Mathematics

Planning Guide

Grade 2 *Equality and Inequality*

Patterns and Relations

Specific Outcomes 4, 5

This Planning Guide can be accessed online at:
http://www.learnalberta.ca/content/mepg2/html/pg2_equalityandinequality/index.html

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Strand: Patterns and Relations
Specific Outcomes: 4, 5

This *Planning Guide* addresses the following outcomes from the Program of Studies:

Strand: Patterns and Relations	
Specific Outcomes:	4. Demonstrate and explain the meaning of equality and inequality, concretely and pictorially. 5. Record equalities and inequalities symbolically, using the equal symbol or the not equal symbol.

Curriculum Focus

This sample targets the following changes to the curriculum:

- The general outcome: "Represent algebraic expressions in multiple ways" has changed in that it is being applied from Grade 1 through Grade 12 in the new curriculum; whereas in the 1997 curriculum it had no specific outcomes from Kindergarten through Grade 5. The 2007 curriculum widens the scope of the study of relationships. "The search for possible relationships involves collecting and analyzing data and describing relationships visually, symbolically, orally and in written form" (Alberta Education 2007, p. 8). The latter part regarding the various modes in which relationships could be described is identical to the previous curriculum; however, the addition of "the search for possible relationships" and "describing relationships" expands the field, as can be seen when examining the changes to the specific outcomes.
- The specific outcomes in Grade 1 through Grade 5 are new. The specific outcomes address equality and inequality and writing equations using variables. The former addresses an area of serious need, as research reveals that students' misconceptions of the equal sign were preventing them from successfully understanding and solving for equalities. Students were interpreting the equal sign as "do something," not as "the same as" (Van de Walle and Lovin 2006). Writing equations with variables was often happening in classrooms, but was not always recognized. When students were asked to solve for the box in an equation such as 5 + □ = 9, the box was a variable. So as early as Grade 1 students were often faced with equations with variables. By Grade 3, problem solving sometimes included using a single variable in an algebraic expression. The 2007 curriculum recognizes the students' need to have a clear understanding of equality and inequality right from the start; therefore, it sets out standard expectations for students in elementary grades for the concepts of equality and inequality and the use of variables.

What Is a Planning Guide?

Planning Guides are a tool for teachers to use in designing instruction and assessment that focuses on developing and deepening students' understanding of mathematical concepts. This tool is based on the process outlined in *Understanding by Design* by Grant Wiggins and Jay McTighe.

Planning Steps

The following steps will help you through the Planning Guide:

Step 1: Identify Outcomes to Address

Guiding Questions

- What do I want my students to learn?
- What can my students currently understand and do?
- What do I want my students to understand and be able to do based on the Big Ideas and specific outcomes in the program of studies?

Big Ideas

Mathematics is often referred to as the science of patterns. Patterns permeate every aspect of mathematics. The brain is a pattern seeker, so the development of the students' ability to recognize, analyze and generally become proficient pattern seekers will not only pave the way for their success in mathematics, but in all learning.

- Numbers can have an infinite number of names or expressions that represent them. For example, 5 can be referred to as 4 + 1, 10 – 5, 3 + 2, 100 – 95, and many other names.
- Various expressions can be compared and found equal or unequal, the former are called equalities and the latter inequalities.
- The equal sign refers to a relationship: "the same as." It does not mean that the answer is coming or to take some action.
- Putting a line through the equal sign, like this ≠, means the expressions on either side are unequal or that the relationship is an inequality.
- A balance scale will show both sides at the same level if the quantities on both sides are of equal value or mass. If the quantities are unequal, the greater or heavier side will be lower than the lesser or lighter side.

Sequence of Outcomes from the Program of Studies

See http://education.alberta.ca/teachers/core/math/programs.aspx for the complete program of studies.

Grade 1

Specific Outcomes

4. Describe equality as a balance and inequality as an imbalance, concretely and pictorially (0 to 20).

5. Record equalities, using the equal symbol.

Grade 2

Specific Outcomes

4. Demonstrate and explain the meaning of equality and inequality, concretely and pictorially.

5. Record equalities and inequalities symbolically, using the equal symbol or the not equal symbol.

Grade 3

Specific Outcomes

4. Solve one-step addition and subtraction equations involving a symbol to represent an unknown number.

Step 2: Determine Evidence of Student Learning

Guiding Questions

- What evidence will I look for to know that learning has occurred?
- What should students demonstrate to show their understanding of the mathematical concepts, skills and Big Ideas?

Using Achievement Indicators

As you begin planning lessons and learning activities, keep in mind ongoing ways to monitor and assess student learning. One starting point for this planning is to consider the achievement indicators listed in the *Mathematics Kindergarten to Grade 9 Program of Studies with Achievement Indicators*. You may also generate your own indicators, and use them to guide your observation of the students.

The following indicators may be used to determine whether students have met these specific outcomes. Can students:

- determine whether two given quantities of the same object (same shape and mass) are equal by using a balance?
- construct and draw two unequal sets, using the same object (same shape and mass), and explain the reasoning?
- demonstrate how to change two given sets, equal in number, to create inequality?
- choose from three or more given sets the one that does not have a quantity equal to the others and explain why?
- determine whether two sides of a given number sentence are equal (=) or not equal (≠), write the appropriate symbol and justify the answer?
- model equalities, using a variety of concrete representations, and record the equalities symbolically?
- model inequalities, using a variety of concrete representations, and record the inequalities symbolically?

Sample behaviours to look for related to these indicators are suggested for some of the activities listed in **Step 3, Section C: Choosing Learning Activities** (p. 11).

Step 3: Plan for Instruction

Guiding Questions

- What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should I use?
- How will I meet the diverse learning needs of my students?

A. Assessing Prior Knowledge and Skills

Before introducing new material, consider ways to assess and build on students' knowledge and skills related to equalities and inequalities. In Grade 1, students were introduced to equality as balance and inequality as imbalance. They will have used the equal sign with sums or differences on either the left or right of the sign. With numbers from zero to twenty, they should have recorded different representations of the same quantity as equalities, such as $3 + 6 = 5 + 4$. Some examples of ways to assess student retention and comprehension of these Grade 1 outcomes are as follows.

Can students:

- tell you what the equal sign means?
- understand and solve equations in which the sum or difference is on the left of the equal symbol with the same proficiency as those with the sum or difference on the right?
- tell you how a balance pan scale works? Are they clear about the relative position of the pans when items on both sides are equal? Do they understand that when one side is higher than the other, the items there must have a lesser value or lighter weight than those on the lower side?
- record other names for a given number?
- understand equalities with different representations for the same number on each side of the equal symbol?
- prove equality and inequality using a balance?
- determine if two given concrete sets are equal or unequal, and explain the process used?

