Mathematics

Planning Guide

Grade 9 *Polynomials*

Patterns and Relations
(Variables and Equations)
Specific Outcomes 5, 6 and 7

This Planning Guide can be accessed online at:
http://www.learnalberta.ca/content/mepg9/html/pg9_polynomials/index.html

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Planning Guide: *Grade 9 Polynomials*

Strand: Patterns and Relations (Variables and Equations)
Specific Outcomes: 5, 6 and 7

This *Planning Guide* addresses the following outcomes from the Program of Studies:

Strand: Patterns and Relations (Variables and Equations)	
Specific Outcomes:	5. Demonstrate an understanding of polynomials (limited to polynomials of degree less than or equal to 2). [C, CN, R, V] 6. Model, record and explain the operations of addition and subtraction of polynomial expressions, concretely, pictorially and symbolically (limited to polynomials of degree less than or equal to 2). [C, CN, PS, R, V] 7. Model, record and explain the operations of multiplication and division of polynomial expressions (limited to polynomials of degree less than or equal to 2) by monomials, concretely, pictorially and symbolically. [C, CN, R, V]

Curriculum Focus

The changes to the curriculum targeted by this sample include:

- The general outcome focus is to represent algebraic expressions in multiple ways.
- The specific outcomes focus on understanding polynomials as well as developing procedures to add and subtract polynomials and multiply and divide polynomials by monomials.
- Multiplying of binomials and factoring of trinomials, which was included in the previous curriculum, has now been moved to Mathematics 10 Combined.

What Is a Planning Guide?

Planning Guides are a tool for teachers to use in designing instruction and assessment that focuses on developing and deepening students' understanding of mathematical concepts. This tool is based on the process outlined in *Understanding by Design* by Grant Wiggins and Jay McTighe.

Planning Steps

The following steps will help you through the Planning Guide:

Step 1: Identify Outcomes to Address

Guiding Questions

- What do I want my students to learn?
- What can my students currently understand and do?
- What do I want my students to understand and be able to do based on the Big Ideas and specific outcomes in the program of studies?

Big Ideas

- Algebraic expressions are composed of terms using numbers and variables to represent a number or relations between numbers.
- A polynomial is a collection of terms that are linked together by addition or subtraction.
- Algebra tiles can be used to model and explore polynomials.
- Polynomials can be added and subtracted by combining like terms.
- Multiplication and division of polynomials can be modelled with algebra tiles and area models.
- The distributive property is used to multiply polynomials.
- Dividing polynomials is the inverse operation of multiplying polynomials.

Sequence of Outcomes from the Program of Studies

See http://education.alberta.ca/teachers/program/math/educator/progstudy.aspx for the complete program of studies.

Grade 8	Grade 9	Grade 10
Patterns and Relations	**Patterns and Relations**	**Algebra and Number**
Specific Outcomes	**Specific Outcomes**	**Specific Outcomes**
2. Model and solve problems concretely, pictorially and symbolically, using linear equations of the form: • $ax = b$ • $\frac{x}{a} = b, a \neq 0$ • $ax + b = c$ • $\frac{x}{a} + b = c, a \neq 0$ • $a(x + b) = c$ where a, b and c are integers. [C, CN, PS, V] .	5. Demonstrate an understanding of polynomials (limited to polynomials of degree less than or equal to 2). [C, CN, R, V] 6. Model, record and explain the operations of addition and subtraction of polynomial expressions, concretely, pictorially and symbolically (limited to polynomials of degree less than or equal to 2). [C, CN, PS, R, V] 7. Model, record and explain the operations of multiplication and division of polynomial expressions (limited to polynomials of degree less than or equal to 2) by monomials, concretely, pictorially and symbolically. [C, CN, R, V]	4. Demonstrate an understanding of the multiplication of polynomial expressions (limited to monomials, binomials and trinomials), concretely, pictorially and symbolically. [CN, R, V] 5. Demonstrate an understanding of common factors and trinomial factoring, concretely, pictorially and symbolically. [C, CN, R, V]

Step 2: Determine Evidence of Student Learning

Guiding Questions

- What evidence will I look for to know that learning has occurred?
- What should students demonstrate to show their understanding of the mathematical concepts, skills and Big Ideas?

Using Achievement Indicators

As you begin planning lessons and learning activities, keep in mind ongoing ways to monitor and assess student learning. One starting point for this planning is to consider the achievement indicators listed in the *Mathematics Kindergarten to Grade 9 Program of Studies with Achievement Indicators*. You may also generate your own indicators and use them to guide your observation of the students.

The following indicators may be used to determine whether or not students have met the specific outcomes for understanding polynomials. Can students:

- create a concrete model or a pictorial representation for a given polynomial expression?
- write the expression for a given model of a polynomial?
- identify the variables, degree, number of terms and coefficients, including the constant term, of a given simplified polynomial expression?
- describe a situation for a given first degree polynomial expression?
- match equivalent polynomial expressions given in simplified form; e.g., $4x - 3x^2 + 2$ is equivalent to $-3x^2 + 4x + 2$?

The following indicators may be used to determine whether or not students have met the specific outcome for adding and subtracting polynomials concretely, pictorially and symbolically. Can students:

- model addition of two given polynomial expressions concretely or pictorially, and record the process symbolically?
- model subtraction of two given polynomial expressions concretely or pictorially, and record the process symbolically?
- identify like terms in a given polynomial expression?
- apply a personal strategy for addition or subtraction of two given polynomial expressions, and record the process symbolically?
- refine personal strategies to increase their efficiency?
- identify equivalent polynomial expressions from a given set of polynomial expressions, including pictorial and symbolic representations?
- identify the error(s) in a given simplification of a given polynomial expression?

