Mathematics

# Planning Guide

Grade 9

*Working with Linear Equations*

Patterns and Relations
(Variables and Equations)
Specific Outcome 3

**This Planning Guide can be accessed online at:**
http://www.learnalberta.ca/content/mepg9/html/pg9_linearequations/index.html

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# **Planning Guide:** *Grade 9 Working with Linear Equations*
**Strand:** Patterns and Relations (Variables and Equations)
**Specific Outcome:** 3

This *Planning Guide* addresses the following outcome from the program of studies:

**Strand:** Patterns and Relations (Variables and Equations)

**Specific Outcome:** 3. Model and solve problems, using linear equations of the form:
- $ax = b$
- $\frac{x}{a} = b,\ a \neq 0$
- $ax + b = c$
- $\frac{x}{a} + b = c,\ a \neq 0$
- $ax = b + cx$
- $a(x + b) = c$
- $ax + b = cx + d$
- $a(bx + c) = d(ex + f)$
- $\frac{a}{x} = b,\ x \neq 0$

where *a, b, c, d, e* and *f* are rational numbers.

## Curriculum Focus

This sample targets the following changes to the curriculum:

- The general outcome is similar to the previous program of studies.
- The specific outcome consists of the following differences:
  - multiplying binomials and factoring trinomials has been moved to Grade 10
  - the form of linear equations represented in this outcome include additional forms of linear equations, including:
    - $ax = b$
    - $\frac{x}{a} = b, a \neq 0$
    - $ax + b = c$
    - $\frac{x}{a} + b = c, a \neq 0$.

## What Is a Planning Guide?

**Planning Guides** are a tool for teachers to use in designing instruction and assessment that focuses on developing and deepening students' understanding of mathematical concepts. This tool is based on the process outlined in *Understanding by Design* by Grant Wiggins and Jay McTighe.

## Planning Steps

The following steps will help you through the Planning Guide:

## Step 1: Identify Outcomes to Address

### Guiding Questions

- What do I want my students to learn?
- What can my students currently understand and do?
- What do I want my students to understand and be able to do based on the Big Ideas and specific outcomes in the program of studies?

### Big Ideas

Mathematics is sometimes referred to as the "science of patterns." Exploring patterns and then translating and representing these patterns into words, symbols, expressions, equations and graphs help Grade 9 students better understand linear functions. Connecting functions to prior learning and to real-life situations facilitates student learning. There are several important Big Ideas that are addressed in this outcome:

- Algebra is a way to represent and explain mathematical relationships and is used to describe and analyze change (Small 2009, p.7).
- Equations are used to express relationships between two quantities (Van de Walle 2001, p. 384).
- The equal sign means that the quantity on the left-hand side of the equation is the same as the quantity on the right-hand side.
- A variable is a symbol that can stand for any one of a set of numbers or other objects and can be represented by boxes or letters (Van de Walle and Lovin 2006, p. 274).

## Sequence of Outcomes from the Program of Studies

See http://www.education.alberta.ca/teachers/program/math/educator/progstudy.aspx for the complete program of studies.

| Grade 7 | Grade 8 | Grade 9 | Grade 10 |
|---|---|---|---|
| **Variables and Equations** | **Variables and Equations** | **Variables and Equations** | **Relations and Functions** |
| **Specific Outcomes** | **Specific Outcome** | **Specific Outcome** | **Specific Outcome** |
| 3. Demonstrate an understanding of preservation of equality by:<br>• modelling preservation of equality, concretely, pictorially and symbolically<br>• applying preservation of equality to solve equations.<br><br>4. Explain the difference between an expression and an equation.<br><br>5. Evaluate an expression, given the value of the variable(s).<br><br>6. Model and solve, concretely, pictorially and symbolically, problems that can be represented by one-step linear equations of the form $x + a = b$, where $a$ and $b$ are integers. | 2. Model and solve problems concretely, pictorially and symbolically, using linear equations of the form:<br>• $ax = b$<br>• $\frac{x}{a} = b,\ a \neq 0$<br>• $ax + b = c$<br>• $\frac{x}{a} + b = c,\ a \neq 0$<br>• $a(x + b) = c$<br>where $a$, $b$ and $c$ are integers. | 3. Model and solve problems, using linear equations of the form:<br>• $ax = b$<br>• $\frac{x}{a} = b,\ a \neq 0$<br>• $ax + b = c$<br>• $\frac{x}{a} + b = c,\ a \neq 0$<br>• $ax = b + cx$<br>• $a(x + b) = c$<br>• $ax + b = cx + d$<br>• $a(bx + c) = d(ex + f)$<br>• $\frac{a}{x} = b,\ x \neq 0$<br>where $a$, $b$, $c$, $d$, $e$ and $f$ are rational numbers. | 9. Solve problems that involve systems of linear equations in two variables, graphically and algebraically. |

## Step 2: Determine Evidence of Student Learning

### Guiding Questions

- What evidence will I look for to know that learning has occurred?
- What should students demonstrate to show their understanding of the mathematical concepts, skills and Big Ideas?

### Using Achievement Indicators

As you begin planning lessons and learning activities, keep in mind ongoing ways to monitor and assess student learning. One starting point for this planning is to consider the achievement indicators listed in *The Alberta K–9 Mathematics Program of Studies with Achievement Indicators*. You also may generate your own indicators and use them to guide your observation of the students.

The following indicators may be used to determine whether or not students have met this specific outcome. Can students:

- model the solution of a given linear equation, using concrete or pictorial representations, and record the process?
- verify, by substitution, whether a given rational number is a solution to a given linear equation?
- solve a given linear equation symbolically?
- identify and correct an error in a given incorrect solution of a linear equation?
- represent a given problem, using a linear equation?
- solve a given problem, using a linear equation, and record the process?

