Mathematics

Planning Guide

Grade 9
Rational Numbers and Square Roots

Number
Specific Outcomes 3, 4, 5 and 6

This Planning Guide can be accessed online at:
http://www.learnalberta.ca/content/mepg9/html/pg9_rationalnumberssquareroots/index.html

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Planning Guide: *Grade 9 Rational Numbers and Square Roots*

Strand: Number
Specific Outcomes: 3, 4, 5 and 6

This *Planning Guide* addresses the following outcomes from the program of studies:

Strand: Number	
Specific Outcomes:	3. Demonstrate an understanding OF rational numbers by: • comparing and ordering rational numbers • solving problems that involve arithmetic operations on rational numbers. 4. Explain and apply the order of operations, including exponents, with and without technology. 5. Determine the square root of positive rational numbers that are perfect squares. 6. Determine an approximate square root of positive rational numbers that are non-perfect squares.

Curriculum Focus

This sample targets the following changes to the curriculum:

- The general outcome focus, to develop number sense, has not changed from the previous program of studies.
- The specific outcomes focus is on comparing and ordering rational numbers, operations with rational numbers and order of operations. The focus is also on determining square roots of rational numbers.
- In the previous program of studies, the general outcome focused on the use of a scientific calculator or computer to solve problems involving rational numbers.

What Is a Planning Guide?

Planning Guides are a tool for teachers to use in designing instruction and assessment that focuses on developing and deepening students' understanding of mathematical concepts. This tool is based on the process outlined in *Understanding by Design* by Grant Wiggins and Jay McTighe.

Planning Steps

The following steps will help you through the Planning Guide:

Step 1: Identify Outcomes to Address

Guiding Questions

- What do I want my students to learn?
- What can my students currently understand and do?
- What do I want my students to understand and be able to do based on the Big Ideas and specific outcomes in the program of studies?

Big Ideas

- Between any two rational numbers there exists an infinite number of other rational numbers.
- Procedures for operations with decimals, fractions and integers are used for operations with rational numbers.
- The order of operations with whole numbers extends to exponents and can be applied to rational numbers.
- Squaring a number and taking its square root are inverse operations.
- Perfect squares and their square roots extend to rational numbers.
- The square root of a rational number that is not a perfect square can be approximated as a decimal.

Sequence of Outcomes from the Program of Studies

See http://www.education.alberta.ca/teachers/program/math/educator/progstudy.aspx for the complete program of studies.

Grade 8

Number

Specific Outcomes

1. Demonstrate an understanding of perfect squares and square roots, concretely, pictorially and symbolically (limited to whole numbers).

2. Determine the approximate square root of numbers that are not perfect squares (limited to whole numbers).

6. Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially and symbolically.

7. Demonstrate an understanding of multiplication and division of integers, concretely, pictorially and symbolically.

Grade 9

Number

Specific Outcomes

3. Demonstrate an understanding of rational numbers by:
   - comparing and ordering rational numbers
   - solving problems that involve arithmetic operations on rational numbers.

4. Explain and apply the order of operations, including exponents, with and without technology.

5. Determine the square root of positive rational numbers that are perfect squares.

6. Determine an approximate square root of positive rational numbers that are non-perfect squares.

Grade 10

Algebra and Number

Specific Outcomes

1. Demonstrate an understanding of factors of whole numbers by determining the:
   - prime factors
   - greatest common factor
   - least common multiple
   - square root
   - cube root.

2. Demonstrate an understanding of irrational numbers by:
   - representing, identifying and simplifying irrational numbers
   - ordering irrational numbers.

Step 2: Determine Evidence of Student Learning

Guiding Questions

- What evidence will I look for to know that learning has occurred?
- What should students demonstrate to show their understanding of the mathematical concepts, skills and Big Ideas?

Using Achievement Indicators

As you begin planning lessons and learning activities, keep in mind ongoing ways to monitor and assess student learning. One starting point for this planning is to consider the achievement indicators listed in *The Alberta K–9 Mathematics Program of Studies with Achievement Indicators*. You may also generate your own indicators and use them to guide your observation of the students.

The following indicators may be used to determine whether or not students have met the specific outcomes for understanding rational numbers. Can students:

- order a given set of rational numbers in fraction and decimal form by placing them on a number line; e.g., $\frac{3}{5}$, $-0.666\ldots$, 0.5, $-\frac{5}{8}$, $-\frac{3}{2}$?
- identify a rational number that is between two given rational numbers?
- solve a given problem involving operations on rational numbers in fraction or decimal form?

The following indicators may be used to determine whether or not students have met the specific outcome for applying the order of operations, including exponents to rational numbers. Can students:

- solve a given problem by applying the order of operations without the use of technology?
- solve a given problem by applying the order of operations with the use of technology?
- identify the error in applying the order of operations in a given incorrect solution?

The following indicators may be used to determine whether or not students have met the specific outcome for determining the square root of rational numbers that are perfect squares (principle square roots only). Can students:

- determine whether or not a given rational number is a square number, and explain the reasoning?
- determine the square root of a given positive rational number that is a perfect square?
- identify the error made in a given calculation of a square root; e.g., is 3.2 the square root of 6.4?
- determine a positive rational number, given the square root of that positive rational number?

The following indicators may be used to determine whether or not students have met the specific outcome for approximating the square root of positive rational numbers that are not perfect squares. Can students:

- estimate the square root of a given rational number that is not a perfect square, using the roots of perfect squares as benchmarks?
- determine an approximate square root of a given rational number that is not a perfect square using technology; e.g., calculator, computer?
- explain why the square root of a given rational number, as shown on a calculator, may be an approximation?
- identify a number with a square root that is between two given numbers?

Sample behaviours to look for related to these indicators are suggested for some of the activities listed in **Step 3, Section C: Choosing Learning Activities** (p. 14).

Step 3: Plan for Instruction

Guiding Questions

- What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should I use?
- How will I meet the diverse learning needs of my students?

A. Assessing Prior Knowledge and Skills

Before introducing new material, consider ways to assess and build on students' knowledge and skills related to comparing and ordering numbers, procedures for operations with decimals, fractions and integers, order of operations and determining the square root outlined in earlier grades. For example:

Activity 1: Ordering Rational Numbers

Order the following numbers in each set from least to greatest. Show, in two different ways, how you know.