The following indicators may be used to determine whether or not students have met the specific outcome for multiplication and division of polynomials expressions by monomials, concretely, pictorially and symbolically. Can students:

- model multiplication of a given polynomial expression by a given monomial concretely or pictorially, and record the process symbolically?
- model division of a given polynomial expression by a given monomial concretely or pictorially, and record the process symbolically?
- apply a personal strategy for multiplication and division of a given polynomial expression by a given monomial?
- refine personal strategies to increase their efficiency?
- provide examples of equivalent polynomial expressions?
- identify the error(s) in a given simplification of a given polynomial expression?

Sample behaviours to look for related to these indicators are suggested for some of the activities listed in **Step 3, Section C: Choosing Learning Activities** (p. 12).

Step 3: Plan for Instruction

Guiding Questions

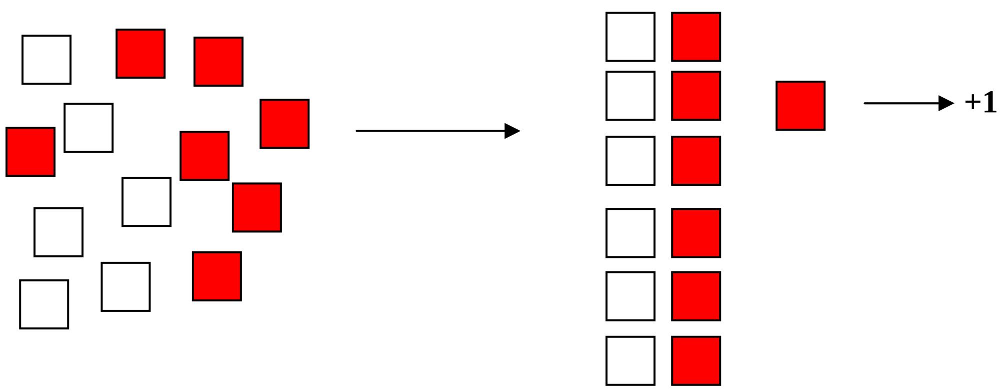
- What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should I use?
- How will I meet the diverse learning needs of my students?

A. Assessing Prior Knowledge and Skills

Before introducing new material, consider ways to assess and build on students' knowledge and skills related to representing operations with integers, writing and modelling algebraic expressions and substitution for a variable in an algebraic expression outlined in earlier grades. For example:

Activity 1: Representing Integers

Provide students with integer tiles. Place the following tiles on the overhead projector and have students explain which integer the tiles represent. If necessary, remind students to make pairs of one positive and one negative to make zeros.



Activity 2: Adding and Subtracting Integers

Find the answers to the following addition and subtraction of integers. Explain your thinking by drawing diagrams as indicated for each question.

a. (+4) + (–2) = ? Draw integer tiles to represent this question.
b. (–4) + (–2) = ? Draw a number line and arrows to represent this question.
c. (–2) – (+4) = ? Draw integer tiles to represent this question.
d. (+2) – (+4) = ? Draw a number line and arrows to represent this question.
e. (+4) – (–2) = ? Draw integer tiles to represent this question.
f. (–4) – (–2) = ? Draw a number line and arrows to represent this question.

Activity 3: Game: Operation Integers

Have students play the game "Operation Integers." Consider having students play just using the operations of multiplication and division.

Players: Two to four.

Materials: A deck of cards (no face cards).

Description: Deal all the cards face down on the table. Black suits are positive and red suits are negative. Each player turns over two cards and decides whether to add, subtract, multiply or divide the two numbers on the cards. The player who has the greatest result wins all the cards that are face up.

Goal: The play continues until one person (the winner) has all the cards.

Variations:

- Use fewer cards or cards with only certain numbers.
- Use fewer operations.
- Turn over three or four cards instead of two cards for each player.
- The player who has the least sum, difference, product or quotient wins all the cards that are face up.
- Each player rolls two (or more) dice with integers on each face rather than using playing cards. The player with the greatest (or least) number resulting from the operations scores one point. The winner is the player with the most points.

Activity 4: Modelling Expressions with Algebra Tiles

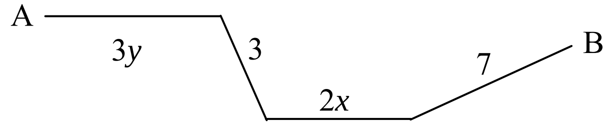
Use algebra tiles to model the expression $3x - 5$.

Change the model of $3x - 5$ if:

a. $3x$ changes to $2x$
b. -5 changes to -2
c. $3x$ is doubled
d. $3x$ changes to $-4x$
e. the entire expression is tripled.

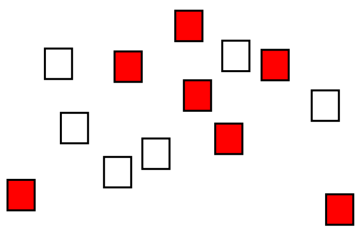
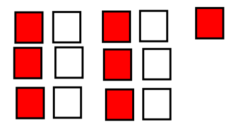
Activity 5: Expressions and Substitution

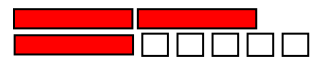
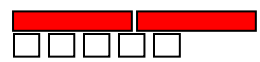
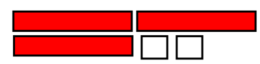
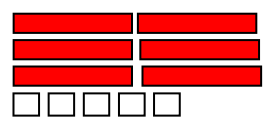
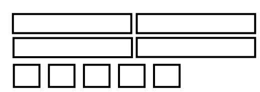
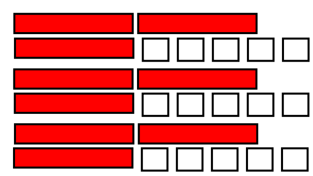
Write an expression for the length of the path from A to B.