Sample behaviours to look for related to these indicators are suggested for some of the activities listed in **Step 3, Section C: Choosing Learning Activities** (p. 13).

# Step 3: Plan for Instruction

## Guiding Questions

- What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should I use?
- How will I meet the diverse learning needs of my students?

## A. Assessing Prior Knowledge and Skills

Before introducing new material, consider ways to assess and build on students' knowledge and skills about their understanding of variables, equality and solving equations. You may choose to use indicators from prior grades to help to determine what students should know or understand. Consider using open-ended questions to more accurately assess what students are able to communicate and do.

### 1. Variables

Research has shown that students often have misconceptions about the concept of variables. Usiskin (1988) identified three different ways that variables are used:

1. As a specific unknown with a single value; e.g., $2n + 15 = 5n + 3$. Students are often asked to solve for the unknown value.
2. As a pattern generalizer; e.g., $2n + 5$ where $n$ can have an infinite number of values.
3. Variables that change in relation to one another; e.g., $C = 2\pi r$, where the measure of the radius ($r$) affects the value of the circumference ($C$).

It is important that students have an opportunity to think about the different ways that variables are used to represent an unknown. Students may also need to be reminded that other symbols like boxes and letters can serve the same purpose. Students may be confused by the meaning of letters as they often are used to represent different ideas in mathematics. The following activities should help diagnose student understanding of the use of letters and variables.

**Activity 1.1: Using Letters as Variables**

Explain what is the same and what is different about the way that the letter *m* has been used in the following cases:

- Martin ran 100 m
- $2m + 15 = 23$
- $3m + 2$.

Students should begin to discuss that, in the first case, the *m* represents an abbreviation for metre, which is different from the second two uses, where *m* represents a value.

- The difference between the second and third cases is that one represents a single value while the other represents an infinite number of values. Students may also require clarification that a numerical coefficient in front of the variable implies multiplication. Ask them to show $2m + 15 = 23$ and $3m + 2$ in a different (expanded) form. They may share answers like:

  – $2 \cdot m + 15 = 23$ or $2 \times m + 15 = 23$ or $2(m) + 15 = 23$ or $15 + 2 \times m = 23$ …
  – $3 \cdot m + 2$ or $3 \times m + 2$ or $3(m) + 2$ or $2 + 3 \times m$ or $2 + 3(m)$ …

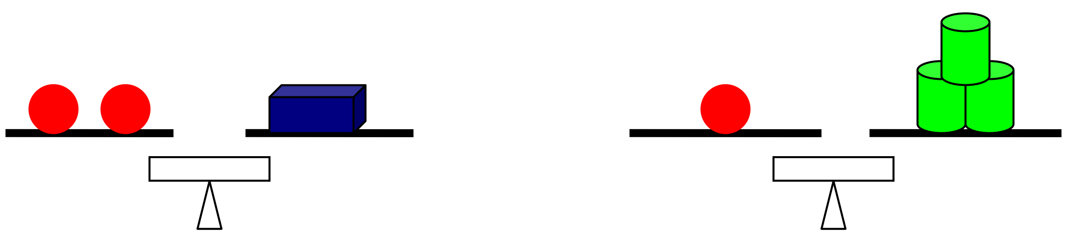
If students do not bring up the operation shortcuts during their discussion, ask them to explain what operations are represented in the last two cases and how they could be written differently.

**Activity 1.2: Scales**

Scales and balances help students make sense of algebraic relationships as well as encourage thinking, reasoning, discussion and debate. Many of these questions do not include operation signs, so students need to infer and interpret to arrive at a strategy to solve equations.

A single shape has the same value or mass.

Which shape has the greatest mass?
Which shape has the least mass? How do you know?



Ideas derived from Van de Walle 2001, p. 399.

**Activity 1.3: Algebra Number Line**

Materials:

- 5 m length of masking or adding machine tape marked with 20 evenly spaced increments. This longer length allows students to stand along the line as they participate in the activity.
- Individual cards marked with the following: 0, 2, –2, 5, –5, 10, –10, $x$, $y$, $y + 3$, $y - 4$, $x + 2$, $x - 2$, $3x$ and $\frac{x}{2}$.

Instructions:

- Hold up the *0* card and ask a student to place it on the line.

Ask the student to justify why he or she chose to place it there. Ask the class if this is the only choice of places and why?

Once there is agreement on placement of the *0* card, ask students to place the integer cards one by one and discuss how the placement of the first integer card determines the interval. Pick up their card and place it on another increment closer or further away from zero. For example, if the *2* card is placed four increments away from *0*, ask students, "What does each interval represent now?"

- In this case, the response should be 0.5.

Determine a place for the *2* card. The goal is to have students discuss and understand that once this number is placed, it determines the position for the subsequent placement of numbers. Ask students to place the remaining integer cards one by one.

- Hold up the $x$ card and ask a student to place it (**Note**: it can go anywhere).

  Ask the following questions:

  – How do you know where to place $x$?
  – Is that the only place that it can be placed?
  – What do you know about the value of $x$?

- Hold up the $x + 2$ card and ask a student to place it.

  Ask the following questions:

  – How do you know where to place $x + 2$?
  – Is that the only place it can be placed?
  – What do you know about the value of $x + 2$?
  – What happens if I place $x + 2$ at $-5$?

- Continue placing the cards with variables, ask similar questions to the ones above and ask students what questions they might ask.

- Consider adding cards like $5y < 12$, or $2x + 3$ or $\frac{y}{2}$ to extend the activity.

- At the end of the activity, have students reflect on or discuss in small groups:

  – something they learned or something that became clearer during the activity
  – a question they have
  – something that is still confusing.

