**Mathematics**

# Planning Guide

## Grade 9
## *Patterns in Linear Equations*

## Patterns and Relations (Patterns)
Specific Outcomes 1 and 2

**This Planning Guide can be accessed online at:**
http://www.learnalberta.ca/content/mepg9/html/pg9_patternsinlinearequations/index.html

## Table of Contents

# **Planning Guide:** *Grade 9 Patterns in Linear Equations*

**Strand:** Patterns and Relations (Patterns)
**Specific Outcomes:** 1 and 2

This *Planning Guide* addresses the following outcomes from the Program of Studies:

| | |
|---|---|
| **Strand:** Patterns and Relations (Patterns) | |
| **Specific Outcomes:** | 1. Generalize a pattern arising from a problem-solving context, using a linear equation, and verify by substitution. [C, CN, PS, R, V] |
| | 2. Graph a linear relation, analyze the graph, and interpolate or extrapolate to solve problems. [C, CN, PS, R, T, V] [ICT: C7–3.1, P2–3.3] |

## Curriculum Focus

The changes to the curriculum targeted by this sample include:

- The general outcome has an increased focus on connecting and contextualizing linear equations, whereas the previous mathematics curriculum focused on generalizing, designing and justifying mathematical procedures.
- The specific outcomes focus on writing expressions and equations that represent a variety of patterns. Students are asked to solve equations and graph linear equations. Students then are asked to extrapolate information from the patterns that are represented.
- The previous curriculum focused on logic, divergent thinking and manipulation of rational algebraic expressions.

## What Is a Planning Guide?

**Planning Guides** are a tool for teachers to use in designing instruction and assessment that focuses on developing and deepening students' understanding of mathematical concepts. This tool is based on the process outlined in *Understanding by Design* by Grant Wiggins and Jay McTighe.

## Planning Steps

The following steps will help you through the Planning Guide:

## Step 1: Identify Outcomes to Address

### Guiding Questions

- What do I want my students to learn?
- What can my students currently understand and do?
- What do I want my students to understand and be able to do based on the Big Ideas and specific outcomes in the program of studies?

### Big Ideas

- Patterns represent identified regularities. Types of patterns include complex repeating patterns, increasing and decreasing patterns, and recursive patterns (Small 2009, p.3).
- A variable is a symbol that can stand for any one of a set of numbers or other objects and can be represented by boxes or letters (Van de Walle and Lovin 2006, p. 274).
- Patterns are key factors in understanding mathematical concepts. The ability to create, recognize and extend patterns is essential for making generalizations, seeing relationships and understanding the order and logic of mathematics (Burns 2000, p. 112).
- Patterns can be recognized, extended and generalized into symbols, words, diagrams, number lines, charts and graphs (Van de Walle and Lovin 2006, p. 265).

## Sequence of Outcomes from the Program of Studies

See http://education.alberta.ca/teachers/program/math/educator/progstudy.aspx for the complete program of studies.

| Grade 8 | Grade 9 | Grade 10 |
|---|---|---|
| **Patterns and Relations** | **Patterns and Relations** | **Relations and Functions** |
| **Specific Outcome** | **Specific Outcomes** | **Specific Outcomes** |
| 1. Graph and analyze two-variable linear relations. [C, ME, PS, R, T, V] [ICT: P2–3.3] | 1. Generalize a pattern arising from a problem-solving context, using a linear equation, and verify by substitution. [C, CN, PS, R, V]<br><br>2. Graph a linear relation, analyze the graph, and interpolate or extrapolate to solve problems. [C, CN, PS, R, T, V] [ICT: C7–3.1 P2–3.3] | 1. Interpret and explain the relationships among data, graphs and situations. [C, CN, R, T, V] [ICT: C6–4.3, C7–4.2]<br><br>4. Describe and represent linear relations. using<br>• words<br>• ordered pairs<br>• table of values<br>• graphs<br>• equations.<br>[C, CN, R, V]<br><br>5. Determine the characteristics of the graphs of linear relations, including the:<br>• intercepts<br>• slope<br>• domain<br>• range.<br>[CN, PS, R, V] |

## Step 2: Determine Evidence of Student Learning

### Guiding Questions

- What evidence will I look for to know that learning has occurred?
- What should students demonstrate to show their understanding of the mathematical concepts, skills and Big Ideas?

