

Learning Objective:

- Recognize right triangles as having 2 sides and a hypotenuse, 2 acute angles; and a right angle.
- Recognize and identify a specific reference angle along with its related opposite side, adjacent side, and hypotenuse.
- Identify which trigonometric ratio should be used to solve for an unknown based on the information given.
- Recognize the difference between the ratios that define sine, cosine, and tangent.
- Understand the mathematical calculations required to solve for the unknown.

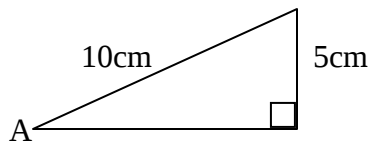
Understanding the Meaning of Trigonometry

Tri --- gono --- metry
3 --- sides --- measurement

Trigonometry is the study of measurement of three-sided figures. More specifically, trigonometry is the study of the measurements of the sides and angles in triangles.

Ancient Greek mathematicians discovered that if two different right triangles had two matching sides in the same ratio, the angle forming those two sides is equal.

Example:

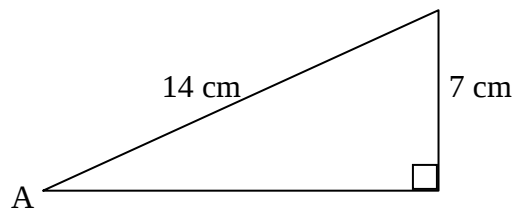


$$\text{ratio} = \frac{\text{height}}{\text{hypotenuse}}$$

$$\text{ratio} = \frac{5\text{cm}}{10\text{cm}}$$

$$\text{ratio} = \frac{1}{2}$$

$$\angle A = 30^\circ$$



$$\text{ratio} = \frac{\text{height}}{\text{hypotenuse}}$$

$$\text{ratio} = \frac{7\text{cm}}{14\text{cm}}$$

$$\text{ratio} = \frac{1}{2}$$

$$\angle A = 30^\circ$$

Since both ratios are equal, the matching angles are also equal. The ratio defined in this example is called sine. There are two other basic ratios called cosine and tangent.

Definition of the Trigonometric Ratios:

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

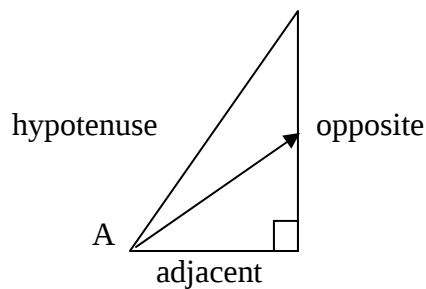
$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

Helpful Hints

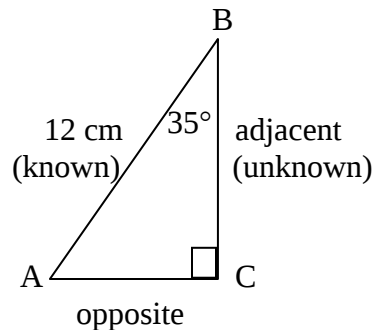
How to Chose the Appropriate Ratio:

1. Begin by selecting the acute angle from which the diagram is “viewed”.
2. Name the sides of the diagram: opposite, adjacent, and hypotenuse as they relate to the selected acute angle. Imagine “backing” into the corner of the angle and pointing to the:
 - opposite side – across from you
 - adjacent side – beside you
 - hypotenuse – the longest side, also across from the right angle



3. From the perspective of the selected acute angle, all information is labeled on the diagram including the unknown. Based on what is known and what must be found, choose the correct ratio.

Example: Given that $\angle B = 35^\circ$, $AB = 12\text{cm}$, find BC.



Because the unknown is the adjacent side and the known is the hypotenuse, we choose the cosine ratio to solve for the measurement of the adjacent side.

$$\begin{aligned}\cos 35^\circ &= \frac{\text{adjacent}}{\text{hypotenuse}} \\ \cos 35^\circ &= \frac{\text{adjacent}}{12\text{cm}} \\ \text{adjacent} &= (\cos 35^\circ)(12\text{cm}) \\ \text{adjacent} &= (0.819)(12\text{cm}) \\ \text{adjacent} &= 9.8\text{cm}\end{aligned}$$

Therefore, in a right triangle with an acute angle of 35° and hypotenuse of 12 cm, the side adjacent to that angle would measure 9.8 cm.

