Mathematics

Planning Guide

Grade 2

Adding and Subtracting Number to 100

Number

Specific Outcomes 8, 9

This Planning Guide can be accessed online at:
http://www.learnalberta.ca/content/mepg2/html/pg2_addingandsubtractingnumberto100/index.html

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Adding and Subtracting Number to 100

Strand: Number
Specific Outcomes: 8, 9

This *Planning Guide* addresses the following outcomes from the Program of Studies:

Strand: Number	
Specific Outcomes:	8. Demonstrate and explain the effect of adding zero to, or subtracting zero from, any number. 9. Demonstrate an understanding of addition (limited to 1- and 2-digit numerals) with answers to 100 and the corresponding subtraction by: • using personal strategies for adding and subtracting with and without the support of manipulatives • creating and solving problems that involve addition and subtraction • using the commutative property of addition (the order in which numbers are added does not affect the sum) • using the associative property of addition (grouping a set of numbers in different ways does not affect the sum) • explaining that the order in which numbers are subtracted may affect the difference.

Curriculum Focus

This sample targets the following changes to the curriculum:

- The specific outcome in 2007 focuses on demonstrating an understanding of addition and the corresponding subtraction for numbers to 100, using personal strategies with or without manipulatives. The previous mathematics curriculum specified using manipulatives, diagrams and symbols to demonstrate and describe the processes of addition and subtraction. The current reference to "corresponding subtraction" indicates the importance of students' awareness of subtraction as the inverse of addition and the option students have of solving subtraction by thinking addition.

- It is of note that the 1997 curriculum included a specific outcome that stated students should "apply and explain multiple strategies to determine sums and differences on 2-digit numbers, with and without regrouping." This has only changed in the wording "personal strategies," rather than "multiple strategies"; the detailed description of some strategies such as those in specific outcome 6 of Grade 3 gives teachers a clear indication of the kinds of personal strategies that they might expect and structure lessons to encourage.
- The specific outcome in 2007 includes the commutative property of addition and the associative property of addition. The previous mathematics curriculum did not specify these properties at this level. Grade 2 students should recognize that the commutative property does not hold for subtraction.
- The 2007 curriculum specifies the students be able to "demonstrate and explain the effect of adding zero to, or subtracting zero from, any number." Zero is the identity element for addition and subtraction. This was not specified in the 1997 curriculum, although it is an expectation of students.
- The previous curriculum specified that students should have recall of addition and subtraction facts to 10 in Grade 2. There is no mention of an expected time of proficiency with facts to 18 in terms of accuracy and speed. The limitation of addition and subtraction facts being addressed each year (to 10 in Grade 1 and to 18 in Grade 2, with an emphasis on mental mathematics strategies to calculate these), will insure most students either commit them to memory through usage or have efficient ways to calculate them whenever needed.
- The old curriculum heavily emphasized the importance of learning operations in the context of problem solving. Although this is still a component of the new curriculum, it also includes the importance of creating problems to solve.

What Is a Planning Guide?

Planning Guides are a tool for teachers to use in designing instruction and assessment that focuses on developing and deepening students' understanding of mathematical concepts. This tool is based on the process outlined in *Understanding by Design* by Grant Wiggins and Jay McTighe.

Planning Steps

The following steps will help you through the Planning Guide:

Step 1: Identify Outcomes to Address

Guiding Questions

- What do I want my students to learn?
- What can my students currently understand and do?
- What do I want my students to understand and be able to do based on the Big Ideas and specific outcomes in the program of studies?

Big Ideas

- Addition and subtraction are related, with subtraction being the inverse of addition.
- The order of the numbers does not matter when you add, but does when you subtract.
- Traditional algorithms are often not the most efficient methods of computing. They are also not naturally invented by students. If the traditional algorithms are not taught to students early on, students will invent or adopt personal strategies that vary with the numbers and the situation. What is important is that the methods used are understood by the user.
- Personal strategies depend on taking apart and combining numbers in a variety of ways and recognizing relationships between numbers.
- Models, be they manipulatives or diagrams, can help a person recognize the operation involved and make sense of a problem.
- Examining any problems for the parts and the whole helps students make sense of the problem and identify the operation required. (See **Categories of Addition and Subtraction Problems Based on Structure**, p. 50.)

Principles and Standards for School Mathematics states that computational fluency is a balance between conceptual understanding and computational proficiency (NCTM 2000, p. 35). Conceptual understanding requires flexibility in thinking about the structure of numbers (base-ten system), the relationship among numbers and the connections between addition and subtraction. The ability to generate equivalent representations of the same number provides a foundation for using personal strategies to add and subtract, recognizing that for some problems either operation may be used. Computational proficiency includes both efficiency and accuracy. Personal strategies must be compared and evaluated so students adopt methods that are efficient, as well as accurate.

Addition and subtraction problems include four main types:

- join problems involving an initial amount, a change amount (the amount being added or joined) and the resulting amount
- separate problems also involve change as in join problems, but the whole is the result in the join problems, whereas the whole is the initial amount in the separate problems
- part–part–whole problems consider two static quantities either separately or combined
- comparison problems determine how much two numbers differ in size.

What is crucial is that students are familiar with the relationship between addition and subtraction and all the possible forms these operations take in problems.

By using a variety of problems, students will construct their own meaning for the inverse relationship between addition and subtraction and for the following properties:

- commutative property of addition (numbers can be added in any order), which does not function for subtraction
- associative property of addition (grouping a set of numbers in different ways does not affect the sum) and
- the identity element for addition and subtraction, that is adding zero to or subtracting zero from a number will result in the original or start number.

Students in Grade 2 generally do not know the names of these properties, but certainly learn to recognize and describe them. Grade 2 teachers who have students using the traditional algorithm will note that too many times their students subtract upside down or backwards, as if the commutative property of addition could be equally applied to subtraction. For example, if the equation to be solved is:

$$\begin{array}{r} 53 \\ \underline{-29} \end{array} \qquad \text{or} \qquad 53 - 29 =$$

students, upon noting that one cannot subtract 9 from 3, without even being aware they are inverting the numbers, may subtract 3 from 9. This creates a difference that is untrue for the equation given. This situation does not occur when students use invented personal strategies. When using the traditional algorithm, the manipulatives will prevent students from making this type of error. Some students will need to be made aware that they unconsciously do this when solving problems without manipulatives, so they can guard against this error.

The use of manipulatives or models helps students understand the structure of the story problem and also connects the meaning of the problem to the number sentence (Van de Walle 2001, p. 108). To develop their understanding of the meaning of operations, students connect the story problem to the manipulatives, connect it to the number sentence and then use personal strategies to solve the problem.

Reflect upon the student who adds

$$\begin{array}{r} 28 \\ \underline{+\,47} \\ 615 \end{array}$$

and prints this incorrect answer: 615 (as all too frequently happens with the traditional algorithm). Errors of this magnitude do not happen when students use personal strategies. Seldom will students use a personal strategy they do not understand. The number sense that students are developing in Grade 2 is critical to their ability to progress to estimating the answer, necessary in Grade 3. Students' understanding of addition and subtraction is enhanced as they develop their own methods and share them with one another, explaining why their strategies work and are efficient (NCTM 2000, p. 220).

Sequence of Outcomes from the Program of Studies

See http://education.alberta.ca/teachers/core/math/programs.aspx for the complete program of studies.

Grade 1

Specific Outcomes

8. Identify the number, up to 20, that is:
   - one more
   - two more
   - one less
   - two less

   than a given number.
9. Demonstrate an understanding of addition of numbers with answers to 20 and their corresponding subtraction facts, concretely, pictorially and symbolically, by:
   - using familiar mathematical language to describe additive and subtractive actions
   - creating and solving problems in context that involve addition and subtraction
   - modelling addition and subtraction, using a variety of concrete and visual representations, and recording the process symbolically.

Grade 2

Specific Outcomes

8. Demonstrate and explain the effect of adding zero to, or subtracting zero from, any number.
9. Demonstrate an understanding of addition (limited to 1- and 2-digit numerals) with answers to 100 and the corresponding subtraction by:
   - using personal strategies for adding and subtracting with and without the support of manipulatives
   - creating and solving problems that involve addition and subtraction
   - using the commutative property of addition (the order in which numbers are added does not affect the sum)
   - using the associative property of addition (grouping a set of numbers in different ways does not affect the sum)
   - explaining that the order in which numbers are subtracted may affect the difference.

Grade 3

Specific Outcomes

6. Describe and apply mental mathematics strategies for adding two 2-digit numerals, such as:
   - adding from left to right
   - taking one addend to the nearest multiple of ten and then compensating
   - using doubles.
7. Describe and apply mental mathematics strategies for subtracting two 2-digit numerals, such as:
   - taking the subtrahend to the nearest multiple of ten and then compensating
   - thinking of addition
   - using doubles.
9. Demonstrate an understanding of addition and subtraction of numbers with answers to 1000 (limited to 1-, 2- and 3-digit numerals), concretely, pictorially and symbolically, by:
   - using personal strategies for adding and subtracting with and without the support of manipulatives
   - creating and solving problems in context that involve addition and subtraction of numbers.

Step 2: Determine Evidence of Student Learning

Guiding Questions

- What evidence will I look for to know that learning has occurred?
- What should students demonstrate to show their understanding of the mathematical concepts, skills and Big Ideas?

Using Achievement Indicators

As you begin planning lessons and learning activities, keep in mind ongoing ways to monitor and assess student learning. One starting point for this planning is to consider the achievement indicators listed in the *Mathematics Kindergarten to Grade 9 Program of Studies with Achievement Indicators*. You may also generate your own indicators and use them to guide your observation of the students.