Ask the class members to share their discussion as a large group; this activity should help assess where students are at in terms of their understanding of the number line and variables. To differentiate or reinforce the learning in this activity, you may wish to show a number line on a transparency or whiteboard and have students place and move the integers and variables in this medium.

## 2. Equality

Pose the following question to your students and ask them to record their solutions on paper. Ask them to share why they selected their answers.
$8 + 4 = \square + 5$

Research shows that the majority of students in elementary grades will not answer this question correctly and will justify their answers by:

- saying the answer is: $8 + 4 = 12 + 5$
- using all the numbers to find a sum: $8 + 4 = 17 + 5$
- incorrectly adding another equal sign: $8 + 4 = 12 + 5 = 17$.

From Carpenter, Franke and Levi, 2003, p. 4.

## 3. Solving Equations

In Grade 8, students should have become familiar with and should be able to solve equation formats shown in the activity below. Put each equation on an individual recipe card and ask students to sort the cards into the following three categories:

a. easy to solve in my head
b. harder, but can be solved in my head
c. easier to use a model or pen and paper to solve.

1. $n + 2 = 7$
2. $3n = 21$
3. $2n + 7 = 19$
4. $4n - 8 = 6n + 2$
5. $5 = 2n - 7$
6. $n - 2 = 7$
7. $\frac{n}{3} = 21$
8. $3n - 5 = n + 1$
9. $6(n - 4) = 2x + 4$
10. $5n - 2 = 9$

Ask students to discuss the following questions with a partner or in a small group:

- how did you decide which questions would be easy to solve mentally?
- what process did you use to solve these equations?
- what are the similarities and differences in the methods that each person used?

If a student appears to have difficulty with these tasks, consider further individual assessment, such as a structured interview, to determine the student's level of skill and understanding. See **Sample Structured Interview: Assessing Prior Knowledge and Skills** (p. 11).

## Sample Structured Interview: Assessing Prior Knowledge and Skills

| Directions | Date: | |
|---|---|---|
| | **Not Quite There** | **Ready to Apply** |
| Read the following to the student: **"A number is doubled and then increased by 5."**<br><br>Translate this into an algebraic expression and explain a situation in real life that models this expression. | The student is unable to translate the operations into English.<br><br>The student is not aware of what one or both of the operation signs represent or may reverse the subtraction.<br><br>The student cannot think of a context or identifies the context incorrectly. | The student is able to translate the expression into $2n + 5$.<br><br>The student is able to articulate a context like, "I got paid the same amount to mow my neighbour's lawn two times and then, the second time, my neighbour gave me a bonus of $5." |
| Ask the student to show a solution using algebra tiles or a symbolic pencil and paper solution:<br><br>• $3x = 6$<br>• $2x = –6$<br>• $–3x = 6$<br>• $–2x = 6$. | The student is not able to identify the operation on the left as multiplication.<br><br>The student is not able to solve some of the situations involving negative coefficients or constants.<br><br>The student is unable to show dividing both sides of the equation by a common divisor.<br><br>The student can use one representation more competently than the other. | The student solves the questions correctly with tiles and is able to show the steps.<br><br>The student is able to translate the pictorial or concrete steps into a symbolic representation. |
| Ask the student to show a solution using algebra tiles or a symbolic pencil and paper solution for:<br><br>• $2(n + 1) = 8$<br>• $2(n + 2) = 3(n + 3)$. | The student is not able to use the distributive property correctly.<br><br>The most common error is that the student does not multiply the coefficient by the second value in the brackets.<br><br>The student is not able to show the correct steps for the solutions using tiles.<br><br>The student is not able to show the correct steps for the solutions using a symbolic representation. | The student uses the distributive property competently.<br><br>The student shows a correct solution using tiles.<br><br>The student shows a correct solution using numbers. |

| | | |
|---|---|---|
| Show the student the following three equations and ask the student to:<br><br>• explain why they are different<br>• explain how each is solved.<br><br>– $2x = 10$<br>– $\frac{x}{2} = 10$<br>– $\frac{2}{x} = 10$ | The student is not able to clarify the differences in the operations.<br><br>The student is unable to see a difference between the two division questions.<br><br>The student has difficulty solving one or more of the equations. | The student is able to articulate that the first equation involves multiplication while the other two involve division.<br><br>The student may predict that the third equation will involve a rational value.<br><br>The student is able to solve all three situations. |

## B. Choosing Instructional Strategies

Consider the following guidelines for teaching patterns in Grade 9.

- Students should be provided with regular opportunities to engage in conversations and discuss open-ended questions to build confidence and competence with mathematics.
- When students are given opportunities to communicate their thinking (either orally or in writing), explain their reasoning and listen to the strategies used by other students, there are more opportunities to deepen their understanding.
- Have students model equations using balances (actual or pictorial) or algebra tiles. The concrete materials will help reinforce the understanding of abstract concepts, even if the student is already a capable abstract thinker.
- Provide opportunities for students to represent equations in multiple ways, including using manipulatives, pictures, numbers and variables. The more flexible students are in using all of these representations, the better their understanding will be.

## C. Choosing Learning Activities

The following learning activities are examples of activities that could be used to develop student understanding of the concepts identified in Step 1.

