Mathematics

Planning Guide

Grade 8 *Surface Area and Volume*

Shape and Space (Measurement)

Specific Outcomes 3, 4

This Planning Guide can be accessed online at:
http://www.learnalberta.ca/content/mepg8/html/pg8_surfacearea/index.html

Table of Contents

Planning Guide: *Grade 8 Surface Area and Volume*

Strand: Shape and Space (Measurement)
Specific Outcomes: 3, 4

This *Planning Guide* addresses the following outcomes from the Program of Studies:

Strand: Shape and Space (Measurement)	
Specific Outcome:	3. Determine the surface area of: • right rectangular prisms • right triangular prisms • right cylinders to solve problems. 4. Develop and apply formulas for determining the volume of right rectangular prisms, right triangular prisms and right cylinders.

Curriculum Focus

The changes to the curriculum targeted by this sample include:

- The general outcome focuses on using direct and indirect measurement to solve problems; whereas the previous mathematics curriculum focused on generalizing patterns and procedures and solving problems involving surface area and volume.
- The specific outcome for surface area focuses on determining the surface area of right rectangular prisms, right triangular prisms and right cylinders to solve problems; whereas the previous mathematics curriculum focused on estimating, measuring and calculating the surface area of any right prism or cylinder and composite 3-D objects.
- The specific outcome for volume focuses on developing and applying formulas for determining the volume of right rectangular prisms, right triangular prisms and right cylinders; whereas the previous mathematics curriculum focused on estimating, measuring and calculating the volume of any right prism or cylinder and composite 3D object.

What Is a Planning Guide?

Planning Guides are a tool for teachers to use in designing instruction and assessment that focuses on developing and deepening students' understanding of mathematical concepts. This tool is based on the process outlined in *Understanding by Design* by Grant Wiggins and Jay McTighe.

Planning Steps

The following steps will help you through the Planning Guide:

Step 1: Identify Outcomes to Address

Guiding Questions

- What do I want my students to learn?
- What can my students currently understand and do?
- What do I want my students to understand and be able to do based on the Big Ideas and specific outcomes in the program of studies?

Big Ideas

- Measurement is a number comparing an item that is being measured with a unit that has the same attribute (length, volume, weight and so on) (Van de Walle 2001, p. 277).
- Area is the measure of the surface of a two-dimensional object and is expressed in square units. Surface area is the sum of the area of the faces of a three-dimensional object and is also expressed in square units.
- The surface area of a three-dimensional object is calculated by analyzing the two-dimensional faces that make up the three-dimensional object. For example, the surface area of a rectangular prism is calculated by finding the area of each of the six rectangles that make up the faces of the object and adding them together.
- Volume refers to the amount of space filled by three-dimensional objects, for example a prism. Prisms have two congruent polygons as the bases and the lines joining corresponding points on the two bases are always parallel.
- The space filled by a 3-D object is compared by using the attribute of the measure of length. Volume is measured in cubic units, such as cubic centimetres or cubic inches.
- Volume formulas can be developed and applied using the attribute of length.
- The relationship between volume and surface area is similar to the relationship between area and perimeter. Prisms that are more cubelike have less surface area than those prisms with the same volume that are less cubelike (Van de Walle and Lovin 2006, p. 246).

Sequence of Outcomes from the Program of Studies

See http://education.alberta.ca/teachers/core/math/programs.aspx for the complete program of studies.

Grade 7	Grade 8	Grade 9
Specific Outcomes 2. Develop and apply a formula for determining the area of: • triangles • parallelograms • circles.	**Specific Outcomes** 3. Determine the surface area of: • right rectangular prisms • right triangular prisms • right cylinders to solve problems. 4. Develop and apply formulas for determining the volume of right rectangular prisms, right triangular prisms and right cylinders.	**Specific Outcomes** There are no directly related specific outcomes in Grade 9.

Step 2: Determine Evidence of Student Learning

Guiding Questions

- What evidence will I look for to know that learning has occurred?
- What should students demonstrate to show their understanding of the mathematical concepts, skills and Big Ideas?

Using Achievement Indicators

As you begin planning lessons and learning activities, keep in mind ongoing ways to monitor and assess student learning. One starting point for this planning is to consider the achievement indicators listed in the *Mathematics Kindergarten to Grade 9 Program of Studies with Achievement Indicators*. You may also generate your own indicators and use them to guide your observation of the students.