The following indicators may be used to determine whether or not students have met specific outcomes 8 and 9. Can students:

- add zero to a given number, and explain why the sum is the same as the given number?
- subtract zero from a given number, and explain why the difference is the same as the given number?
- model addition and subtraction, using concrete or visual representations, and record the process symbolically?
- create a problem involving addition or subtraction given a number sentence?
- create an addition and subtraction number sentence and corresponding story problem for a given solution?
- solve a given problem involving a missing addend, and describe the strategy used?
- solve a given problem involving a missing minuend or subtrahend, and describe the strategy used?
- refine personal strategies to increase their efficiency?
- match a number sentence to a given missing addend problem?
- match a number sentence to a given missing subtrahend or minuend problem?
- explain or demonstrate the commutative property for addition: the order of addition does not affect the sum; e.g., $5 + 6 = 6 + 5$?
- add a given set of numbers, using the associative property of addition (grouping a set of numbers in different ways does not affect the sum), and explain why the sum is the same; e.g., $2 + 5 + 3 + 8 = (2 + 3) + 5 + 8$ or $5 + 3 + (8 + 2)$?
- solve a given addition or subtraction computation in either horizontal or vertical formats?
- identify what each number in the problem means in relation to a part or a whole?
- recognize that some strategies are more efficient than others in particular cases?
- recognize that subtraction is not commutative, and so do not subtract upside down or backwards?
- explain how a strategy for adding and subtracting works and apply it to another similar problem (limited to 1- and 2-digit numerals)?

- create a different personal strategy for adding and subtracting and decide which strategy is more efficient when solving problems?
- analyze a personal strategy created by another person and decide if it makes sense in solving an addition or a subtraction problem?
- solve problems that involve addition and/or subtraction of more than two numbers with a sum or subtrahend of no more than 100?

Sample behaviours to look for related to these indicators are suggested for some of the activities listed in **Step 3, Section C: Choosing Learning Activities** (p. 14).

Step 3: Plan for Instruction

Guiding Questions

What learning opportunities and experiences should I provide to promote learning of the outcomes and to permit the students to demonstrate their learning?

- What teaching strategies and resources should I use?
- How will I meet the diverse learning needs of my students?

A. Assessing Prior Knowledge and Skills

Before introducing new material, consider ways to assess and build on students' knowledge and skills related to counting. For example:

- Can the student read number sentences such as 4 + 5 = ☐, 7 – 5 = ☐ or variations such as 4 + ☐ = 9, ☐ + 5 = 9, ☐ – 5 = 2, 7 – ☐ = 2? (Does the student use the terms: "plus," "minus," "add," "subtract", "equal" and, when appropriate, a term for the variable, such as "something", "blank," "box," or "what/some number"?)
- Can the student demonstrate the meaning of such number sentences to sums of 20 or with minuends no larger than 20, by dramatizing a problem with manipulatives, with pictures or with diagrams? If not, can the student do so to answers of 10? Clarify whether the problem is with the size of the numbers, the concepts of addition and subtraction or the vocabulary. If the student is not yet able to demonstrate competence with addition and subtraction facts to ten, check for rote counting and one-to-one correspondence to 10 and then 20.
- Model the addition of 12 and 4 using concrete or visual representations and record the process symbolically. Can the student create addition models (to sums of 20) and write the corresponding symbolic representations?
- Model the subtraction of 8 from 13 using concrete or visual representations and record the process symbolically. Can the student create subtraction models and record the process symbolically (using numbers no larger than 20 for the minuend)?
- Can the student create an addition or subtraction story problem for various number sentences such as: 13 – 8 = ☐ or 8 + ☐ = 13?
- How accurately and how rapidly does the student solve these problems?
- Can the student verbalize the strategy used?

If a student appears to have difficulty with these tasks, consider further individual assessment, such as a structured interview (see sample), to determine the student's level of skill and understanding. The Kathy Richardson books listed in the bibliography contain excellent structured interviews for investigating concept development. They also indicate how a teacher can address the development of missing concepts. See **Sample Structured Interview: Assessing Prior Knowledge and Skills** (p. 11).

If the student is very proficient in producing correct answers quickly and can verbalize the strategies being employed, you may need to consider additional challenges for this student while others are developing their skills to this level. Challenges could range from developing recognition of alternate personal strategies and flexibly using them, to employing the skills in more complex story problems. Students can be challenged to improve their explanations of personal strategies used. Searching for relationships amongst problems is another way to challenge students who are proficient with these operations and the related concepts. For example, students could be asked to solve a series of problems and examine their solutions to find a relationship or pattern and explain why it exists. One such series of problems is: 25 – 13, 26 – 14 and 27 – 15. There are many opportunities to challenge students in this area of the curriculum.

Sample Structured Interview: Assessing Prior Knowledge and Skills

<table>
<tr><th rowspan="2">Directions</th><th colspan="2">Date:</th></tr>
<tr><th>Not Quite There</th><th>Ready to Apply</th></tr>
<tr><td>Provide a variety of counters, as well as paper and pencil. Say, "Make up an addition problem and show me how to solve it with counters. Then write the number sentence that goes with the problem."
If the student creates a problem with a sum beyond 20 and errors in the addition, ask the student to create a problem with smaller numbers.</td><td>• Story problem is not addition or does not match the action or equation.
• Shows the wrong number of counters.
• Does not use the addition sign and/or equal sign in the equation.</td><td>• Creates an addition scenario, represents the numbers with the counters and records the corresponding equation correctly.</td></tr>
<tr><td>Provide a variety of counters, as well as paper and pencil. Say, "Make up a subtraction problem and show me how to solve it with counters. Then write the number sentence that goes with the problem."
If the student creates a problem with a minuend beyond 20 and makes errors in the subtraction, ask the student to create a problem with smaller numbers.</td><td>• Story problem is not subtraction or does not match the action or equation.
• Shows the wrong number of counters.
• Does not use the subtraction sign and/or equal sign in the equation.
• Writes the subtrahend number as the minuend and vice versa.</td><td>• Creates a subtraction scenario, represents the numbers with the counters and records the corresponding equation correctly.</td></tr>
<tr><td>"Create a story problem for this number sentence."
Show the student:
8 + 3 = 11</td><td>• Creates a story problem using some of the numbers provided, but not all, such as 11 + 3 = 14.
• Creates a story problem that uses the family of numbers, but with a different operation, such as 11 – 3 = 8.
• Cannot create a story for the equation.</td><td>• Creates a story problem that is represented by the number sentence.</td></tr>
</table>

"Create a story problem for this number sentence." Show the student: **10 – 3 = 7**	• Creates a story problem using some of the numbers provided but not all, such as 7 – 3 = 4. • Creates a story problem for 10 – 7 = 3. • Creates a story problem that uses the family of numbers, but with a different operation, such as 7 + 3 = 10. • Cannot create a story for the equation.	• Creates a story problem that is represented by the number sentence.
Read the student several story problems for addition and subtraction and say, **"Tell me whether you would add or subtract to find the answers for these problems:** **1. There were 18 students on the playground and 7 went home. How many students are still on the playground?** **2. There were 18 students playing on the climbing equipment and 7 playing in the sand. How many more were playing on the climbing equipment than in the sand?** **3. There are 10 pairs of boots on the boot rack and 8 pairs of shoes. How many pairs of shoes and boots are on the boot rack?** **4. There are 10 pairs of boots on the boot rack and 8 pairs of shoes. How many fewer pairs of shoes than boots are there?** **5. Frank has 6 toy cars and 8 toy trucks. How many toy vehicles does he have?"**	• Student gives the wrong operation for two or more of the problems.	• Student identifies the correct operation for at least four of the story problems presented.

B. Choosing Instructional Strategies

Consider the following guidelines for teaching addition and subtraction:

- Use "subtract" or "minus," but not "take away," so as not to reinforce a narrow, incomplete definition of subtraction.
- Interchange "is the same as" with "equal" frequently to reinforce the meaning of "equal."
- Teach frequently in a problem-solving context. Research shows that by solving problems using addition and subtraction, students create personal strategies for computing and develop understanding about the relationship between the operations and their properties (NCTM 2000, p. 153). Overly simple problems make it easy for some students to develop strategies such as identifying two large numbers in a problem means that you add, whereas a large and a small number in a problem indicate subtraction.
- Provide time for students to create personal strategies to solve the problem and share these strategies with members of their groups or with the entire class.
- Choose problems that relate to the student's own lives. Contextual problems might derive from recent classroom experiences, including literature sharing, a field trip, or learning in other subject areas such as art or social studies.
- Provide a variety of problems representing the different addition and subtraction situations with varying degrees of difficulty to differentiate instruction.
- Work with the whole group initially and have the students paraphrase the problem to enhance understanding and to recognize which numbers in a problem refer to a part or to a whole.
- Provide a variety of manipulatives, including base ten blocks and others that can be grouped in tens and ones, such as tiles, pennies, stir sticks, straws, ice cream sticks, Unifix cubes, multilinking cubes or beans for students to use as needed.
- Guide the discussion about the operations of addition and subtraction by asking questions to encourage thinking about number relationships, the connections between addition and subtraction and their personal strategies. Such questions could include:

    1. When we add two numbers, is the sum usually bigger or smaller than either of the numbers? We have to be careful not to say "always bigger," since when students add negative numbers eventually, this is not so. Also, when we add zero, the number stays the same.
    2. When we subtract two numbers, is the difference usually less than or more than the number we subtracted from?
    3. Does the difference change if we subtract a subtrahend one larger than the last from a minuend that is one larger than the last? What about if we decrease both numbers in the subtraction problem by one? What if we were to increase or decrease both numbers in the problem by 2? What if we increase the minuend by 1 and the subtrahend is decreased by 1?
    4. Can we break numbers up in ways that make it easier for us to add or subtract the parts in order to arrive at the answer mentally and efficiently?