1. –42, –76, +84, –41, +60, 0

2. 2.6, 6, 2.69

3. $\frac{1}{3}, \frac{7}{10}, \frac{3}{8}, \frac{5}{12}$

Activity 2: Game—Integer Operations

Create cards with 20 different integers. Place integer cards in a box. Create a card of a different size and colour for each of the operations +, –, ×, and ÷.

The first player picks two of the integer cards and chooses an operation card that will give him or her the largest result. The player completes the operation and records the number sentence; the result is his or her score for that round. Each operation must be used at least once and division can only be used if the quotient is an integer.

Repeat the selection and scoring for each player until all of the cards have been removed. Each player finds his or her total score. The winner is the player with the greatest score.

Activity 3: **Adding and Subtracting Fractions**

Fill the empty square with a number that, when added to the numbers in the other three squares, will result in a total of $5\frac{5}{8}$.

$1\frac{1}{8}$	
$1\frac{3}{8}$	$2\frac{1}{4}$

Activity 4: Multiplying Fractions

Find three fractions that could have been multiplied to get a product of $1\frac{3}{4}$.

Activity 5: **Dividing Fractions**

Rita is serving pizza at her party! She has five pizzas; each serving will be $\frac{3}{8}$ of a pizza.

Illustrate this division with a model or a diagram. Find the quotient and explain what you did? How many servings of pizza can Rita make?

Activity 6: **Adding and Subtracting Decimals**

3.5 □ + □.51 + 0. □ = 12.23. What might the missing numbers be? Justify your answer.

Activity 7: **Multiplying and Dividing Decimals**

1. Two numbers are multiplied together to give a product of 16.8. What might these numbers be? Justify your answer.

2. Sheena divided 8.36 by 4 and got the answer 2.9. What error did Sheena make and what other similar questions might she get wrong?

Activity 8: **Order of Operations**

Use at least three of the numbers 16, 10, 7, 5 and 2 (each number may only be used once) and any of the operations +, –, ×, and ÷, as well as squaring a number and taking the square root of a number, to find as many different expressions as you can to reach the target number 20.

Activity 9: **Finding Square Roots**

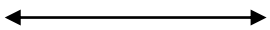
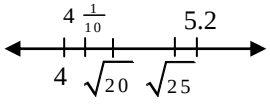
Place one square root, one integer, one fraction and one decimal on the number line below. How did you decide where to place your numbers?

If a student appears to have difficulty with these tasks, consider further individual assessment, such as a structured interview, to determine the student's level of skill and understanding. See **Sample Structured Interview: Assessing Prior Knowledge and Skills** (p. 11).

Sample Structured Interview: Assessing Prior Knowledge and Skills

Directions	Date:	
	Not Quite There	**Ready to Apply**
Ask the student to order the following numbers in each set from least to greatest. Ask the student to explain how he or she decided on the correct order. a. –42, –76, +84, –41, +60, 0 b. 2.6, 6, 2.69 c. $\frac{1}{3}, \frac{7}{10}, \frac{3}{8}, \frac{5}{12}$	The student is unable to order the three sets of numbers. OR The student is able to order one or two sets of numbers, such as the integers and the decimals, but is unable to order the set of fractions. OR The student orders the sets of numbers correctly but cannot explain his or her method.	The student orders the sets of numbers correctly and can give an appropriate method for ordering. a. –76,–42, –41, 0, +60, +84 b. 2.6, 2.69, 6 c. $\frac{1}{3}, \frac{3}{8}, \frac{5}{12}, \frac{7}{10}$
Create cards with 20 different integers. Place integer cards in a box. Create a card of a different size and colour for each of the operations +, –, ×, and ÷. Ask the student to choose two integer cards and an operation that he or she thinks will give the largest result and record the number sentence. Ask the student to repeat the process until he or she has used all of the operations. Each operation must be used at least once and division can only be used if the quotient is an integer.	The student is able to complete some of the computation correctly but makes some errors. OR The student is not able to choose the operation that will give the largest result, based on the restrictions that he or she must use each operation at least once and division can only be used if the quotient is an integer.	The student is able to complete all of the computations correctly, order the integers appropriately, and choose the operation that will give the largest result based on the restrictions.

Fill the empty square with a number that, when added to the numbers in the other three squares, will result in a total of $5\frac{5}{8}$. $1\frac{1}{8}$ | $1\frac{3}{8}$ | $2\frac{1}{4}$	The student is not able to add the three mixed numbers in the squares correctly and then subtract the result from $5\frac{5}{8}$.	The student correctly determines the sum of the fractions in the squares as $4\frac{3}{4}$ and determines the missing number as $\frac{7}{8}$.
Ask the student to find the answer: Find three fractions that could have been multiplied to get $1\frac{3}{4}$.	The student is unable to correctly determine three fractions with a product of $1\frac{3}{4}$. OR The student is able to determine correctly two fractions that multiply to $1\frac{3}{4}$.	The student correctly determines three fractions with a product of $1\frac{3}{4}$, such as: $\frac{7}{4} \times \frac{4}{2} \times \frac{1}{2}$ OR $\frac{7}{3} \times \frac{3}{2} \times \frac{1}{2}$
Ask the student to find the answer to the following problem and show how his or her answer makes sense with a model or a diagram. Rita is serving pizza at her party! She has five pizzas; each serving will be $\frac{3}{8}$ of a pizza. How many servings of pizza can Rita make?	The student is unable to divide 5 by $\frac{3}{8}$ correctly. OR The student is able to divide 5 by $\frac{3}{8}$ correctly, but is unable to show how the division makes sense with a model or a diagram.	The student is able to complete the computation $5 \div \frac{3}{8} = 13\frac{1}{3}$ correctly, and is able to model the division with a diagram or a model.
Ask the student to determine the missing numbers in the following addition sentence and explain how he or she found the missing numbers. 3.5 □ + □.51 + 0. □ = 12.23.	The student attempts to determine the missing numbers through guess and test but is unable to do so correctly.	The student is able to find a correct solution to the problem. The student uses a method, such as assigning a value to one of the unknown numbers and then subtracting it from 12.23, to see what is left to work with. One possible solution is 3.52 + 8.51 + 0.2 = 12.23.