### Using Achievement Indicators

As you begin planning lessons and learning activities, keep in mind ongoing ways to monitor and assess student learning. One starting point for this planning is to consider the achievement indicators listed in the *Mathematics Kindergarten to Grade 9 Program of Studies with Achievement Indicators*. You may also generate your own indicators and use them to guide your observation of the students.

The following indicators may be used to determine whether or not students have met specific outcomes 1 and 2. Can students:

- write an expression representing a given pictorial, oral or written pattern?
- write a linear equation to represent a given context?
- describe a context for a given linear equation?
- solve, using a linear equation, a given problem that involves pictorial, oral and written linear patterns?
- write a linear equation representing the pattern in a given table of values, and verify the equation by substituting values from the table?
- describe the pattern found in a given graph?
- graph a given linear relation, including horizontal and vertical lines?
- match given equations of linear relations with their corresponding graphs?
- extend a given graph (extrapolate) to determine the value of an unknown element?
- interpolate the approximate value of one variable on a given graph, given the value of the other variable?
- extrapolate the approximate value of one variable from a given graph, given the value of the other variable?
- solve a given problem by graphing a linear relation and analyzing the graph?

Sample behaviours to look for related to these indicators are suggested for some of the activities listed in **Step 3, Section C: Choosing Learning Activities** (p. 10).

## Step 3: Plan for Instruction

### Guiding Questions

- What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should I use?
- How will I meet the diverse learning needs of my students?

### A. Assessing Prior Knowledge and Skills

Before introducing new material, consider ways to assess and build on students' knowledge and skills related to patterns. You may choose to use indicators from prior grades to help to determine what students should know or understand. Consider using open-ended questions to more accurately assess what students are able to communicate and do.

Research has shown that students often have misconceptions about the concept of variables. Usiskan (1988) identified three different ways that variables are used:

1. As a specific unknown with a single value; e.g., $2n + 15 = 5n + 3$. Students often are asked to solve for the unknown value.
2. As a pattern generalizer; e.g., $2n + 5$ where $n$ can have an infinite number of values.
3. Variables that change in relation to one another; e.g., $C = 2\pi r$, where the measure of the radius ($r$) affects the value of the circumference ($C$).

It is important that students have an opportunity to think about the different ways that variables are used, as in these outcomes, possibilities 2 and 3 are both used.

**Activity 1: Assessing Student Understanding of Math Language**

Determining student understanding of two important terms used in this unit will help you assess prior knowledge and understanding. These two formative assessment activities can be used as opener activities (alone, together or as a differentiated or choice option for students). Choice A is more open-ended, while Choice B is supported with more guiding examples or questions.

**Choice A**. Ask students to work with a partner to write a definition of equations and expressions. Have them show at least one similarity and one difference between these two terms. Ask students to support their definitions with examples.

**Choice B**. Provide students with examples of expressions and equations.

These are equations:
$2 + 5 = 7$
$3a - 4 = 17$
$x + 5 = 12 - x$
$15 = 15$.

These are expressions:
$7x - 5$
$y^2$
$5x - 8y$

(7) (8).
Ask students to categorize the examples below and to explain their reasoning.

$n^3$
$b + 7 = 42$
$7 < x$
$35 - 23 = a$
$\frac{21}{y}$
$3x^2 + 2y - 1$
$2 = 7 - 5$

**Activity 2: Use Achievement Indicators from Prior Grades to Determine Student Readiness**

Grade 8 Achievement Indicator: Determine the missing value in an ordered pair for a given equation.

a. For the equation $y = 5x$, fill in the missing values for the following ordered pairs: (5, ___); (___, 5); (1, ___); (___, 1).
b. For the equation $y = 5x$, find three other ordered pairs that will make the equation true.
c. Graph the ordered pairs from the prior two activities. Explain how you set up the graph and located each of the ordered pairs. Describe any pattern that you see in your graph.