The following indicators may be used to determine whether or not students have met the specific outcome for surface area. Can students:

- explain, using examples, the relationship between the area of 2-D shapes and the surface area of a given 3-D object?
- identify all the faces of a given prism, including right rectangular and right triangular prisms?
- identify all the faces of a given right cylinder?
- describe and apply strategies for determining the surface area of a given right rectangular or right triangular prism?
- describe and apply strategies for determining the surface area of a given right cylinder?
- solve a given problem involving surface area?

The following indicators may be used to determine whether or not students have met the specific outcome for volume. Can students:

- determine the volume of a given right prism, given the area of the base?
- generalize and apply a rule for determining the volume of right cylinders?
- explain the connection between the area of the base of a given right 3-D object and the formula for the volume of that object?
- demonstrate that the orientation of a given 3-D object does not affect its volume?
- apply a formula to solve a given problem involving the volume of a right cylinder or a right prism?

Sample behaviours to look for related to these indicators are suggested for some of the activities listed in **Step 3, Section C: Choosing Learning Activities** (p. 13).

Step 3: Plan for Instruction

Guiding Questions

- What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should I use?
- How will I meet the diverse learning needs of my students?

A. Assessing Prior Knowledge and Skill

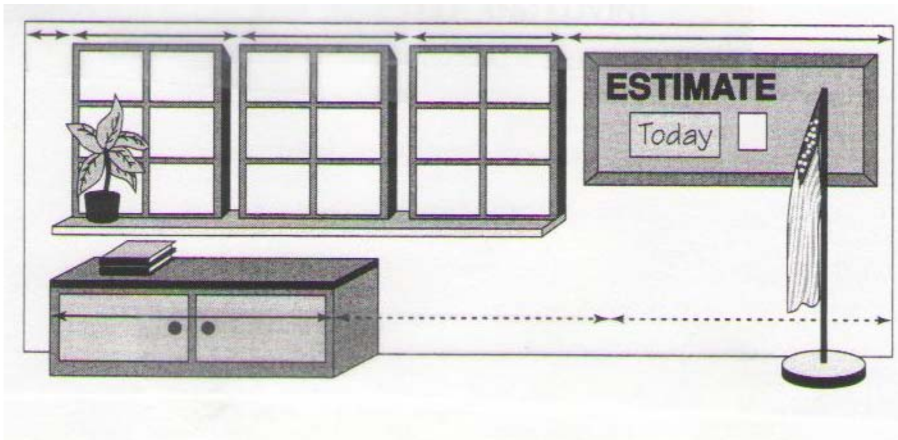
Before introducing new material, consider ways to assess and build on students' knowledge and skills related to the understanding of the attributes of perimeter and area. It is critically important that students understand the attribute that they are to measure. Students should also know that measurement is not precise—that it is estimation and so must have opportunities to practise estimating. Students should be given as much opportunity as possible to participate in hands-on activities that allow them to practise estimating, discover the characteristics of perimeter, area and volume, and develop appropriate formulas. Activities could include some of the following:

Activity 1: Do triangles with the same base have the same area? Give reasons for your solution, including diagrams.

Activity 2: The formula for finding the area of a rectangle and the area of a parallelogram is the same. What relationship is there between these two polygons that allows this to happen? Give reasons for your explanation, including diagrams.

Activity 3: Estimation Quickie: Select a single object such as a box, a watermelon, a jar or even the principal. Each day, select a different attribute or dimension to estimate. For example, students can estimate the length, weight or circumference around the middle section of a watermelon.

Activity 4: This activity compares the length of the room using non-standard units.



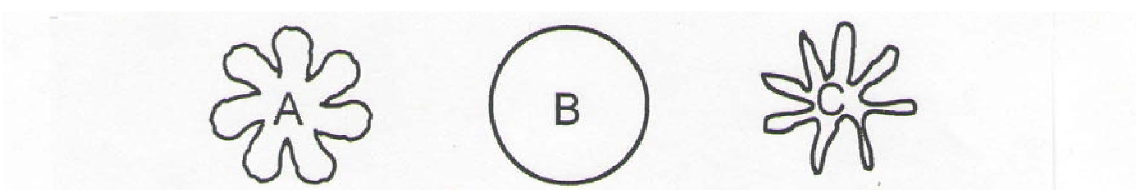
Estimate the length of the room pictured above in terms of:

- windows
- bulletin board
- cabinet

Note: The length of the room is approximately 3 cabinets plus a little more.

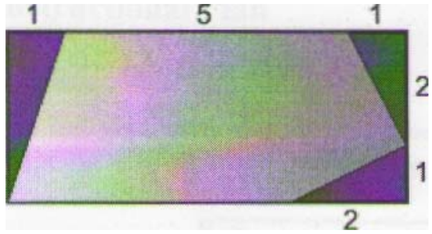
Activity 5:

Consider the following diagrams.



a. Which shape has the greatest area? Why?
b. How could you find the area of figure A?