Ask the student to do the following questions. Two numbers are multiplied together to give a product of 16.8. What might these numbers be? Justify your answer. Sheena divided 8.36 by 4 and got the answer 2.9. What error did Sheena make and what other similar questions might she get wrong?	The student attempts to determine the missing numbers through guess and test but is unable to do so correctly. The student is unable to determine the error or is unable to predict when this error would occur again.	The student is able to find a correct solution to the problem. The student assigns a value to one number and uses division to determine the second number. One possible solution is $2 \times 8.4 = 16.8$. The student is able to explain that when the number in the tenths position is divided by 4, the quotient is 0 with three remaining. (Sheena forgot to record the zero in the tenths place of the quotient. She would get a question wrong where the divisor is larger than any digit in the dividend.)
Ask the student to try to use at least three of the numbers 16, 10, 7, 5 and 2 (each number can only be used once) and any of the operations +, –, × and ÷, as well as squaring a number and taking the square root of a number, to find several different expressions to reach the target number 20.	The student is unable to create an expression to determine the target number. OR The student is able to determine only one or two straightforward ways with three numbers to determine the target number of 20, such as $5 \times 2 + 10 = 20$.	The student is able to determine several ways to determine the target number of 20, such as: • $5 \times 2 + 10 = 20$ • $\frac{10^2}{5} = 20$ • $\frac{\left(\sqrt{16} \times 10\right)}{2} = 20$ • $2\left(\frac{5^2 \times \sqrt{16}}{10}\right) = 20$.
Ask the student to place one square root, one integer, one fraction and one decimal on the number line below and explain how he or she decided where to place the numbers. 	The student is unable to place any or a few of the numbers appropriately on the number line. OR The student places numbers correctly on the number line but is unable to explain his or her decisions.	The student is able to place numbers appropriately on the number line and to explain his or her decisions. A possible solution is: 

B. Choosing Instructional Strategies

Consider the following guidelines for teaching rational numbers and square roots:

- Build on students' understanding of integer, decimal and fraction computation.
- Provide a variety of hands-on activities using manipulatives, such integer tiles, 1-cm grid paper and cards, to represent positive and negatives.
- Provide opportunities for students to work with various strategies to estimate and calculate rational numbers.
- Provide opportunities for students to develop number sense by choosing activities that allow them to practise rational number computation.
- Build on students' understanding of perfect squares and square roots to develop their understanding of perfect squares of positive rational numbers and approximating square roots of positive rational numbers that are not perfect squares.

C. Choosing Learning Activities

The following learning activities are examples of activities that could be used to develop student understanding of the concepts identified in Step 1.

Sample Activities:

1. **Exploring Rational Numbers** (p. 15)
2. **Comparing and Ordering Rational Numbers** (p. 16)
3. **Numbers In-between** (p. 17)
4. **Adding and Subtracting Rational Numbers** (p. 18)
5. **What's the Pattern?** (p. 19)
6. **Multiplying Rational Numbers** (p. 21)
7. **Dividing Rational Numbers** (p. 22)
8. **Rationals—Two-ways** (p. 23)
9. **Where's the Error?** (p. 25)
10. **Order of Operations and Magic Squares** (p. 26)
11. **Perfect Squares** (p. 27)
12. **Perfect Squares—How Many Ways?** (p. 28)

Sample Activity 1: Exploring Rational Numbers

The following activity connects students' understanding of combining integers using the zero property to combining rational numbers.

Materials: Two-coloured tiles or two-coloured counters (approximately 10 per small group) and charts to record.

Directions: Have students determine which colour will represent positive numbers and which colour will represent negative numbers. Each group of students places their counters or tiles in a paper cup. Students take turns tossing the tiles from the cup onto their desks. Remind students that the sum of opposite integers is zero. Students should take turns until everyone has had three tosses. Students will use a chart to record their toss; for example:

Number of positive 1 tiles	Number of negative 1 tiles	Resulting integer	Number sentence
6	4	2	6 + –4 = 2
1	9	–8	1 + –9 = –8

Students will repeat the activity but now the tiles will represent $+\frac{1}{2}$ and $-\frac{1}{2}$.

Students could use a chart to record their tosses; for example:

Number of positive $\frac{1}{2}$ tiles	Rational number being represented	Number of negative $\frac{1}{2}$ tiles	Rational number being represented	Resulting rational number	Number sentence
3	$1\frac{1}{2}$	2	–1	$\frac{1}{2}$	$\left(1\frac{1}{2}\right)+(-1)=\frac{1}{2}$
3	$1\frac{1}{2}$	7	$-3\frac{1}{2}$	–2	$\left(1\frac{1}{2}\right)+\left(-3\frac{1}{2}\right)=-2$

Discussion

- What was the same and what was different between the two activities?
- How do procedures for operations with integers help to build a procedure for the addition of rational numbers?

Look For …

Do students:

☐ correctly determine the sign of the solution?

Sample Activity 2: Comparing and Ordering Rational Numbers

The following activity connects students' understanding of comparing and ordering positive fractions to comparing and ordering rational numbers.

Use the digits 1 through 9 to write as many positive and negative proper and improper fractions as you can. Both the numerators and denominators should be single digits. Make a list of the fractions you compose.

Part A
Order your fractions on a number line from least to greatest.

Answer the following questions:

- Which fractions are less than –1?
- Which fractions are between –1 and 0?
- Which fractions are between 0 and 1?
- Which fractions are greater than 1?

Look For …
Do students:
- ☐ reduce fractions when possible?
- ☐ have a strategy to compare the size of fractions with different denominators?

Part B
Write each fraction as a decimal.
Use the decimals to check the accuracy of your number line.

Sample Activity 3: Numbers In-between

Part A
Choose any two positive or negative fractions that are beside each other on a number line. Try to find at least five more fractions that are between these two numbers. Sketch a number line to show all the fractions.

Part B
Choose any two positive or negative decimals that are beside each other on a number line. Try to find at least five more decimals that are between these two numbers. Sketch a number line to show all the decimals.