**Activity 3: Looking for Patterns**

Asking students to find similarities and differences between two sets of number patterns helps them explore the patterns more deeply (Van de Walle 2001, p. 391). For example:

a. Extend each of the following patterns by at least five more numbers.
b. Write an explanation about how the following patterns are alike and how they are different.
   2, 4, 6, 8, 10, … and 3, 5, 7, 9, 11, …
   2, 4, 6, 8, 10, … and 2, 4, 8, 16, 32, …

If students appear to have difficulty with these tasks, consider further individual assessment, such as a structured interview, to determine students' levels of skill and understanding. See **Sample Structured Interview: Assessing Prior Knowledge and Skills** (p.11).

## Sample Structured Interview: Assessing Prior Knowledge and Skills

| **Directions** | Date: | |
|---|---|---|
| | **Not Quite There** | **Ready to Apply** |
| Read the following to the student:<br>"If $k$ represents the number of kids in the gym, think of a situation that might represent:<br>a. $k - 7$<br>b. $\frac{k}{2}$." | The student cannot think of a context. The student is unable to translate the operations into English.<br><br>The student is not aware of what one or both of the operation signs represent or may reverse the meaning. | The student is able to articulate a context, such as seven kids had to go back to class or half the class got to go on a field trip.<br><br>The student understands the use of the subtraction and division operation. |
| Ask the student to write a number expression for each of the following English expressions:<br>a. a number, $x$, is increased by nine<br>b. half the number of DVDs in a store were sold on Saturday<br>c. your mom's age is triple your age decreased by seven. | The student is not able to translate the operations in the math expressions correctly.<br><br>The student is able to determine the operation but is unable to translate the expression completely.<br><br>The student reverses the operation or correct order of the expression. | The student translates the expressions correctly.<br><br>The student may provide alternatives. For example:<br>• in question b., the student might use:<br>$\frac{d}{2}$<br>$d - \frac{1}{2}d$<br>$d - \frac{d}{2}$<br>• in question c., the student might say "if my age is $n$, then my mom's age is $3n - 7$." |
| Provide students with graph paper and ask them to draw a Cartesian plane and label the $x$ and $y$ axis.<br><br>Then ask them to plot the following three points:<br>A = (5, 1) B = (5, 7)<br>C = (–1, 7).<br><br>Ask them to plot and label point D, so that they build a square ABCD. | The student may reverse the $x$ and $y$ values.<br><br>The student may not be able to locate and/or identify point D correctly. | The student is able to place the points in the correct location and is able to complete the square. |

## B. Choosing Instructional Strategies

Consider the following general strategies for teaching patterns in Grade 9.

- Students should be engaged in conversations and discussions of open-ended questions to build confidence and competence with mathematics.
- When students are given opportunities to communicate their thinking (either orally or in writing), explain their reasoning and listen to the strategies used by other students, there are more opportunities to deepen their understanding.
- Have students model patterns using hands-on material or drawings that help them understand the connections between different representations.
- Students should examine how geometric and numeric patterns can be represented in multiple formats, including oral descriptions, symbols, tables and graphics. They should also be able to verify their answers by substitution.

## C. Choosing Learning Activities

The following learning activities are examples that could be used to develop student understanding of the concepts identified in Step 1.

**Sample Activities:**

1. **Starter** (p. 11)
2. **Guess My Rule** (p. 12)
3. **Translating English and Building Patterns with Variables** (p. 14)
4. **Using Technology** (p. 15)
5. **Contextualizing a Problem with Several Possible Representations to the Solution** (p. 16)
6. **Extending Growing Patterns** (p. 17)
7. **Findings Patterns in Graphs** (p. 18)
8. **Solve Problems** (p. 19)

## Sample Activity 1: Starter

Represent each of the following in at least two different ways:

a. a number increased by five
b. three more than half the number of students in a class
c. your age, if your cousin is twice your age decreased by five.

You may want to consider having larger size paper such as chart paper or 11" x 17" paper, so that, when students share, their solutions are easy for the rest of the class to see. If students cannot think of first or second representations, then suggest that they consider a math expression, a table of values or a graph. How is the pattern for each situation similar and how is it different?