- Challenge the students to solve the problem another way, do a similar problem without models or clarify the explanation of their personal strategies.
- Have the students evaluate their personal strategies as well as those of their classmates to decide which strategy works best for them and why.
- Limit the number of story problems that students are solving a day to one or two and leave half a page where each solution can be shown and explained.

C. Choosing Learning Activities

The following learning activities are examples that could be used to develop student understanding of the concepts identified in Step 1.

Sample Activities:

Sample Activity 1: Zero, the Identity Element for Addition and Subtraction

Begin by using story problems in which zero is added or subtracted. Students may use manipulatives if you think they need concrete representations. Have the students work with you to record the corresponding equations or number sentences. Then study them and look for a pattern. Ask the students, "What do you notice about the answers and the start numbers? Are they related in some way?"

Possible stories could be as follows:

1. There was a hungry bear who went fishing one spring morning and caught nothing. That evening he was so glad that the salmon were running because he caught and ate 3 fish. He wasn't so hungry then and had a good sleep that night for 10 hours. How many fish did the bear eat that day?
2. Five students were playing after school on the climbing equipment. Another group of 6 students came onto the playground, but none came to join them on the climbing equipment. How many students were playing on the climbing equipment altogether that day?
3. Eli had seven dollars saved in his bank last week. When he got his $2 allowance this week, he went to buy his brother a birthday present at the dollar store. The present he found cost exactly $2 and so he didn't have to take any out of his bank. How much does Eli have saved in his bank now?
4. Lanna was eating her 10 small snack crackers when her friend Rob came over to play with her. She gave him none since he has a wheat allergy and the crackers would not be good for him. How many crackers did Lanna have left to eat after she gave Rob none?

If you have students who still are having problems recognizing that the start number does not change when you add or subtract zero, have them place a number of counters in one hand and zero in the other. Then ask them to put their hands together and shake up the counters from both hands together, so all the counters from both hands are mixed together. Then ask them to open their hands and count the counters. If they have trouble with this, have them work in pairs with one person placing counters in the other person's hand. Record what they find each time. Counters that are not too large, like beans or pennies, are ideal for little hands. Similarly, have them represent subtraction of zero by placing a number of counters in one hand and reaching in to subtract zero. Then check how many counters remain. Students will have a quick way of checking the result of adding zero to or subtracting zero from any number by imagining a number of counters in their hands and the sum or difference resulting.

Look For …

Do students:

- ☐ find sums that are larger than the original number when adding zero, such as stating 5 + 0 is 6? They may be under the misconception that the sum is always larger than the addends.
- ☐ find differences that are smaller than the minuend when subtracting zero? The student may mistakenly believe that the difference must always be smaller than the minuend.

If either or both of these are the case, use manipulatives until the misconception is corrected.

Sample Activity 2: The Commutative Property of Addition

Students generally find this concept easy to grasp. There are some activities that can help students become very aware of this property. Ask the students to make two quantities with counters. Using counters that come in two different colours makes it easier for students to keep track of each quantity as it is moved. Ask the students to place the first quantity, for example four, on the left of the second quantity, seven. Have them calculate the sum. Ask them to move the four to the right of the seven. Have them calculate the sum. Did it change? Then ask them to position the four above the seven and calculate the sum. Did it change? Have them then pull the four down below the seven and calculate the sum. Again, reinforce that the sum has not changed. Ask them if there is any way you could position the two numbers to change the sum. Why not? As you do these examples, record them so students see the symbolic representations: 4 + 7 = 11, 7 + 4 = 11,

$$\begin{array}{r} 4 \\ \underline{+\ 7} \\ 11 \end{array}, \quad \begin{array}{r} 7 \\ \underline{+\ 4} \\ 11 \end{array}.$$

Another way to reinforce that order when adding two numbers does not impact the sum is with Unifix or Multilink cube trains. Place a train of four in one hand and add it to the train of seven in the other and the train that results from this joining of two parts is eleven cars long. Now separate the two parts again and place them in the opposite hands. Again join them together. Does it change the total length of the train from the first demonstration? It helps if each of the original trains to be joined together are made up of different coloured cubes. The colours make it easy for the students to see the two parts in the whole train that results from the joining of the two smaller trains. It is also makes it easy for you to explain, "It doesn't matter if the 4 orange cars are first and then 7 black cards follow or if the 7 black train cars come first and the 4 orange ones follow, the combination train that they make up together is still 11 cars long." This is a quick way to reinforce that the order of the numbers when adding does not change the sum.

Also consider having the students compare their results to a static cube train or cube stick for the total amount. Each time they compare, they will note that the totals match the static train.

Have the students check out whether it matters what order the numbers are in when you subtract. It may help to put it in a context of owing someone money or trying to pay back someone a quantity from what you have. For example, if you have 9¢ and owe your mother 5¢. How much do you have left? If you have 5¢ and owe your mother 9¢, can you give her what you owe her? Students will either say that they can't pay back their mother yet or that they can give her the 5¢ and will still owe her 4¢. As you record these number sentences, 9 – 5 = 4 but 5 – 9 ≠ 4, you can help students become aware that subtraction does not have a commutative property. Some students tend to subtract upside down or backwards when the number in the ones' column is smaller than the number to be subtracted instead

Look For …

Do students:

- ☐ recognize that they can change the order of addends in a problem or a basic fact to make it easier for them to solve the problem or fact?
- ☐ subtract backwards or upside down, rather than taking a ten and adding it to the ones number of the minuend before subtracting the ones number of the subtrahend?

of adding a ten to the number before subtracting. This occurs when students are focused on subtracting using the traditional algorithm, rather than focusing on the meaning of the problem and using personal strategies to solve it.

Sample Activity 3: Associative Property of Addition

This property of addition highlights that grouping a set of numbers in different ways does not affect the sum. To develop this concept, have the students add three or more addends. Ask them to share their strategies. Do any of the students add in any other order than what is given and can they explain why they would choose to change the order of the addends? Encourage the students to explain their strategies that aid in computation. They may not always select two digits to make ten. Sometimes they look for other combinations that they know well. In modelling, have the students demonstrate how it can make the computation faster and easier if they look for combinations of ten first and then add the other numbers. For example, in the following addition equation:

$$32 + 14 + 28 = ?$$

adding the 2 and the 8 first to make ten and then adding 4 to arrive at 14 is for most people more efficient than simply proceeding in the sequence of 2 + 4 = 6 plus 8 = 14. Problems with more than three addends may have multiple opportunities to reorganize numbers for efficiency. A real-life situation in which this might be done is adding the costs of various items in a bill. At the Grade 2 level, these may need to be items that are in whole numbers well under 100. A sample problem might be:

> John bought a container of bubbles for his birthday party that cost $6. He also bought a package of 6 yo-yos for $12 and a package of 6 balls for $9, so that every student would get one in a loot bag. The paper plates cost $8. The loot bags cost $4. How much did he spend on the birthday party?

Students following the sequence of the story problem, might write the equation:

$6 + $12 + $9 + $8 + $4 = ?

Showing the students how the 6 plus the 4 would make one ten and 2 plus 8 make another ten allows the students to add the 9 to the 2 tens for a total of 29, which can then be added to the 10 left from the 12 for a grand total of 39. Other students may add the 8 to the 12 to make 20, then add 6 + 4 to make 10, subtotal those as 30 and finally add the 9 for a tally of 39. When students draw lines to show their choices of order of addition or reorganize a problem for ease in addition, it is clear they understand the associative property of addition. To make this an easy performance task, give the students the price tags and ask them to sequence them in the way they would like to add them on the bill. Ask them to explain their reasoning for placing the costs in that particular order.

Look For …

Do students:

- ☐ reorder the addends in equations to expedite their computation?
- ☐ substitute a combination of numbers equal to a given number to make the computation easier?

If you need additional manipulative work with students to have them practise looking for their preferred order of addition with multiple addends, have them make the numbers on ten frames and order them as they wish. Alternatively, have them make the numbers with connecting cubes. Provide a template showing a ten on their work space, so that they can bring together the numbers they wish to add first to complete as many ten sticks as possible. The same thing can be done with other small manipulatives such as beans or pennies and portion cups.

The full usefulness of this property will be realized as students recognize that they can regroup or rename a number to make their calculations easier. For example, they may find the fact 7 + 8 hard to recall, but recognize that 7 is the same as 2 + 5 and so add the 2 to 8 to make ten, which they can now add to 5 for a sum of 15. When faced with adding two-digit numbers, such as 48 + 35, they will see that if they could have 2 more to add to the 48 it would make 50, a much easier number to add to another. To get the 2, they would like to combine with 48, they just need to regroup 35 as 30 + 2 + 3. This allows them to reconfigure the question to (48 + 2) + 30 + 3. The addition can now be 50 + 30 + 3 or 50 + 33, both easily done mentally. Once students begin to use this application of the associative property of addition, they can fully take advantage of its benefits.

Sample Activity 4: Baby Steps to Personal Strategies

There is no one best way to introduce students to problems that lead them to create personal strategies. What follows is one suggested way, building on some of the simpler computations to those that are more challenging.

- **Adding Pairs of Numbers that Do Not Bridge a Ten**
  The following examples may be used in story problems or not, as suits your lesson:
  - 30 + 42
  - 63 + 20
  - 24 + 53
  - 57 + 22.

- **Adding Tens, Adding Ones; Combining**
  Students can solve these with a variety of strategies, which may include the hundred chart, ten frames, models such as base ten or mental math. Some students will share the idea of adding the tens and the ones and then recombining, such as solving 24 + 53 by adding 20 + 50, adding 4 + 3, then combining the subtotals of 70 + 7 for a grand total of 77. It is highly unlikely that any of the students are starting with the ones and progressing in the manner of traditional algorithms unless they are using or envisioning a place value addition mat (with or without counters) being used in a manner that they have been taught.