Activity 6: Have the students determine the area of a shape that is drawn within a rectangle, such as the pentagon in the rectangle below.



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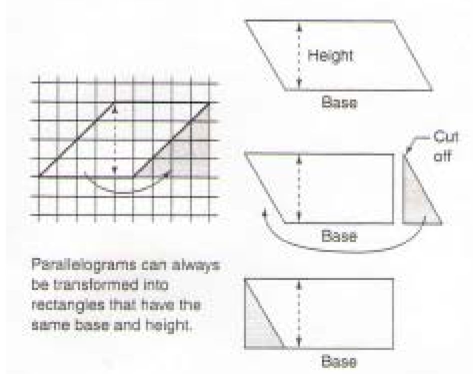
If a student appears to have difficulty with these tasks, consider further individual assessment, such as a structured interview, to determine the student's level of skill and understanding. See **Sample Structured Interview: Assessing Prior Knowledge and Skills** (p. 10).

Sample Structured Interview: Assessing Prior Knowledge and Skills for Activities 1 and 2

<table>
<tr><th rowspan="2">Directions</th><th colspan="2">Date:</th></tr>
<tr><th>Not Quite There</th><th>Ready to Apply</th></tr>
<tr><td>1. Do triangles with the same base have the same area? Give reasons for your solution, including diagrams.</td><td>1. Student responds, "yes."</td><td>1. Student responds with clear diagrams that show as the height of the triangle changes, so does the area, regardless of whether the base is the same or not.</td></tr>
<tr><td>2. The formula for finding the area of a rectangle and the area of a parallelogram is the same. What relationship is there between these two polygons that allows this to happen? Give reasons for your explanation, including diagrams.</td><td>2. Student does not understand the relationship between a rectangle and a parallelogram. Student simply applies formulas without any understanding.</td><td>2. Student responds with the understanding that a parallelogram can always be transformed into a rectangle with the same base and height as pictured below.


Adapted from John A. Van de Walle, LouAnn H. Lovin, Teaching Student-Centered Mathematics: Grades 5–8, 1e (p. 255). Published by Allyn and Bacon, Boston, MA. Copyright © 2006 by Pearson Education. Reprinted by permission of the publisher.</td></tr>
</table>

Sample Structured Interview: Assessing Prior Knowledge and Skills for Activities 3 to 6

Directions	Date:	
	Not Quite There	**Ready to Apply**
3. Estimation Quickie: Select a single object such as a box, a watermelon, a jar or even the principal. Each day, select a different attribute or dimension to estimate. For example, students can estimate the length, weight or circumference around the middle section of a watermelon. Adapted from John A. Van de Walle, LouAnn H. Lovin, *Teaching Student-Centered Mathematics: Grades 5–8*, 1e (p. 235). Published by Allyn and Bacon, Boston, MA. Copyright © 2006 by Pearson Education. Reprinted by permission of the publisher.	3. Student is unsure of which attribute is to be used and has difficulty connecting attribute to proper units; e.g., length of watermelon to centimetres, volume of watermelon to cubic centimetres.	3. Student clearly understands which attribute to use and clearly connects attribute to correct units; e.g., weight of watermelon to kilograms.
4. Estimate the length of the room pictured in Step 3, Part A, Activity 4 using the attributes of: • windows • bulletin board • cabinet	4. Student does not understand how to compare the attributes of the objects given to the length of the room.	4. Student clearly answers that the room is approximately four $\frac{1}{2}$ windows long, three bulletin boards long and three and a little bit cabinets long.
5. a. Which shape shown in Step 3, Part A, Activity 5 has the greatest area? Why?	5. a. Student chooses A or C. Student could be confusing area and perimeter.	5. a. Student chooses Shape B. Explanation: All three shapes appear to have similar diameters. Shape B is a full circle; whereas shapes A and C appear to have pieces missing.