## Sample Activity 2: Guess My Rule

a. Have students try to "Guess My Rule …"

| Starting Value | Rule | Final Value |
|---|---|---|
| 4 | | 12 |

There are many solutions to this rule including $3n$, $n + 8$, $2n + 4$ and $5n–8$.

Ask students how many values they would require to more accurately guess the rule (Schuster and Canavan Anderson 2005, p. 105).

b. You can continue this exploration by presenting students with questions that ask them to write down a secret number. You ask them a series of questions and then, finally, "magically" guess their secret number.

For example:

- Choose a number
- Add 5
- Double the result
- Subtract 4
- Divide the result by 2
- Subtract the number you started with
- The result is … 3.

Ask students to try to prove why the answer is always 3. Ask students to substitute to prove their answer. Here is a possible solution:

| | | |
|---|---|---|
| Choose a number | $x$ | 7 |
| Add five | $x + 5$ | $7 + 5 = 12$ |
| Double the result | $2(x + 5)$ or $2x + 10$ | $2(7) + 10 = 24$ |
| Subtract 4 | $2x + 10 – 4 = 2x + 6$ | $2(7) + 6 = 20$ |
| Divide by 2 | $\frac{2x+6}{2} = x + 3$ | $20 \div 2 = 10$ |
| Subtract the original number | $x + 3 – x = 3$ | $10 – 7 = 3$ |

(Briston 1996)

c. There are investigations for guessing rules in Wheatley and Abshire's *Developing Mathematical Fluency: Activities for Grades 5–8*.

Provide students with activities like:

| $x$ | $y$ |
|---|---|
| 10 | 20 |
| 5 | 25 |
| 3 | 27 |
| 16 | |
| | 40 |

Students complete the table and try to find the rule. In this case, it is 30–$n$. You should also provide examples with rational values where the rule might be $x + \frac{1}{2}$, $x - \frac{1}{4}$ or $x + 0.005$.

Ask students to try to develop their own "Guess My Number" or "Number Trick" and use verification to prove the result. Have students share their work with one another.

## Sample Activity 3: Translating English and Building Patterns with Variables

Each student shown in the table below selects a different value for $x$. Try to find all of the missing values:

| **English** | **Algebra** | Student 1 | Student 2 | Student 3 | Student 4 | Student 5 |
|---|---|---|---|---|---|---|
| Think of a number | $\boldsymbol{x}$ | **2** | | | | |
| Add 8 | $\boldsymbol{x + 8}$ | **10** | | | | |
| Triple it | $\boldsymbol{3x + 24}$ | **30** | | **84** | | |
| Subtract 8 | | | | | **190** | |
| Add the original number | | | | | | **20** |
| Divide by 4 | | | | | | |
| Subtract 4 | | | **14** | | | |

Ask students to share their strategies for filling in the table. You may want to ask questions like:

- What was your first step?
- What strategies did you use as you completed this task?
- Where did you run into difficulty with this task?

**Look For …**

Do students:

☐ use the algebra or try to work backward to "undo" what was done to the original number?

| **English** | **Algebra** | Student 1 | Student 2 | Student 3 | Student 4 | Student 5 |
|---|---|---|---|---|---|---|
| Think of a number | $\boldsymbol{x}$ | **2** | 14 | 20 | 58 | 1 |
| Add 8 | $\boldsymbol{x + 8}$ | **10** | 22 | 28 | 66 | 9 |
| Triple it | $\boldsymbol{3x + 24}$ | **30** | 66 | **84** | 198 | 27 |
| Subtract 8 | $3x + 16$ | 22 | 58 | 76 | **190** | 19 |
| Add the original number | $4x + 16$ | 24 | 72 | 96 | 248 | **20** |
| Divide by 4 | $x + 4$ | 6 | 18 | 24 | 62 | 5 |
| Subtract 4 | $x$ | 2 | **14** | 20 | 58 | 1 |

## Sample Activity 4: Using Technology

a. If you have access to an electronic whiteboard or students have access to computers, there are some effective online activities such as Interactivate: Activities (Shodor) Function Machine at http://www.shodor.org/interactivate/activities/FunctionMachine/?version=1.6.0_05&browser=MSIE&vendor=Sun_Microsystems_Inc.