  Adding Groups of Ten
  Students adding 30 to 42 may add groups of tens, such as 30 + 40. Students may count up by tens, for example 30, 40, 50, 60, 70, keeping track of how many tens they have added with their fingers or base ten rods. Other students may use the strategy of always starting with the largest number and do the same thing. Other students may say that these numbers are 3 tens and 4 tens and since they know 3 + 4 = 7 that the sum is 7 tens or 70. Some students feel comfortable with counting on by tens no matter what the start number is. So if the problem had been written 42 + 30, they would have started at 42 and counted on by ten three times: 42, 52, 62, 72. Whatever strategies they suggest, keep records of them showing the students how they can write a description of their thinking. Ask them if those strategies will work for other pairs of numbers, one of which is a multiple of ten? Check some out.

Look For …

Do students:

- ☐ add numbers without bridging ten with ease? If so, move on to bridging ten situations.
- ☐ require base ten manipulatives to add two-digit numbers that do not bridge ten? If so, work on place value concepts and counting by tens.
- ☐ Count both numbers by ones or from the highest number by ones? If so, work on facts and place value. Students might work with ten frames to find the missing amount to go with given numbers to make the sum of 50 or 100.

- **Bridging a Ten**

  Begin work on bridging ten by having the students add a one-digit number to a two-digit number that requires them to bridge over a ten. Allow the students to use manipulatives, as needed. For example, 35 + 8 = ? To solve this equation, students may think 35 is 30 + 5 add 8, so another way to show it is 30 + 13 = 43. Alternatively, students may think of the ten frames and how to make tens. They may envision 8 as 5 + 3 and so add the 5 to 35 to make 40 and then add the remaining 3 from the 8 to 40 for a total of 43. If students are not coming up with these strategies, but are simply counting on by ones, try using the hundred chart. Ask students, "How many will it take to go from 35 to 40? How many of the 8 does that use up? How many of the 8 are not used yet? If we put those 3 more with 40, what number will that bring us to?" You can also encourage the thinking in the first strategy above by having the students build the number with ten frames. Be sure to leave enough time for the students to solve the problem individually or in small groups and then share their strategies. Ask the students to try the strategy with another pair of numbers to see how it works. Talk about their findings. Then move on to addition problems with two two-digit addends requiring a ten to be bridged.

  As students move from doing this addition with concrete materials to mental mathematics, they can do pictorial and symbolic combinations first. For addition with addends such as 36 + 45, prepare a worksheet or overhead bearing both numerals and illustrations of base ten blocks drawn simply as lines and dots. On the overhead or worksheet include a number of rectangles with various numerals within a circle on each of the rectangles. With each numeral would be base ten blocks to be added to each amount. Direct the students to write the sum of the numeral and the base ten blocks shown beside each rectangle. Ask students to share their strategies for finding the sums.

Look For …

Do students:

- ☐ have difficulty with bridging ten? If so, check on their combinations for ten and abilities to find other names for numbers. You may work with manipulatives using different coloured counters for each number and break one of the numbers into sets to allow the making of another ten.

Sample Activity 5: Personal Strategies You Might Encounter

You do not want to teach these strategies, but rather structure the problems that you give students to solve so that they face some of these easier computations prior to more difficult ones. You also want to be prepared to understand what students are describing in terms of their personal strategies. Give the students sufficient time to work in small groups to solve the problems given. Within their small groups, students should share their ideas, so they can work through the language needed to explain their thinking. Also, not every student will invent strategies, but they will use those that they learned from others if they suit them. Then small groups can share their personal strategies, which you record. Following the possible problems are some common personal strategies that you might encounter. When observing the groups solving the problems, do not interject or give them a strategy. If you do, they will expect you to be the dispenser of strategies or think they are supposed to try to guess your strategy.

Possible Problem:

There are 38 students in Grade 1 and 44 students in Grade 2. All the Grade 1 and Grade 2 students are going to the library to hear an author read her new book. If all the students are at school that day, how many students will be in the library for the story?

Common Personal Strategies Invented for Addition:

1. Add Tens, Add Ones, Combine
   38 + 44 = ? Regroup as (30 + 8) + (40 + 4)
   Solve by combining the tens: 30 + 40 = 70
   Then combine ones: 8 + 4 = 12
   Combine these subtotals: 70 + 12 = 82

2. Add on Tens, Then Add Ones
   38 + 44 = ? Regroup as 38 + (40 + 4) = ?
   First add the tens from the second addend to the complete first addend: 38 + 40 = 78
   Then to add the remaining 4 as (2 + 2), since you need 2 to add to 8 to make 80 and still have 2 more to add to 80 for a total of 82.

3. Move Some to Complete a Ten
   38 + 44 = ?
   To make 38 to the closest ten, you need 2 more, so take 2 from 44, making the equation:
   (38 + 2) + (44 – 2) =
   40 + 42 = 82

4. Use a Nice Number and Compensate
   38 + 44 = ?
   A nicer number to add to would be 40, instead of 38, so add 2 and you have the number you like, so now the equation reads: 40 + 44 = 84, but you know that is 2 more than were actually there, because you loaned the one set two and so now you have to take it back:
   84 – 2 = 82

Common Personal Strategies for Subtracting By Counting Up

This is the way change was counted prior to cash registers that told cashiers how much change to give. It is a great way for students to subtract, since it has them solve subtraction situations by thinking addition, which usually they have mastered or find easier. To have students think addition for subtraction, present them with join problems with the change unknown or missing part problems. Examples are as follows:

Join with the Change Unknown Problem

Luke had 57 hockey cards. His Uncle came to visit and brought him some more. Now Luke has 82 hockey cards. How many cards did his Uncle bring him?

82 – 57 = ? Think 57 + ? = 82.

Missing Part Problem

Maria collected all the markers in her house and checked to see which ones were still good and which were dried out. She found 82 in all. Of those, 57 still worked well. How many were too dried out and needed to be thrown out? 82 – 57 = ? the standard form. Think 82 = 57 + ? semantic version (its meaning).

When we expect students to write the standard form for each problem, it makes solving the problem difficult for some students. For some problems it means solving the problem mentally, then reorganizing the equation. Students should be allowed to write equations with missing parts to follow the problem meaning, such as, 5 + ? = 12 or 12 - ? = 5. When allowed to do so, some students have a greater chance of success with problems of these types (Willis 2004).

Personal Strategies Commonly Invented for Subtracting by Counting Up:

1. Add Tens Until Close, Then Ones
   82 – 57 = ? Think 57 + ? = 82
   57 + **20** = 77, if 30 was added to 57, it would be 87 and that is more than 82
   77 + **3** = 80
   80 + **2** = 82.
   Sum of additions: 20 + 3 + 2 = 25

2. Add Tens Until Just Past Number, Then Back Up
   82 – 57 = ? Think 57 + ? = 82
   57 + 30 = 87, which is 5 more than 82, so to find the correct number.
   30 – 5 = 25 five needs to come off the 30,
   which means that 57 + 25 = 82 or 82 – 57 = 25.

3. Add Ones to Make a Ten, Then Add Tens and Ones
   82 – 57 = ? Think 57 + ? = 82
   57 and **3** would be 60
   60 and **20** would be 80
   80 and **2** would be 82, which means that in total

3 + 2 + 20 was added on or 25 in total.

Some students will do these last two steps as one,
That is 3 added to 57 will make it 60.
Then add 22 to 60 to get 82.
So 3 + 22 = 25 had to be added to 57 to make 82.

Personal Strategies for Take-Away Subtraction Problems
The strategies for take-away subtraction problems are harder to do mentally, but common due to the heavy reliance on the traditional algorithm to solve subtraction problems. A sample problem would be:

> There were 82 milk cartons in the lunchroom fridge before lunch. Fifty-seven students each bought one carton for lunch. How many cartons of milk are still in the fridge?

Assumptions
Note that if this problem had read, “Fifty-seven students bought milk for lunch,” students would have needed to talk about assumptions. In that case, students would need to assume that each student only bought one carton of milk. Several more problems with assumptions to be discussed are:

> There are 12 pairs of rain boots and four umbrellas in the boot room. How many students came prepared for the rain?
> The data we collected on pets shows that 15 students have dogs and 11 have cats. There are only 22 students in the class. How can there be 26 students who have dogs or cats? (Students discuss that some students may have both and others neither. Did the data collected reflect how many students had these pets or how many of these pets they had? Some students may have more than one dog or cat, another consideration.)

It is important that students’ attention is drawn to assumptions that must be made when the problem does not give you these details.

1. Subtract Tens from Tens, Then Ones From Ones
   This method and the next are similar to the approach students use when working independently with base ten blocks.
   82 – 57 = ?
   First, take the 8 tens or 80 of 82 and subtract the 5 tens or 50 from 57 and subtract: 80 – 50 = 30
   You needed to subtract 7 more than 50, so 30 – 7 = 23
   But you had 2 more than 80 to start, so add 2 back on, that is 23 + 2 = 25
   OR
   80 – 50 = 30 as above, but the student thinks, "I can take 2 away from the 2 that was with 80, but I needed to take 7 away. So, since 2 + 5 = 7, I need to take another 5 away from 30 for a final difference of 25."

2. Take Away Tens, Then Ones
   82 – 57 = ?
   Leaving the minuend intact, only the subtrahend is changed to the last multiple of ten before subtracting, so 82 – 50 = 32.
   Then take away the 7, which can also be 2 + 5, so taking off the 2 leaves 30 and then taking off 5 leaves 25.

