, using materials and strategies such as square shapes, grid paper or prime factorization, and explain the reasoning?
- determine the square root of a given perfect square and record it symbolically?
- determine the square of a given number?

The following indicators may be used to determine whether or not students have met the specific outcome for approximating square roots. Can students:

- estimate the square root of a given number that is not a perfect square, using the roots of perfect squares as benchmarks?
- approximate the square root of a given number that is not a perfect square, using technology; e.g., calculator, computer?
- explain why the square root of a number shown on a calculator may be an approximation?
- identify a number with a square root that is between two given numbers?

Sample behaviours to look for related to these indicators are suggested for some of the activities listed in **Step 3, Section C: Choosing Learning Activities** (p. 10).

Step 3: Plan for Instruction

Guiding Questions

- What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should I use?
- How will I meet the diverse learning needs of my students?

A. Assessing Prior Knowledge and Skills

Before introducing new material, consider ways to assess and build on students' knowledge and skills related to using factors of numbers, determining area, and comparing and ordering numbers outlined in earlier grades. For example:

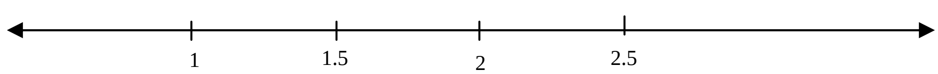
Activity 1: Use the divisibility rules for 2 and 5 to organize the numbers 215, 150, 80, 460 and 256 into the following Carroll diagram.

	Divisible by 5	Not divisible by 5
Divisible by 2		
Not divisible by 2		

Activity 2: Have students write the factors of whole numbers; e.g., 10, 24. Ask them to explain how they determined the factors.

Activity 3: Using 1 cm dot paper, have students draw all the possible rectangles that will have an area of 36 cm^2.

Activity 4: Use a number line to order the following numbers: 1.92, 2.04, 1.97 and 2.14.



Activity 5: Order the following numbers in each set from least to greatest and explain how you know:

a. 5.771, 5.32, 3.55, 7.1
b. 4.2, 1.22, 3.518, 0.34.

If a student appears to have difficulty with these tasks, consider further individual assessment, such as a structured interview, to determine the student's level of skill and understanding. See **Sample Structured Interview: Assessing Prior Knowledge and Skills** (p. 8).

Sample Structured Interview: Assessing Prior Knowledge and Skills

<table>
<tr><th rowspan="2">Directions</th><th colspan="2">Date:</th></tr>
<tr><th>Not Quite There</th><th>Ready to Apply</th></tr>
<tr>
<td>Have the student explain how he or she knows if a number is divisible by 2 or 5. Then use the divisibility rules for 2 and 5 to organize the numbers 215, 150, 80, 460 and 256 into a Carroll diagram.

<table>
<tr><th></th><th>Divisible by 5</th><th>Not divisible by 5</th></tr>
<tr><td>Divisible by 2</td><td></td><td></td></tr>
<tr><td>Not divisible by 2</td><td></td><td></td></tr>
</table></td>
<td>The student is unable to explain the divisibility rules for 2 and/or 5.

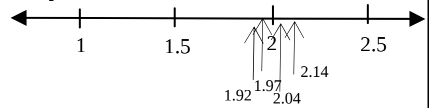
The student is able to explain the divisibility rules for 2 or 5 but is unable to apply the rules to determine where the numbers 215, 150, 80, 460 and 256 fit into the Carroll diagram.</td>
<td>The student responds that if a number is even, it is divisible by 2 or if the number ends in 0 or 5, the number is divisible by 5.

The student completes the Carroll diagram correctly.

<table>
<tr><th></th><th>Divisible by 5</th><th>Not divisible by 5</th></tr>
<tr><td>Divisible by 2</td><td>150, 80, 460</td><td>256</td></tr>
<tr><td>Not divisible by 2</td><td>215</td><td></td></tr>
</table></td>
</tr>
<tr>
<td>Ask the student to determine all the factors of 10 or 24. Ask the student to explain how he or she determined the factors.</td>
<td>The student responds with a partial list of factors for 10 or 24.

