Fractions Math Help

Learning Objective:

- Recognize that a fraction is a rational number.
- Identify the numerator and denominator of a fraction.
- Identify the multiples of a number.
- Identify the lowest common denominator of two fractions from the multiples of the denominators.
- Represent fractions pictorially, symbolically and concretely.
- Convert fractions to an equivalent form.
- Convert improper fractions to mixed fractions.
- Convert mixed fractions to improper fractions.
- Apply the arithmetic operations of addition, subtraction, multiplication and division to fractions in ways that are concrete, pictorial and symbolic.

Helpful Hints:

Definitions:

**Fraction**: a rational number that represents part of a whole or part of a set.

**Rational Number**: a number that can be written in the form \( \frac{a}{b} \) where \( a \) and \( b \) are both integers and \( b \neq 0 \).

Hint: the word “rational” contains the root word “ratio” therefore rational numbers are those that can be written as ratios or fractions.

**Numerator**: the top numeral of a fraction, tells how many parts are being considered.

**Denominator**: the bottom numeral of a fraction, tells how many parts into which the whole has been divided.

Eg. \( \frac{3}{4} \) → numerator \( 3 \) denominator \( 4 \)

**Improper Fraction**: a fraction with a numerator greater than the denominator.

Eg. \( \frac{5}{3} \)

**Mixed Fraction**: consists of both a whole number and a fraction.

Eg. \( 2\frac{1}{6} \)

**Multiples**: a set of numbers that is divisible by a given number.

Eg. the multiples of 4 are: 4, 8, 12, 16, 20, 24, etc.
**Lowest Common Denominator**: the smallest common multiple that 2 numbers will both divide into.

Eg. the multiples of 3 are: 3, 6, 9, 12, 15, 18, etc
   the multiples of 5 are: 5, 10, 15, 20, 25, etc
   The lowest common denominator (LCD) of the numbers 3 and 5 is 15.

**Fraction Strip**: a “ruler” showing the division of a whole into parts used in the manipulation of fractions.

Eg. Quarters

```
-3  -2  -1  0  1  2  3
```

```
Negative Fractional Parts
Positive Fractional Parts
```

4 parts = quarters

Eg. Sixths

```
-3  -2  -1  0  1  2  3
```

```
Negative Fractional Parts
Positive Fractional Parts
```

6 parts = sixths

**Fraction Grid**: a 2 dimensional scale used to show the operations of multiplication and division of fractions.

```
0  1
```

```
2 parts = parts = halves quarters
4 parts = quarters
```

3 parts = thirds

```
0  1
```

```
2 parts = parts = halves quarters
4
```

1 by1 grid = 4 x 2
1 whole = 8 boxes

1 by 1 grid = 3 x 4
1 whole = 12 boxes
Simplification of Fractions

1. Reducing to the Lowest Terms:

   Fractions should be reduced to their lowest terms at the completion of an operation.
   - divide the numerator and denominator by the highest common factor of both.
   - reduction is complete when 1 is the only common factor for both numerator and denominator.

   Eg. \(\frac{18}{24}\) =
   \[
   \frac{18}{24} = \frac{6 \times 3}{6 \times 4} \quad \text{(The highest common factor is 6.)}
   \]
   \[
   \frac{18}{24} = \frac{18 \div 6}{24 \div 6} \quad \text{(Numerator and denominator are divided by 6.)}
   \]
   \[
   \frac{18}{24} = \frac{3}{4} \quad \text{Reduced form.}
   \]

2. Mixed and Improper Fractions:

   Fractions may be converted from one form to another as required for an operation.
   - Mixed → Improper: The whole number must be changed to the same denominator as the fraction.

   Eg. \(1\frac{2}{5} = 1 + \frac{2}{5}\)
   \[
   = \frac{5}{5} + \frac{2}{5} 
   = \frac{7}{5}
   \]

   - Improper → Mixed: The whole parts must be identified and separated from the fraction.

   Eg. \(\frac{7}{3} = \frac{3}{3} + \frac{3}{3} + \frac{1}{3}\)
   \[
   = 1 + 1 + \frac{1}{3} 
   = 2\frac{1}{3}
   \]
3. **Converting to an Equivalent Form:**

- Equivalent forms of all fractions are formed by multiplying or dividing the numerator and denominator by the same value.