3. Take Extra Tens, Then Add Back the Extra
   82 – 57 = ?
   Move the subtrahend up to the next ten, so 57 becomes 60:
   82 – 60 = 22.
   However, 60 is actually 3 more than the number to be subtracted, so add 3 to the difference of 22 for a total difference of 25, or students might say that since you took away 3 too many, you have to give back 3.

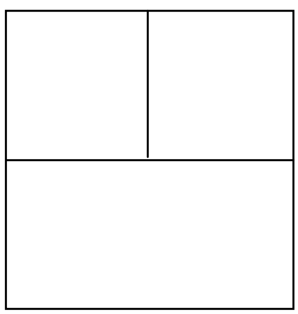
4. Add to the Whole or Subtrahend When Needed
   82 – 57 = ?
   By giving 5 to 82, it would make the minuend 87 and the subtraction very easy: 87 – 57 = 30.
   However, 5 was loaned to 82 to make the minuend 87, so now to play fair you have to take back 5, so 30 – 5 = 25, the actual difference once the loan is paid off.

It is not necessary that students know and use every possible variation of these personal strategies. Students should have several strategies that work on many different problems, are understood and are fairly efficient. Students should be able to understand those strategies preferred by other students and be able to critique them.

Sample Activity 6: Recognizing the Parts and the Whole in Addition and Subtraction Problems

All types of addition and subtraction problems can be analyzed as parts and the whole. Doing so can help students make sense of a problem and recognize whether the unknown will be a part or the whole. Based upon the definitions of addition and subtraction, the knowledge of whether the unknown is a part or a whole can point the student to which operation is required. This analysis also reinforces the inverse relationship between these operations. It is not necessary for students to know the categories of problems of this planning guide, but they do need to be able to solve all the variations. You can help by taking a copy of the categories each month and making note which types you have included in your lessons. At the end of each month, your goal will be to have included all the variations in each category, so that students have opportunities to learn how to solve all of them.

Identifying the parts and the whole with manipulatives can be done with an addition/subtraction mat. A line separates the mat in half, top and bottom. The top half of the page or mat is separated in half again vertically, making two quarters. These quarter regions should be large enough for the students to place the manipulatives representing the two parts. The other half of the page is for the whole.



Look For …

Do students:
- ☐ identify parts and the whole correctly?
- ☐ place the counters which represent the numbers for the parts and the whole correctly on mats?
- ☐ show their understanding of the inverse relationship of these two operations by turning the mat as needed?
- ☐ show the correct action in using manipulatives to be combined or separated?

The same mat is used to show subtraction. The student just turns the mat upside down so that the half for the whole number is on top and the two quarters for the parts are below. The amount left is now counted and can be moved to the other quarter to show the two remaining parts that originally comprised the whole. In this way, the students begin to visualize addition and subtraction operations and recognize their inverse relationship. For Grade 2 students it is like the inverse relationship of doing and undoing, joining together into a whole and separating into parts. It is ideal if, during addition, students can use two different coloured manipulatives to make the two addends, so that when they are joined it is still easy to see the two parts that comprise the whole. Story problems give context to their work and might be as follows:

1. There were 4 library books in Anita's book bag and 2 more on her shelf. How many library books does she have altogether to return to the library? This question provides the students with the two parts, 4 and 2, and asks them to find the whole, 6.

2. Graham's class ordered 5 cartons of white milk and 8 cartons of chocolate milk for lunch. How many cartons of milk will the person filling their class order need to place on their tray?

3. Amy's box of markers holds 24. She took the 8 colours out that she needed to do her assignment. How many markers are still in her box?

4. Yehia had collected 17 small vehicles to play with in the sand box. He carried his 9 favourites with him to school one day. How many were left at home?

Have the students add and subtract 2-digit numbers using materials that can be joined and separated to further reinforce the concepts of addition as joining together parts and subtraction as separating the whole into parts. Popsicle sticks, stir sticks or straws with elastic bands can be used to gather tens into groups and empty cans or plastic containers can be used as the receptacles for each quantity. Then the student using tens and ones can make the numbers in the two cans or plastic containers (or more, depending on the number of addends you want them to experience). This way, the students may take note of whether they are joining or bundling subgroups into a whole or separating or undoing the whole into subsets. Use problems such as:

1. Hope had saved $45 by her birthday. For her birthday, she received $20 from her grandma and $8 from her aunt, who always gives her the number of dollars that matches her age. How much money does Hope have now?

2. Arthur had 97¢ when he went shopping at the garage sale. He bought a book on making paper airplanes for 75¢. How much money does he have now?

Look for whether students can describe addition as joining sets or parts together into a larger whole group or "result." Inversely, can the students describe the concept of subtraction as the separating of the whole into smaller groups or parts? This refers to the use of subtraction for taking away, not for subtraction used in comparison and complementary numbers. The categories for the structure of addition and subtraction problems describe addition as "joining" and subtraction as "separating." The three parts in addition are referred to as the "start" (the number in the initial group); the "change" (as the amount being joined to the "start") and the sum is the "result" (enlarged group/whole). For subtraction in which the numbers "separate," the "result" is a part. The "initial" number was the whole or the minuend. The "change" is the quantity that was separated from the "initial" amount. For examples of questions asking students to solve for each of the numbers in an equation, see Appendix A of this planning guide.

Sample Activity 7: The Influence of Manipulatives

Your choice of manipulatives can influence what students will do. If you continue to use a mat as shown in the previous section, the students are more likely to focus on personal strategies. If you change to place value mats that have a column on the right for ones and a tens column on the left, students are likely to continue to add the tens first. This tends to be a natural inclination unless you or their families have taught them to do the traditional algorithm.

Using Place Value Mats

- Use different colours of counters for each number so students can easily keep track of the origin of the regrouped sets.
- Students do not always grasp that one base ten stick that does not come apart is equivalent to ten units. They also may not understand that 10 "longs" are equivalent to a hundred "raft."
- To help students visualize, have students draw standards showing the size of a ten stick made from linking ones cubes together and a hundred standard can be formed using 10 'longs."
- Using ten frames helps students visualize the combinations of ten and the numbers in the set without actually counting them by ones.
- Time must be taken for students to construct their understanding of equivalencies. Begin with units that can be seen as individuals within the groups of ten. This can be done by using portion cups with pennies (not traded for dimes) or beans. Bundles of sticks or straws could also be used.
- Subtraction can be done as the inverse operation by dismantling groups of ten as needed. Students need to dump or unbundle a group of ten and place it with any other ones.

Give careful thought to the influence of your choice of manipulatives on the lesson outcomes. Your choice of manipulatives can be as important as the structure of the problems used.

Sample Activity 8: The Traditional Algorithms

Students will not normally invent the traditional algorithms. They may know them from home or previous classes. Note that there are alternatives to the algorithms used in Canada. The Japanese, for example, traditionally add from left to right both on the abacus and on paper. When and if you must teach the traditional algorithms, plan to teach them for understanding.

Teaching Traditional Strategies

As a general rule, when teaching by demonstrating as with the traditional algorithms, model the activity, then have the students give you direction as to how to model, followed by students undertaking the task with your close supervision as they complete each step, and finally moving to independently competing the task. An overhead projector or Smart Board can allow the students to see your work without being crowded into a group around you. As the numbers become too large for the overhead to accommodate tens and ones with the appropriate addition and subtraction activity, using plastic sheets of squares made for embroidery, cut into tens, ones and hundreds will allow you to fit representations of larger quantities on the overhead. Beans will often show through plastic portion cups on the overhead or use empty plastic containers from jam portions or clear plastic containers for fishing tackle. The students may find these small plastic pieces too little to handle effectively and should use larger manipulatives, such as base ten blocks, connecting cubes or beans and cups. It is through this manipulative work that students gain the knowledge to apply to pictorial work and finally to the symbolic representations.

Terminology

When using the traditional algorithm, terms such as "borrowing" and "carrying" are obsolete and conceptually misleading. Instead the terms "regrouping" and "trading" are commonly used. For Grade 2 students, the term "regrouping" may be hard to understand, so "trading" is often preferred.

Build on Personal Strategies

Students learn that addition is still combining parts to make the whole. Present the students with addition problems with and without regrouping right from the start. When presented first with only those that do not require regrouping, students tend to make more errors, such as 28 + 47 = 615, when they are faced later with addition problems requiring regrouping. Students may learn to show addition as the combination of partial sums, for example:

$$\begin{array}{r} 28 \\ \underline{+\,47} \\ 60 \\ \underline{+15} \\ 75. \end{array}$$

This method follows closely what some students would do when following a personal strategy and uses place value knowledge. In the traditional algorithm, students learn the rule that when one more than nine accumulates in the ones place, it is bundled or grouped into a "ten," which is moved to the next column to the left, where the tens reside. The same applies to groups of ten. When one more than nine tens accumulate, they are to be bundled into ten groups of ten and moved to the next column to the left as a hundred. The biggest question students learning the traditional algorithm have is why they must start from the right column, or the ones. To connect the written symbols to the actions with manipulatives, students need to record the two addends before combining them. Then when they are combined students calculate and record their sum. This method parallels multiplication by a two-digit number based upon the distributive property, which will follow in later grades.