The student is unable to determine all the factors in a systematic way.</td>
<td>The student responds with the list of factors.
• The factors of 10 are 1, 2, 5 and 10.
• The factors of 24 are 1, 2, 3, 4, 6, 8, 12 and 24.
• The student can find all the factors by creating an organized list of the factor pairs and then listing the factors:
1 × 24
2 × 12
3 × 8
4 × 6.</td>
</tr>
<tr>
<td>Have the student use dot paper to draw all the possible rectangles that will have an area of 36 cm^2.</td>
<td>The student confuses perimeter and area and draws shapes that have a perimeter of 36 units.

The student is not able to draw all the possible rectangles of 36 square units, or repeats rectangles that are in a different orientation.</td>
<td>The student creates the following rectangles: 1 × 36, 2 × 18, 3 × 12, 4 × 9 and 6 × 6.</td>
</tr>
</table>

Ask the student to arrange the numbers 1.92, 2.04, 1.97 and 2.14 on the given number line.	The student is unable to place the numbers correctly on the number line.	The student places the number correctly on the number line. 
Ask the student to order the following numbers in each set from least to greatest and ask the student to explain his or her thinking: a. 5.771, 5.32, 3.55, 7.1 b. 4.2, 1.22, 3.518, 0.34.	The student is unable to order the numbers correctly.	The student is able to order the following numbers correctly from least to greatest: a. 3.55, 5.32, 5.771, 7.1 b. 0.34, 1.22, 3.518, 4.2 and explain his or her thinking.

B. Choosing Instructional Strategies

Consider the following guidelines for teaching perfect squares and square roots:

- Build on the students' understanding of factors and area to model and recognize perfect squares and square roots.
- Provide a variety of hands-on activities using manipulatives, such as tiles, 1 cm grid paper, rulers and calculators.
- Provide various strategies to estimate and calculate square roots.
- Provide opportunities for students to develop number sense by choosing activities that allow them to use benchmarks to facilitate calculating whole number square roots.

C. Choosing Learning Activities

The following learning activities are examples that could be used to develop student understanding of the concepts identified in Step 1.

Sample Activities:

Sample Activity 1: Modelling Square Numbers

The following activity involves modelling perfect squares and determining the square root as the side length of the square.

Directions: Each group of students will need 100 square tiles and 1 cm grid paper.

Use the tiles to model squares with side lengths of 1, 2, 3 …
Sketch the squares on the grid paper.
Use models and diagrams to complete the chart.

Side length	Number of squares in the area
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

Look For …
Do students:
- ☐ confuse finding the square and the square root of a number?

Discussion:
How is the side length of each square related to its area?
Why would numbers like 1, 4, 9, 16 … be called perfect squares?

The product of number by itself is called a **perfect square**.⋆
When we multiply a number by itself, we say we *square* the number.
Since $3 \times 3 = 3^2 = 9$, the number 3 is called the *square root* of 9.
We write $\sqrt{9} = 3$.

How do the squares you have drawn show the square roots of 9, 16 and 25?
What is the square root of 100?

⋆ A perfect square for whole numbers is any whole number that can be expressed as the square of another whole number.

Sample Activity 2: Recognizing Perfect Squares

The following activity provides opportunities for students to recognize perfect squares and square roots.

Materials: Use tiles and/or grid paper.

Try and find all of the perfect squares between 100 and 200 or 200 and 300.
Complete the table.

Side length	Number of squares in the area

Look For …
Do students:
- ☐ establish that perfect squares can be generated by squaring whole numbers?

Sample Activity 3: Patterns in Perfect Squares

The following activity encourages students to recognize the patterns in the final digits of sequential perfect squares.

Fill in the rest of the numbers in the chart. Circle the digit in the *ones* place.

n^2	Square number
0^2	0
1^2	1
2^2	4
3^2	9
4^2	16
5^2	
6^2	
7^2	
8^2	
9^2	
10^2	

Look For …

Do students:

- ☐ see another pattern between sequential perfect squares? (The difference between the perfect squares is increasing and forms the set of odd numbers.)