If a student appears to have difficulty with these tasks, consider further individual assessment, such as a structured interview, to determine the student's level of skill and understanding. See **Sample Structured Interview: Assessing Prior Knowledge and Skills** (p. 7).

Sample Structured Interview: Assessing Prior Knowledge and Skills

Directions	Date:	
	Not Quite There	**Ready to Apply**
Show the student an equal sign. Ask, **"What does this symbol mean?"** If the student replies "equal." Ask, **"What does 'equal' mean?"** If the student still gives no answer, put the symbol in context. Ask, **"What does this sign mean?"** Point to the equal sign in the equation: 3 + 4 = 7	• Is not able to answer, "What does 'equal' mean? • Says that the equal sign means to find the answer or the answer is coming next.	• Correctly defines the equal sign as meaning "is the same as" or "has the same value as."
Show the student the following equations. Say, **"Please solve these number sentences or equations."** Have manipulatives available and also a balance pan scale, if you wish. **4 + 4 = _____** **_____ = 5 + 5** **10 – 7 = _____** **_____ = 9 – 3**. Watch for counting backwards from 10 and 9 to solve the subtraction problems as opposed to using addition: 7 + ? = 10 or 3 + ? = 9 to solve these last two.	• Is not able to solve the equations. • Answers correctly or easily only those with the solution on the right of the equal sign. • Requires manipulatives to solve all these equations. • Required the balance pan scale to solve and solved by adding one at a time for all or some of the solution numbers.	• Solved all the equations accurately and in most cases did so without using fingers or manipulatives.

Have a variety of manipulatives available. Show the student a balance pan scale and ask, **"How does this balance pan scale work?"**	• Can not show you how it works. • Can not describe the way it works in words. • Used different sized or shaped manipulatives in the demonstration, such as two triangles and a trapezoid on one side and a number of Multilink cubes on the other.	• Shows and describes the way in which it works accurately and completely, including mention of where the greater and lesser amounts are in tilted positions.
Have counters available. **"Give me some other names for eight."** If the student is unable to respond to this direction, ask, **"How many ways can you make eight?"**	• Says there are no other names for eight. • Can only give one or two addition combinations that make eight. • Only responded after you asked how many ways you can make eight.	• Accurately gives you the combinations: $1 + 7$, $2 + 6$, $3 + 5$ and $4 + 4$. If the response goes on to include subtraction or multiplication alternatives such as $10 - 2$ or 2×4, the student is very well prepared.
Present the student with two sets of counters and ask, **"How could you make these sets equal or unequal?"** When they have shown you a correct possibility, ask, **"Could you do it another way?"**	• Cannot make unequal sets equal in at least two ways. • Cannot make equal sets unequal in two or more ways.	• The student accurately and flexibly demonstrates his or her knowledge of various ways to make the sets unequal or equal. If this is done verbally rather than physically, the student is well prepared.
Show the student the following equation and say, **"Tell me what this means."** **$4 + 5 = 7 + 2$**	• Adds all the numbers from both sides for a total of 18 or some error in an attempted sum. • Tells you the equation is wrong because $4 + 5 = 9$, not 7. • In some other way explains the equation incorrectly.	• Explains that $4 + 5 = 9$ and $7 + 2 = 9$ so both sides are the same or the same value and that is why they are equal. The student might explain that both sides are other names for nine and so are equal.

Given two sets of counters and a balance pan scale, direct the student, **"Show me how these sets can be proved equal or unequal and explain it to me."** Students may find it easy to count the manipulatives and compare numbers in the two sets above. If you want to make this less likely, present the quantities to be determined as equal or unequal as marbles in two identical plastic bags. Students are less able to count each set reliably.	• Does not know how to show you on the scale how they are equal or unequal. • Places the sets on the two pans and watches the scale either balance or tilt, but does not provide a clear explanation of what that proves or how.	• Places the counters on the two pans. Either pronounces the sets equal when it balances or unequal when it tilts and then goes on to explain. The explanation includes equal as balancing because the value or weight is the same or if they are unequal, that the lesser or lighter amount is higher while the heavier or greater amount tilts lower.

B. Choosing Instructional Strategies

Consider the following general strategies for teaching the basic concepts of equality and inequality.

- Begin with concrete materials such as a balance scale and manipulatives. Interlocking cubes demonstrate this well, since each number can be represented by a different cube colour. This makes the combinations on each side of the scale still obvious even after the addends have been joined and the multi-coloured sticks from each pan compared by standing them beside one another.
- Draw comparisons to other things in the students' experiences or the world in general that work the same way and reinforce the sense making in mathematics. For example, compare the way the equal symbol is made into the "not equal" symbol to the way that signs show no parking, no littering, no biking or no smoking. It then makes sense to students that drawing a diagonal line through a sign means the inverse of what was originally communicated. Likewise, students understand the tilting of a balance pan scale more easily if they have had the opportunity to use a see-saw and know that the heavier person may sit on the ground and hold the lighter person aloft.
- Allow sufficient time for students to grasp and internalize these concepts. It may take an average student thirty to forty exposures before they own a concept. Having a balance pan scale in the room once and demonstrating how it works will not be enough. Some students will require more opportunities to experiment with the balance pan scale to gain an understanding of how equalities and inequalities affect the tilt or balance of the scale. Reinforce daily that the equal sign means "is the same as."
- Move from the concrete to the pictorial by showing what was done with the scale in diagrams.
- Take the next connecting step and add the symbolic representations to the concrete actions and the illustrations.
- Have the students practise translating from one representation to the other. When shown an equality or inequality, can they draw a diagram that illustrates it? Can they demonstrate what it means with a balance pan scale? Given the illustration, can they provide a concrete and symbolic representation?

C. Choosing Learning Activities

The following learning activities are examples that could be used to develop student understanding of the concepts identified in Step 1.

Sample Activities:

1. **Introduction of Equal and Unequal using the Balance Pan Scale with Pattern Blocks** (p. 12)
2. **Development of the Concept of Combinations Being Equal or Unequal Using the Balance Pan and Multilink Cubes** (p. 14)
3. **Using a Mathematical Balance Scale** (p. 17)
4. **Constructing Unequal Sets and Explaining the Reasoning** (p. 18)
5. **Which One Doesn't Belong Here?** (p. 19)
6. **Using the = or ≠ Symbols** (p. 20)
7. **Translating Symbolic and Pictorial to Concrete** (p. 21)
8. **Reasoning with Pattern Block Equalities and Inequalities** (p. 22)

Sample Activity 1: Introduction of Equal and Unequal Using the Balance Pan Scale with Pattern Blocks

Using various pattern block pieces, the goal is to ensure that students know how the balance pan scale works; however, before achieving this, it is important to check that your students understand conservation of number and mass. That means that they recognize that the number or mass/weight is not altered by the arrangement or subdivision of a group of items. Place the same number of orange squares on both pans. The scale will balance. Regroup the set of orange square pattern blocks on one side of the pan close together and the equal set on the other side spread around the perimeter of the pan. Students who still have not made the transition to conservation of number will view the spread out squares as being a greater quantity than those compactly grouped. If you have pans that can be lifted off the scale, ask the students to indicate what they think the outcome will be when you place the pan back on the scale before actually doing so. If your pans do not lift off the scale, show the equal sets piled in the same way on the table before setting them on the scale. Then take them off and show them on the table arranged in a compact group and a spread out group and ask for student predictions before placing them on the scale. A good way to ask students to predict is to ask them to show thumbs up if they think the amounts on each side of the scale are equal and thumbs down if they think that the amounts are unequal. Establish with the students that the number of blocks on each side is in fact equal regardless of how spread out their arrangement. Take note of the students who demonstrated they still do not understand conservation of number and mass and arrange for them to do further activities so they can confront their developmental misconceptions and attain conservation of number and mass (Lowery 1986). Follow-up testing of these individuals can take place once they have done some activities that encouraged them to confront the need to change their thinking.