Problems Using Traditional Algorithms

The concept of subtraction does not vary with the traditional algorithm; however, students have to learn to start on the right with the ones. If there are insufficient ones to subtract, they must bring over a bundle of ten and dismantle it. Then they can subtract. Before the students separate the subtrahend from the minuend, they need to record the minuend. The unbundling of tens also takes step-by-step recording of the symbols that mirror the actions so that students learn how the symbolic record is kept. Too many students find this tedious and want the teacher to just show them the procedure. If they learn to subtract as a procedure, they are set up for future failure. Just think about facing a problem such as 1000 – 999. If a student in Grade 3 is asked to do this from knowledge of place value, it is easily answered as 1. The recording of the traditional algorithm, however, has the potential for error in any of a number of steps.

$$\begin{array}{r} 9\;9\;9 \\ 1\,\cancel{0}\,\cancel{0}\,\cancel{0} \\ \underline{-\;9\;9\;9} \\ 1 \end{array}$$

Sample Activity 9: Sample Problems for Developing Personal Strategies

The following are examples of problems that could be presented to students and some of the possible strategies they might use for solving the problems. As strategies are accumulated, record them on a class list, then discussions can follow about personal preferences for the "best" strategy or strategies in problems being solved and why students perceive them as such. You may ask if anyone knows the answer before they place the manipulatives on their mats. Then ask them to explain how they reached the sum. Allowing time to use the manipulatives to prove their sums and methods will help those students just being introduced to the strategy by giving them the time and support to visualize how it works.

1. Mitchel read 52 pages of his new book at home last night and 28 pages during free reading time at school this morning. How many pages has he read so far?

   Guide discussion as to whether the numbers in the problem refer to parts or the whole, and which operation could be used to solve the problem. Students may work in partners to make the numbers with manipulatives to speed up the process. Suppose the students are using Unifix cubes and when they come to combine the two numbers some note that they added the tens and then the ones to make 50 +20 +10 (being the combination of 2+8 from the ones cubes). These are students who prefer working from left to right and are very proficient at seeing tens and using groupings of tens for easy calculation.

   Other students may say that they like adding tens so they put 2 extra on the 8 Unifix of the 28. If students are still using two different coloured cubes to make the two parts, then when they add the two extra cubes, they should use a third colour. This will make it easy for them to see the two cubes that must be removed to compensate after the initial joining. Now they have 3 ten sticks to join with the 50 + 2 for a total of 30 + 50 + 2 = 82. However, they have to compensate for adding the two extra cubes by removing them from the final total, so 82 – 2 = 80.

2. Star wants to purchase a toy ring for 19¢ and the matching necklace for 27¢. She has 50¢ and wants to figure out if she has enough money to buy both.

   Ask the students, "How could you figure this out if you were at the store without any manipulatives?" When they give you a possible solution, ask, "Can you prove your strategy works with your manipulatives?"

   Students may say that they would pretend the ring cost 20¢ to make it easier to add in their heads: 20 + 20 + 7 for a total of 47¢. Then they can either compare this to 50¢ being 5 tens and thus larger than 4 tens and 7 ones and so conclude that Star has enough money to make the purchase. They may be more particular about the exact cost and wish to compensate for the extra penny cost they added to the ring and take 1¢ away from the 47¢ total to arrive at the precise sum of 46¢ before they compare it to her 50¢.

Be sure students experience problems in which the person does not have enough money. Can students use this strategy to solve the following problems: 39 + 19
68 + 21
48 – 19?

Ask the students if they changed the strategy in anyway. For example, for 39 + 19 the student may have opted to move each number up one to an even ten, thus adding 40 and 20 mentally for an easy calculation of 60, but then subtracted 2 from this total to balance the score, since the student had added 2 to the original figures.
For 68 + 21 the students may have chosen to just move the 1 from 21 over to the 68, making the question read 69 + 20, which then became a total of 8 tens and 9 ones or 89.
For 48 – 19 the students may have simply added 1 to 48 to mentally calculate 49 – 19 = 30, but then compensated for adding 1 to 48 by subtracting 1 from the difference to make it 29.
There are many variations to these strategies based upon personal preferences.

You may also encourage students to write related number facts such as 48 – 19 = 49 – 20, which is yet another strategy of balancing or "playing fair" by adding 1 to both the numbers in a subtraction problem. Students may have to discuss and experience how it is that the differences are the same despite the problem changing. Can they see that if they owe you an amount, but before you collect you give them one more, but also collect one more, they are left with the same amount remaining in either case? What happens if you give them two extra and then take away two extra, will the difference remain constant? Try it with three extra given and three extra taken away. Is there a pattern? Can they make the generalization that if you give and take away the same amount, the difference remains constant?

Step 4: Assess Student Learning

Guiding Questions

- Look back at what you determined as acceptable evidence in Step 2.
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

In addition to ongoing assessment throughout the lessons, consider the following sample activities to evaluate students' learning at key milestones. Suggestions are given for assessing all students as a class or in groups, individual students in need of further evaluation, and individual or groups of students in a variety of contexts.

A. Whole Class/Group Assessment

Provide base-ten materials and other counters that are easily grouped in tens available for the students to use as needed. You may wish to subdivide the following based on your needs.

Name : ________________________ Date: _______________________

Whole Class/Group Assessment – Grade 2: Number Sense
Addition and Subtraction – Part A

You may use manipulatives, if you wish, on any of the following.

1. Show what happens when you add zero to or subtract zero from any number. Explain why.

   ____

   ____

   ____

   ____

   ____

2. Tell or show how you would teach a student to do the following addition question so that the student could do it faster and make fewer errors.

$$\begin{array}{r} 17 \\ 22 \\ 24 \\ 18 \\ \underline{+13} \end{array}$$

3. Use a personal strategy to solve the following problem. Be sure you show your thinking so others can understand the personal strategy you used.

   Before lunch the students sold 32 Popsicles. After lunch they sold 46 Popsicles. How many Popsicles did the students sell that day?

4. Solve the following question using a different personal strategy than the one you used in Question 3.

   There were 73 fish in a school when a shark swam by and scared some of them off. Now there are only 46 fish left in the school. How many fish swam away from the school when the shark swam by?

5. Solve the following problem using a personal strategy.

   John emptied his bank and counted 59 pennies. Then his father gave him the 37 pennies from his dresser. How many pennies does John have now?

Name: _____KEY ___________________ Date: _______________________

Whole Class/Group Assessment – Grade 2: Number Sense
Addition and Subtraction – Part A

You may use manipulatives, if you wish, on any of the following.

1. Show what happens when you add zero to or subtract zero from any number. Explain why.

   4 + 0 = 4, 5 + 0 = 5, 6 + 0 = 6 and 5 – 0 = 5, 6 – 0 = 6, 7 – 0 = 7 are some examples.
   When you add zero to any number, the answer is always the same number you started with.
   When you subtract zero from any number, the answer is always the same number that you started with.

2. Tell or show how you would teach a student to do the following addition question so that the student could do it faster and make fewer errors.

17	Student loops or draws lines between the 7 of 17 with the 3 of 13
22	Student loops or draws lines between the 2 of 22 with the 8 from 18
24	Student shows 10 + 10 + 4 or 24 resulting from the ones.
18	adds the 24 to the tens by counting: 24, 34, 44, 54, 64, 74, 84, 94
+13	or student adds the 2 tens to 1 + 2 + 2+ 1 + 1 ten for a total of 9 tens, which are then added to the 4 for a final total of 94.

   Student may rewrite the equation as 17 + 13 + 22 + 18 + 24, so that the ten combinations are beside each other.

3. Use a personal strategy to solve the following problem. Be sure you show your thinking so others can understand the personal strategy you used.

   Before lunch the students sold 32 Popsicles. After lunch they sold 46 Popsicles. How many Popsicles did the students sell that day?
   One possible strategy could be: Add tens, add ones, combine.

   30 + 2 + 40 + 6
   30 + 40 = 70
   2 + 6 = 8
   70 + 8 = 78

   The students sold 78 Popsicles that day.

4. Solve the following question using a different personal strategy than the one you used in Question 3.

   There were 73 fish in a school when a shark swam by and scared some of them off. Now there are only 46 fish left in the school. How many fish swam away from the school when the shark swam by?
   Counting Up
   73 – ? = 46
   46 + ? = 73 Thinking addition for subtraction
   46 + 4 = 50
   50 + 20 = 70
   70 + 3 = 73
   Adding all the amounts: 4 + 20 + 3 = 27 (or 4 + 3 + 20 or 20 + 4 + 3)
   When the shark came by, 27 fish swam away from the school.

5. Solve the following problem using a personal strategy.

   John emptied his bank and counted 59 pennies. Then his father gave him the 37 pennies from his dresser. How many pennies does John have now?

   Adding on, then compensating:
   59 + 37 = ?
   (59 + 1) + 37=
   60 + 37 = 97
   97 – 1 (to compensate) = 96

   or

   Transfer of some of the number:
   59 + 37 = ?
   Give one from 37 to 59, making 60 + 36 = 96

   John has 96 pennies now.

Rubric for Whole Class Assessment Part A–
Addition and Subtraction: Properties and Personal Strategies

Concept/Skill	Not Yet	Needs More Instruction/Practice	Achieved
The identity element for addition and subtraction: zero in Question 1.	• Gives incorrect examples, such as 5 + 0 = 6 or 5 – 0 = 4 • No response • Explains mistakenly that when you add the sum is larger and when you subtract the difference is smaller than the start numbers.	• Gives correct examples, but no explanation or an incomplete or inadequate explanation.	• May or may not show examples, but gives a clear explanation of the fact the start numbers remain unchanged by adding or subtracting zero.
Associative property in Question 2, the order of adding numbers does not affect the sum.	• Describes adding or proceeded to add in the sequence given	• Shows one grouping of a combination of ten, but not the other. • The sum may be correct or incorrect.	• Explains that finding combinations of ten make the addition faster and more likely to be accurate or shows the grouping of the two combinations of ten to find the correct sum.
Flexibility with personal strategies to add and subtract in Questions 3, 4 and 5	• Cannot solve the three problems. • Solves three problems correctly, using the traditional algorithm.	• Solves one to three of the problems correctly, but uses only one strategy for all three. • May have used more than one personal strategy in the two or three solutions found, but is not yet showing the steps clearly enough for someone to follow.	• Solves all three questions accurately and the equations and/or descriptions make it clear that at least two different personal strategies were used competently.