5. b. How could you find the area of Figure A shown in Step 3, Part A, Activity 5?	5. b. Student makes reference to distance around "fingers"—indicating confusion with area and perimeter.	5. b. Possible answers: • Place a grid on top of Shape A and count units. • Find the area of one of the fingers from the grid and multiply by 7.
6. Have the students determine the area of a shape that is drawn within a rectangle, such as the pentagon in the rectangle shown in Step 3, Part A, Activity 6 (p. 9).	6. Student: • is able to recognize shapes of a rectangle, triangle and pentagon • is able to calculate the area of the rectangle and explain answer • is able to recognize triangular shapes, but is unable to find area and cannot explain why OR attempts to break pentagon into a parallelogram and a triangle; gives a correct response and explanation for the area of the parallelogram, but guesses at the height of the triangle formed.	6. Student: • is able to calculate the area of the rectangle to be 21 square units and can explain why • is able to calculate the area of the three triangles to be 1 square unit + 1 square unit + 1.5 square units • calculates the total area of the triangles to be 3.5 square units and can explain why • is able to calculate area of pentagon by subtracting area of triangles from area of rectangle to get 17.5 square units and can explain why.

B. Choosing Instructional Strategies

Consider the following general strategies for teaching surface area/volume and surface area/volume formula development.

Concept Knowledge to Develop	Type of Activity to Use
1. Understand the attribute being measured.	1. Make comparisons based on the attribute.
2. Understand how filling, covering, matching or making other comparisons of an attribute with units produces what is called a *measure.*	2. Use physical models of measuring units to fill, cover, match or make the desired comparison of the attribute with the unit.
3. Understand the way measuring instruments work.	3. Make measuring instruments and use them along with actual unit models to compare how each works.

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C. Choosing Learning Activities

The following learning activities are examples of activities that could be used to develop student understanding of the concepts identified in Step 1.

Sample activities:

1. **Development of Understanding of Area of Two Dimensional Objects** (p. 14)
2. **Surface Area of Rectangular and Triangular Prisms** (p.16)
3. **Surface Area of a Cylinder** (p.17)
4. **Modified Frayer Model – Surface Area** (p.18)
5. **Surface Area – Application** (p.19)
6. **Comparing Volume – Cylinders** (p.20)
7. **Discovering Volume** (p.21)
8. **Volume** (p.22)
9. **Modified Frayer Model – Volume** (p.23)

Sample Activity 1: Development of Understanding of Area of Two Dimensional Objects

Research indicates that even if students have had prior exposure to the development of the area of a rectangle formula, this activity is well worth the time to develop again.

a. Rectangle Comparison—Square Units

Give the students a pair of rectangles that are either the same in area or are very close, a model or drawing of a single square unit, and an appropriate ruler. The students are not permitted to cut out the rectangles or even draw on them. The task is to compare, in any way that they can, the areas of the two rectangles. They should use words, drawings and numbers to explain their decisions. Some suggested pairs are as follows:

4×10 and 5×8, 5×10 and 7×7, 4×6 and 5×5

b. Area of a Parallelogram

Give the students two or three parallelograms either drawn on grid paper or, for a slightly harder challenge, drawn on plain paper. If drawn on plain paper, provide all dimensions—the length of all the sides and the height. Their task is to use what they have learned about the area of rectangles to determine the area of these parallelograms. Encourage the students to find a method that will work for all parallelograms.

c. Area of a Triangle

Give students a piece of grid paper with at least two triangles drawn on it—not right triangles. The task if for students to find the area of the triangles using what they have just learned about the area of a parallelogram. The method chosen must work for all triangles and should be justified for all the triangles they have been given as well as one more that they draw.

d. The Real Story Behind the "Area of a Circle."

One sunny afternoon, Dominic Candalara asks Archimedes if he is interested in walking down to a very popular pizza shop in downtown Syracuse, Sicily, for lunch. Even though Archimedes is extremely busy, he does not want to upset his friend so he accepts his luncheon invitation.

Look For…
Do students:

- explain solutions for area using their own words and without the use of formulas?
- understand the meaning of square units and their origins?
- understand that the formulas for the area of a rectangle $A = l \times w$ and $A = b \times h$ are interchangeable?
- recognize the similarities and differences between rectangles and parallelograms?
- recognize that a parallelogram is always made up of two congruent triangles?
- recognize that as the pieces of "pizza" (circle) become smaller, the near-parallelogram become closer and closer to forming a near-rectangle?
- recognize the dimensions of the near-rectangle to be the radius of the circle and half the circumference?

They place an order for a large, 14-inch round pizza, but just as it was served they hear the sounds of fireworks. Dominic immediately jumps from his seat, hoping to catch a glimpse of what was taking place outside, leaving Archimedes alone with the pizza.

Not interested in the activities on the street and not really hungry, Archimedes calls to the chef, “Hey, Rosseti, bring me a big sharp knife!” The chef obliges and watches while Archimedes begins to cut the round pizza into very thin slices. Each slice of pizza is uniform in size and the slices are even in number.