**Sample Activities:**

1. **What Is the Mass of Each Shape Below?** (p. 14)
2. **Equations from Geometric Patterns** (p. 15)
3. **Using Algebra Tiles to Solve Equations** (p. 16)
4. **Mathematical Debate** (p. 19)
5. **Representing and Solving Problems with Linear Equations** (p. 20)
6. **Error Analysis** (p. 21)
7. **Connecting Algebra to Area** (p. 22)
8. **Representing an Area Question with** $ax = b$, $\frac{b}{x} = a$ **and** $\frac{b}{a} = x$ (p. 23)

As you work through these activities with students, reinforce the meaning of the vocabulary of algebra terms that arise, such as:

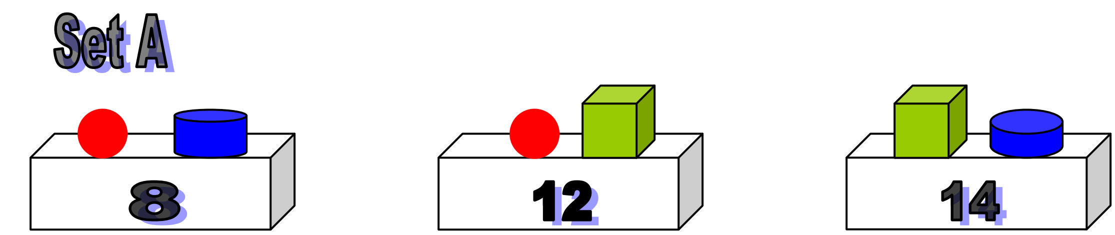
- variable
- coefficient
- constant
- distributive property
- simplify
- evaluate
- isolate.

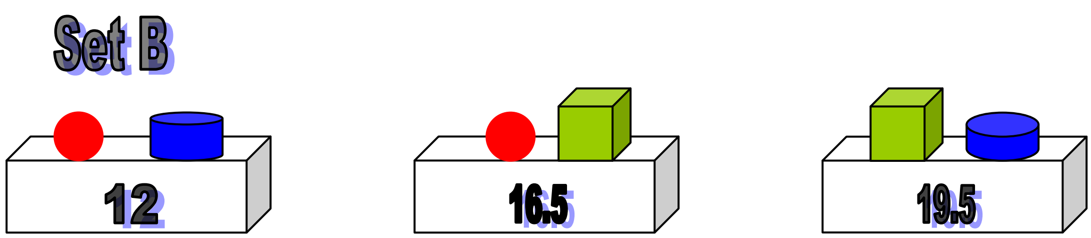
## Sample Activity 1: What Is the Mass of Each Shape Below?

The combined mass for each pair of shapes is shown on the scale below the shapes. Ask students to determine the mass of each shape if the shapes that are the same colour and size have the same mass. Ask them to represent their answer in at least two ways or show at least two strategies.

Set A has integral answers, while Set B has rational answers. Ask students to solve Set A first and share their solution strategies. Then provide them with Set B and ask them to solve for the unknowns. Was their strategy the same for both sets?

Many students will use guess and check to solve the missing values. They may choose to represent the unknowns with letters like *x*, *y* and *z* or they may show their solution through guess and check. Students should share their process in solving these questions with the rest of the class. Differentiate these activities by using different sets of numbers, including natural and rational numbers.

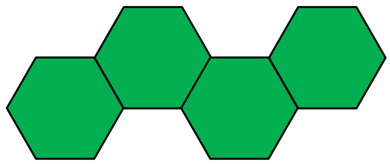


## Sample Activity 2: Equations from Geometric Patterns
(Van de Walle 2001, p. 409)

Have students build a progressive linear geometric pattern using different types of pattern blocks. Different groups may work with different polygons and develop their patterns in tables of value; then, translate this information into an expression or rule that explains the pattern.

For example, if students build a string of hexagons,



the table of values should look like the one below:

| # of Hexagons | # of Sides | Perimeter |
|---|---|---|
| 1 | 6 | 6 (2.5 cm) |
| 2 | 10 | 10 (2.5 cm) |
| 3 | 14 | 14 (2.5 cm) |
| 4 | 18 | 18 (2.5 cm) |

Ask students using different shapes to graph their data on a class graph to see the similarities and differences between the linear functions. All pattern blocks have sides that measure 2.5 cm, except for the trapezoid, which has a long side of 5 cm.

Once students translate their data into an algebraic expression, challenge them to write their expression in at least two other equivalent ways. Some of the possibilities are:

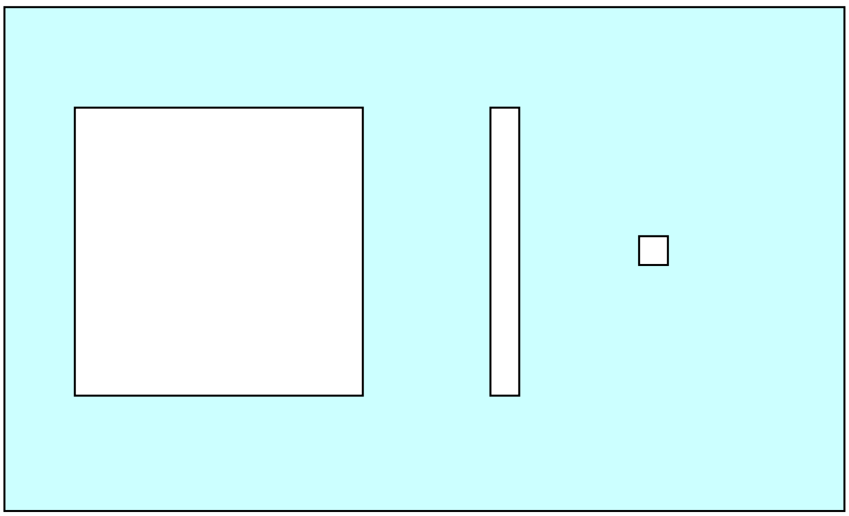
- $4n + 2$
- $2(2n + 1)$ [use of distributive property]
- $2n + 2n + 4$
- $2n + 2n + 2 + 2$
- $2(n + 1) + 2(n + 1)$ [use of distributive property]
- $4(n – 1) + 6$.