Look For …

Do students:

- ☐ signal that the amounts are unequal based on the proximity of the blocks in the one group? If so, make note of those students who seem not to have conservation of number. Activities to help these students make the transition to Conservation of Number can be found in *It's the thought that Counts*.

Now you are ready to make sure that the students also know how the balance pan scale works when faced with inequality.

- Draw the students' attention to the position of the pans in relation to each other while the amounts in each pan are equal.
- Next, add or remove a square.
- As soon as you change the number of squares on one side, the pans tilt.
- Ask questions to make sure that students notice whether the greater number of blocks is lower or higher.
- Ask if students can explain why this is so.
- Repeat the procedure with other blocks such as the red trapezoids, yellow hexagons and the green triangles. This may seem redundant, but for some students who may not be familiar with the balance pan scale or have not had enough experience handling it in Grade 1, this repetition could be essential.

Your students should now understand how the balance pan scale demonstrates whether the contents of its pans are equal or unequal.

Clarifying the Meaning of the Equal Sign

Look For …

Do students:

- ☐ state that the equal sign means "the same as"?
- ☐ recognize that the scale will balance the pans when the amounts in each pan are the same?
- ☐ use the same manipulatives on both sides of the pan when comparing amounts?

- Show various groupings on the two pans. For example, place the orange squares on one pan in a stack of five and a stack of three.
- Then on the other pan, place a stack of four and a second stack of four.
- Ask the students what they see. Illustrate it with a simple balance diagram on the board, showing the two pans and the corresponding stacks of square orange blocks.
- Ask the students how we could write this with numbers instead of a drawing.
- When the students suggest it could be recorded as 5 + 3 on one side and 4 + 4 on the other, ask how we can let people know what the balance pan scale did? They will likely suggest the equal sign.

Now is a great time to point out that the equal sign means that what's on one pan is the same or has the same value as what is on the other pan. In this case, it means the same number in total are on both sides and as a result both sides have the same amount of mass or weight. A clear understanding of the equal sign is critical to the students' ability to solve for variables in future work. Too many students deduce that the equal sign means the answer comes next or to do something. As a result of these misconceptions, they are not able to solve for variables correctly in all positions.

Students need to have the term "unequal" and its symbol to use as the alternative to "equal" and the equal sign.

Look For …

Do students:

- ☐ recognize that the symbol ≠ means "not equal"?
- ☐ relate the diagonal line through the equal sign to other signs they have seen, such as a bike with a diagonal line through it to mean no bike riding here?
- ☐ understand and use the term "unequal"?
- ☐ make equalities into inequalities and vice versa in more than one way.

- Add some more orange blocks to one side and observe the pans of the scale.
- Ask the students what has happened. Is it balanced now? Why not?

If the number of blocks is no longer equal, what could we say they are? Can students come up with the term "unequal"? Perhaps hints such as if someone is no longer happy, how can we change the word "happy" to express what they are feeling? If your shoe is no longer tied, how can we change the word "tied" to describe it? When students come up with "unequal," you can ask how people make signs to show something is not allowed.

Give specific examples to students if this is too abstract. For example, if dogs are not allowed in the park, how do they show it in a picture sign? In this way, introduce the not equal sign: ≠. Follow this with examples using other pattern block groups that demonstrate equalities. Ask how the students could make the relationships unequal. Then give some inequalities and ask what possible ways there are to make the relationships equal. Students can tell you how many you would have to add or subtract from one pan or the other to re-establish balance and equality. Students will now be prepared to state whether expressions are equal or unequal with words and symbols.

Sample Activity 2: Development of the Concept of Combinations Being Equal or Unequal Using the Balance Pan and Multilink Cubes

Warm Up

In this lesson, all of the development in the previous lesson is reinforced with the addition of an easy physical comparison of the total cubes from each pan due to the colouring of the manipulatives and the fact they will link together securely. Instead of piling the pattern blocks in two groups in each pan, use a different colour of cubes on each side to represent each number. So, if you were demonstrating two addends on one side compared to two addends on the other side, you would have four colours of cubes altogether on the scale. This makes it easy for students to see the combinations that composed the equality or inequality. These multilink cubes are preferable over Unifix cubes, since they weigh more and don't come apart as easily. The two colours of cubes from each side can be joined into a stick. The two sticks can be held up beside each other to reflect what the scale has indicated, that the combinations from each pan are in fact equal or unequal.

Look For …

Do students:

- ☐ represent each number in an equation with a different colour of Multilink Cubes?
- ☐ combine the cubes on each pan into a stick and compare them physically to prove that they are equal or unequal?

Equality and Inequality

The next goal is to introduce the terms "equality" and "inequality." Develop some concrete examples of equalities and inequalities and model the transfer of these to pictorial and symbolic records, asking the students to direct you as much as they are able. It is advisable to use two addends in one pan and a total group in the other pan at this beginning stage; however, you can place the total group in the first pan and the two addends in the second pan to help prepare the students to solve both $a + b = c$ and $c = a + b$ formats. Show the students how in the new term "equality" the word "equal" is hiding. Help them recognize that we make some words have the opposite meaning by adding the prefix or beginning "in-," such as "inappropriate" behaviour or dress, "incorrect" or "incomplete." Now that the students can understand the terms "equalities" and "inequalities," it is just a matter of using them frequently enough with the students so that they can adopt them.

Look For …

Do students:

- ☐ relate the term "equality" to "equal"?
- ☐ relate the term "inequality" to "unequal"?
- ☐ understand the power of the prefix, "in-" to create the inverse of the word to which it is added?
- ☐ understand concrete equations regardless of the pan on which the sum is placed?