Sample Activity 4: Adding and Subtracting Rational Numbers

When adding or subtracting rational numbers, use procedures developed for adding and subtracting integers.

Students work in pairs. Each pair will require 10 red squares and 10 white squares, all of equal size, made of construction paper. The red squares represent positive rational numbers and the white squares represent negative rational numbers. Students can use two of the red squares and two of the white squares and cut them into halves and quarters respectively.

Part A
Each student represents either a positive or negative fraction with the red and white pieces and then records the fraction represented by his or her model. Use the model to combine the pieces through addition. Write the addition sentence the model represents. Repeat this with two more equations.

Part B
Repeat the activity in Part A, this time subtracting the two fractions. Write the subtraction sentence each model represents.

Discussion

- Have students examine their addition and subtraction sentences from the above activity to discuss how procedures for addition and subtraction of rational numbers are the same as and how they are different from procedures for addition and subtraction of integers and fractions.

Sample Activity 5: What's the Pattern?

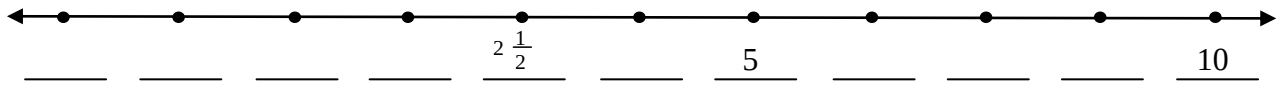
Complete the sequences by filling in the blanks.
The same number is added or subtracted each time.

Look For …

Do students:

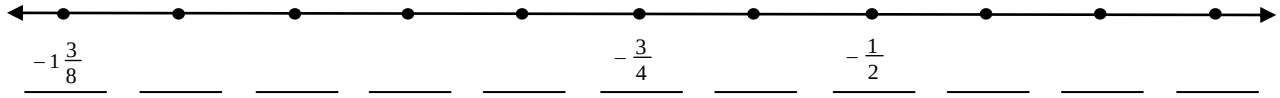
- ☐ get a common denominator when trying to determine what is added or subtracted?
- ☐ develop a strategy to calculate what is added or subtracted when there is more than one blank between two given values?

1.



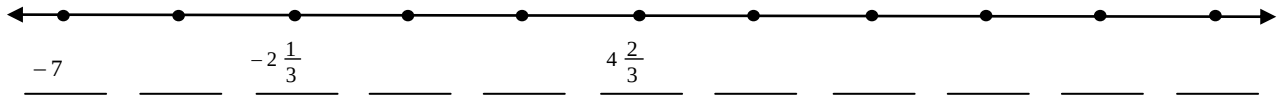
What is the pattern? _____

2.



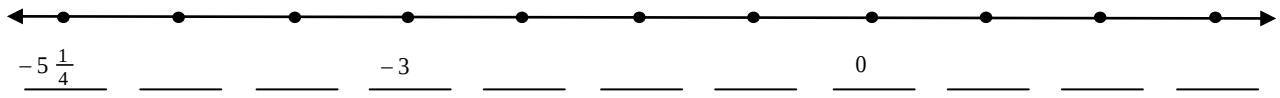
What is the pattern? _____

3.



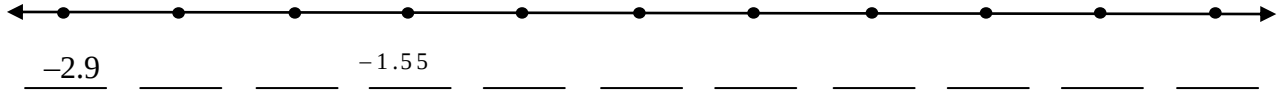
What is the pattern? _____

4.

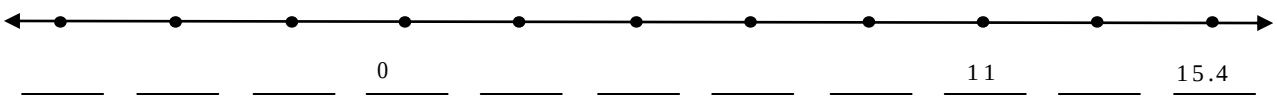


What is the pattern? _____

5.

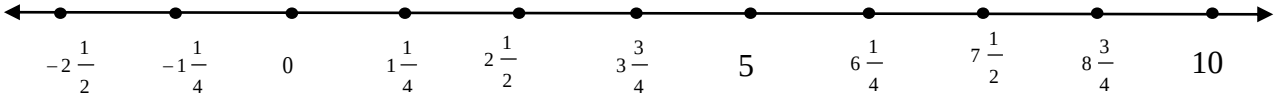


What is the pattern? _____

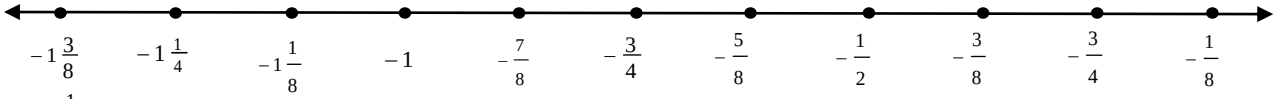
6. 

What is the pattern? _____

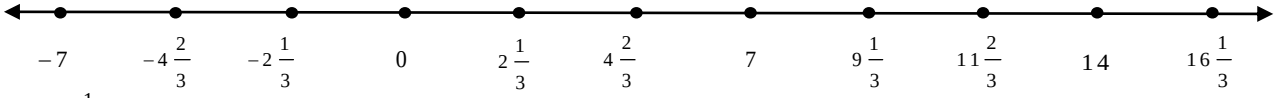
Solutions

1. 

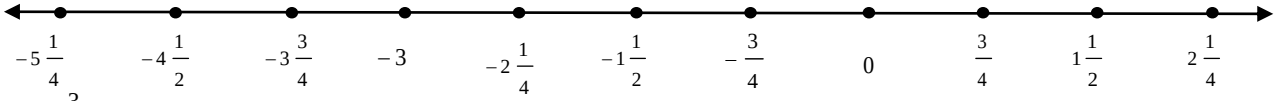
Add $1\frac{1}{4}$ each step as you move right.