Work involving brackets is often confusing to students and reinforcing the application of distributive property is essential.

## Sample Activity 3: Using Algebra Tiles to Solve Equations

Algebra tiles will help reinforce the steps that students should use to simplify equations with integral values; however, they will need to solve equations with rational values in Grade 9, so understanding the steps in the solution process is important. Using a balance template also reinforces the need for students to understand the concept of equality. This activity provides a graphic organizer for students to build some understanding using tiles to solve questions and then to apply their algorithms to a question with rational values.

An effective transitional tool for algebra tiles is to provide students with a recipe card, have them trace the tiles on the card and then cut out holes for the three shapes, thereby making a stencil.



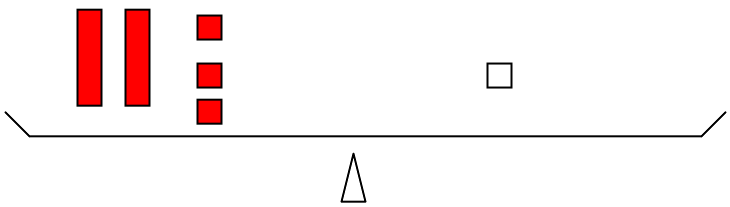
You should proportionally reduce the size of the tile cutouts (a good mathematics activity for students) in order for students to be able to compact their algebra tile drawings and use less paper when using the template. This method of drawing tiles increases the efficiency of drawing and Grade 9 students generally appreciate graduating from concrete materials (algebra tiles) to the pictorial representation. In addition, consider using websites where students can explore questions electronically, such as the National Library of Virtual Manipulatives at http://nlvm.usu.edu/.

This online balance generates algebraic equations for students to solve and begins by having students represent the equations pictorially on the balance. They then select a solve option and select one of four operations to begin the solution. While they continue to solve each step of the equation, the online applet shows a theme, both the pictorial (concrete) and algebraic representations of the steps that they have selected. This applet will only allow students to complete equations that have integral solutions.

Students may also use this applet to create their own equations to solve. Ask students to solve the equations below using the online applet and/or using the algebra tile template above.
Important hint: They will need to use two colours in their drawings when working with positive and negative values.

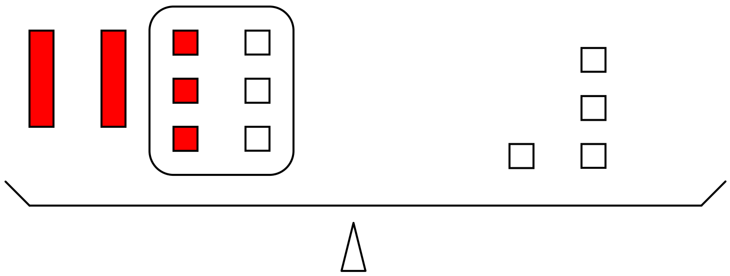
| Solve using algebra tiles (show all steps) | Solve algebraically (show all steps and verify your answer) | More difficult: Solve (show all steps and verify your answer) |
|---|---|---|
| $3x = -6$ | $3n = -6$ | $\frac{1}{2}n = 12$ |
| $2x + 3 = -1$ | $2n + 3 = -1$ | $0.4n + 3.6 = 12.8$ |
| $2(x - 3) = 4$ | $2(n - 3) = 4$ | $\frac{3}{4}(n-4) = 5$ |
| $3x + 2 = x - 4$ | $3x + 2 = x - 4$ | $\frac{3}{4}n - \frac{1}{2} = \frac{2}{3}n - \frac{2}{3}$ |
| $3(2x + 2) = 2(2x - 4)$ | $3(2n + 2) = 2\ (2n - 4)$ | $0.8(\ 6n + 6) = 0.6(4n - 12)$ |
| $\frac{x}{3} = 2$ | $\frac{x}{3} = 2$ | $\frac{x}{2} = 0.8$ |
| $\frac{12}{x} = 3$ | $\frac{12}{x} = 3$ | $\frac{2.5}{n} = 0.5$ |
| $\frac{x}{3} + 5 = -2$ | $\frac{x}{3} + 5 = -2$ | $\frac{x}{0.4} + 8.2 = 12.8$ |

Sample Solutions for $2x + 3 = -1$:



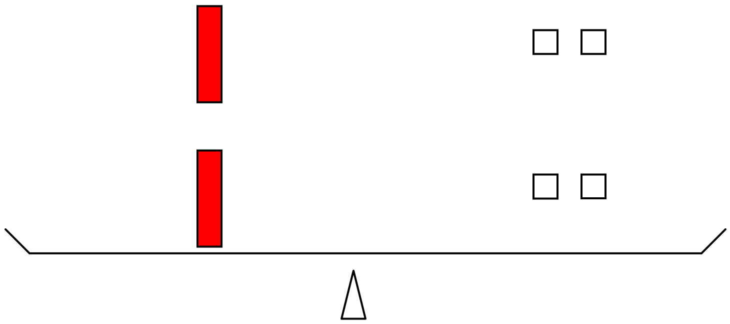
Add negative 3 (–3) tiles to both sides and create zero on the left.