Students can now learn to associate a variety of concrete equalities and inequalities using a pan balance scale with their symbolic form. Show a number of relationships with two colours of blocks in two piles on the one side and a single group on the other. Translate these into pictorial and symbolic versions as you proceed. Ask the students to tell you how to do this as much as they are able after the first one, so that they are more and more prepared to do this independently. When you are comparing the amounts, they can verify their interpretation of the tilt or balance of the scale by combining all the cubes on the side that has two groups and holding the sticks from each side up to one another to compare their lengths. Since the two groups from the one side can still be identified by their colours, it will help the students connect it to the equation. After demonstrating some equalities and inequalities with three groups of cubes or two addends opposing a single group, start comparing two groups on each pan. If you are not using Multilink cubes, it will help the students visualize the groupings if you place the quantity for each amount in a small clear plastic lid from the top of a card or jewellery box. This way the students can match the groupings with the numbers in the corresponding equation. The students should now be very familiar with concrete equalities and inequalities on the balance pan scale.

Look For …

Do students:

- ☐ draw the balance pan scale correctly in pictorial representations?
- ☐ print the equations correctly for the symbolic versions of each concrete comparison?
- ☐ recognize that a comparison can be made between two or more addends on both pans?

Variables and Equations

Students can now learn how variables are related to the equations shown on the balance pan scale. After doing more equalities and inequalities, such as 5 + 6 on one side and 3 + 8 on the other, or 2 + 6 on one side and 4 + 5 on the other, you will be able to leave one area or clear plastic lid empty. Then ask the students what could be placed in the empty space or lid to make an equality. For example, if one side of the scale has five orange cubes and seven blue ones and the other pan has three yellow cubes and an empty plastic lid or space, the equation looks like this: 5 + 7 = 3 + ◯. Ask the students how they could figure out how many cubes should be placed in the empty lid on the right pan. Students may suggest that you keep adding one cube at a time until the scale pans balance or are even. Other students may tell you that since the cubes on the left side are twelve and three and nine make twelve, you need to place twelve/nine on the empty lid on the right side for the scale to balance. In all of these cases, ask the students how they knew how many cubes there were or how many were needed. Their answers can be quite revealing. If the students want to add blocks one at a time until the scale pans balance, you know that they have the concept of equality related to evenly positioned pans on the scale, but they are not proficient yet with the number facts and missing addends. In your modeling of the symbolic representations, you can show them the work or thinking of students who are able to mentally calculate the total on one side and ask themselves what is missing on

Look For …

Do students:

- ☐ know how to symbolically show the scale before the empty space is filled with a marker for the missing variable or amount?
- ☐ select the number of cubes needed to balance the scale and place them in the lid on the scale or do they add one cube at a time until the scale balances?

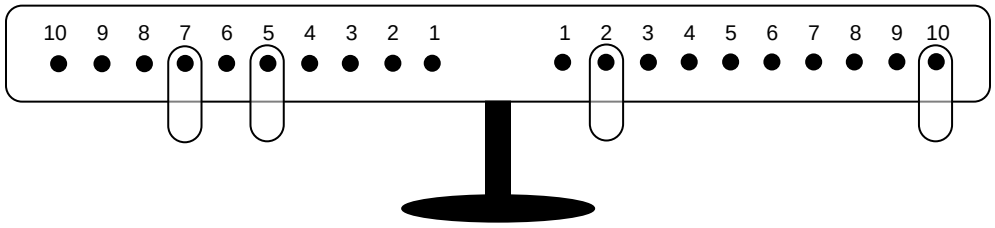
Look For …

Do students:

- ☐ balance the equal arm scale with a single number on one side and two or more on the other?
- ☐ balance two or more numbers on each side?
- ☐ draw the balance with equal numbers on each side?
- ☐ translate these into their symbolic forms, that is, write the corresponding equations?
- ☐ transcribe the imbalances as inequalities using the not equal sign?

Sample Activity 3: Using a Mathematical Balance Scale

This T-shaped scale has numbers one through ten down each side of the arm from the pivotal point. Underneath the numbers are pegs. Some have plastic numbers that vary in size and thus mass to hang on each side. So if you hung a seven on one side, to balance the arm of the scale you would need to place a three and a four, or a two and a five or a one and a six on the other side. Other versions have rectangular plastic weights that are hung under the appropriate number. For example, if you have a weight hung on one side from the three and the five, to balance them you need a weight hung under the eight on the opposite side. These scales are a very worthwhile investment for students in grades one and two who are investigating the concept of equalities and inequalities. There are also activity books available to go with them that have ideas for their use appropriate for students from grades 2 through 8. If you can access one of these scales, show it to the students and develop some equalities and inequalities using it. Connect those you make to the corresponding pictorial and symbolic representations.



7 + 5 = 2 + 10

Sample Activity 4: Constructing Unequal Sets and Explaining the Reasoning

Give the students counters and ask them to construct two unequal sets, draw a picture of them and explain how they knew they were unequal. By not specifying whether each group has to be made up of one pile or colour, you may see the various levels of development of your students' mathematical thinking in terms of equalities and also their language and reasoning sophistication. Some students will show you only one pile of counters in each set and the symbols for equal or not equal between them. Their reasoning may simply state, "Seven is more than three, so these sets are unequal." Other students may consider that each set may be made up of two parts or colours and draw the diagrams as such on a balance pan and show you the correct tilt. Their reasoning may include the exact difference or may be a comparison. In the case of the latter, it might show $5 + 7 \neq 6 + 5$. The reasoning might explain that five is the same on each side, but six is one less than seven so the side that has $6 + 5$ is one less than the other side. Your next lesson might be aimed at developing the concept of comparing components on both sides using relational thinking, rather than actually adding the totals and comparing them.

Look For …

Do students:

- ☐ show unequal sets as combinations or single numbers?
- ☐ compare with exact differences or relational thinking?

Sample Activity 5: Which One Doesn't Belong Here?

Show the students three sets, two of which are equal and one of which is not. Ask the students to pick out the one that is unequal and explain why. After doing this concretely with manipulatives and hearing the students' explanations, giving all students the opportunity to see and hear what is expected, this may be assigned as a written task in which the three possibilities become more sophisticated. The first examples can be pictorial. The next series of examples can be addition equations only, such as 4 + 7, 3 + 9 or 5 + 6. The next series can be all subtraction facts and, for a challenge, some mixed number facts, such as 15 – 7, 2 + 6 or 4 + 5.

Sample Activity 6: Using the = or ≠ Symbols

Another written activity can be assigned in which students place the correct sign = or ≠ between the given numbers. After some pictorial examples showing the balance pan scale, the task may begin very simply with two addends on one side and a single number on the other and progress to a number fact on each side including both addition and subtraction.