   Students interact with the game by entering a number and trying to guess the function or rule that is provided.

b. Students also can use the functionality in a graphing program such as Microsoft Excel to record patterns. They can electronically create the graph for the expressions they are investigating in the Function Machine.

c. There is another function game on the National Library of Virtual Manipulatives at http://nlvm.usu.edu/en/nav/frames_asid_191_g_3_t_2.html?from=category_g_3_t_2.html.

   This game helps students look for a pattern and then complete the function. An enhancement to this activity would be to have students translate the tables of values from this activity into a graph.

## Sample Activity 5: Contextualizing a Problem with Several Possible Representations to the Solution

Students are setting up tables for the grade awards celebration at the school, and they want everyone to sit around square tables. If there are exactly 144 people attending the event, determine all of the square tables arrangements that can be used to evenly seat everyone. A table with a side length of one unit can fit one seat per side and a table with a side length of two units will fit two chairs per side.

Provide dot paper or geoboards and graph paper for students. You may also want to consider letting students represent their solution on an electronic whiteboard (if there is access to one). If there is access to technology, consider having students explore the problem at this Web site: http://www.shodor.org/interactivate/activities/Chairs/.

**Questions to pose to students**

- What is the smallest table that can be used? How many will it seat?
- Can you predict how many possible solutions there may be?
- How many possible answers did you find? How many are not possible?
- What strategy will you use to find your answer? (Encourage students to share all possible representations of the solution, including diagrams, charts, expressions, equations or graphs.)
- What is the largest possible table?

Record your solution and represent at least two patterns that you found.

One possible representation of the solution:

| Length side | # seats | # tables required |
|---|---|---|
| 1 | 4 | 36 |
| 2 | 8 | 18 |
| 3 | 12 | 12 |
| 4 | 16 | 9 |
| 5 | 20 | 7.2 not a solution |
| 6 | 24 | 6 |
| 7 | 28 | 5.14 not a solution |
| 8 | 32 | 4.5 not a solution |
| 9 | 36 | 4 |
| 10 | 40 | 3.6 not a solution |
| 11 | 44 | 3.27 not a solution |
| 12 | 48 | 3 |
| 13 | 52 | 2.77 not a solution |
| 14 | 56 | 2.57 not a solution |
| 15 | 60 | 2.4 not a solution |
| 16 | 64 | 2.25 not a solution |
| 17 | 68 | 2.18 not a solution |
| 18 | 72 | 2 |
| 36 | 144 | 1 |
| | | Total possible solutions: 9 |

## Sample Activity 6: Extending Growing Patterns

One of the important parts of this outcome is to have students extrapolate information from a pattern or graph. Building geometric patterns with manipulatives or using pencil and paper helps students see the pattern in a concrete representation, thereby facilitating the abstraction of the pattern. Provide students with various regular polygon tiles or pattern blocks. Ask students to show diagrams in their journals to capture their representations and thinking.

**Sample Tasks:**

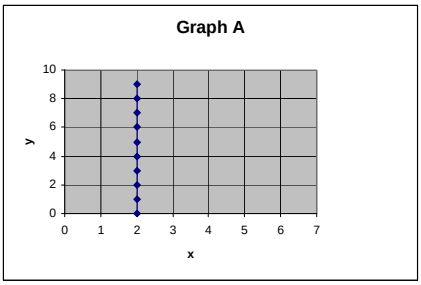
a. begin with one tile; count the sides and then add another tile and count the unconnected sides
b. extend and draw at least three more steps to the pattern created
c. build a table of values to represent how many sides can be counted for each step of the pattern
d. look for a pattern and represent the pattern with an expression
e. graph the results in the table of values
f. predict how many items there are in the $100^{th}$ step
g. repeat steps two more times with two different polygons.