$2x + 3 + -3 = -1 + -3$



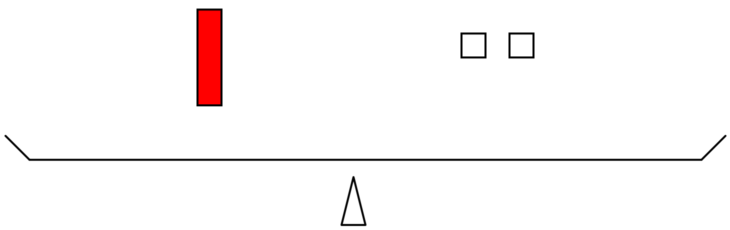
$2x = -4$

Divide both sides of the equation by 2.



$$\frac{2x}{2} = \frac{-4}{2}$$

$x = -2$

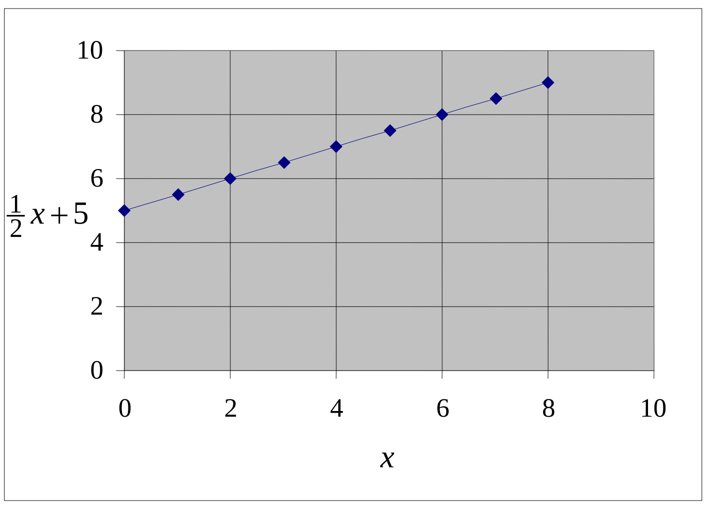


## Sample Activity 4: Mathematical Debate

Three students are having a debate about the equation $y = \frac{1}{2}x + 5$. Student A thinks that every time $y$ changes by 1, $x$ changes by 2. Student B thinks that every time $y$ changes by 1, $x$ will change by $\frac{5}{2}$. Student C thinks that every time $y$ changes by $\frac{1}{2}$, $x$ changes by 1. Who do you think is correct and why? Use at least two representations to prove your case.

Encourage students to create a table of values and a graph to help them contextualize and represent the problem in other ways. They should find that both Student A and Student C are correct if they look at the progression of values in either a table or on a graph.

| $x$ | $\frac{1}{2}x + 5$ |
|---|---|
| 0 | 5 |
| 1 | 5.5 |
| 2 | 6 |
| 3 | 6.5 |
| 4 | 7 |



## Sample Activity 5: Representing and Solving Problems with Linear Equations

There are a variety of problems that can be solved using linear equations. Ask students to solve these equations using a personal strategy that is then shared with the class. Examples include:

- Age problems
  - The sum of Adrien's age and his mother's age is 62. Adrien's mom is three times and four years older than he is. How old are they?
- Number problems
  - 10 less than $\frac{1}{4}$ of a number is 5. What is the number?
  - Mario's weight is 2 kg less than twice his sister Jayne's weight. Together they weigh 97 kg. How much do they each weigh?
- Money problems
  - For a school play, 680 tickets sold for $1,620. Tickets in the front half sold for $3, while tickets in the bleachers sold for $2. How many tickets of each price sold?
- Distance and travel problems
  - Taylor has 88 cm of trim to place around a triangular sign. Two sides of the triangle are 10 cm shorter than the third side. If Taylor uses all of the trim, what are the side lengths of the triangle?

Consider using open-ended problems (with small group discussion and then class sharing) in addition to more traditional questions to help students expand their repertoire of problem-solving approaches. You may also want to set up scenarios where students need to compare situations to arrive at a solution. For example:

Yolanda's school is holding an evening fundraiser where students set up various games and competitions. There are two ways that students can participate: 1) they can pay a $5 entry fee and play games for 75 cents each, or 2) they can pay no entry fee and play games for $1.25 each.

- If Yolanda has $15 to spend on games that evening, what choice will allow her to play the maximum number of games?

- If Yolanda has an unlimited amount of money, would one choice be better than the other?

## Sample Activity 6: Error Analysis

Error analysis is an effective way to improve understanding of common mistakes and engage students in higher-level thinking activities. Develop and use questions like:

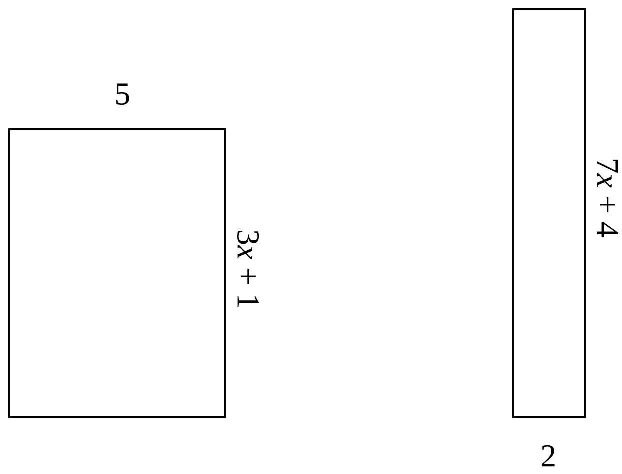
Mercedes has made some errors in her solution to the question. Can you find them?

$$
\begin{aligned}
3(3x + 12) &= -2(3x - 6) \\
9x + 12 &= -6x - 6 \\
9x - 6x + 12 &= -6 \\
3x &= -6 + -12 \\
3x &= 18 \\
\frac{3x}{3} &= \frac{18}{3} \\
x &= 6
\end{aligned}
$$

## Sample Activity 7: Connecting Algebra to Area

The two rectangles below have the same area. Ask students to find:

- the value of $x$
- the area of each rectangle.