Name : ________________________ Date: _______________________

Whole Class/Group Assessment – Grade 2: Number Sense
Addition and Subtraction – Part B

You may use manipulatives, if you wish, on any of the following questions.

1. Create an addition problem and solve it using a personal strategy. Show and explain your personal strategy well enough for others to understand it.

   My problem: ____

   ____

   ____

   ____

2. Create a subtraction problem and solve it using a personal strategy. Show and explain your personal strategy well enough for others to understand it.

   My problem: ____

   ____

   ____

   ____

Rubric for Whole Class Assessment Part B –
Addition and Subtraction – Creating Problems and Solving with Personal Strategies

Concept/Skill	Not Yet	Needs More Instruction/Practice	Achieved
Creating an addition or subtraction problem.	• Does not create a problem that corresponds to the operation specified. • In the subtraction problem, the larger number is being taken from the smaller number.	• The problems created correspond to the operation specified, but are incomplete. For example, the question is not stated. • The numbers may be reversed in a subtraction problem, such that the minuend is smaller than the subtrahend, but the work done shows the correct numbers in each position.	• Problems correspond to the operation specified and at least contain all the basic elements of a joining and separating problem.
Personal Strategy	• No evidence of a personal strategy being used in either solution.	• Evidence of a personal strategy being used in one or both of the solutions. • Work shown may be incomplete or make leaps that have to be filled in by the reader.	• Both solutions show efficient use of appropriate strategies that are clearly enough detailed for the reader to follow without having to fill in steps that weren't recorded.

You will find it easy to spot those students whose problems are beyond basic and whose solutions are more carefully explained or whose strategies are more advanced than the majority of your students. This is due in part to their mathematics development and their language and writing skill sophistication. If the problems created by the student have a more complex structure than just change problems, joining or separating, this indicates a greater depth of understanding of addition and subtraction, as well as good oral and written language skills. If the strategies go beyond counting up and adding tens and ones separately and then combining to accurate use of compensation strategies, you know that the student has a well developed sense of logic and reason. If the thought process is detailed carefully in symbols, it reflects a strong logical mind with a disciplined systematic approach. If it is equally well detailed in words, it reflects strong language skills as well.

Name : ________________________ Date: _______________________

Whole Class/Group Assessment – Grade 2: Number Sense
Addition and Subtraction – Part C

You may use manipulatives, if you wish, on any of the following questions.

1. Draw a line between the problem below and the number sentence that belongs to it.

   a. Yari had some hockey cards. His friend gave him 27 more. Now Yari has 62. How many did Yari have to start? — $62 + 27 = \square$

   b. Juanita had 27 sea shells. Her friend had 62 sea shells in her collection. How many more sea shells does her friend have than Juanita? — $62 - 27 = \square$

   c. Terry had made a space ship with 27 pieces and another with 62 pieces. How many pieces did the two ships use? — $\square + 27 = 62$

   d. Sharon had 62¢ when she went to the dollar store. She had 27¢ after her purchase there. How much did she spend at the dollar store? — $62 - \square = 27$

2. Circle the numbers that are parts in the above equations. Put an X through the numbers that represent the whole in the above equations.

3. Add or subtract the following.

   a. $41 + 37 = \square$ b. $85 - 52 = \square$ c. $65 - 26 = \square$

   d. $19 + \square = 70$ e. $36 + 45 = \square$ f. $83 - \square = 48$

   g. $\begin{array}{r} 43 \\ \underline{+\ 32} \end{array}$ h. $\begin{array}{r} 73 \\ \underline{-\ 36} \end{array}$ i. $\begin{array}{r} 27 \\ 19 \\ 33 \\ \underline{+\ 21} \end{array}$ j. $\begin{array}{r} 92 \\ \underline{-58} \end{array}$ k. $\begin{array}{r} 38 \\ \underline{+\ 26} \end{array}$

Rubric for Whole Class Assessment Part C –
Addition and Subtraction – Matching Problems and Equations,
Recognizing Parts and the Whole, Computation Accuracy and Strategies

Concept/Skill	Not Yet	Needs More Instruction/Practice	Achieved
Matching an addition or subtraction problem with its equation in Question 1.	• Matches one or none accurately.	• Matches two accurately.	• Matches three or all accurately.
Recognizing parts and the whole in Question 2.	• Correctly identifies the parts in four or less of the eight possibilities. • Correctly identifies the whole in two or less of the four equations. • Does not complete this part. • Marks numbers as the parts when they are the whole and vice versa in more than one instance.	• Correctly identifies the parts in five or six of the eight possibilities. • Identifies the whole in three of the four equations. • Errors are more often of omission, rather than citing a number or variable incorrectly. For example, the student may not have marked the variables.	• Correctly marks all the numerals and variables as parts or at least made no other errors than not indicating the role of each variable.
Computation accuracy and strategies in Question 3.	• Made more than two errors in computation. • Are few indications of any personal strategies being used. • The majority of computations appear to have been done by the traditional algorithm.	• Made two or fewer computation errors. • Is some indication of strategies such as adding the tens and then the ones or finding combinations of ten, but strategies are not apparent for all or are not discernable from the student's work.	• Made one or no computation errors. • Student work indicates that the student has at least two strategies that are being applied beyond adding tens and then ones or finding combinations of ten.

B. One-on-one Assessment

Directions	Date:	
	Not Quite There	**Ready to Apply**
Provide a variety of counters that can be used to represent ones and tens easily, as well as paper and pencil. Say, **"I am going to read with you the problems on this page. Please show me with any of the manipulatives you would like to use what the problem means. Then write the equation that goes with what you did."** Use several problems similar to what you have been using in class. Include at least one each of addition and subtraction. If you are aware the student is struggling at that level, use smaller numbers and the most direct forms first and proceed to more difficult forms and or larger quantities to establish what concepts or skills are causing the difficulties.	• Story problem does not match the action or equation. • Shows the wrong number of counters. • Shows the right number of counters, but makes a calculation error of the sum or difference. • Does not use the addition or subtraction sign and/or equal sign in the equation appropriately.	• Dramatizes with counters the problem scenario correctly. • Records the corresponding equation.

If you still need more information about the student, do the same creation problems as in the written whole class assessment, but orally. Provide a variety of counters, as well as paper and pencil. Say, **"Tell me an addition problem that you have made up and I will write it down."** When a problem has been created, say, **"Show me how to solve it with counters. Then write the number sentence that goes with the problem."** When the student completes that, if it has not been obvious what personal strategy the student used, say, **"Tell me what personal strategy you used to solve the problem and how it worked."** If the student creates a problem with a sum beyond 100 and makes errors in the addition, ask the student to create a problem with smaller numbers.	• Story problem does not call for addition or does not match the action or equation. • Shows the wrong number of counters. • Miscalculates with the counters. • Does not use the addition sign and/or equal sign in the equation. • Does not have a personal strategy. • Attempts an appropriate, recognizable personal strategy, but makes an error, such as compensating by adding instead of subtracting.	• Creates an addition scenario, represents the numbers with the counters and records the corresponding equation correctly. • The personal strategy used was apparent and effective or the strategy described when prompted was.

Provide a variety of counters, as well as paper and pencil. Say, **"Make up a subtraction problem and I will write it down. Then show me how to solve it with counters. Lastly, write the number sentence that goes with the problem."** If the student creates a problem with a minuend beyond 100 and makes errors in subtracting, ask the student to create a problem with smaller numbers. If the student's personal strategy is not apparent, say, **"Tell me what personal strategy you used to solve the problem and how it worked."**	• Story problem does not call for subtraction or does not match the action or equation. • Shows the wrong number of counters. • Miscalculates with the correct number of counters. • Does not use the subtraction sign and/or equal sign in the equation. • Writes the minuend number as the subtrahend and vice versa. • Does not have a personal strategy. • Attempts a personal strategy, but makes an error.	• Creates a subtraction scenario, represents the numbers with the counters, and records the corresponding equation correctly. • The personal strategy used was apparent and effective or the strategy described when prompted was.
If the student cannot create a story problem without prompts, say, **"Create a story problem for the number sentence:** **28 + 33 = 51"** If the student is successful at creating a problem to match this equation, you might ask, **"What strategy would you use to solve this problem?"**	• Creates a story problem using some of the numbers provided but not all. • Creates a story problem that uses the family of numbers, but with a different operation, such as 51 – 33 = 28. • Cannot create a story for the equation. • Cannot give an appropriate strategy for solving or cannot explain it well enough to be understood.	• Creates a story problem that is represented by the given number sentence. • Gives a possible personal strategy explained clearly enough to be understood.
"Create a story problem for the number sentence: **55 – 18 = 37"** If the student is successful at creating a problem to match this equation, you might ask, **"What strategy would you use to solve this problem?"**	• Creates a story problem using some of the numbers provided but not all. • Creates a story problem for 55 – 37 = 18. • Creates a story problem that uses the family of numbers, but with a different operation, such as 18 + 37 = 55. • Cannot create a story for the equation.	• Creates a story problem that is represented by the given number sentence. • Gives a possible personal strategy explained clearly enough to be understood.

If the student was unable to create problems with the equation as a prompt, try checking the student's ability to discern addition and subtraction situations. Give the student several story problems for addition and subtraction and say, **"Tell me whether you would add or subtract to find the answers for these problems:** **1. There were 38 students on the playground and 17 went home. How many students are still on the playground?** **2. There are 40 pairs of boots on the boot rack and 28 pairs of shoes. How many pairs of shoes and boots are on the boot rack?** **3. Frank has 16 model cars and 7 model airplanes. How many model vehicles does he have?** **4. Frank has 16 model cars and 7 model airplanes. How many more model cars than model airplanes does he have?"**	• Student gives the wrong operation for one or more of the problems.	• Student identifies the correct operation for each of the story problems presented.