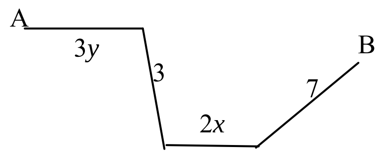


Evaluate the length of the path when the value of $x = 1.5$ cm and the value of $y = 2.3$.

Sample Structured Interview: Assessing Prior Knowledge and Skills

Directions	Date:	
	Not Quite There	**Ready to Apply**
Show the student the following tiles and have the student explain which integer the tiles represent. If necessary, remind the student to make pairs of one positive and one negative to make zeros. 	The student is unable to organize the tiles to make zero pairs.	The student organizes the tiles so that zero pairs are evident and recognizes that the one tile that is not part of a zero pair represents +1. 
Ask the student to find the answers to the following addition and subtraction of integers. Ask the student to explain his or her thinking by drawing diagrams as indicated for each question. a. (+4) + (–2) = ? Draw integer tiles to represent this question. b. (–4) + (–2) = ? Draw a number line and arrows to represent this question. c. (–2) – (+4) = ? Draw integer tiles to represent this question. d. (+2) – (+4) = ? Draw a number line and arrows to represent this question. e. (+4) – (–2) = ? Draw integer tiles to represent this question. f. (–4) – (–2) = ? Draw a number line and arrows to represent this question.	The student is unable to represent the situation with integer tiles and/or with number lines. OR The student is able to complete some of the drawings and computations correctly but makes some errors.	The student is able to complete all of the drawings and computations correctly. a. +2 b. –6 c. –6 d. –2 e. +6 f. –2

Use a deck of cards. Black suits are positive and red suits are negative. Ask the student to turn over two cards and decide whether to add, subtract, multiply or divide the two numbers on the cards to arrive at the highest value. Continue with the activity until the student has used all of the operations.	The student is unable to choose the correct operation that will produce the largest value. OR The student is unable to complete some or all of the operations with integers correctly.	The student chooses the correct operation, which will result in the largest value and also correctly determines the result.
Ask the student to use algebra tiles to model the expression $3x - 5$. Have the student change the model of $3x - 5$ if: a. $3x$ changes to $2x$ b. -5 changes to -2 c. $3x$ is doubled d. $3x$ changes to $-4x$ e. the entire expression is tripled.	The student is unable to complete the model of $3x - 5$ correctly. OR The student is unable to change the model correctly to fit the specified conditions.	The student is able to complete the model of $3x - 5$ correctly.  The student changes the model correctly to represent the requirements. a.  b.  c.  d.  e. 

Write an expression for the length of the path from A to B.  Have the student evaluate the length of the path when the value of $x = 1.5$ cm and the value of $y = 2.3$.	The student is unable to write an expression for the length of the path from A to B. OR The student is unable to substitute correctly the values of $x = 1.5$ cm and $y = 2.3$ to find the length of the path.	The student is able to find a correct expression for the length, $3y + 2x + 10$. The student also substitutes the values of $x = 1.5$ cm and $y = 2.3$ [3(2.3) + 2(1.5) + 10] to find the length of the path to be 19.9 units.

B. Choosing Instructional Strategies

Consider the following guidelines for teaching polynomials:

- Students need to be engaged in learning activities that involve models that will enhance their understanding of algebraic concepts.
- Students should be given many opportunities to make sense of the way that algebra uses symbols and letters to represent mathematical ideas. It is important for students to connect the differences and the similarities between operations with polynomial notation and operations with whole numbers, integers and rational numbers.
- The use of models such as algebra tiles help students to visualize concepts. Visual representations build lasting and meaningful understandings in algebra.
- Students should practise procedures for operations with polynomials only after they have learned to make sense of operations with concrete materials.
- Students must be given opportunities to communicate their thinking with other students and with the teacher. Students learn by explaining their reasoning and by listening to the strategies of other students.
- Students develop an understanding of concepts through problem solving.

C. Choosing Learning Activities

The following learning activities are examples of activities that could be used to develop student understanding of the concepts identified in Step 1.

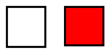
Sample Activities:

1. **Exploring Polynomials** (p. 13)
2. **Adding and Subtracting Polynomials Part A** (p. 15)
3. **Adding and Subtracting Polynomials Part B** (p. 16)
4. **Multiplying Polynomials by a Monomial** (p. 17)
5. **Multiplying Polynomials by a Monomial Using Personal Strategies** (p. 18)
6. **Dividing Polynomials by a Monomial** (p. 19)
7. **Dividing Polynomials by a Monomial Using Personal Strategies** (p. 20)
8. **Multiplying and Dividing Polynomials by a Monomial** (p. 22)

Sample Activity 1: Exploring Polynomials

Models can be used to help you understand polynomials**.**

When working with integers, one red unit tile and one white unit tile combine to model zero.
These two unit tiles form a zero pair.