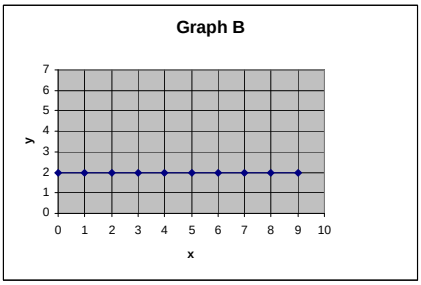
Student questions for reflection or journals:

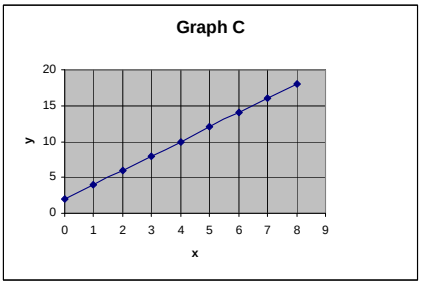
- How do the three lines on the graph compare? Explain why.
- How did you predict what the side count for the $100^{th}$ step is?

## Sample Activity 7: Finding Patterns in Graphs

Look at and analyze the following three graphs. Think about at least two other ways to express the patterns in these graphs. Suggest similarities and differences. How does the table of values for each set of graphs differ? What equation might represent each situation?




**Look For …**

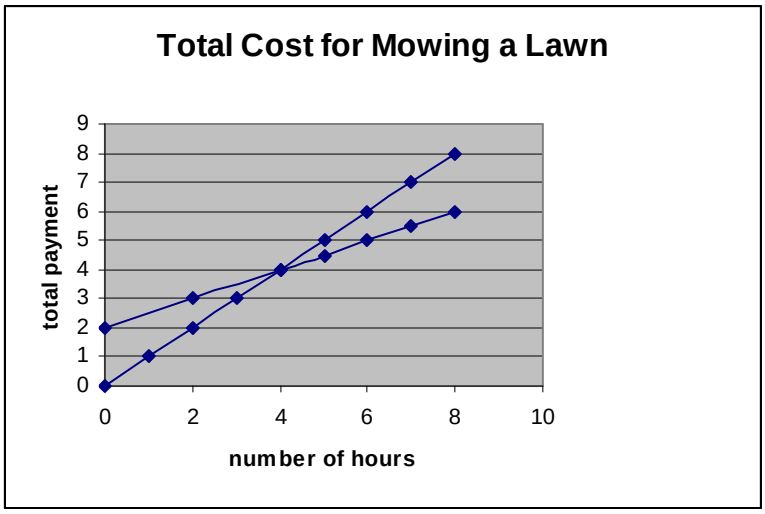
Do students:

- ☐ reverse the form of the equation for horizontal and vertical lines?

## Sample Activity 8: Solve Problems

A student spends his or her summer mowing lawns to earn extra money. He or she offers his or her customers two different payment options.

a. If the lines on the graphs below represent the two payment options, explain what they might be.
b. What would he charge if it took him nine hours to mow the lawn and if it took him 10 hours?




**Look For …**

Do students:

- ☐ identify the slope as the rate of pay per hour?
- ☐ know the significance of the intersection of the lines at four hours?

## Step 4: Assess Student Learning

### Guiding Questions

- Look back at what you determined as acceptable evidence in Step 2.
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

In addition to ongoing assessment throughout the lessons, consider the following sample activities to evaluate students' learning at key milestones. Suggestions are given for assessing all students as a class or in groups, individual students in need of further evaluation, and individual or groups of students in a variety of contexts.

### A. Whole Class/Group Assessment

Formative assessment or assessment for learning focuses on student learning and understanding during the instruction process and occurs naturally in learning activities. Formative assessment does not require grading; instead, it helps teachers better understand student thinking and determine:

- in what ways a student understands
- to what degree a student understands a math problem or concept.

Teachers may use a variety of tools to assist with formative assessment, including observation, listening to student conversation and responses to open-ended questions, feedback, interaction and reflection with students or groups of students. Evidence of learning may be gathered using photographs, videos, observation checklists, student journals or rough drafts.

Formative assessment informs teaching decisions and provides the tools to evaluate and redesign lessons, as necessary.