1. Describe any patterns you see.
2. Which square numbers have the same *ones* digit?
3. Which square numbers have a *ones* digit that is different from that in any other square number?

   Continue the table and record the squares of the numbers from 11 to 20.
   Again, examine the *ones* digit in each square number. Describe any patterns you see.
4. Does the pattern continue? Explain.

Sample Activity 4: Determining Square Roots from Factors

The following activity explores using factors to determine the square roots of perfect squares.

In this list of whole numbers, circle the numbers that are perfect squares:

- 24
- 25
- 45
- 59
- 64
- 100.

Look For …
Do students: ☐ find all of the factors for each number? ☐ have a personal strategy to find factors?

Determine all of the factors by determining factor pairs (the product of the factor pair is equal to the number) for each of the numbers above.

Discussion:

- Which numbers have an even number of factors?
- Which numbers have an odd number of factors?
- What do you notice about the factors of perfect squares?
- How could you determine the square root of a perfect square from your list of factor pairs?

Using Factors to Draw Rectangles
Use 1 cm dot paper to complete the following activity:

- use all of the factors of 24 as dimensions of a rectangle having an area of 24 square units
- use all of the factors of 25 as dimensions of a rectangle having an area of 25 square units.

The number 25 is a called a *square number* or a *perfect square* and 24 is not; explain using your diagrams why these names are used for 25 but not for 24.

Which diagram shows the square root of 25? Explain how you know.

Sample Activity 5: Estimating Square Roots of Non-perfect Squares

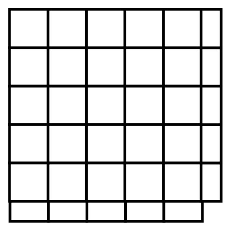
The following activity explores estimation of square roots that are not perfect squares.

From the previous activities, you know that squares of whole numbers are called perfect squares; therefore, you have been finding the square roots of perfect squares.

What if you are asked to find the square root of a number such as 30, which is not a perfect square?

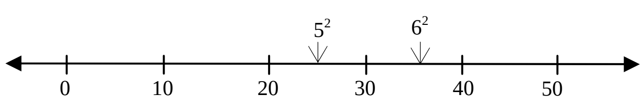
Method 1:

- Use 30 square tiles to make the biggest square possible. State the dimensions of the square. How many squares are left over?
- Create your square on grid paper and cut out the square. Cut out a strip that represents the leftover tiles.
- Cut the strip of leftover tiles in two narrow strips and line them up against the square.
- Use the diagram to estimate the $\sqrt{30}$.



Method 2:

Using a number line is another way to estimate $\sqrt{30}$.



- 30 lies between the perfect squares of 25 and 36.
- $\sqrt{25}=5$ and $\sqrt{36}=6$.
- 30 is between 25 and 36, but is slightly closer to 25.
- So $\sqrt{30}$ is between 5 and 6, but slightly closer to 5.
- Estimate the $\sqrt{30}$.
- Check your estimate using the $\sqrt{\ }$ key on your calculator to find $\sqrt{30}$.
- Use one of the methods above to estimate the $\sqrt{18}$.
- Check your estimate using the $\sqrt{\ }$ key on your calculator to find $\sqrt{18}$.

Look For …

Do students:

- ☐ use perfect squares to check estimates and calculator results? (It is important for students to verify that results generated with calculators are reasonable.)

Step 4: Assess Student Learning

Guiding Questions

- Look back at what you determined as acceptable evidence in Step 2.
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

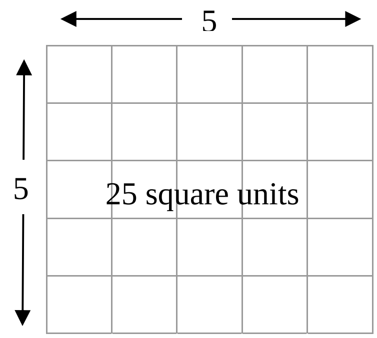
In addition to ongoing assessment throughout the lessons, consider the following sample activities to evaluate students' learning at key milestones. Suggestions are given for assessing all students as a class or in groups, individual students in need of further evaluation, and individual or groups of students in a variety of contexts.