## Sample Activity 8: Representing an Area Question with $ax = b$, $\dfrac{b}{x} = a$, and $\dfrac{b}{a} = x$

A rectangle has an area of 15.3 $cm^2$. The length is 5.1 and the width is $x$. Show at least three different ways to write this equation. Ask students to solve each equation.

Have students share their equations and discuss which equations correctly represent the problem.

Possible correct equations:

- $5.1x = 15.3$
- $\frac{15.3}{x} = 5.1, x \neq 0$
- $\frac{15.3}{5.1} = x$

Ask students to:

- share the steps for their solutions
- discuss why all equations that represent the problem arrive at the same solution
- discuss why $x \neq 0$ is included for the second sample solution.

## Step 4: Assess Student Learning

### Guiding Questions

- Look back at what you determined as acceptable evidence in Step 2.
- What are the most appropriate methods and activities for assessing student learning?
- How do I align my assessment strategies with my teaching strategies?

In addition to ongoing assessment throughout the lessons, consider the following sample activities to evaluate students' learning at key milestones. Suggestions are given for assessing all students as a class or in groups, individual students in need of further evaluation, and individual or groups of students in a variety of contexts.

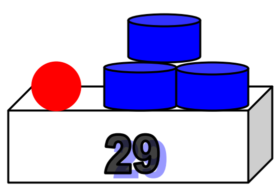
### A. Whole Class/Group Assessment

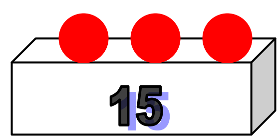
**Exit Cards and Specific Feedback**

As you listen and observe your students discussing solutions to the questions, opportunities to assess student thinking are increased. As the first five minutes of class and the last five minutes of class are often the most critical learning times in a class, consider using an exit card with a question that addresses the outcome(s) covered in the class. Use the last five to 10 minutes of class to have students respond to questions like the ones below:

**Activity 1**

What is the value of each unknown? Show how you know.




**Activity 2**

Write three equivalent expressions for $4(n + 2)$.

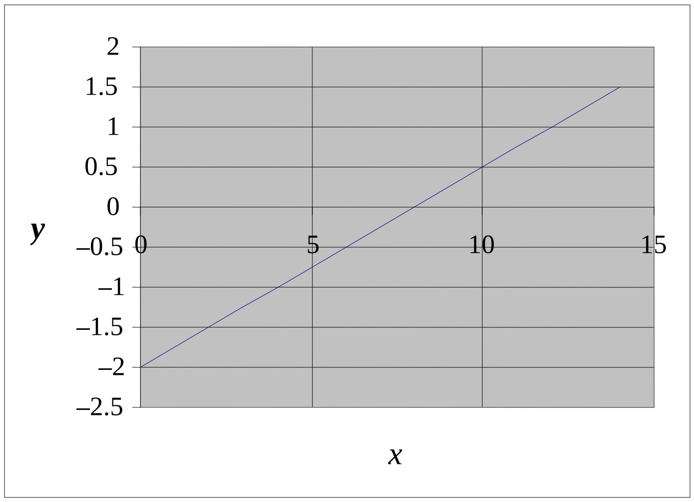
**Activity 3**

Compare the method of solution for the following two sets of equations:

- $\frac{x}{5} = 2.5$ and $\frac{5}{x} = 2.5$
- $2(3x + 1) = 4(3x + 8)$ and $-2(3x + 1) = -4(3x + 8)$

**Activity 4**

Create a function in Excel. Provide students with a copy of the graph and ask them to find the equation that describes the linear function. They must show steps or give an explanation proving their solution.




**Activity 5**

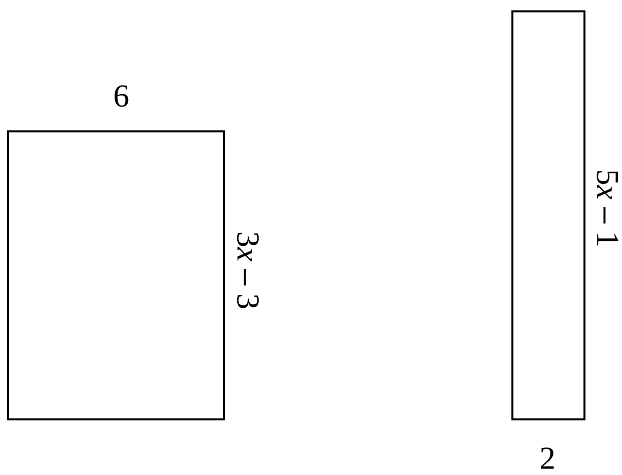
Have students select a question from the practice section of their resource and have them solve the same question but include one or two errors. Students can analyze another student's questions to try to locate the errors that were made.

The regular use of an exit question helps guide your lesson planning and is a good opportunity to provide short comments and feedback to students. Research shows that if you provide specific feedback, without a grade, student understanding and performance on summative assessments are improved.

**Activity 6**

The two rectangles below have the same area. Ask students to find:

- the value of $x$
- the area of each rectangle.




**Activity 7**

Prove that $\frac{18}{x} = 6$ and $6x = 18$ are related equations.

**Student Self-assessment: Rating Scale**

Student self-assessment is a very effective assessment strategy. Students must be aware of the goals and outcomes they are trying to meet as well as how well they meet these goals. Self-assessment involves four stages:

1. Students and the teacher negotiate and co-create the criteria for assessing the work.
2. Teacher assists students in defining levels of performance through the development of a rubric or rating scale.
3. Teacher coaches students in applying the measures.
4. Teacher provides specific feedback to students about their self-assessment.

Consider using descriptors to complement or replace a numerical scale. For example:

- novice or not yet there
- proficient or on target
- advanced or above and beyond.

Here is a possible example of a self-assessment.