**Feedback**
Observations of student learning during learning activities and through written work provide opportunities to assess student understanding and provide questions and comments to help students improve their understanding. Feedback should be timely and include references to what students already know and what they have done well, in addition to what needs improvement. You may want to consider having students redo an assignment or question so that they have a chance to clear up misconceptions or errors.
For example:
"*Your work shows a good understanding of translating math sentences into math expressions and tables of value. You need to continue working on graphing ordered pairs as you occasionally reverse the* x *and* y *values. Please redo your graph and resubmit.*"

**Open-ended Problems or Questions**

Use of open-ended problems helps students move away from memorization-based drill and practice toward developing strategies for solving problems in a non-routine manner. This conceptual approach helps students deepen their understanding. This process often presents a struggle for both students and teachers, but with persistence, students often develop enough confidence so that they do not want to be told the steps or the rules. Developing a culture of open discussion and risk taking is important for both teachers and students as they begin to solve open-ended questions. Use of starter activities is an effective way to help teachers focus on observing student conversations, pose appropriate questions to help students move forward in their thinking and listen to student explanations and viewpoints. Questions that you may consider posing include:

- Solutions A and B have the same answer but the process looks different. Which one is correct? Why?
- How are solutions A and B similar? How are they different?
- What makes a solution efficient? Are some of our solutions more efficient than others?
- Can you classify your solutions into categories of strategies, such as using a table and working backward?
- Can you make up a similar problem to solve?

**Checklists**

Consider using a checklist of student names and indicators that you may be looking for during a learning activity. For the learning activities, you may want to develop a specific checklist or use the generic one shown below:

Yes (✓) or Not Yet (×)

| Student name | Communicates suggestions for solution | Justifies solution process and result | Contributes alternate solutions or strategies | Understands the concept |
|---|---|---|---|---|
| | | | | |
| | | | | |
| | | | | |
| | | | | |

**Student Self-assessment: Rating Scale**

Student self-assessment is a very effective assessment strategy. Students must be aware of the goals and outcomes that they are trying to meet as well as how well they meet these goals. Self-assessment involves four stages:

1. Students and the teacher negotiate and co-create the criteria for assessing the work.
2. The teacher assists students in defining levels of performance through the development of a rubric or rating scale.
3. The teacher coaches students in applying the measures.
4. The teacher provides specific feedback to students about their self-assessment.

Consider using descriptors to complement or replace using a numerical scale. For example:
1. novice or not yet there
2. proficient or on target
3. advanced or above and beyond.

Here is a possible example of a self-assessment.

Have students select one of the three following categories for each of the criteria and comment on why their learning best fits into that category:

| I am able to: | Novice | Proficient | Advanced | Comments |
|---|---|---|---|---|
| draw/build and extend geometric patterns | | | | |
| represent the pattern in a table of values | | | | |
| represent the pattern in an algebraic expression | | | | |
| extend the pattern to predict the $100^{th}$ value | | | | |

**Journal and Exit Card Prompts**
Journals provide an excellent opportunity for students to reflect on their own learning and think about what they know and understand and what they still struggle with. Journal prompts help students focus on a particular outcome or learning activity.

Exit cards provide a quick assessment of what parts of a learning outcome students know or need more work on. Some ideas for prompts are:
- What patterns can you see in the following graph?
- How would you explain to a new student in your class the different ways to represent a horizontal and vertical line?
- Write a word problem or describe a situation that might be represented by the expression $4x + 30$.

## B. Small Group/Individual Assessment

Summative assessment usually occurs at the end of the learning cycle and provides an opportunity for students to demonstrate what they know and understand. Summative assessment usually includes the assignment of a score or grade to the student's work. You may want to provide opportunities for choice in terms of the types of products that students can use to demonstrate their understanding

In small groups, have students create a product or presentation that shows their understanding of a particular concept. For example:

- Does the graph of $y = 2x + 5$ ever intersect the graph of $y = {}^{-}2x + 5$? Explain and show how you know.
- Does the graph of $x = 2$ ever intersect the graph of $y = 2$? Explain and show how you know.
- Create three different line graphs (linear functions) where the ordered pair (2, 10) is one of the values. Explain how you solved this problem and represent your answer in at least two different ways.
- Create a geometric pattern by showing the first three elements. Demonstrate a solution to your question in at least three different ways. What would the $100^{th}$ value be?