Using the Ratios for Trigonometry

The correct selection and usage of the trigonometric ratios allows you to find:

- every possible angle formed by all possible ratios of sides.
- every possible side measurement formed by all possible angles.

Although trigonometry can be used to solve for unknown sides or angles in non-right angled triangles, only right triangles will be investigated in the **Explore It** mode.

Examples from the Explore It mode:

1. Using **R** as the reference angle, **sin R** is selected to find the length of the **opposite** side. The right portion of the screen shows the solution for the measurement of the opposite side.

The screenshot shows an interactive trigonometry tool. On the left, there are controls for 'Function' (sin R, cos R, tan R) and 'Unknown' (Opposite, Adjacent, Hypotenuse, Angle R, Angle S). The 'Reference Angle' is set to R. The main area displays a right triangle with vertices S, T, and R. Angle R is 59.7 degrees, and the hypotenuse SR is 50.51 cm. The opposite side ST is labeled with a red question mark. On the right, the calculation is shown: $\sin R = \frac{\text{opposite}}{\text{hypotenuse}}$, $\sin 59.7^\circ = \frac{\text{opposite}}{50.51 \text{ cm}}$, $\text{opposite} = \sin 59.7^\circ \times 50.51 \text{ cm}$, $\text{opposite} = 0.863 \times 50.51 \text{ cm}$, and $\text{opposite} = 43.6 \text{ cm}$. A note at the bottom right states: 'Note the displayed values are rounded and may not match calculated values.'

2. Using **R** as the reference angle, **cos R** is selected to find the length of the **hypotenuse**. The right portion of the screen shows the solution for the measurement of the hypotenuse.

Instructions

Reset

Function

☐ sin R

☒ cos R

☐ tan R

Unknown

☐ Opposite

☐ Adjacent

☒ Hypotenuse

☐ Angle R

☐ Angle S

Reference Angle

☒ R

☐ S

Scale 1 1.5 2

Rotate

$$\cos R = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\text{hypotenuse} = \frac{\text{opposite}}{\sin R}$$

$$\text{hypotenuse} = \frac{25.5 \text{ cm}}{\cos 58.9^\circ}$$

$$\text{hypotenuse} = \frac{25.5 \text{ cm}}{0.517}$$

$$\text{hypotenuse} = 49.31 \text{ cm}$$

Note the displayed values are rounded and may not match calculated values.

3. Using **S** as the reference angle, **tan S** is selected to find the length of the **adjacent** side. The right screen shows the solution for the measurement of the adjacent side.

Instructions

Reset

Function

☐ sin S

☐ cos S

☒ tan S

Unknown

☐ Opposite

☒ Adjacent

☐ Hypotenuse

☐ Angle R

☐ Angle S

Reference Angle

☐ R

☒ S

Scale 1 1.5 2

Rotate

$$\tan S = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 60.5^\circ = \frac{45 \text{ cm}}{\text{adjacent}}$$

$$\text{adjacent} = \frac{45 \text{ cm}}{\tan 60.5^\circ}$$

$$\text{adjacent} = \frac{45 \text{ cm}}{1.765}$$

$$\text{adjacent} = 25.5 \text{ cm}$$

Note the displayed values are rounded and may not match calculated values.

4. With the known opposite and adjacent sides, $\angle S$ is be found by choosing **tan S**. The right portion of the screen shows the solution for the measurement of $\angle S$.

Instructions

Reset

Function

☐ sin S

☐ cos S

☒ tan S

Unknown

☐ Opposite

☐ Adjacent

☐ Hypotenuse

☐ Angle R

☒ Angle S

Reference Angle

☐ R

☒ S

Scale 1 1.5 2

Rotate

$$\tan S = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan S = \frac{41.8 \text{ cm}}{46 \text{ cm}}$$

$$\tan S = 0.909$$

$$S = 42.3^\circ$$

Note the displayed values are rounded and may not match calculated values.