2. 

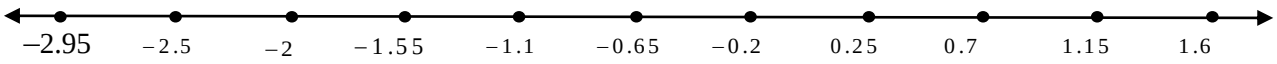
Add $\frac{1}{8}$ each step as you move right.

3. 

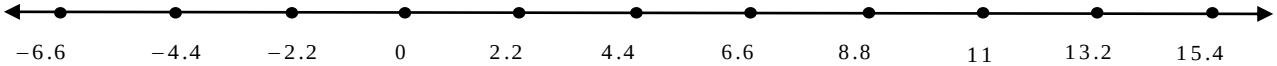
Add $2\frac{1}{3}$ each step as you move right.

4. 

Add $\frac{3}{4}$ each step as you move right.

5. 

Add 0.45 each step as you move right.

6. 

Add 2.2 each step as you move right.

Sample Activity 6: Multiplying Rational Numbers

You can multiply rational numbers by applying what you have learned about multiplying integers, decimals and fractions.

How do you multiply integers? $-7 \times 2 = ?$ or $-7 \times -2 = ?$
How do you multiply decimals? $6.13 \times 0.5 = ?$
How do you multiply fractions? $\frac{4}{5} \times \frac{2}{3} = ?$

How are the following examples the same and how are they different?

$\frac{5}{8} \times \frac{1}{2} = \frac{5}{16}$ $\frac{-5}{8} \times \frac{1}{2} = \frac{-5}{16}$ $\frac{-5}{8} \times \frac{-1}{2} = \frac{5}{16}$

$12.3 \times 0.21 = 2.583$ $-12.3 \times 0.21 = -2.583$ $-12.3 \times -0.21 = 2.583$

Discussion

- Have students summarize their procedures for multiplying rational numbers.
- Discuss with students how their knowledge of multiplying integers, decimals and fractions allowed them to learn how to multiply rational numbers.
- Since integers, fractions and decimals are rational numbers, the rules for multiplying rational numbers are the same.

Sample Activity 7: Dividing Rational Numbers

You can divide rational numbers by applying what you have learned about dividing integers, decimals and fractions.

How do you divide integers? $-5 \div 2 = ?$ or $-5 \div -2 = ?$
How do you divide decimals? $4.2 \div 0.6 = ?$
How do you divide fractions? $\frac{3}{8} \div \frac{1}{4} = ?$

Describe how the following examples are the same and how they are different? Then, write a summary.

$4 \div \frac{2}{5} = 10$ $\quad -4 \div \frac{2}{5} = -10$ $\quad -4 \div -\frac{2}{5} = 10$

$7.5 \div 0.5 = 15$ $\quad -7.5 \div 0.5 = -15$ $\quad -7.5 \div -0.5 = 15$

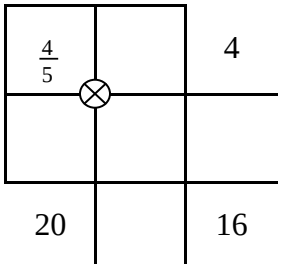
Discussion

- Have students summarize their procedures for dividing rational numbers.
- Discuss with students how their knowledge of dividing integers, decimals and fractions allowed them to learn how to divide rational numbers.
- Since integers, fractions and decimals are rational numbers, the rules for dividing rational numbers are the same.

Sample Activity 8: Rationals—Two-ways

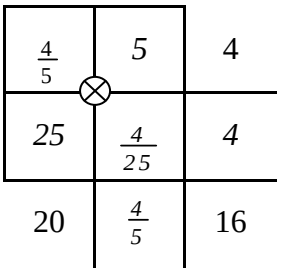
In two-ways, the numbers in a row must multiply to make a number at the right and the numbers in each column must multiply to a number at the bottom. Then, the two numbers at the right must multiply to make the number in the bottom right corner and the numbers along the bottom must also multiply to make the same number. Thus, there is a self-check built into the design of the activity.

For example:



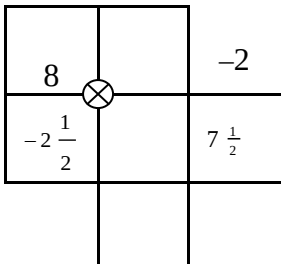
In the third column, 4 × 4 = 16. That was easy. Across the first row, I needed five $\frac{4}{5}$ to make 4 so the answer is 5. Across the third row, I thought that $\frac{1}{5}$ of 20 is 4 and 4 × 4 = 16 so the answer is $\frac{4}{5}$ (multiplying by 4 and dividing by 5). In the first column, I saw that $5 \times \frac{4}{5}$ is 4 and multiplying by 5 again gives 20, so the number is 25. In the second column, "What can I multiply 5 by to get $\frac{4}{5}$?" $\frac{1}{5}$ of 5 is 1 and $\frac{1}{5}$ of that is $\frac{1}{5}$. I need 4 of those to make $\frac{4}{5}$ so the answer is $\frac{4}{25}$. Now to check, I looked across the second row.

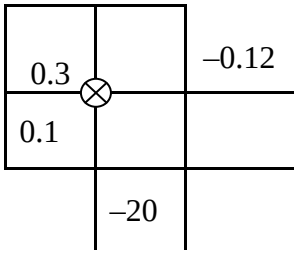
25 × ____ = 4. 4 divided by 25 is $\frac{4}{25}$. Since I got $\frac{4}{25}$ both ways, I know all my answers are correct.



Note: Solving multiplication two-ways will provide students with rich computational experiences and opportunities to relate the mathematical operations of multiplication and division.

Examples to try:





Sample Activity 9: Where's the Error?

1. Bethany and Carl completed the following question: $\frac{3}{5} - \frac{3}{8} \times \frac{5}{6} \div \frac{-5}{16}$. Bethany's answer is $\frac{8}{5}$ and Carl's answer is $\frac{-3}{5}$? Who is correct? Explain where the error was made by the other student.