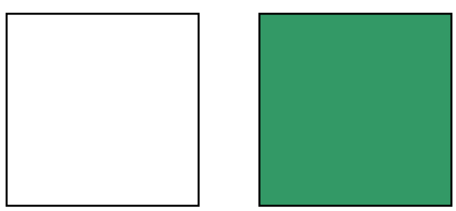
The red variable tile represents *x* and the white variable tile represents –*x*.
One red variable tile and one white variable tile also combine to model zero.
The two variable tiles form a zero pair.

Algebra tile kits also contain square tiles.

The green square tile measures *x* units on each side; it has an area of $x \times x$ or x^2 square units.

This tile is labelled as an x^2 tile. The opposite of x^2 is $-x^2$; the white square tile represents $-x^2$.

One green square tile and one white square tile also combine to model zero.
The two square tiles form a zero pair.



Look For …

Do students:

- ☐ simplify the expression by forming zeros with tiles that represent the same quantity but have different colours?
- ☐ write the polynomial by combining like terms? When writing polynomials, the tiles can only model one, two or three term expressions. The like terms must be combined. The number of terms does not equal the number of tiles in the model. All the tiles of the same size combine to form one term.

We can use the square tiles, variable tiles and unit tiles to represent polynomials.

When using algebra tiles, we often use the variable *x* to describe the polynomial; however, any variable could be used.

Students will work in twos or threes using the algebra tiles to create examples of polynomials.

- The polynomials should include both positive and negative terms.
- Polynomials should be recorded pictorially by creating a sketch of the tiles including the colours. Polynomials should also be recorded symbolically.
- Have students share the models and diagrams of the polynomials with other groups of students.
- Have students look for similarities and differences.

Use one or two of the examples of the students' polynomials to initiate the conversation involving the language of polynomials. Initial terminology students will need to become proficient with are *terms*, *variables*, *coefficients* and *constants*.

Polynomials could then be classified as monomial, binomial and trinomial.

Students will need to understand and be comfortable with using the terminology, whether they are working in concrete, pictorial or symbolic modes.

Extension 1: Identifying Equivalent Polynomials

- State the tiles students are to use. They can use up to a specified number and either the positive or negative sides of the tiles.
- They are asked to create as many different monomials, binomials or trinomials as possible.
- Students record their list of polynomials both pictorially and symbolically.
- When students compare their polynomials and the number of different polynomials they created, they will be learning to identify equivalent polynomials.

Extension 2: Identifying Equivalent Polynomials

- Students are given a description of the polynomial and asked to create as many polynomials as they can think of to meet the description.
  (Sample description: Each polynomial must have four terms; the polynomial must include the coefficients 1, –2 and 4, in any order.)
- Students record their list of polynomials both pictorially and symbolically.
- When students compare their polynomials and the number of different polynomials they created, they will be learning to identify equivalent polynomials.

Sample Activity 2: Adding and Subtracting Polynomials Part A

- Students will work with a partner using the algebra tiles.
- Each person represents a polynomial with an algebra tile model and then records the polynomials pictorially and symbolically.
- The two students find the sum of the two polynomials and record the addition sentence.
- The two students compare with another pair of students how they found the sum of the two polynomials.
- The four students now subtract the two polynomials created by each group.
- The four students should then reverse the order of the polynomials and then subtract and note the result.

Look For …

Do students:

- ☐ develop a strategy for the subtraction of polynomials? Students may need to add zero pairs in order to subtract the polynomials. Students may choose to add the opposite to subtract.

Sample Activity 3: Adding and Subtracting Polynomials Part B

A display of tiles representing a polynomial such as $–2x^2 + 3x – 5$ represents the sum or difference of two polynomials.

Identify two polynomials whose sum is $–2x^2 + 3x – 5$.
Identify three polynomials whose sum is $–2x^2 + 3x – 5$.
Identify two polynomials that have a difference of $–2x^2 + 3x – 5$. Find as many answers as you can.

Sample Activity 4: Multiplying Polynomials by a Monomial

Multiplication can be thought of as repeated addition.

6×4 is the same as adding four sixes: $6 + 6 + 6 + 6$
As a sum: $6 + 6 + 6 + 6 = 24$
As a product: $6 \times 4 = 24$

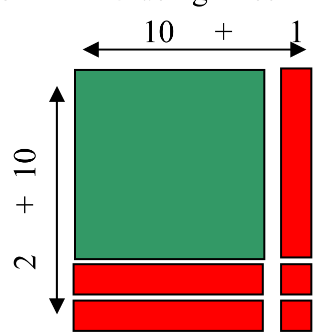
Students will work with a partner using algebra tiles to represent the following multiplications:

- $5(3x)$
- $2(4x + 3)$
- $4(x – 2)$.

The products should be recorded pictorially as well as symbolically.
Students should be able to explain how the model represents both repeated addition and multiplication.

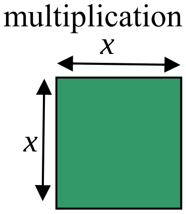
Area models can be used to represent multiplication.
This area model represents the multiplication of 12×13 using Base -10 Blocks.

This picture shows the product:
$11 \times 12 = (10 + 1)(10 + 2)$
$= 100 + 10 + 20 + 2$
$= 132$.

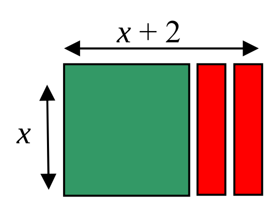


The concept of using an area model can also apply to multiplication of polynomials.