Sample Activity 7: Translating Symbolic and Pictorial to Concrete

In most of the lessons, the progression is from concrete to pictorial and then symbolic. Students are seldom asked to work in other directions. It is helpful for students to have to translate other ways. Give the students some equalities and inequalities and ask them to show them in pictorial form. As the students work on their pictorial representations, move around the class, asking each student to use manipulatives to show you one in concrete form. Alternatively, you could invite the students to a centre where there is a balance pan scale and a variety of manipulatives. Ask them to demonstrate one of the equalities and one of the inequalities. This will help students develop flexibility in translating between all three forms: concrete, pictorial and symbolic.

Sample Activity 8: Reasoning with Pattern Block Equalities and Inequalities

For students who need more challenge, use the equivalencies of the pattern blocks to make interesting problems for them to solve and allow them to use the balance pan scale as needed. Ask the students to write their explanations of why these groups are equalities or inequalities. Three green triangles are equivalent to one red trapezoid. Two red trapezoids are equivalent to one yellow hexagon. Present the students with problems such as:

1. Are 6 green triangles equal or not equal to a yellow hexagon? Explain your reasoning.
2. Are 2 red trapezoids equal or not equal to a 5 green triangles? Explain your reasoning.
3. Are 4 red trapezoids equal or not equal to 2 yellow hexagons? Explain your reasoning.
4. Are 10 green triangles equal or not equal to 2 yellow hexagons? Explain your reasoning.

Step 4: Assess Student Learning

Guiding Questions

- Look back at what you determined as acceptable evidence in Step 2.
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

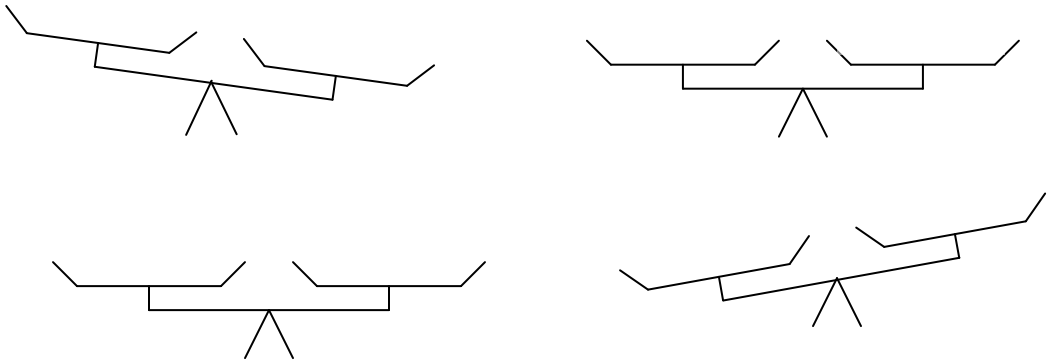
In addition to ongoing assessment throughout the lessons, consider the following sample activities to evaluate students' learning at key milestones. Suggestions are given for assessing all students as a class or in groups, individual students in need of further evaluation, and individual or groups of students in a variety of contexts.

A. Whole Class/Group Assessment

Equality and Inequality

Name : _____ Date: _____

1. Circle the balance pan scales that show an equality in blue and those that show an inequality in red. Explain how you know which have inequalities or equalities when you can't see what is in the pans.



2. Draw three balance pan scales below. Each one should look different than the other two. Show the sets that are in the pans and write the equations that go with them beside each one.

a. _____

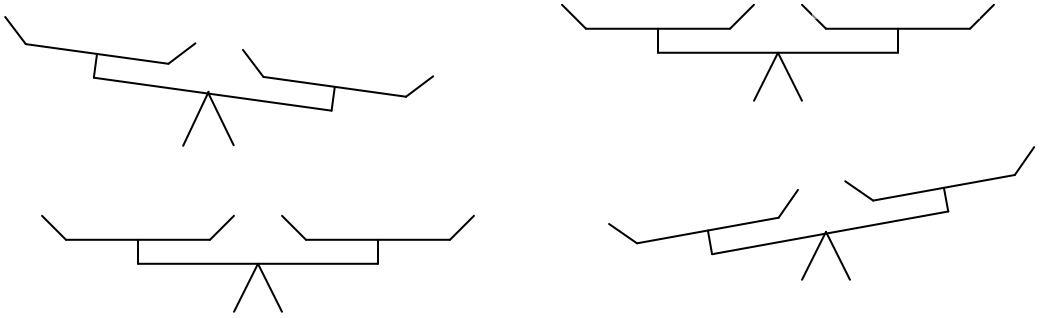
b. _____

c. _____

Whole Class/Group Assessment ANSWER KEY

Name : _____ Date: _____

1. Circle the balance pan scales that show an equality in blue and those which show an inequality in red. Explain how you know which have inequalities or equalities when you can't see what is in the pans. Print more, less or equal on the pans to tell what amount each pan must contain.



Give four marks, one each for the appropriately coloured circle drawn around the scale. The two scales balanced should be circled in blue and the two imbalanced should be circled in red. If the students were unable to do this correctly, you know there is a lack of understanding of the balance pan scale and/or equalities and inequalities. A structured interview with any student not accomplishing the circling correctly should follow.

Then students should explain that if the pans on the scales are even, it means they both have the same (an equal) amount in each pan (two marks). That means they are equalities because they have equal amounts in both pans. The scales that are tipping tell you that the high side is lighter or has less in it than the heavy side that tilts down lower (three marks). The marks are not nearly as important as identifying which students do not understand the concepts of equality and inequality or have reversed which pan represents more and which represents less. This diagnostic information will inform your planning and teaching.

If marks are not necessary, rubrics can be used.

Rubric for Number 1 – Understanding Equalities and Inequalities on a Balance Pan Scale			
Not Yet	**Needs More Instruction**	**Achieved**	**WOW!**
Does not circle the scales correctly to show which are equalities and inequalities.	Circles the balance pan scales correctly in red or blue, indicating the equalities and inequalities, but labels the higher pans as "more" and lower pans as "less." May have circled and labelled correctly, but does not explain why.	Circled and labelled correctly, as well as provided an explanation that included when the pans are even, level or balanced you know that they have the same amount or value.	Has all of the indicators of "Achieved" plus his or her explanation includes that a scale that is tipped has the greater or heavier quantity in the lower pan. The explanation may then note the inverse, that the lesser quantity is in the higher pan.

2. Draw three balance pan scales below. Each one should look different than the other two. Show the sets that are in the pans and write the equations that go with them beside each one.

   a. _____

   b. _____

   c. _____

   Give one mark in each of the three cases if the scales are balanced in one, tipped with the right side lower in another and the left side lower in the third case. Then check that the numbers of items drawn in each pan correspond to the position of the scales. If they do, give another mark to each drawing. Finally, check the equations written. Give two marks each to the equality and inequalities, one for the appropriate equal or not equal sign and one for the equation if its numbers correspond to the diagrams. This allows four marks per letter or twelve in total. This activity is marked out of a total of 29.

   If marks are not necessary, rubrics can be used.