  Eg. \[ \frac{2}{3} = \frac{2 \times 5}{3 \times 5} \quad \frac{16}{12} = \frac{16 \div 4}{12 \div 4} \]

  Eg. \[ \frac{2}{3} = \frac{10}{15} \quad \frac{16}{12} = \frac{4}{3} \]

**Note:** Adding or subtracting the same number does not yield an equivalent fraction.

  Eg. \[ \frac{1}{2} \neq \frac{1+3}{2+3} \quad \frac{5}{6} \neq \frac{5-2}{6-2} \]

  Eg. \[ \frac{1}{2} \neq \frac{4}{5} \quad \frac{3}{4} \neq \frac{6}{4} \]

- For the operations of addition and subtraction all fractions must have the same (common) denominator. That common denominator is the lowest common multiple of the denominators in the question.

  Eg. \[ \frac{3}{4} + \frac{5}{6} = \]

  Multiples of 4: 4, 8, 12, 16, etc.

  Multiples of 6: 6, 12 18, etc.

  The lowest common denominator (LCD) is 12.

  Convert both fractions to a denominator of 12.

- Converting to a common denominator:

  Determine what each denominator must be multiplied by to equal 12.

  Multiply the numerator by the same value.

  \[ \frac{3}{4} = \frac{3 \times 3}{4 \times 3} = \frac{9}{12} \quad \frac{5}{6} = \frac{5 \times 2}{6 \times 2} = \frac{10}{12} \]

  Multiply by 3 = 12 \quad \text{Multiply by 2 = 12}

  Each numerator is then multiplied by the same number.

- Equivalence:

  \[ \frac{3}{4} = 9 \text{ parts out of 12} \quad \frac{5}{6} = 10 \text{ parts out of 12} \]
Since both fractions are parts of the same whole (12) they can be added.

- **Addition:**
  \[
  \frac{3}{4} + \frac{5}{6} = \frac{9}{12} + \frac{10}{12} = \frac{19}{12}
  \]

**Operations on Fractions:**

1. **Addition:** requires fractions to have a common denominator.
   (Convert any mixed fractions to improper fractions first.)

   Eg. \( \frac{2}{3} + \frac{3}{4} = \)

Pictorial Representation:

- Select each fraction.
- Find the LCD using Multiples of the Denominators:
  The LCD for 3 and 4 is 12.
- Pink Fraction: Shaded area shows \( \frac{2}{3} \) as 8 parts out of 12.
- Blue Fraction: Shaded area shows \( \frac{3}{4} \) as 9 parts out of 12.
- Sum: Pink + Blue = 8 parts + 9 parts = 17 parts out of 12 = 1 whole and 5 parts out of 12.
Concrete Solution:

- Addition requires fractions to have a common denominator.
- Use multiples of the denominators to find the LCD.
- Convert each fraction to the LCD. (See above: 3. Converting to an Equivalent Form)

\[
\begin{align*}
\frac{2}{3} &= \frac{2 \times 4}{3 \times 4} = \frac{8}{12} \\
\frac{3}{4} &= \frac{3 \times 3}{4 \times 3} = \frac{9}{12}
\end{align*}
\]

- Add the numerators of the common denominators to see how many parts of 12 there are.

\[
\frac{2}{3} + \frac{3}{4} = \frac{8}{12} + \frac{9}{12} = \frac{17}{12} = 1 \frac{5}{12}
\]

2. **Subtraction:** requires fractions to have a common denominator.
   (Convert any mixed fractions to improper fractions first.)

   i. Eg. \( \frac{3}{2} - \frac{5}{4} = \)

Pictorial Representation:

- Select each fraction then shade the pink fraction forward and shade the blue fraction back.
- Find the LCD using Multiples of the Denominators:
  The LCD for 2 and 4 is 4.
• Pink Fraction: Shaded area shows $\frac{3}{2}$ as 6 parts of 4.
• Blue Fraction: Shaded area shows $\frac{5}{4}$ as 5 parts of 4.
• Difference: 1 part of 4 remains.

Concrete Solution:

• Subtraction requires fractions to have a common denominator.
• Use multiples of the denominators to find the LCD.
• Convert each fraction to the LCD. (See above: 3. Converting to an Equivalent Form)

\[ \frac{3}{2} = \frac{3 \times 2}{2 \times 2} = \frac{6}{4} \quad \frac{5}{4} = \frac{5}{4} \]

• Subtract the numerators of the common denominators to see how many parts of 4 are left.

\[ \frac{3}{2} - \frac{5}{4} = \]
\[ \frac{6}{4} - \frac{5}{4} = \frac{1}{4} \]

ii. Eg. $\frac{2}{3} - \frac{3}{4} =$

• Pink fraction: shade 8 parts of 12 forward.
• Blue fraction: shade 9 parts of 12 backward.
• Difference: -1 parts of 12.
3. **Multiplication:**
   - Used to find a part of another fraction.
   - Used to replace the word “of”: \( \frac{1}{2} \) of 10 = \( \frac{1}{2} \times 10 \)

   Eg. \( \frac{5}{4} \times \frac{1}{2} = \)

   **Pictorial Representation:**
   - 1 by 1 grid represents the number of parts in 1 whole: \( 4 \times 2 = 8 \) parts.
   - The area of overlapped shading is the result of the multiplication.
   - Pink Fraction: 5 parts of 4 are shaded down.
   - Blue Fraction: 1 part of 2 is shaded across.
   - Product of Pink x Blue: 5 parts x 1 part = 5 parts of 1 whole are overlapped.
   - Result: 5 parts of 8.