You have already used a model for x^2.
The dimensions of the square are x by x units.



The product of $x(x+2)$ can also be represented by the area model of a rectangle x units by $x + 2$ units:
$x(x+2) = x^2 + 2x$.



Represent each of the following products as an area. Record the tiles you used with a sketch. Write a multiplication statement for each part.

a. $3(x + 2)$ b. $x(x + 5)$ c. $x(2x + 4)$ d. $2x(4x \quad 1)$

Sample Activity 5: Multiplying Polynomials by a Monomial Using Personal Strategies

Draw on prior knowledge by reviewing how the distributive property was applied when solving linear equations. Have students share their ideas on how the distributive property could be used for multiplying a polynomial by a monomial. Have an algebra tile model available for students to link the concrete and the symbolic. Have students share their ideas.

Look For …

Do students:

- ☐ have difficulty representing two different variables concretely? The factors in the multiplication statement may need to be sketched (see sample personal strategy).

Explain that students will explore various ways to multiply polynomials by a monomial.
Present the following questions:

- $4(5x)$
- $–3(5n)$
- $2(3x – 1)$
- $2(m^2 + m + 3)$
- $–3(x^2 – x + 1)$
- $-5z(-3z)$
- $2d(4d – 3)$
- $-2x(4x + 3y – 2)$.

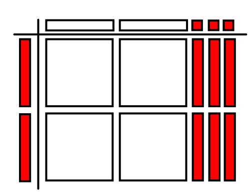
Ask students to solve the questions using any strategy or materials they wish. Provide enough time for students to complete the questions. They should be writing number sentences and also summarizing their personal strategy to show how they found the product for each case. Have students explain how their symbolic strategies work by relating to the concrete representations. To address different learning styles and abilities, have grid paper and algebra tiles available for students who need the concrete of the visual representation before using symbolic strategies.

Have a class discussion on the personal strategies students used to find the products. Have students think about the strategies that have been discussed and have students decide which strategy is most efficient for them to use in multiplying polynomials by a monomial. Students should then apply their personal strategies to find similar products.

Sample personal strategies:

Use algebra tiles.
$2x(–2x + 3)$
Form a rectangle, using guiding tiles for the dimensions
- two x-tiles one side
- two $–x$-tiles and three 1-tiles along the other side.

Four $–x^2$-tiles and six x-tiles fill the rectangle. The area is $–4x^2 + 6x$.
$2x(–2x + 3) = –4x^2 + 6x$

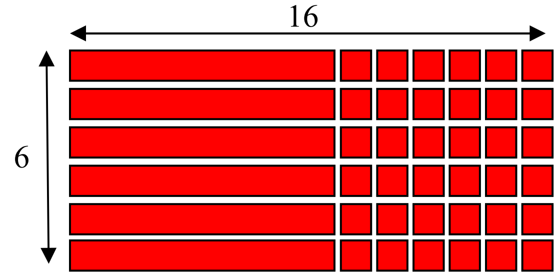


Use the distributive property.
Multiply each term of the polynomial inside the brackets by the monomial outside the brackets.

$$2x(–2x + 3) = 2x\,(–2x) + 2x\,(3)$$
$$= –4x^2 + 6x$$

Sample Activity 6: Dividing Polynomials by a Monomial

Multiplication and division are inverse operations. Just as we used algebra tile models to multiply, we can also use algebra tile models to divide.



From the area model we can see that
$16 \times 6 = 96$.

The related division statements would be
$96 \div 6 = 16$ or $96 \div 16 = 6$.

The area of a rectangle is $6x^2 + 12x$ and can be arranged into different rectangular shapes.

Use algebra tiles to create a model.
Change the model, as needed, to help answer the following questions:

- What is the length when the width is $2x$, $3x$ and $6x$?
- Can you create a rectangle with a width of 2, 3 and 6?
- Could you model division by 2, 3 and 6 another way?
- How can you use what you have learned to divide each polynomial? For example:
  - $16x^2 \div 2x$
  - $(16x^2 + 8) \div 2$
  - $(16x^2 + 8x) \div 2x$.

Look For …
Do students:
☐ have trouble representing the question using tiles? It may be helpful to represent the divisor using tiles to assist in the creation of rectangular shapes.

Sample Activity 7: Dividing Polynomials by a Monomial Using Personal Strategies

Draw on prior knowledge by reviewing how division is the opposite operation of multiplication. Have students share their ideas how the application of the distributive property in multiplying a polynomial by a monomial could be interpreted when dividing a polynomial by a monomial. Have an algebra tile model available for students to link the concrete and symbolic. Have students share their ideas.

Explain that students will explore various ways to divide polynomials by a monomial. Present the following questions:

- $(6x + 15) \div 3$
- $(6x^2+12x) \div 3x$
- $(8y^2 + 4x + 4) \div 4$
- $(-8h^2+6h-4) \div 2h$

Ask students to solve the questions using any strategy or materials they wish. Provide enough time for students to complete the questions. They should be writing number sentences and also summarizing their personal strategy to show how they found the quotient for each case. Have students explain how their symbolic strategies work by relating to the concrete representations. To address different learning styles and abilities, have grid paper and algebra tiles available for students who need the concrete of the visual representation before using symbolic strategies.