2. Megan and André completed the following question: $4^2 \div \left[(9.75 + 4.15 + 2.1) \times 1.38 - 2.08\right]$. Megan's answer is 0.8 and André s answer is –0.7? Who is correct? Explain where the error was made by the other student.

Sample Activity 10: Order of Operations and Magic Squares

Calculate each of the expressions in the Magic Square.

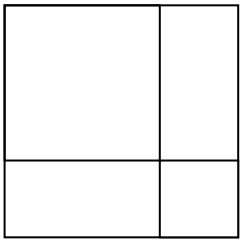
Determine the magic number.

Use the magic number to find any missing values.

		$\left(2\frac{3}{4}+1\frac{1}{8}\right)-\left(1\frac{1}{6}+\frac{5}{24}\right)$
$1-\left(\frac{11}{12}+\frac{5}{8}\right)+\frac{1}{24}$	$2\times\left(\frac{5}{6}-\frac{1}{12}\right)$	
$\left(\frac{5}{6}+\frac{-4}{3}\right)+\left\lceil\left[\left(1\frac{3}{10}-\frac{1}{5}\right)\times\frac{10}{11}\right]\right\rceil$		

Sample Activity 11: Perfect Squares

These diagrams illustrate 1.5^2. Explain how they show that $1.5^2 = 2.25$.



Use grid paper to draw diagrams that illustrate 2.1^2 and 3.5^2.

Sample Activity 12: Perfect Squares—How Many Ways?

For each set of numbers, find as many results as you can by combining the numbers by either squaring, finding the square root or using any of the operations +, –, × and ÷.

- $\frac{1}{2}$ $\frac{1}{4}$ 80 2
- 1 0.25 0.5 10
- $\frac{1}{2}$ $\frac{4}{9}$ $\frac{1}{16}$ 100
- 6.25 0.4 20 1.2

Step 4: Assess Student Learning

Guiding Questions

- Look back at what you determined as acceptable evidence in Step 2.
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

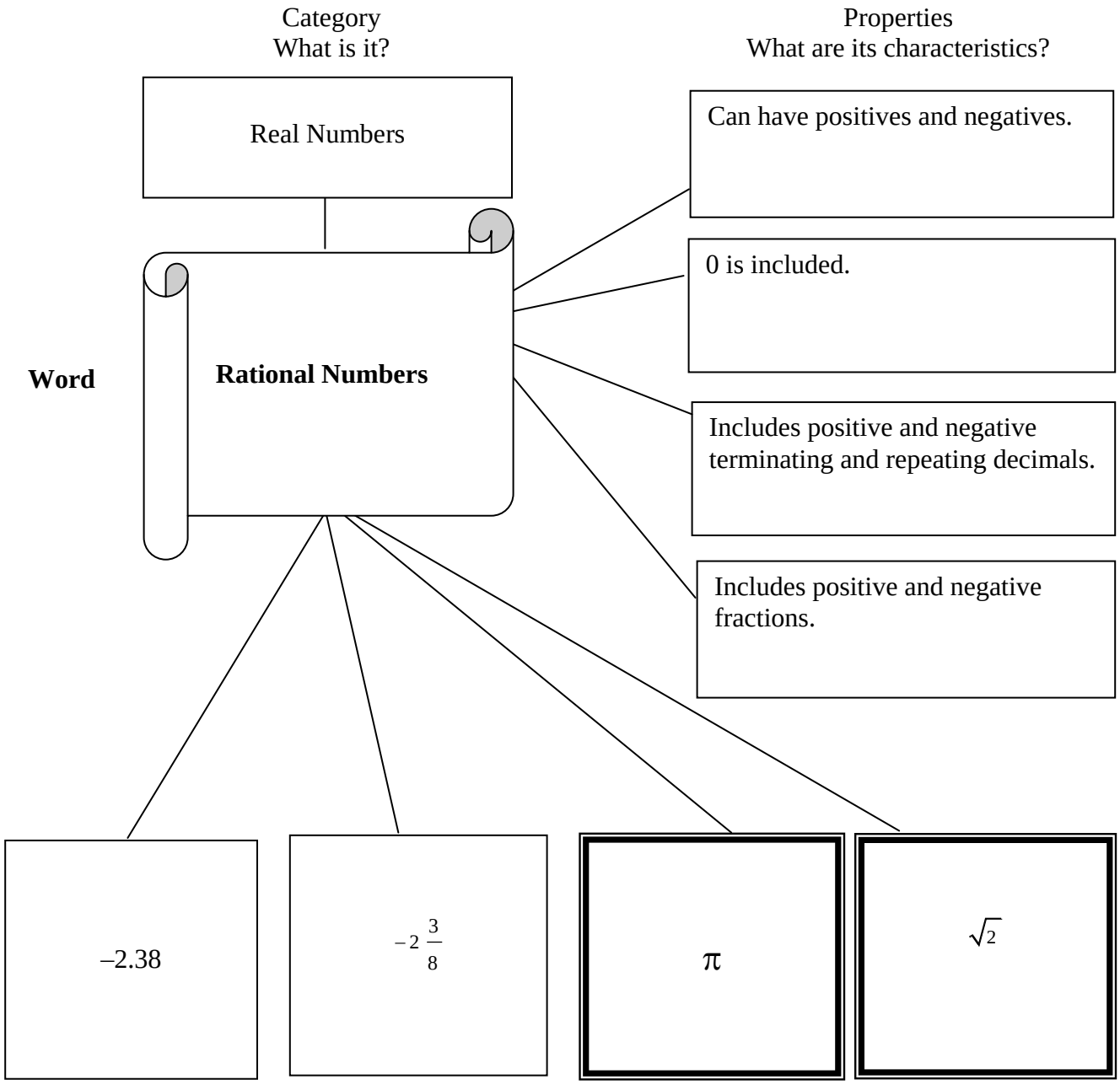
In addition to ongoing assessment throughout the lessons, consider the following sample activities to evaluate students' learning at key milestones. Suggestions are given for assessing all students as a class or in groups, individual students in need of further evaluation, and individual or groups of students in a variety of contexts.