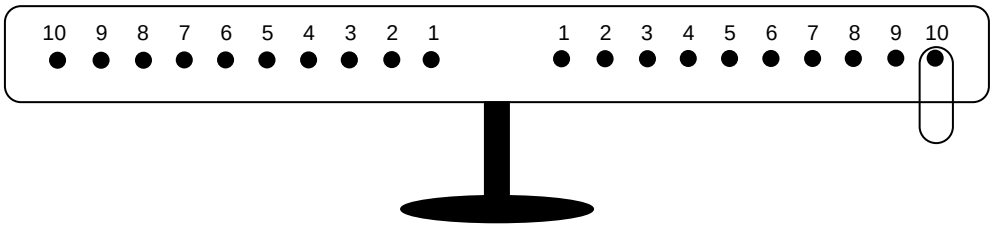
Rubric for Number 2 – Composing and Demonstrating Equalities and Inequalities on a Balance Pan Scale Pictorially and Concretely			
Not Yet	**Needs More Instruction**	**Achieved**	**WOW!**
Does not draw the scales in all three possible positions. Does not draw the sets in the diagrams. Does not produce equations or writes some that do not correspond with the scales and set drawn in each case.	May have the balance pan scales correctly drawn in two positions and the corresponding sets drawn are correct and match the equations written. The scales are shown in all three positions and have sets on them that correspond to the positions, but the equations are missing or do not correspond to the diagrams. All is correct except for placing the smaller numbers on the lower pans and the larger numbers on the higher pans.	Draws scales in all three possible positions. Shows sets on the pans that correspond to their positions. Writes the equations that correspond to each using the = and ≠ signs or =, > or < signs. All of the equations have a single amount on one or both pans.	Draws scales in all three possible positions. Shows sets on the pans that correspond to their positions. Writes the equations that correspond to each using the = and ≠ signs or =, > or < signs. Plus the equations include at least two addends on one or both pans.

A. Whole Class/Group Assessment for Equalities and Inequalities

Name : _____ Date: _____

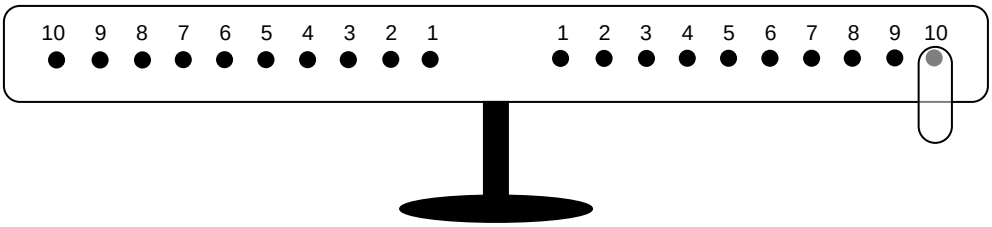
1. On the following balance scales, show where two missing weights could be placed to make the sets equal. Then complete the equation below the equal arm balance to show the equality on it. Make each one different.

a.



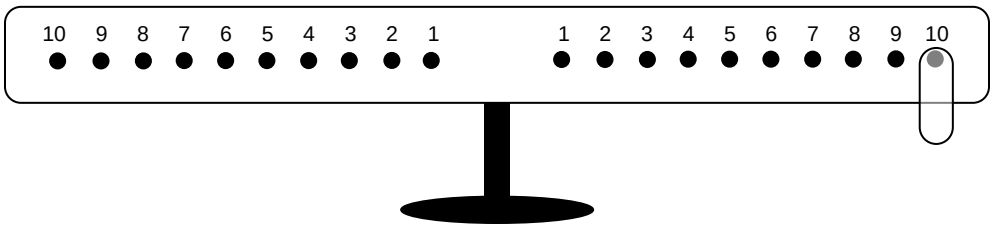
___ + ___ = 10

b.



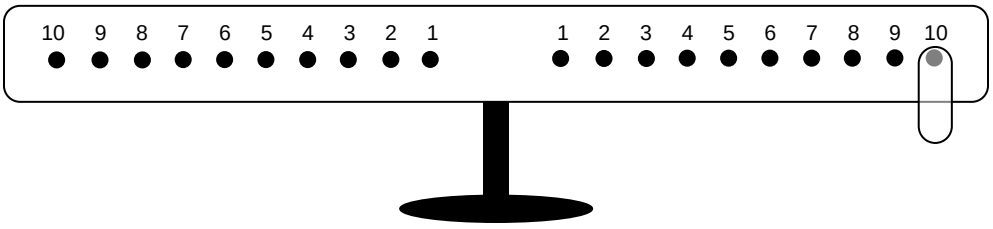
___ + ___ = 10

c.



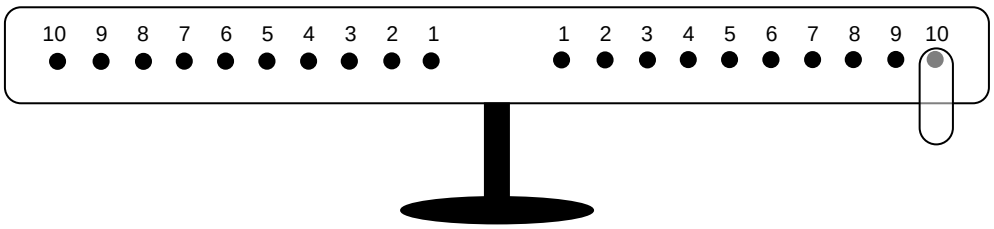
___ + ___ = 10

d.



___ + ___ = 10

e.



___ + ___ = 10

2. Read the following equations, find any that are untrue and change the equal sign to a not equal sign.

a. 3 + 6 = 9

b. 17 = 8 + 8

c. 16 – 7 = 7

d. 5 + 4 + 3 = 12

e. 14 – 6 = 8

f. 13 = 18 – 6

g. 3 + 9 = 7 + 4

h. 14 – 6 = 4 + 4

i. 9 + 6 = 6 + 6 + 3

j. 13 – 7 = 18 – 6

3. Circle the one that does not belong in each set of equations.

a. 4 + 5	12 – 3	6 + 2
b. 16 – 8	15 – 8	4 + 4
c. 13 – 7	7 + 6	19 – 6
d. 17 – 4	9 + 4	5 + 7
e. 9 + 5	8 + 7	19 – 4

4. Solve the following equations.

a. 7 + _____ = 13

b. _____ + 9 = 14

c. 9 + 8 = _____

d. _____ = 4 + 7

e. 14 – 8 = _____

f. _____ = 12 – 7

g. 16 – _____ = 9

h. _____ – 11 = 7

5. Think about what you have learned about pattern blocks. Do you remember:
   - 3 green triangles equal 1 red trapezoid?
   - 2 red trapezoids equal 1 yellow hexagon?