A. Whole Class/Group Assessment

Activity 1: Creating a Frayer Model

Have students work in groups to complete a modified **Frayer Model** for the concepts of perfect squares and square roots. See the following sample on page 17.

Modified Frayer Model for Perfect Squares

Definition	Visual and Numeric Representation
The number that is the square of a whole number.	
Examples	**Non-examples**
$4 \times 4 = 16$: 16 is a perfect square. I can draw a square that is 4 by 4 units. $7 \times 7 = 49$: 49 is a perfect square. I can draw a square that is 7 by 7 units.	20 is not a perfect square, since there is no number that, when multiplied by itself, will result in exactly 20. Although you can create a rectangle of 20 square units, you cannot create a square.

Format adapted from D. A. Frayer, W. C. Frederick and H. J. Klausmeier, *A Schema for Testing the Level of Concept Mastery*. (Working Paper/Technical Report No. 16) (Madison, WI: Research and Development Center for Cognitive Learning, University of Wisconsin, 1969). Adapted with permission from the Wisconsin Center for Education Research, University of Wisconsin-Madison.

Modified Frayer Model for Square Roots

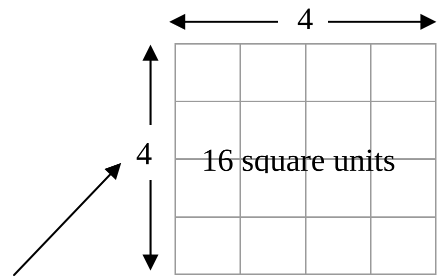
Definition

The number which, when multiplied by itself, results in a given number.

When a number is listed as pairs of factors, the square root is the factor that is repeated.

Squaring a number and taking its square root are inverse operations.

Visual and Numeric Representation



The side length of 4 is the square root of 16.

Examples

$\sqrt{81}=9$: 9 is the square root of 81 because 9 × 9 = 81.

Non-examples

$\sqrt{60}\neq 6$: When 6 is multiplied by itself, the product is not 60. Therefore, 6 is not the square root of 60.

Format adapted from D. A. Frayer, W. C. Frederick and H. J. Klausmeier, *A Schema for Testing the Level of Concept Mastery*. (Working Paper/Technical Report No. 16) (Madison, WI: Research and Development Center for Cognitive Learning, University of Wisconsin, 1969). Adapted with permission from the Wisconsin Center for Education Research, University of Wisconsin-Madison.

Modified Frayer Model for ___________

Definition	Visual and Numeric Representation
Examples	**Non-examples**

Activity 2: Game – Approaching the Root

Materials: playing cards, number lines and calculators.

- Play this game with up to four players.
- Remove the queens and kings from a deck of playing cards. Let the jacks represent 0 and aces represent 1.
- Player 1 draws two cards and uses those two cards to form a 2-digit target number.
- If Player 1 can form a perfect square with the two cards, Player 1 is awarded 10 points and Player 2 takes his or her turn.
- If Player 1 does not draw a perfect square, then each player estimates the square root of the target number without a calculator. Each player records his or her first estimate on a number line.
- Player 1 uses the calculator to determine the square root of the target number. All players compare their estimates to see who is closest.
- Scores are awarded based on the closeness of the estimate but only if the closest estimate is placed correctly on the player's number line.

Look For …

Do students:

☐ use correct vocabulary, such as *perfect square*, *square number* and *square root*?

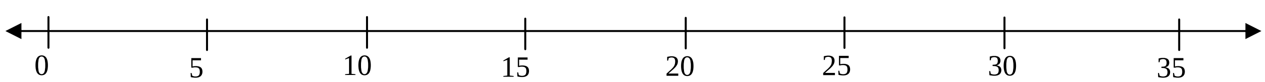
☐ determine the square root of a perfect square?

☐ estimate the square root of a given number that is not a perfect square, using the roots of perfect squares as benchmarks?