For every slice, there is a top and a bottom. Placing a top and a bottom together, side by side, Archimedes discovered the circle becomes a square. The area of the near rectangular shape is approximately the radius of the circle time one half the circumference of the circle.

Sample Activity 2: Surface Area of Rectangular and Triangular Prisms

- Supply the students with nets of rectangular and triangular prisms—either photocopies of nets or a variety of cardboard boxes, such as cereal boxes, cracker boxes, Toblerone boxes.
- Have the students find the areas of the nets showing all calculations.

Look For …

Do students:

- recognize that the surface area of the rectangular prism is found by adding the areas of the six rectangular faces together?
- recognize that the surface area of the triangular prism is found by calculating the area of the two triangular faces and the three rectangular faces and adding them together?

Sample Activity 3: Surface Area of a Cylinder

- Supply the students with nets of cylinders or round cardboard containers, such as cocoa cans or potato chip containers.
- Have the students measure and find the surface area of the cylinder net.
- Have the students try to develop a formula for calculating the surface area of a cylinder.

Look For …

Do students:

- recognize that the net of the cylinder is made of two circles and a rectangle?
- recognize that the base of the rectangle is the circumference of the circle and that the height of the rectangle is the height of the prism?
- recognize that the area of the rectangle can be calculated by finding the circumference of the circle multiplied by the height of the cylinder?
- recognize that the formula for finding the surface area of a cylinder is:
  $SA = 2\pi r^2 + 2\pi rh$?

Sample Activity 4: Modified Frayer Model – Surface Area

Have the students complete a modified Frayer model for the concept of surface area.

<table>
<tr><td>Definition

Surface Area
• the measure of the total area of the surface of a 3-D object
• calculated by finding the area of each face of the 3-D object and adding them together
• measured in square units, such as square centimetres, square metres or square inches</td><td>Examples

The surface area of a rectangular prism is found by calculating the area of the six rectangles that make up the prism and adding the areas together.

Non-examples

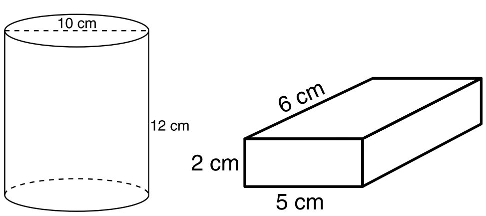
Perimeter—the measure of the distance around a 2-D object—expressed in units.

Area—the measure of surface of a 2-D object—expressed in square units.

Volume—the measure of the amount of space occupied by a three-dimensional object—expressed in cubic units.</td></tr>
</table>

Surface Area

<table>
<tr><td>Visual and Numeric Representation



Cylinder:
$SA = 2\pi r^2 + 2\pi rh$
$= 2(3.14 \times 5 \times 5) + (2 \times 3.14 \times 5 \times 12)$
$= 533.8\text{ cm}^2$

Rectangular Prism:
$= 2(2\text{ cm} \times 5\text{ cm}) + 2(2\text{ cm} \times 6\text{ cm})$
$+ 2(5\text{ cm} \times 6\text{ cm})$
$= 124\text{ cm}^2$</td><td>Real-life Application

Your parents are building a new home and have chosen to have the outside finished with siding. Siding costs about $12.00 per square metre. How much will the siding cost for your parents' new home?</td></tr>
</table>

Format adapted from D. A. Frayer, W. C. Frederick and H. J. Klausmeier, *A Schema for Testing the Level of Concept Mastery* (Working Paper/Technical Report No. 16) (Madison, WI: Research and Development Center for Cognitive Learning, University of Wisconsin, 1969). Adapted with permission from the Wisconsin Center for Education Research, University of Wisconsin-Madison.

Sample Activity 5: Surface Area – Application

- Supply the students with a can of juice and this task:
  You are working for a juice company. The management has decided to spray coat their cans with a plastic to improve taste and shelf life. Your job is to calculate the surface area of one juice can so that they can find out how much spray it will take.

Sample Activity 6: Comparing Volume – Cylinders

Give each pair of students two sheets of construction paper and tape. They are to make a tube shape (cylinder) with one sheet by taping the two long edges together. Using the second sheet, they are to make a shorter, fatter tube by taping the short edges together. The task is for the students to compare the volumes of the two cylinders when the cylinders are placed upright.