Look For …

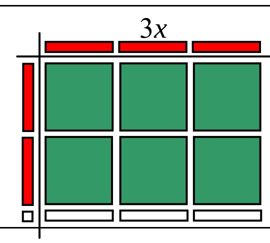
Do students:

- ☐ make a connection between multiplying and dividing? The solution to a divisor question can be verified by multiplying its answer and the value the polynomial was being divided by. Check: Dividend=(Divisor) (Quotient)

Have a class discussion on the personal strategies students used to find the quotients. Have students think about the strategies that have been discussed and have students decide which strategy is most efficient for them to use in dividing a polynomial by a monomial. Students should then apply their personal strategies to find similar quotients.

Sample personal strategies:
When a polynomial is divided by a monomial, the process of multiplying is reversed.

Use algebra tiles.
$(6x^2-3x) \div 3x$
Arrange six x^2–tiles and three $-x$ tiles in a rectangle with one dimension, $3x$. The guiding tiles along the other side are:
$(6x^2-3x) \div 3x = 2x - 1$.



Think of division as being the opposite of multiplication.
$3x \times ? = 6x^2 - 3x$
$3x \times (2x) = 6x^2$ and $3x \times (-1) = -3x$
So, $3x \times (2x - 1) = 6x^2 - 3x$
$(6x^2-3x) \div 3x = 2x - 1$

Dividing each term of the polynomial.
Write the division statement as the difference of two fractions using the monomial divisor as the denominator.

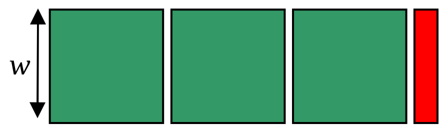
$$\left(6x^2 - 3x\right) \div 3x = \frac{6x^2}{3x} - \frac{3x}{3x}$$

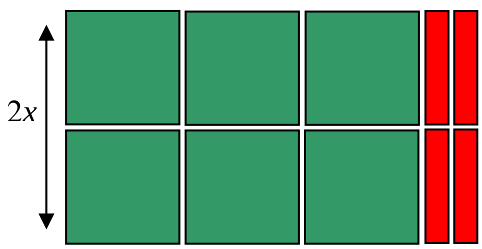
Complete the division of each fraction.

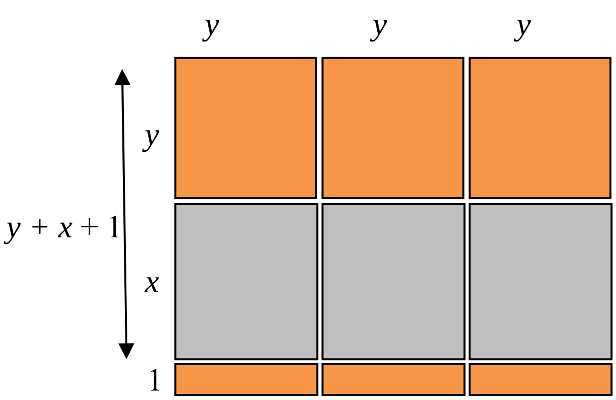
$$\frac{6x^2}{3x} - \frac{3x}{3x} = 2x - 1$$

Sample Activity 8: Multiplying and Dividing Polynomials by a Monomial

Write the multiplication sentence modelled by each set of algebra tiles.







Look For …

Do students:

- ☐ confuse the dimensions of the representation with the polynomial? The introduction of a second variable and the *xy* tile may cause confusion. It may be necessary to write the dimensions of tiles in sketches to assist in finding solutions.

Write a division sentence that each model represents.

Step 4: Assess Student Learning

Guiding Questions

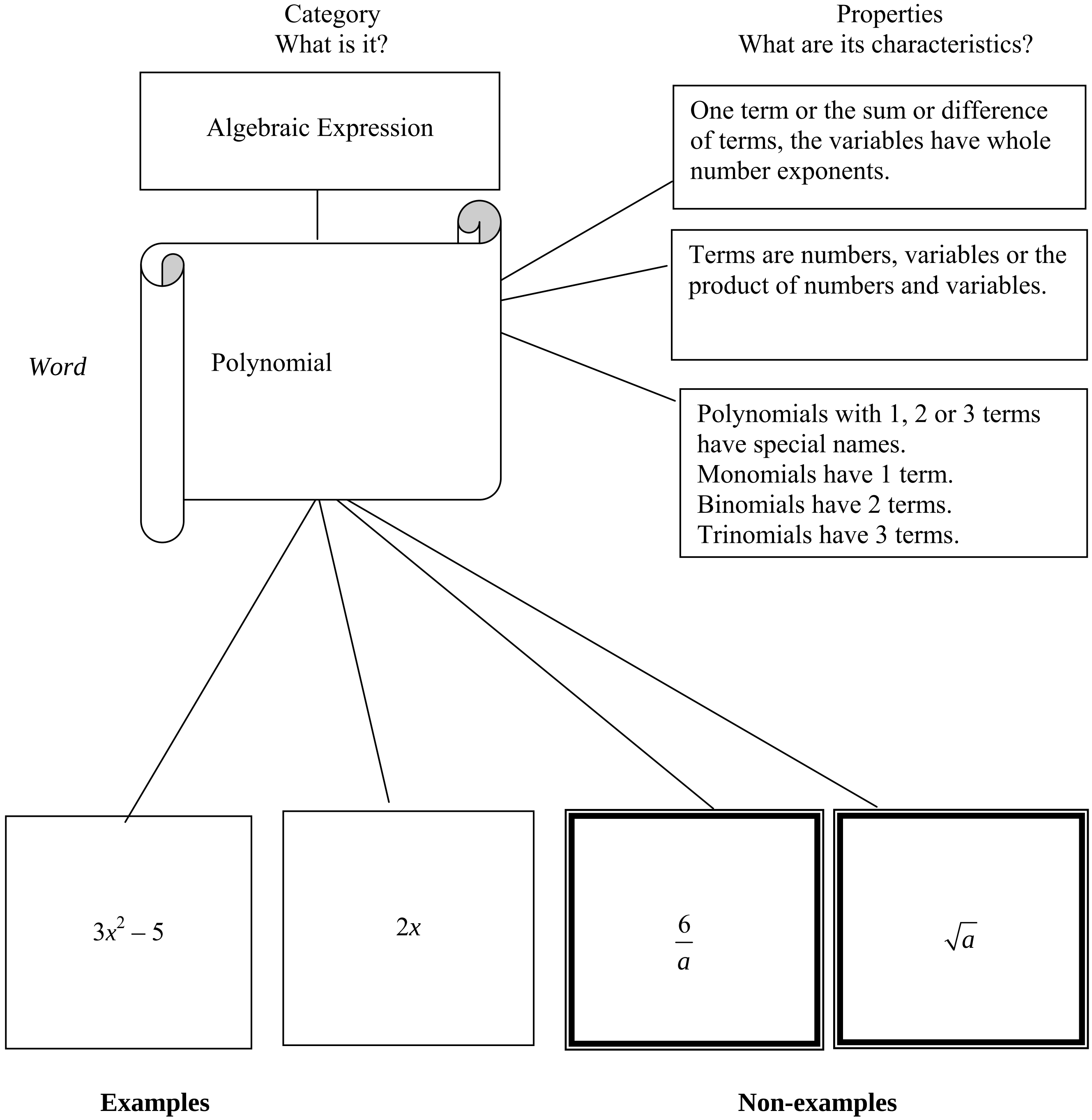
- Look back at what you determined as acceptable evidence in Step 2.
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