Have students select one of the three following levels for each of the criteria and comment on why their learning best fits into that category:

| I am able to: | Novice | Proficient | Advanced | Comment |
|---|---|---|---|---|
| • solve linear equations using a balance or algebra tiles | | | | |
| • algebraically solve linear equations numerically and verify my answer | | | | |
| • translate a line on a graph into a table of values and a linear equation | | | | |
| • translate word problems into linear equations and solve them | | | | |

**Using Technology**

There are a number of activities that can be used to reinforce and assess student learning on the Interactivate: Activities (Shodor) website at http://www.shodor.org/interactivate/activities/. Notes for students and instructors are given and a help button for the game is provided.

The two activities in the links below allow the student to select options to differentiate according to:

- time allowed to arrive at a solution (10 seconds to unlimited)
- difficulty level
- equation structure, including:
  - variables on both sides
  - using the distributive property
  - one-step
  - two-step
  - quadratic (**Note**: do not select as this is a senior high school outcome).

The first activity is a game that two students can play and the second is an individual quiz activity. See the following websites:

- http://www.shodor.org/interactivate/activities/AlgebraFour/
- http://www.shodor.org/interactivate/activities/AlgebraQuiz/.

### B. One-on-one Assessment

When students appear to be having difficulty with concepts or outcomes, providing specific questions through a structured interview may help assess the source of difficulties. These one-on-one assessments should help the teacher gain insight into the student's thinking as well as provide the opportunity to clear up misunderstandings.

### C. Applied Learning

Provide opportunities for students to create and solve linear equation problems that involve travel, money, age or number. Some ideas that may help students get started in the creation of questions is to have them investigate:

- the impact that increasing mass has on the length of an elastic. Using the same elastic, students should measure its original length and then hang increasing masses on the elastic and measure the changes
- how increasing the length and width of a square or rectangle by two or four units changes the value of the unknowns in the equation
- fuel consumption on a family trip by comparing a small and large vehicle
- earnings for different jobs; e.g., a bus person or server in a restaurant makes a base salary plus tips (students could look at the relationship between hours worked and total earnings based on anticipated tips)
- data from cafeteria or snack machines where students might set up proportional situations based on beverage consumption (in litres) for the cafeteria or school population and extrapolate this information to a larger context.

## Step 5: Follow-up on Assessment

### Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction?

### A. Addressing Gaps in Learning

Consider the following strategies to assist students who are having difficulty:

- Begin working with one-step equations that involve addition, subtraction, multiplication and division, such as:
  - $n + 3 = 6$
  - $n - 3 = 6$
  - $2n = 6$.

  Ask students to show or talk about how they can solve the equations. Encourage them to try using the balance model, algebra tiles or pictures of tiles. Listen and observe to see if there is one model that is easiest for them to use.

- Introduce two-step equations, such as:
  - $3n + 2 = 14$
  - $4n - 3 = 9$.

  Ask students to show or talk about how they can solve the equations. Encourage them to try using the balance model, algebra tiles or pictures of tiles. Listen and observe to see if there is one model that is easiest for them to use.

### B. Reinforcing and Extending Learning

Students who have achieved or exceeded the outcomes will benefit from ongoing opportunities to apply and extend their learning. These activities should support students in developing a deeper understanding of the concept and should not progress to the outcomes in subsequent grades.

Consider strategies, such as the following.

**Activity 1**

With students in small groups or pairs, display questions, one at a time, and have students solve them with tiles; then, use the following template to record the steps pictorially and numerically. The progression from concrete to abstract and then from integral to rational should help scaffold the skills students learned in Grade 8 and help them to apply their learnings to solve equations with rational coefficients and constants.

| Solve using algebra tiles and a balance model (show all steps) | Solve (show all steps and verify your answer) | More difficult: Solve (show all steps and verify your answer) |
|---|---|---|
| $3x = -9$ | $3n = -9$ | $\frac{1}{2}n = 12$ |
| $2x + 3 = -1$ | $2n + 3 = -1$ | $-0.4n + 3.6 = 12.8$ |
| $2(x - 3) = 4$ | $2(n - 3) = 4$ | $\frac{3}{4}(n-4) = 5$ |
| $3x + 2 = x - 4$ | $3x + 2 = x - 4$ | $\frac{3}{4}n - \frac{1}{2} = \frac{2}{3}n - \frac{2}{3}$ |
| $3(2x + 2) = 2(2x - 4)$ | $3(2n + 2) = 2\ (2n - 4)$ | $0.8(6n + 6) = 0.6(4n - 12)$ |
| $\frac{x}{3} = 2$ | $\frac{x}{3} = 2$ | $\frac{x}{2} = 0.8$ |
| $\frac{12}{x} = 3$ | $\frac{12}{x} = 3$ | $\frac{2.5}{n} = 0.5$ |
| $\frac{x}{3} + 5 = -2$ | $\frac{x}{3} + 5 = -2$ | $\frac{x}{0.4} + 8.2 = 12.8$ |

**Activity 2: Games**

Using two dice (one for positive numbers and one for negative), have students throw the dice twice. Substitute the value from each die into the two boxes on the left-hand side of the equation on the first toss and into the two boxes on the right-hand side of the equation on the second toss:

- $\square x + \square = \square x + \square$

Students should decide which die colour represents positive values and which die colour represents negative values. For example, if, on the first toss, the student throws a 3 and 4 this could represent –3 and 4; and if, on the second toss, the student throws a 2 and a 3, use –2 and 3. Students decide which box contains which dice value. The first equation to solve could be:

- $-3x + 4 = -2x + 3$

Other equation formats that help students reinforce the concept of subtraction of negative values are:

- $\square x - \square = \square x + \square$
- $\square x + \square = \square x - \square$
- $\square x - \square = \square x - \square$

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