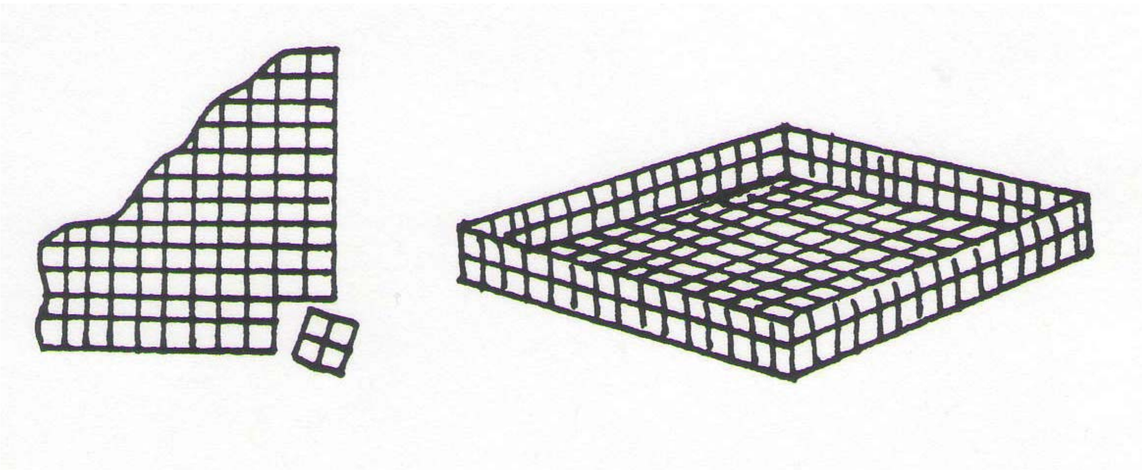
Note: This is a good opportunity to ask for a prediction before the students begin the project and to focus on the techniques of problem-based learning. Stand back, ask questions and allow the students to decide how they are going to solve the problem. Consider having a box of packing peanuts or a bag of puffed rice or wheat on hand.

Sample Activity 7: Discovering Volume

This activity adapted from Burns, Marilyn. *About Teaching Mathematics: A K–8 Resource, Second Edition*, p. 55. Copyright © 2000 by Math Solutions Publications. Adapted by permission. All rights reserved. (Note: This title is now in its third edition, copyright © 2007.)

This activity should lead the students to the understanding that the volume of a prism is the area of the base times the height of the prism: $V = B \times h$, where B is the area of the base of the prism and h is the height of the prism.

Give each pair of students several sheets of 1-centimetre paper (different colours could make it more interesting) and scissors. If you have a piece of 20-by-20 centimetre paper, you can cut a square the same size from each corner and fold up what is left to make a box.



1. How many different sized boxes could you make using this method, making each from a different piece of 20-by-20 centimetre squared paper? Make them.
2. Which of these boxes hold the most? How do you know this? The solution must be explained or justified without the use of a formula.
3. Go back to Sample Activity 6. Have students calculate the volume of each cylinder using what has been learned in this activity. Is the result the same?

Note: If a hint is required, suggest that they cover the base of the box with centimetre-cubed blocks.

Sample Activity 8: Volume

Calculate the volumes of the following three-dimensional objects:

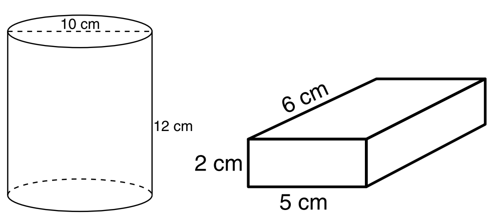
a. a rectangular prism 24 cm high, 18.5 cm wide and 5.6 cm long
b. a cylinder with a radius of 14 cm and 12 cm high.

Sample Activity 9: Modified Frayer Model – Volume

Have the students complete a modified Frayer model for the concept of volume.

Definition	**Examples**
Volume: • the measure of the amount of space occupied by a three-dimensional object • measured in cubic units such as cubic centimetres, cubic metres or cubic inches	$V = B \times h$ where B = area of the base of the 3-D object h = height of the object **Non-examples** Perimeter—the measure of the distance around a 2-D object—expressed in units. Area—the measure of surface of a 2-D object—expressed in square units. Surface Area—the measure of the sum of the areas of all the faces of a 3-D object—expressed in square units.

Volume

Visual and Numeric Representation	**Real-life Application**
 Cylinder: $V = B \times h$ $= 3.14 \times 5 \times 5 \times 12$ $= 942\text{ cm}^3$ Rectangular Prism: $V = B \times h$ $= 6 \times 5 \times 2$ $= 60\text{ cm}^3$	A rectangular box has dimensions 25 cm by 18 cm by 21 cm. Calculate the volume of the box.