A. Whole Class/Group Assessment

Activity 1: Creating a Concept Map

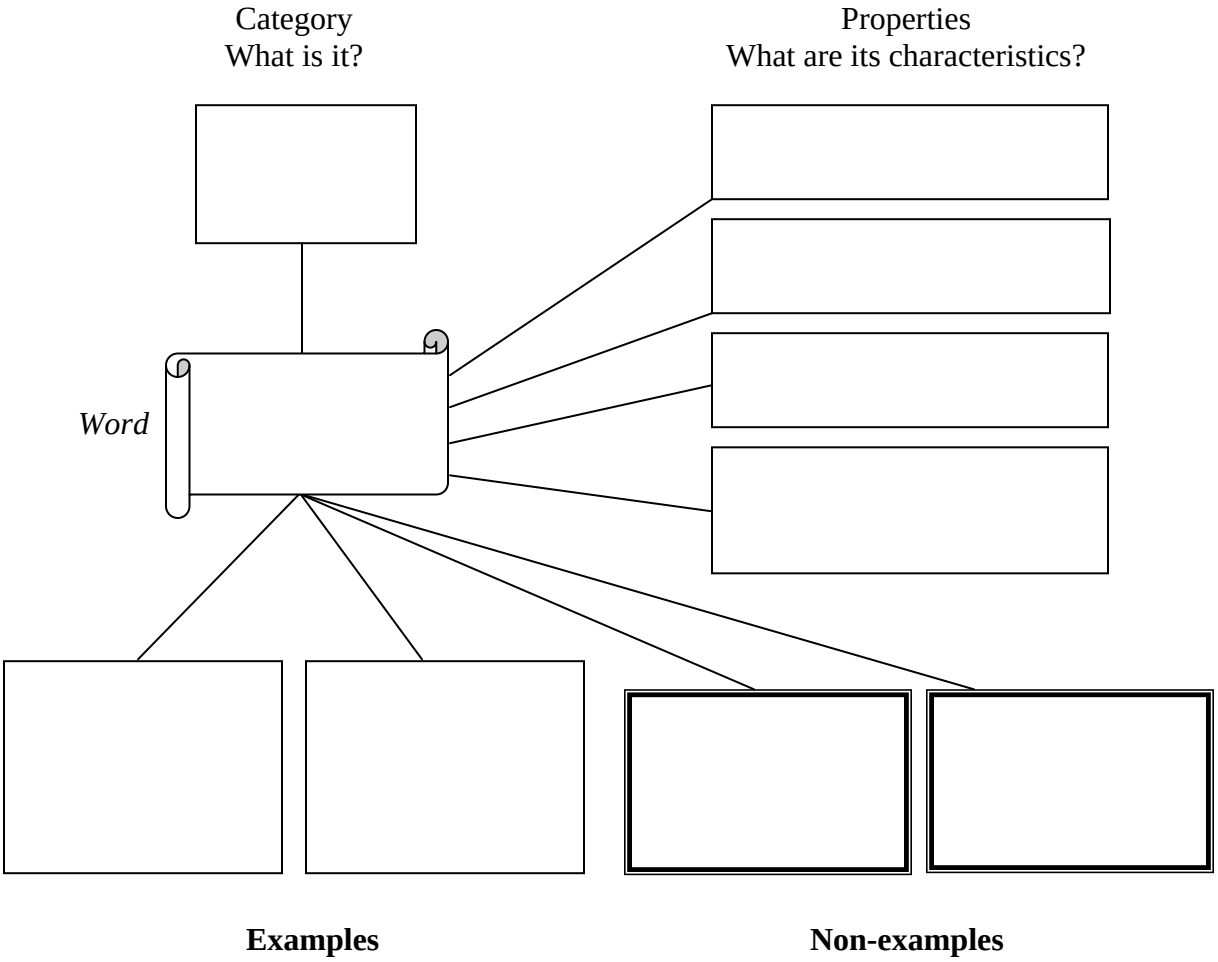
Have students work in groups to complete a Concept Definition Map for the concept of rational numbers. See the following sample.

Concept Definition Map



Format adapted from R. M. Schwartz. "Learning to Learn Vocabulary in Content Area Textbooks." *Journal of Reading* 32, 2 (November 1988), p. 110, Example 1. Adapted with permission from the International Reading Association.

Concept Definition Map



Format adapted from R. M. Schwartz. "Learning to Learn Vocabulary in Content Area Textbooks." *Journal of Reading* 32, 2 (November 1988), p. 110, Example 1. Adapted with permission from the International Reading Association.

Activity 2: Zooming in on the Target

Write the numbers 1 through 10 on the board. Students will work in pairs to eliminate each number listed on the board.

You will need four dice with the following numbers written on each:

- Die 1: $-\frac{1}{2}, -\frac{1}{4}, -\frac{1}{3}, \frac{1}{6}, \frac{2}{3}, \frac{3}{4}$
- Die 2: $\frac{1}{2}, \frac{1}{4}, \frac{1}{3}, -\frac{1}{6}, -\frac{2}{3}, -\frac{3}{4}$
- Die 3: –2, –5, –10, 16, 20, 100
- Die 4: 2, 5, 10, –16, –20, –100.

Roll all the dice and write the numbers rolled on the board, so that students can see what numbers came up.

Pairs of students work for five minutes to come up with as many of the numbers 1 through 10 as possible. The students will use some or all of the numbers that were rolled and any of addition, subtraction, multiplication, division, squaring, finding the square root and brackets to create any of the numbers 1 through 10.

Starting with 1, have students share with the class the number sentences they have used to create the numbers. List all of the possibilities that the class has come up with. Pairs score one point for a correct number sentence. Eliminate 1 off the board once points are awarded and repeat the process for the other numbers. Some numbers may not have been eliminated. Roll the dice again and have student pairs try for the remaining numbers. Before the second or third roll of the dice, you may wish to have a discussion about what numbers would be most helpful.

When all of the numbers have been eliminated, the pair with the most points is the winner.

Adapted from R. Bresser and C. Holtzman. *Minilessons for Math Practice: Grades 3–5,* p.13. Sausalito, CA: Math Solutions, 2006. Marilyn Burns Education Associates.

B. Applied Learning

Provide opportunities for students to use the knowledge they have gained about rational numbers, perfect squares and square roots and notice whether or not this knowledge transfers.