☐ approximate the square root of a given number that is not a perfect square, using technology?

The player who draws and recognizes a perfect square scores 10 points.
The player with the closest estimate scores 10 points.
The player with the next closest estimate scores 5 points.
The first person to have 50 points is the winner.

Number line for each player



B. One-on-one Assessment

Activity 1: Solve the following problems. Solutions should contain diagrams and written explanations.

- Use square tiles to make as many rectangles as possible with 30 square units. Sketch your rectangles on grid paper. Is 30 a perfect square? Explain why or why not.
- How do you determine if a number is a perfect square?
- List the square numbers between 25 and 121. Show how you know each number is a perfect square.
- Why is squaring a number the opposite operation to calculating the square root? Explain using an example.
- $\sqrt{72}$ lies between which two whole numbers? Explain how you know. Sketch a number line to show your answer.
- How would you estimate $\sqrt{45}$?

C. Applied Learning

Provide opportunities for students to use the knowledge they have gained about perfect squares and square roots and notice whether or not this knowledge transfers.

Activity 1: Given the area of a square, find the side of the square and the perimeter.

Activity 2: Apply knowledge of square roots with right triangles and the Pythagorean theorem.

Step 5: Follow-up on Assessment

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction?

A. Addressing Gaps in Learning

- Review the concepts of perfect squares and their square roots by having students work with models, such as tiles and grid paper.

- Provide time for students to reflect on their learning when using graphic organizers, such as the Frayer Models or Concept Definition Map.

- Make sure that students are solving problems by drawing pictures and providing explanations to justify their thinking.

- Remind students to use their divisibility rules to find factor pairs. Encourage students to write the factor pairs in an organized list to help determine the square root. If students are having difficulty identifying factors, you may want to provide them with multiplication tables.

- Provide opportunities for students to develop number sense by encouraging them to refine their estimates of the square roots of non-perfect squares.

B. Reinforcing and Extending Learning

Students who have achieved or exceeded the outcomes will benefit from ongoing opportunities to apply and extend their learning. These activities should support students in developing a deeper understanding of the concept and should not progress to the outcomes in subsequent grades.

Consider strategies, such as the following.

Activity 1: Using 1 cm dot paper and square tiles, have students draw all the possible rectangles that will have an area of 60 square units. Challenge students to determine the number of rows that have to be added or removed to change each rectangular model into a square.

Activity 2: Students could play the game "Approaching the Root" with 3- or 4-digit numbers (see page 20).

Activity 3: Solve the following problems. Solutions should contain diagrams and written explanations.

Fencing
Mattie is planning to fence a portion of her yard to create kennels for her three dogs. She wants the space large enough so there will be 4 m^2 of space for each dog.

- What should be the area of the fenced section?
- Mattie would like the kennel to be a square. Approximate the length of each side of the fence needed to create a square fence.
- How much fencing will be needed to create the kennel?
- Chain-link fencing requires a support post at every corner and at least at every metre. Sketch a diagram of the kennel, including where you place the support posts.
- Metal support posts cost $7.99 each and chain-link fencing costs $12.95/m. Determine the cost of materials to build the dog kennel.

Installing Carpet
A hotel wants to install an area carpet in the lobby of the hotel. The hotel lobby is 12 m by 8 m. They want the carpet to be square and fill not more than 80% of the lobby area.

- What is the maximum area the carpet can cover?
- What are the dimensions of the square carpet, to the nearest centimetre, that the hotel needs to order from the carpet company?

Bibliography

Alberta Education. *The Alberta K–9 Mathematics Program of Studies with Achievement Indicators*. Edmonton, AB: Alberta Education, 2007.

Frayer, D. A., W. C. Frederick and H. J. Klausmeier. *A Schema for Testing the Level of Concept Mastery* (Working Paper/Technical Report No. 16). Madison, WI: Research and Development Center for Cognitive Learning, University of Wisconsin, 1969.

Van de Walle, John A. and LouAnn H. Lovin. *Teaching Student-Centered Mathematics: Grades 5–8*. Boston, MA: Pearson Education, Inc., 2006.