   Make the following into equalities.

   a. _____ green triangles = 1 yellow hexagon

   b. 2 hexagons = _____ red trapezoids

   c. _____ green triangles = 2 yellow hexagons and 1 red trapezoid

6. Look at the following examples. Explain how you can tell they are true or untrue without actually working out the addition or subtraction.

   a. 11 – 5 = 12 – 6

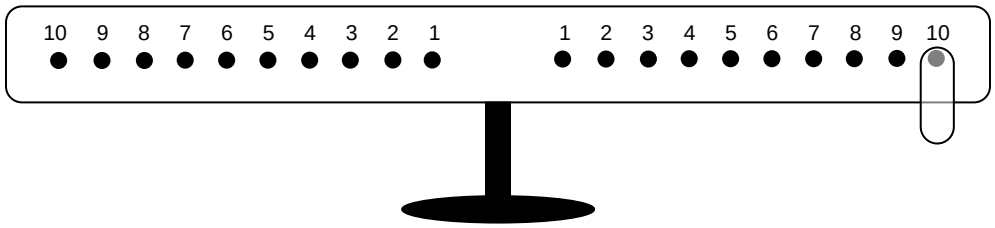
   b. 35 + 17 = 37 + 15

Whole Class/Group Assessment for Equalities and Inequalities Key

Name : _____ Date: _____

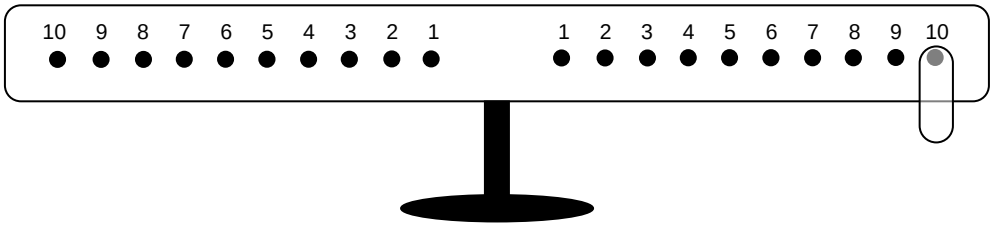
1. On the following balance scales, show where two missing weights could be placed to make the sets equal? Then complete the equation below the equal arm balance to show the equality on it. Make each one different.

a.



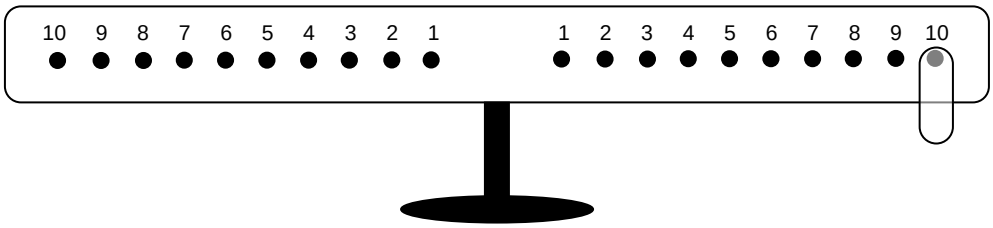
___ + ___ = 10

b.



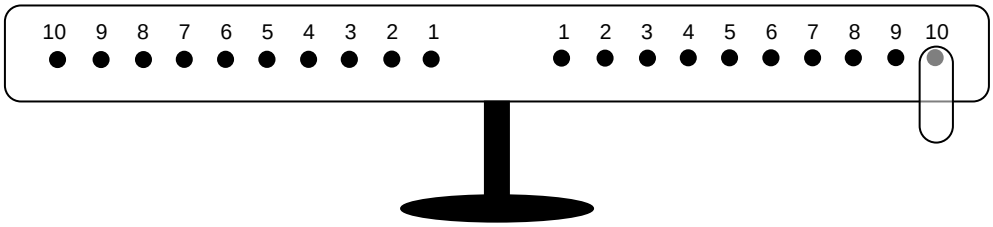
___ + ___ = 10

c.



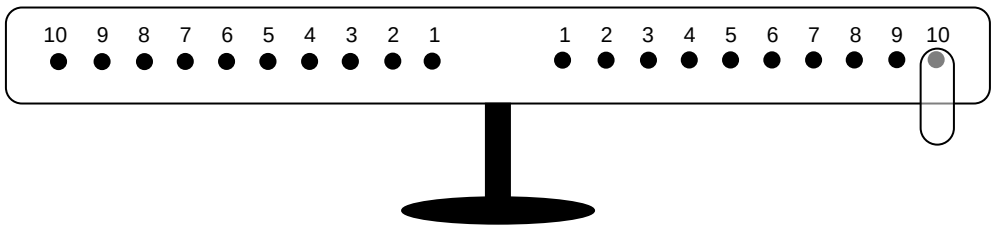
___ + ___ = 10

d.



___ + ___ = 10

e.



___ + ___ = 10

The scales should show the following combinations by having a ring hanging from each of these numbers on the side opposite to the side with the ten bearing a ring on it.

- 1 + 9
- 2 + 8
- 3 + 7
- 4 + 6
- 5 + 5

2. In the following equations, find any which are untrue and change the equal sign to a not equal sign.

a. $3 + 6 = 9$	b. $17 \neq 8 + 8$
c. $16 - 7 \neq 7$	d. $5 + 4 + 3 = 12$
e. $14 - 6 = 8$	f. $13 = 18 - 6$
g. $3 + 9 \neq 7 + 4$	h. $14 - 6 = 4 + 4$
i. $9 + 6 = 6 + 6 + 3$	j. $13 - 7 \neq 18 - 6$

3. Circle the one that does not belong in each set of equations.

a.	$4 + 5$	$12 - 3$	$\underline{6 + 2}$
b.	$16 - 8$	$\underline{15 - 8}$	$4 + 4$
c.	$\underline{13 - 7}$	$7 + 6$	$19 - 6$
d.	$17 - 4$	$9 + 4$	$\underline{5 + 7}$
e.	$\underline{9 + 5}$	$8 + 7$	$19 - 4$

4. Solve the following equations.

   a. 7 + ___6___ = 13

   b. ___5___ + 9 = 14

   c. 9 + 8 = ____17___

   d. ___11___ = 4 + 7

   e. 14 – 8 = ___6___

   f. ___5___ = 12 – 7

   g. 16 – ___7___ = 9

   h. __18____ – 11 = 7

5. Think about what you have learned about Pattern Blocks. Do you remember:
   - 3 green triangles equal 1 red trapezoid?
   - 2 red trapezoids equal one yellow hexagon?