## C. One-on-one Assessment

Assessment activities may require one-on-one interaction with students who are experiencing difficulty with an outcome.

- Modify the questions that are posed to the whole class so that they include more structure and support for students experiencing difficulty. For example, modify and scaffold the original question "*Does the graph of y = 2n + 5 ever intersect the graph of y = –2n +5? Explain and show how you know.*"
  - Show how you would create a linear graph for $y = 2x + 5$.
  - Show how you would find the graph of $y = {}^{-}2x + 5$.
  - Do the graphs intersect? If so, describe the location with an ordered pair.

If the two-step equation is too difficult, have the student look at one-step relationships such as $y = 2x$ and $y = {}^{-}2x$. See if the student understands that these expressions involve multiplication and can be rewritten as $y = 2 \times x$ or $y = 2 \bullet x$.

- Students may not be able to think about how to begin solving the question "*Create three different line graphs (linear functions) where the ordered pair (2, 10) is one of the values. Explain how you solved this problem and represent your answer in at least two different ways.*" Support their learning by:
  - asking them how they might use addition only or addition and multiplication to get 10 as a solution from the original value 2
  - asking them "2 added to what number gives 10?"
  - having students write down the expression in a table of values (with a third column for students to record ordered pairs) and then generate four other values; e.g.,

| $x$ | $y = x + 8$ | *Ordered pair* |
|---|---|---|
| *2* | *10* | *(2, 10)* |
| *3* | *11* | *(3, 11)* |
| *4* | *12* | *(4, 12)* |
| *0* | *8* | *(0, 8)* |
| *–2* | *6* | *(–2, 6)* |

  - having the students consider a negative value or zero for *n* and then having them reorder their *n* values from smallest to largest
  - supporting students in graphing their linear relationship
  - having students generate another linear relationship using two operations such as $y = 3x + 4$; then having them create and graph these new ordered pairs.

- Assist students who are having difficulty with generating a geometric pattern by providing them with a manipulative, such as toothpicks, and asking them to create a shape with the toothpicks. Ask students to:
  - build on this shape by creating the next shape in the pattern
  - articulate what they are changing
  - demonstrate how this change would look in a third shape
  - represent their change in a table of values and then a graph.

If a student has difficulty, guide him or her to begin with a simple regular geometric shape, such as a triangle or square, and ask if the student would like to use the number of sides or perimeter in the geometric relationship. You might have the student look at both number or sides and perimeter and see if there are similarities or differences.

You may also need to assist the student in understanding what parts of the pattern are represented by the *x* and *y* values in the student's pattern. The student also may need additional support in plotting the ordered pairs on the graph.

Review the concept of extending the values to the $10^{th}$, $20^{th}$ or $100^{th}$ value.

## D. Applied Learning

Provide opportunities for students to investigate real-life situations such as comparing cell phone plans or television and satellite packages. Students can compare how various monthly plans are similar or different in terms of how much they cost. They can personalize their investigation in terms of their own activities that may include more text messaging, more talking or particular channels that their families prefer. Cell phone and television providers often show the various plans and costs on their corporate Web sites, so students can access real information. Students might investigate the question "What is the most cost effective plan for my own pattern of cell phone use?"

## Step 5: Follow-up on Assessment

### Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction?

### A. Addressing Gaps in Learning

If students are having difficulty solving the basic facts using strategies, consider the following:

- Work one-on-one with a student as he or she completes a balance problem. Listen carefully to the language the student uses in order to determine where his or her misunderstanding is.
- Encourage the student to use algebra tiles and/or draw pictures to visually represent the problem.
- If a student has difficulty following an algebraic solution, work first to have the student verbally and/or visually represent the solution process, then move to translating the process into the algebraic representation.

### B. Reinforcing and Extending Learning

Students who have achieved or exceeded the outcomes will benefit from ongoing opportunities to apply and extend their learning. These activities should support students in developing a deeper understanding of the concept and should not progress to the outcomes in subsequent grades.

Consider strategies, such as:

- Students could investigate patterns that compare increasing side length of various regular polygons to their areas. This extension allows students to look at how the graphs differ from linear graphs.
- Students can create puzzles or problems and produce solutions to these problems.

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