In addition to ongoing assessment throughout the lessons, consider the following sample activities to evaluate students' learning at key milestones. Suggestions are given for assessing all students as a class or in groups, individual students in need of further evaluation, and individual or groups of students in a variety of contexts.

A. Whole Class/Group Assessment

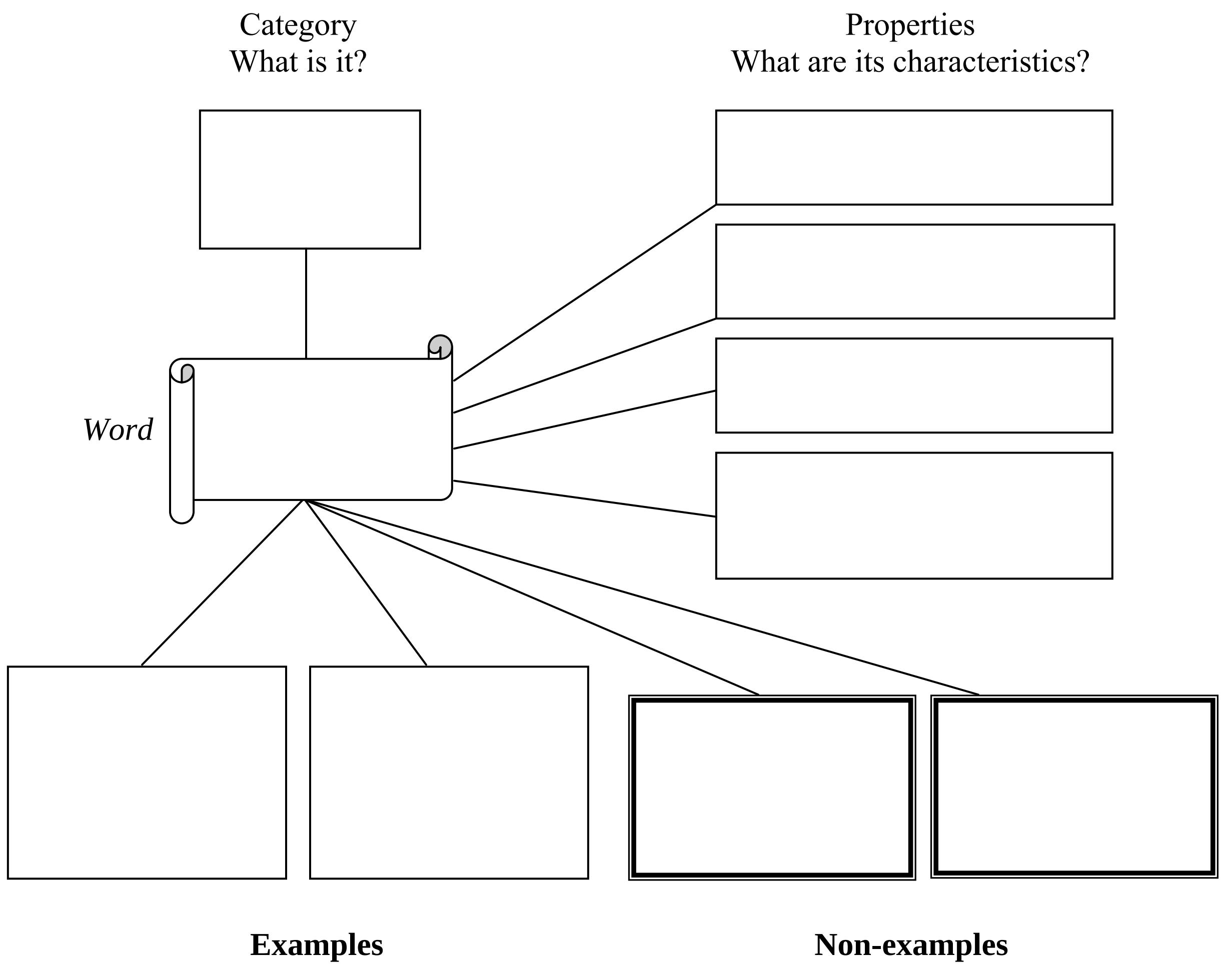
Activity 1: Have students work in groups to complete a Concept Definition Map for the concept of polynomials. See the following sample.