   **Concrete Solution:**
   - No common denominator required.
   - Convert any mixed fractions to improper fractions first.
   - Multiply straight across the top and bottom.

   - Reduce the result.
\[
\frac{5}{4} \times \frac{1}{2} = \frac{5 \times 1}{4 \times 2} = \frac{5}{8}
\]

Multiply straight across.

\[
\frac{5}{8}
\]

Result of multiplication.

4. **Division**: is the opposite or reverse of multiplication.

Using example 3 above:

- **Short Hand**: If the product of two fractions is divided by one of the fractions, it will result in the other.

  
  
  Eg. Since \( \frac{5}{4} \times \frac{1}{2} = \frac{5}{8} \)
  
  Then \( \frac{5}{8} \div \frac{1}{2} = \frac{5}{4} \) and \( \frac{5}{8} \div \frac{5}{4} = \frac{1}{2} \)

- **Long Hand**: Change division of fractions to multiplication by inverting the second fraction. Reduce the final result.

  
  
  Eg. \( \frac{5}{8} \div \frac{2}{2} = \frac{5}{8} \times \frac{2}{2} = \frac{5}{8} \times \frac{2}{2} = \frac{10}{16} = \frac{5}{8} \)

  
  
  \( \frac{5}{8} \times \frac{4}{4} = \frac{20}{32} = \frac{5}{8} \times \frac{4}{4} = \frac{1}{2} \)

  
  Therefore: \( \frac{5}{8} \div \frac{1}{4} = \frac{5}{8} \times \frac{4}{4} = \frac{5}{8} \times \frac{1}{2} \)

- **When dividing mixed fractions**, change to improper fractions first.
5. **Multiplication and Division**: of improper fractions.

**Multiplication:**

\[
\begin{align*}
\text{Eg. } 2\frac{1}{3} \times 1\frac{1}{2} &= \quad \text{Pictorial Representation:} \\
\end{align*}
\]

- 1 by 1 grid = 3 parts x 2 parts = 6 parts in 1 whole.
- Pink fraction: 7 parts of 3 shaded down.
- Blue fraction: 3 parts of 2 shaded across.
- Product of Pink x Blue: 7 parts x 3 parts = 21 parts of 6.

**Concrete Solution:**

- Convert each fraction to its improper form.
  \[
  \frac{7}{3}, \frac{7}{2} = \frac{3}{11}, \frac{3}{12}
  \]

- Multiply straight across and simplify the result.
  \[
  2\frac{1}{3} \times 1\frac{1}{2} = \frac{7}{3} \times \frac{3}{2} = \frac{21}{6}
  \]
  Convert each to an improper fraction.
  \[
  = \frac{21}{6}
  \]
  Multiply straight across.
\[
\frac{7}{2} \quad \text{Reduce by dividing both by 3.}
\]

\[
= \frac{3 \frac{1}{2}}{2} \quad \text{Express as a mixed fraction.}
\]

**Division:**

**Long Hand Concrete Solution:**

\[
3 \frac{1}{2} \div 2 \frac{1}{3} = \frac{7}{2} \div \frac{7}{3}
\]

Convert each to an improper fraction.

\[
= \frac{7}{2} \times \frac{3}{7}
\]

Change to multiplication by inverting the second fraction.

\[
= \frac{21}{14}
\]

Multiply straight across.

\[
= \frac{3}{2}
\]

Reduce by dividing both by 7.

\[
= 1 \frac{1}{2}
\]

Change to a mixed fraction.

\[
3 \frac{1}{2} \div 1 \frac{1}{2} = \frac{7}{2} \div \frac{3}{2}
\]

Convert each to an improper fraction.

\[
= \frac{7}{2} \times \frac{2}{3}
\]

Change to multiplication by inverting the second fraction.

\[
= \frac{14}{6}
\]

Multiply straight across.

\[
= \frac{7}{3}
\]

Reduce by dividing both by 2.

\[
= 2 \frac{1}{3}
\]

Change to a mixed fraction.