Format adapted from D. A. Frayer, W. C. Frederick and H. J. Klausmeier, *A Schema for Testing the Level of Concept Mastery* (Working Paper/Technical Report No. 16) (Madison, WI: Research and Development Center for Cognitive Learning, University of Wisconsin, 1969). Adapted with permission from the Wisconsin Center for Education Research, University of Wisconsin-Madison.

Step 4: Assess Student Learning

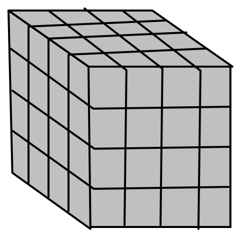
Guiding Questions

- Look back at what you determined as acceptable evidence in Step 2.
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

In addition to ongoing assessment throughout the lessons, consider the following sample activities to evaluate students' learning at key milestones. Suggestions are given for assessing all students as a class or in groups, individual students in need of further evaluation, and individual or groups of students in a variety of contexts.

A. Whole Class/Group Assessment

Activity 1: A block made of small cubes is dipped in paint. The block has four cubes on each edge as shown below. How many small cubes have paint on them?



If students have difficulty with this task, provide some base 10 blocks or some sugar cubes.

Activity 2: Marta made a stack of cubes: 5 cubes across the front, 4 cubes deep and 6 cubes high. The top and the four sides of the stack are painted red. How many cubes have paint on them? How many cubes do not have paint on them?

If students have difficulty with this task, provide base 10 blocks or sugar cubes or have them draw a diagram.

Activity 3: A rectangular box holds 36 cubes. When she opened the box, Marcia could see 12 on the top. If she could open it at an end, how many cubes would she see?

Activity 4: A rectangular box holds 48 cubes. When you open the top, how many might you see? For each possibility, list how many you would see from an end.

These activities reproduced with permission from Grayson H. Wheatley and George E. Abshire, *Developing Mathematical Fluency: Activities for Grades 5–8* (Tallahassee, FL: Mathematics Learning, 2002), pp. 44, 57, 58. www.mathematicslearning.org.

B. One-on-one Assessment

Activity 1:

This activity adapted with permission from Rocky View School Division, "Grade 8 PBT – Lesson 3 – House Surface Area" (Airdrie, AB: Rocky View School Division, n.d.), student sheet and rubric.

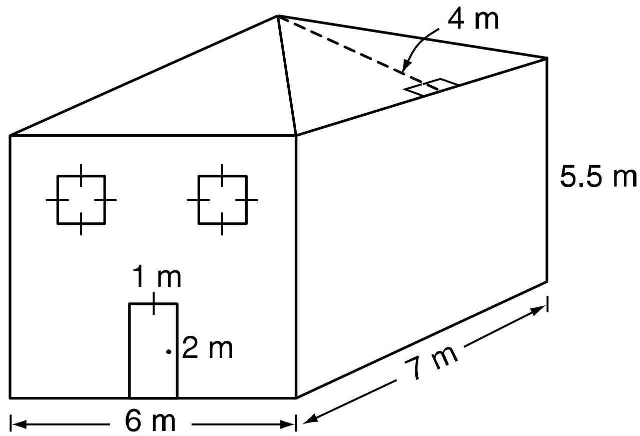
House Surface Area

Congratulations! You are the proud owner of a new home. You purchased the home knowing it needs a new look. You have decided to replace the siding on all sides (excluding the door and windows), as well as the shingles on the roof.

Your job is to purchase the materials needed and calculate the cost to renovate the home. You must be exact in your calculations. All measurements will be in metres. You may want to think about strategies to make your calculations correct. The back of the home is the same as the front and the other one is the same as the side shown.

HINT: You may want to break down the home into nets.

Cost of siding: $5.00 a square metre
Cost of shingles: $8.50 a square metre



Grade 8 – Problem-based Tasks

General Scoring Criteria/Rubric

Scale Score	Criteria
5	• Complete answers to all parts • Final answers are correct • All work is shown • Communication is easy to understand
4	• Complete answers to all parts • Final answers must be reasonable • All work is shown • May have minor errors • Communication may lack some clarity
3	• Complete answers to most parts • Final answers are sometimes unreasonable • All work is shown • Some errors present in calculations but the processes are mostly correct • Communication lacks clarity
2	• *EITHER* complete and correct answers with no work shown, *OR* incomplete and incorrect answers with some correct work shown • Major procedural errors • Communicates some strategy
1	• A significant start to the solution • Demonstrates limited understanding to some part of the problem
0	• No significant start made • Blank paper

Activity 2:

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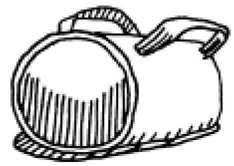
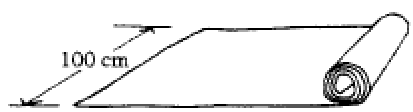
Sports Bag



Each bag has the shape of a cylinder and is described below.