Concept Definition Map



Format adapted from R. M. Schwartz. "Learning to Learn Vocabulary in Content Area Textbooks." *Journal of Reading* 32, 2 (November 1988), p. 110, Example 1. Adapted with permission from the International Reading Association.

Concept Definition Map



Format adapted from R. M. Schwartz. "Learning to Learn Vocabulary in Content Area Textbooks." *Journal of Reading* 32, 2 (November 1988), p. 110, Example 1. Adapted with permission from the International Reading Association.

B. One-on-one Assessment

Have students complete the assessment task. Students use polynomials to create general designs for athletic spaces that can vary to fit different fitness facilities.

Designing Floors for RiverGlen Fitness Centre

The floors of fitness centres are often surfaced with sheet flooring made of recycled ground rubber.
RiverGlen Fitness Centre is planning to resurface the floors in the facility. The new floors will be surfaced with sheet flooring made of recycled ground rubber. The flooring company could use polynomials to create general designs that can vary to fit different fitness spaces.

Question 1

The aerobic area of the fitness centre will be a rectangular shape. The length of the rectangle is $3x + y + 1$ with a width of $3x$.

Use algebra tiles to create a model of the polynomial that represents the rectangular shape for the aerobic area and make a sketch of your model.

Find the area of the rectangular shape for the aerobic area using the polynomials for length and width. Explain how you determined the polynomial that represents the area of that space.

Question 2

The yoga and stretching area will be located in an adjacent rectangular space. This space will be $2y + 3x + 1$ long and $2y$ units wide.

Use algebra tiles to create a model of the polynomial that represents the rectangular shape for the yoga and stretching area and make a sketch of your model.

Find the area of the rectangular shape for the yoga and stretching area using the polynomials for length and width. Explain how you determined the polynomial that represents the area of that space.

Question 3

Determine the polynomials that represent the combined the area of the two rectangular spaces from Question 1 and Question 2. Explain how you determined the polynomial that represents the combined area.

If $x = 2.5$ m and $y = 3$ m, find the area of each rectangular space and the combined area of both spaces.

Question 4

Racquetball courts are also part of the fitness centre. The polynomial $12x^2 + 6xy$ represents the area that the courts will cover.

The length of the rectangular space for the racquetball courts will be $3x$.
Model the rectangular space with algebra tiles. Determine the width of the space.

Explain how the width of the space could be determined through the division of a polynomial by a monomial.

Question 5

Write a paragraph to explain the advantages of designing athletic spaces using polynomials.

Look For …

Do students:

- show sketches of algebra tile models and calculations in detail?
- provide clear explanations of their thinking behind solutions?
- show the correct use of the language of polynomials?
- describe the advantages of using polynomials to design spaces?

C. Applied Learning

Provide opportunities for students to use what they have learned about polynomials and notice whether or not the knowledge is being transferred.

Activity 1: Given the dimensions of a rectangle as a polynomial, find the perimeter and area.

Activity 2: Given the perimeter of a square or equilateral triangle as a polynomial, find the side length.

Activity 3: Given the area and length of a rectangle as polynomials, find the polynomial that describes the width.

Activity 4: Given the perimeter and the length of a rectangle as polynomials, find the polynomial that describes the width.

Activity 5: Given a polynomial that represents the surface area of a cube, find a polynomial expression that represents the area of one face and the length of a side as a polynomial.

Step 5: Follow-up on Assessment

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction?

A. Addressing Gaps in Learning

If students have difficulty in solving the basic facts using strategies, consider the following.

- Review the operations with integers, which apply to operations with polynomials.

- Provide time for students to reflect on their learning by using graphic organizers such as the Frayer Model or Concept Definition Map.

- Make sure that students understand the language and operations with polynomials by using algebra tiles, sketching their results and providing explanations to justify their thinking.

- Remind students that although an area model is one interpretation of multiplication, multiplication can also mean repeated addition or equal groups. When multiplying by a constant, students may not be able to form a rectangle but may be able to form equal groups.

B. Reinforcing and Extending Learning

Students who have achieved or exceeded the outcomes will benefit from ongoing opportunities to apply and extend their learning. These activities should support students in developing a deeper understanding of the concept and should not progress to the outcomes in subsequent grades.

Consider strategies, such as:

Activity 1: Students could use algebra tile models containing three different variables and create two or three polynomials with five or six different terms. Students should make a drawing of the polynomials and record the polynomials symbolically. Students could then combine the like terms using the tiles symbolically.

Activity 2: Have students explore multi-step questions involving both sums and differences. For example: $(5a^2 - 4a - 3) - (2a^2 - 6a) + (9a^2 - 3a - 7)$.

Activity 3: Extend the multiplication of polynomials by monomials to include a series of the sum or difference of several products. For example: $2y(2y + 1) + 3x(2y - 1) - 2x(y + x + 5)$.

Activity 4: Have students apply the algorithm for long division to the division of a polynomial by a monomial.

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