   Make the following into equalities.

   a. __6__ green triangles = 1 yellow hexagon

   b. 2 hexagons = __4__ red trapezoids

   c. __15__ green triangles = 2 yellow hexagons and 1 red trapezoid

6. Look at the following. Explain how you can tell they are true or untrue without actually working out the addition or subtraction.

   a. 11 – 5 = 12 – 6

   Both numbers increased by one in the second equation, so the difference remains the same. Students may explain that when both numbers move up one on the number line, the space in between them is still the same size, or something similar.

   b. 35 + 17 = 37 + 15

   When you compare 35 to 37, it went up 2 and 17 to 15 went down two, so really it is the same amount on each side, the two from 17 were just moved into the pile of 35 to make it all the same number of counters/manipulatives, just reorganized.

Evaluate students based on the following rubric:

Rubric for Equalities and Inequalities			
Not Yet Is not able to do one or more of questions one to four	**Needs More Practice** Has two or more errors in more than one of the questions.	**Achieved** Completes questions one to four with no more than one error in any question. May have completed number five correctly.	**WOW!** Completes questions one to four with no more than one error. Completes numbers five and six demonstrating relational thinking and ability to describe in words the process at least clearly enough to be understood.

B. One-on-one Assessment

See **Sample Structured Interview: Equality and Inequality Outcomes 4, 5 for Grade 2** (p. 35).

Sample Structured Interview: Equality and Inequality Outcomes 4, 5 for Grade 2

Directions	Date:	
	Not Quite There	**Ready to Apply**
Present the student with a balance pan scale and two sets of counters. You may prefer these to be marbles in two identical plastic bags. Ask, **"How can you use this scale to show me whether these are equal amounts?"**	• Counts the manipulatives in each set and compares the two numbers. • Lines the two sets of manipulatives up to check 1:1 correspondence.	• Places the two sets of counters on the two pans and when the pans balance, explains that the sets are equal because the scale tells you both pans are the same or that they weigh the same amount.
Have manipulatives and a balance pan scale available. Say, **"Make two unequal sets and tell me how you know they are unequal."** If the student uses 1:1 correspondence, you can ask the student if there is another way to prove these sets are unequal to check the student's knowledge of the other methods.	• Makes sets that are unequal, but cannot justify the inequality.	• Makes two unequal sets and confirms their inequality through 1:1 correspondence, counts of both sets and comparison of the totals or by placing each set on a pan of the scale and waiting to see if the pans are even or tilted and explains whichever method was used.
Present the student with two sets of manipulatives. These can be on the pans of a balance scale or on a flat surface. Make sure there are extra counters at hand so that the student may change set quantities through addition, as well as subtraction. Ask**, "How can you make this equal set unequal?"** If the student is successful, replace the counters as they were and ask, **"Is there another way you could make the sets unequal?"**	• Can not show you how to make the sets unequal. • Can only do it in one way, such as by taking one away from one of the sets.	• Easily offers a variety of ways to make the sets unequal, including taking varying amounts away and adding varying amounts.

Present the student with three sets of Multilink cubes. These sets may be made up of two sticks of different colours. The sets may be placed on different coloured sheets of paper as mats or in three box lids. Two of the sets should be different combinations for the same number and one a different total, such as 3 + 4 in set one, 2 + 5 in set two and 3 + 3 in set three. Ask, **"Which one of these does not belong with the others and why?"** This can be repeated with a larger sets.	• Can not identify the set with a total sum different than the other two as the one that does not belong. • Identifies correctly the one that does not belong, but is unable to explain why.	• Identifies the one that does not belong correctly and explains that it is unequal to the other sets.

Present the student with two sets of counters and ask them to write the equation that goes with it. Vary the counters used. If you place them on the pans of a scale, students can easily distinguish the sets. If not, place them on two different coloured sheets of paper, so they can easily tell what counters belong in the sets. Begin with two or more addends on one side and a total amount on the other. Then progress to two addends or more on each side. Mix the sets that are equalities and inequalities so that students have to use either the = or the ≠ sign. Do not penalize students who correctly use the < or > signs. This can be complex when students have to watch you and record your action of subtraction. Some samples to use are as follows: a. Three red pattern blocks piled up beside six red pattern blocks on one side and nine red pattern blocks on the other side. b. Fourteen blue Unifix cubes on the left and on the right six green and seven yellow. c. On the left: five black and eight white Unifix cubes. On the right: seven orange and seven blue cubes. d. On the left place fourteen Multilink cubes and on the right place sticks of five and three cubes. Ask the student to record what is there and then to watch you and change the equation to show what happened. Take six cubes away from the group of fourteen on the left.	• Cannot record the equations correctly. • Cannot use the = or ≠ sign appropriately, especially when two or more addends are used on both sides of the equation. • Cannot record subtraction activity on one side or both sides.	• The student accurately records the equations modeled in all formats.

C. Applied Learning

- It is important that the students solve problems that have the variable in all the possible positions for both addition and subtraction.
- Have the students make up problems to go with various equalities or inequalities.

Step 5: Follow-up on Assessment

Guiding Questions

- What conclusions can be drawn from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction?

A. Addressing Gaps in Learning

Once students have shown an area of weakness or a gap, address it through intervention. If the problem shows up in written work, move back to the concrete and check whether the student understands at the concrete level. If so, work on connecting it to the symbolic level directly and with the pictorial representation as an intermediary step. If the student can show the process but is unable to satisfactorily explain it, the problem may be language or the student has only memorized what to do. Be sure that the student's lack of explanation has been checked out orally. If the student can explain the process or justify the answer orally, the problem could be with written language and not the mathematics concepts and skills.

Focus on the basic requirements of the two specific outcomes for this section. What is preventing the student from demonstrating the meaning of equality or inequality with manipulatives? If it is the terms, go back to the section on developing understanding of them and do further work. If it is making the pictorial representations, investigate whether it is a misconception that only equalities can be shown or a lack of knowledge of the symbols to put between the diagrams of the sets. Check whether or not the student has seen inequalities represented in diagrams or pictures. If symbols are an issue, have the student transfer concrete inequalities and equalities on the balance pan scale to their pictorial form, since these do not use the symbols. When these symbols are no longer a problem for the student, start to record the equalities or inequalities as equations, using the symbol.

B. Reinforcing and Extending Learning

Students who have achieved or exceeded the outcomes will benefit from ongoing opportunities to apply and extend their learning. These activities should support students in developing a deeper understanding of the concept and should not progress to the outcomes in subsequent grades.

- As students master addition and subtraction of numbers to one hundred, present them with more equations to assess as equal or unequal based on relational thinking and have the students share their thinking. As an example, students will learn to recognize that 42–23 will have the same difference as 52–33 or 32–13, even though they may not know or have yet worked out the actual difference between these numbers.
- Provide parents with information about the students' work on equalities and inequalities. Include the importance of solving equations with variables in all the possible positions, so that any reinforcement at home includes variety. Parents will especially benefit from understanding that the meaning of the equal sign is "the same as." If parents have been informed of the common misconceptions of an equal sign meaning "the answer is coming" or "do something," they will be better prepared to ensure their children do not maintain such a misconception.

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