Activity 1: Apply knowledge of rational numbers and order of operations in finding the volume and surface area of right prisms and composite shapes.

Activity 2: Given the surface of right prisms, apply knowledge of square roots to determine a missing dimension.

Activity 3: Given a set of data on daily maximum or minimum temperatures, apply knowledge of rational numbers and order of operations to find the average daily temperature.

Activity 4: Apply knowledge of rational numbers, order of operations, squares and square roots when evaluating algebraic expressions.

Activity 5: Apply knowledge of rational numbers, order of operations, squares and square roots when substituting into the formula, such as converting between temperature scales.

Step 5: Follow-up on Assessment

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction?

A. Addressing Gaps in Learning

- Review the operations with integers, fractions and decimals; students should be comfortable with estimating answers.

- Provide time for students to reflect on their learning by using graphic organizers, such as the Frayer Models or Concept Definition Map.

- Make sure that students are solving problems by drawing pictures and providing explanations to justify their thinking.

- Remind students to use divisibility rules to find factor pairs. Encourage students to write the factor pairs in an organized list to help determine the square root. If students are having difficulty identifying factors, you may want to provide them with multiplication tables.

- Provide opportunities for students to develop number sense by encouraging them to refine their estimates of square roots of non-perfect squares.

B. Reinforcing and Extending Learning

Students who have achieved or exceeded the outcomes will benefit from ongoing opportunities to apply and extend their learning. These activities should support students in developing a deeper understanding of the concept and should not progress to the outcomes in subsequent grades.

Consider strategies, such as the following:

Activity 1

Stock Market gains and losses are recorded as rational numbers. When individuals buy stocks in a company, they become part owners of the company. Each share of a stock has a purchase price and a selling price. Have students look at the stock listings in a newspaper and choose a stock; the prices are in cents unless they are marked with a dollar sign ($). Students are to choose a stock and imagine they are purchasing 1000 shares. Students will hold on to the stock for several weeks and watch the change in the price of their stock. They will then need to make a decision on the best time to sell and determine their profit. Continue to review the stock for a few more days, asking "did they sell at the right price or could they have realized larger profits if they had kept the stock longer?" Students could choose a different stock and repeat the process.

Activity 2

A study of the Wind Chill Equivalency Charts provides opportunities to pose many problems involving operations of rational numbers.

Wind chill describes a sensation; e.g., the way we feel as a result of the combined cooling effect of temperature and wind. This feeling can't be measured using an instrument, so scientists have developed a mathematical formula that relates air temperature and wind speed to the cooling sensation we feel on our skin. From the mathematical formula, tables have been created to indicate the temperature you would feel when the actual temperature is combined with the effect of the wind at different speeds.

Wind Chill Calculation Chart
where T air = Air temperature in °C, and V_{10} = Observed wind speed at 10m elevation in km/h.

T air / V_{10}	5	0	-5	-10	-15	-20	-25	-30	-35	-40	-45	-50
5	4	-2	-7	-13	-19	-24	-30	-36	-41	-47	-53	**-58**
10	3	-3	-9	-15	-21	-27	-33	-39	-45	-51	**-57**	**-63**
15	2	-4	-11	-17	-23	-29	-35	-41	-48	-54	**-60**	**-66**
20	1	-5	-12	-18	-24	-30	-37	-43	-49	**-56**	**-62**	**-68**
25	1	-6	-12	-19	-25	-32	-38	-44	-51	**-57**	**-64**	**-70**
30	0	-6	-13	-20	-26	-33	-39	-46	-52	**-59**	**-65**	**-72**
35	0	-7	-14	-20	-27	-33	-40	-47	-53	**-60**	**-66**	**-73**
40	-1	-7	-14	-21	-27	-34	-41	-48	-54	**-61**	**-68**	**-74**
45	-1	-8	-15	-21	-28	-35	-42	-48	**-55**	**-62**	**-69**	**-75**
50	-1	-8	-15	-22	-29	-35	-42	-49	**-56**	**-63**	**-69**	**-76**
55	-2	-8	-15	-22	-29	-36	-43	-50	**-57**	**-63**	**-70**	**-77**
60	-2	-9	-16	-23	-30	-36	-43	-50	**-57**	**-64**	**-71**	**-78**
65	-2	-9	-16	-23	-30	-37	-44	-51	**-58**	**-65**	**-72**	**-79**
70	-2	-9	-16	-23	-30	-37	-44	-51	**-58**	**-65**	**-72**	**-80**
75	-3	-10	-17	-24	-31	-38	-45	-52	**-59**	**-66**	**-73**	**-80**
80	-3	-10	-17	-24	-31	-38	-45	-52	**-60**	**-67**	**-74**	**-81**

Environment Canada's Wind Chill, Wind Chill Calculation Chart. Taken from http://www.msc-smc.ec.gc.ca/contents_e.html.

Many problems relating to rational number operations can be developed from the data.
For example:

a. Suppose the wind is blowing at a speed of 20 km/h, what is the actual temperature if the wind chill factor is –8.5°C?
b. In Fort McMurray, the air temperature was –25°C. Estimate the wind chill factor when the wind started to blow at 12.5km/h. A wind of 30 km/h would cause the wind chill factor in Fort McMurray to drop how many degrees?

Activity 3

Use four different rational numbers that would give an approximate sum of –8.

Activity 4

On Sunday at noon, the temperature was 2.3°C and by midnight, it was –16.9°C. Find the change in temperature. Show two different ways to find the change.

Activity 5

Marcia's monthly bank statement shows a list of her transactions: 4 of –$150, 3 of –$55.25, 6 of $77.89 and 1 of $1589. Marcia started the month with a balance of $695.89. Write a single mathematical statement that calculates Marcia's balance at the end of the month.

Activity 6

The maximum daily temperatures this week in Red Deer were Sunday –11.7°C, Monday –7.9°C, Tuesday –16.5°C, Wednesday –21.2°C, Thursday –8.5°C, Friday –3.1°C and Saturday 2.8°C. Write a single mathematical statement to calculate the average daily maximum temperature in Red Deer this week.

Activity 7

The rectangle is half of a square. If the area of the square is 14.44 cm^2, what are the dimensions of the rectangle?

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