- The body of the bag is 60 cm long.
- The circular ends have a diameter of 25 cm.
- The body is made from a single rectangular piece of heavy fabric.
- The ends are each made from a circular piece of the same kind of fabric. (That makes three pieces altogether: one body, two ends.)
- Remember to add an extra 2 cm all around each piece to allow the piece to be stitched together.
- The strapping will be cut from different material, created and put together by a different department, so you do not need to worry about it.

You will cut the three pieces of your bag from fabric that comes on a long roll. The roll is 100 cm wide.



1. What is the shortest length that you need to cut from the roll to have enough fabric for one bag? Draw a diagram showing how the three pieces will be cut from the roll. Explain your thinking.
2. What is the shortest length that you would need to cut from the same fabric roll to have enough fabric for three bags? Draw a diagram of how the pieces will be cut from the roll and label the dimensions of the pieces. Make sure to explain what you did and how you did it.

Activity 3: How does the volume of a cylinder change when:

a. the height doubles?

b. the diameter doubles?

Activity 4: The area of the floor of a rectangular room is 315 m^2. The area of one wall is 120 m^2 and the area of another is 168 m^2. The floor and ceiling are parallel. What is the volume of the room?

Adapted with permission from Grayson H. Wheatley and George E. Abshire, *Developing Mathematical Fluency: Activities for Grades 5–8* (Tallahassee, FL: Mathematics Learning, 2002), p. 45. www.mathematicslearning.org.

C. Applied Learning

Provide opportunities for students to use what they have learned about surface area and volume in practical situations and notice whether or not the knowledge transfers.

Activity 1: Have the students measure and calculate the amount of material required to cover a small building such as an outside shed, a cabin, a boathouse or a grain bin, excluding the roof. Students should include a diagram, net shapes and all calculations.

Activity 2: Have the students analyze the relationship between the volume and surface area of a cylinder as the radius increases while the height changes and/or analyze the relationship between the volume and the surface area of a cylinder as the height increases and the radius stays the same. Students could work in groups and share their results.

Step 5: Follow-up on Assessment

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction?

A. Addressing Gaps in Learning

- Surface Area

Review the concept of area of two-dimensional objects by providing more hands on experiences, such as:

Fixed Perimeter: Provide the students with a loop of non-stretchy string that is exactly 24 units long. The task is to decide what different-sized rectangles can be made with a perimeter of 24 units. Students may want to place their strings on a 1-unit grid. Each different rectangle can be recorded on grid paper with the perimeter and the area noted. Alternatively, ask the students to use centimetre grid paper to find rectangles with a perimeter of 24 cm. Each different rectangle could also be recorded with perimeter and area noted. What conclusions can the student make about the perimeters and areas of the different rectangles?

Look For …

Do students:
- confuse the concepts of perimeter and area?
- think they have made a mistake when perimeter changes?
- observe patterns as perimeter changes?
- recognize that the area is not changing?

Fixed Area: Provide pairs of students with 36 square tiles, at least two sheets of centimetre paper and a recording sheet.

The task is to see how many different rectangles can be made with 36 tiles. Determine and record the perimeter and area for each rectangle.

What conclusion can the students make about the perimeters and areas of the different-sized rectangles?

Adapted from John A. Van de Walle, LouAnn H. Lovin, *Teaching Student-Centered Mathematics: Grades 5–8*, 1e (pp. 242, 263, 264). Published by Allyn and Bacon, Boston, MA. Copyright © 2006 by Pearson Education. Reprinted by permission of the publisher.

- Volume

Have the students construct a cube using 24 cube-a-links. Discuss with the students the relationship between the number of cubes and the volume. Guide students to understand the relationship between the area of base and the volume. What is the missing piece? What is the height?

B. Reinforcing and Extending Learning

Students who have achieved or exceeded the outcomes will benefit from ongoing opportunities to apply and extend their learning. These activities should support students in developing a deeper understanding of the concept and should not progress to the outcomes in subsequent grades.

Consider activities such as:

1. Find the area of a trapezoid.
2. Develop the formula for the surface area of a sphere.
3. Study the relationship between the area of a circle and the surface area of a sphere.
4. Connect volume to whole number factor triples; e.g., find the factor triples that correspond to a volume of 24 cubic inches.
5. Calculate the volume of a composite figure.

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