Assessment activities can be used with individual students, especially students who may be having difficulty with the outcome.

1. For students who seem to falter with subtraction more so than addition, ask the student to explain to you the connection between addition and subtraction by using manipulatives on the part-whole mat as described in Step 3, number five. Turn it as required to start with the whole for subtraction or to start with the parts for addition. Coach the student to recognize the parts and the whole. Then have the student show how each problem could be transformed into the inverse operation. An activity for practising thinking addition for subtraction is to have students work in pairs with a calculator. Student one enters a secret number into the calculator and then adds to it a number that both students agree upon, such as five. Student one enters the equal sign and shows the sum to student two. Student two tells student one what the original secret number was by subtracting mentally and then takes the calculator and subtracts the five from the sum to verify the secret number.

2. For students who struggle with problems presented with missing addends or minuend or subtrahend, concentrate together on problems with these structures. Read them together. Have manipulatives available to use as needed. Pose the following questions to guide thinking, as necessary:
   - Tell me in your own words what the problem says.
   - What are each of the numbers in the problem, a part or the whole?
   - What is it we have to find out, a part or a whole?
   - What number sentence could you write to show the meaning of the problem?
   - Does the problem use addition or subtraction (or both, if the student is thinking addition to solve for subtraction)? Explain.
   - Use a strategy that makes sense to you to find the answer to the problem. Explain your thinking as you write the numbers, words or draw a diagram.

C. Applied Learning

Provide opportunities for students to use addition and subtraction in a practical situation and notice whether or not the strategies transfer. For example, ask a student to compare heights of students to the heights of each other, their parents or the teacher.

Does the student:

- obtain the two measures using nonstandard units and compare them in some way?
- use a personal strategy that makes sense in comparing the two measures?

There can be many other opportunities to add and subtract in authentic problems. Students can keep data on the daily temperature to compare, on money collected so far for a field trip and how much is still to come in, on the number of library books signed out by their class each week and how many overdue books their class has, on the number of students going home for lunch, staying at school or buying milk. Working on data collection and graphing in interesting ways will encourage students to suggest topics for which they would like to collect data. All of the data collected provides opportunities for mathematics. Some questions and recording methods to get students started might include:

- How many students have an 'o' in their first name? Have the students place an "o"-shaped cereal on one of two sucker sticks or pipe cleaners that are stuck in a ball of Plasticine. In front of one stick place a card that states "Yes" and in front of the other card stating, "No."
- Are you oldest, youngest, only or none of these in your family? Display a chart that states these headings in four columns. Give the students each a self-adhesive coloured dot to place on the chart in the appropriate column.

When an opportunity to add or subtract arises that can be integrated with your mathematics, it is important to observe how the students go about solving the challenges and having them share their personal strategies. It benefits the other students who learn from them and allows you to assess the transfer of their mathematics lessons to general practice.

Step 5: Follow-up on Assessment

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction?

A. Addressing Gaps in Learning

Students who have difficulty solving addition or subtraction problems by using personal strategies will enjoy more success if one-on-one time is provided. This time will allow for open communication to diagnose where the learning difficulties lie. Observing a student solving problems will provide valuable data to guide further instruction. Success in problem solving depends on a positive climate in which the students are comfortable taking risks. Find out which concepts and skills each student already has and build upon them.

If the difficulty lies in understanding the problem, use the following strategies:

- Provide problems that relate to the student's interests and personalize the problem by using the student's name in the problem and/or the names of his or her friends or family members.
- Initially use smaller numbers in the problem.
- Ask the student the following questions about the problem:
  - What are you asked to find out?
  - What do you know?
  - What information do you need?
  - Is some of this information unnecessary?
- Have the student paraphrase the problem.
- Guide the student to determine if the numbers refer to a part or the whole.
- Ask the student if the unknown in the problem refers to a part or a whole.
- Provide manipulatives for the students to represent the problem as needed.
- Have the student act out the problem, using other students and manipulatives as needed.
- Have the student decide which operation should be used and why.
- Ask guiding questions to show the connections between addition and subtraction and the possible option of thinking addition for subtraction.

If the difficulty lies in using personal strategies to solve addition and subtraction problems, use the following strategies:

- Initially use smaller numbers in the problems.
- Review place value, counting by tens beginning with any number (the hundred chart or base ten blocks are useful for this) and number facts.
- Provide counters that can be grouped in tens or base-ten materials as needed.
- Think aloud a personal strategy that you would use to solve the problem and explain why this strategy is more efficient than another one that you describe.

- Emphasize flexibility in choosing a personal strategy; a strategy that is efficient for one student may not be efficient for another student.
- Build on the student's understanding of place value and number facts to guide him or her in finding a strategy that works.
- Provide ample time for students to think and ask questions to clarify their thinking.
- Have students work in groups so that they learn strategies from one another.
- Guide students to critique various personal strategies to find one that can be used on a variety of problems efficiently.
- Have students explain their personal strategies to the class so others can hear how they work in kid-friendly language.
- Post various personal strategies in the classroom for students to share and critique.

B. Reinforcing and Extending Learning

Students who have achieved or exceeded the outcomes will benefit from ongoing opportunities to apply and extend their learning. These activities should support students in developing a deeper understanding of the concept and should not progress to the outcomes in subsequent grades.

Consider strategies, such as the following.

- Provide parents information about the importance of students learning to make sense of addition and subtraction situations and developing their own strategies for solving these problems prior to learning the traditional algorithm. If you need detailed information on the rationale to support invented strategies over traditional algorithms, Van de Walle and Lovin (2006) compare the two and lay out the benefits of invented strategies (pp. 160, 161). The benefits include the enhancement of base ten concepts, fewer errors, a foundation for estimation and mental mathematics, and less time consuming, as previously mentioned. Other benefits include less reteaching required and student proficiency on standard tests is at least equal. Show parents the variations in the structure of story problems that exist. Explain that students need practice with all these types of problems and their variations. Let parents know that using key words is not a successful strategy. Give them some samples of the kinds of strategies that students might invent or adopt as personal strategies, so they understand what to look for and encourage.
- Provide suggestions for parents about opportunities to involve their students in authentic adding and subtracting situations at home or in the community. For example, at home adding might include how many:
  – loads of wash are done in a week, month or year in your household
  – pieces of silverware are on the table if each person has a knife, spoon and fork for various numbers of settings
  – cans of food are left in the cupboard if you use 6 to make chili
  – doors and windows there are in your home
  – more or less doors and windows are there in another home than yours.

In a restaurant, help students use amounts rounded to the closest dollar or up to the next dollar to determine how much various options will be or if they have enough money for a particular order.

Take the students shopping and figure out sums of various purchases and differences between purchase options (costs may have to be rounded to the next dollar due to the limitations of the calculation skills of Grade 2 students).

- Have the students create problems showing the various types of addition and subtraction problems (change, both joining and separating; part–part–whole; comparison and equalizing) and write appropriate number sentences for each one. These problems can be displayed in a chart or on a bulletin board.
- Have the students make their problems more interesting by adding story details or including extraneous information and numbers. Have them write their solutions on the backs of these problems and share their problems with the class.
- Have the students create problems with different contexts but using the same numbers, such as 29 and 21. They could follow this up by having the class decide which of the problems could be solved using a given number sentence, such as 29 + 21 = ?
- Have the students critique other students' personal strategies and explain why they work or not. Which strategy would be the most efficient and why?
- Have the students write explanations of a personal strategy so that everyone in the class can understand it. Challenge the students to solve a problem in a second way.

Categories of Addition and Subtraction Problems Based on Structure

	Unknown Outcome/Result	**Unknown Change**	**Unknown Start or Initial Quantity**
Add to or Join Problems	Jane had 5 pennies. Her dad gave her 3 more. How many pennies does Jane have now?	Brett had 9 hockey cards. Bill gave him some more. Now he has 14. How many did Bill give Brett?	There were some cartons of milk in the fridge when the delivery person put in 45 more. Now there are 83 cartons in the fridge. How many cartons were in the fridge before the delivery?
Separate or Take-Away Problems	Tom had 7 cars and gave 3 to his cousin. How many cars does Tom have left?	Rita had 24 crackers. She gave some to Sam. Rita now has 13 crackers. How many did she give Sam?	Fred had some marbles. He gave 14 to Tim. Now he has 15. How many marbles did Fred start with?
Combining (2 static quantities) Also referred to as Part-part-whole, (misleading since all categories have whole and parts)	Ann has 4 dimes and 3 nickels. How many coins does she have altogether?	Jim has 42 hockey cards and some basketball cards. He has 75 cards altogether. How many basketball cards does he have?	Sara put her pennies in her new bank. Then her Uncle Jack put in 8 pennies. Now there were 14 pennies in the bank. How many pennies did Sara put in?
Compare Problems	Louis has 4 pairs of shoes and Marley has 7. How many more pairs of shoes does Marley have than Louis?	Sasha has 23 fish in his aquarium. He has 12 more than Pat. How many fish does Pat have?	Nash had some computer game points. He had 35 more points than Kareem, who had 61. How many points did Nash have?
Equalize Problems	Jamie has $18. Maria has $12. How many more dollars does Maria have to save to have the same number as Jamie?	Isabelle has 11 pairs of earrings. Susan needs 4 more pair to have the same number as Isabelle. How many pairs of earrings does Susan have now?	Carter has some books. Sandy, who has 48 books, has to get 32 more to have the same number as Carter. How many books does Carter have?

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