Mathematics

Planning Guide

Grade 2 *Place Value to 100*

Number

Specific Outcomes 5, 6, 7

This Planning Guide can be accessed online at:
http://www.learnalberta.ca/content/mepg2/html/pg2_placevalueto100/index.html

Table of Contents

Curriculum Focus 3

What Is a Planning Guide? 4

Planning Steps 4

Step 1: Identify Outcomes to Address 5
- Big Ideas 5
- Sequence of Outcomes from the Program of Studies 6

Step 2: Determine Evidence of Student Learning 7
- Using Achievement Indicators 7

Step 3: Plan for Instruction 8
- A. Assessing Prior Knowledge and Skills 8
  - Sample Structured Interview: Assessing Prior Knowledge and Skills 9
- B. Choosing Instructional Strategies 10
- C. Choosing Learning Activities 11
  - Sample Activity 1: Explaining and Demonstrating with Counters the Meaning of 2-digit Numbers, Including Those with Identical Numerals in Both the Tens and Ones Places ... 12
  - Sample Activity 2: Count the Number of Objects in a Given Set, Using Groups of 10s and 1s, and Record the Result as a 2-digit Numeral Under the Headings 10s and 1s 15
  - Sample Activity 3: Describe a Given 2-digit Numeral in at Least Two Ways 16
  - Sample Activity 4: Illustrate, Using Ten Frames and Diagrams, That a Given Numeral Consists of a Certain Number of Groups of Ten and a Certain Number of Ones 17
  - Sample Activity 5: Illustrate, Using Base 10 Materials, That a Given Numeral Consists of a Certain Number of Tens and a Certain Number of Ones 19
  - Sample Activity 6: Identify and Explain Errors in a Given Ordered Sequence 20
  - Sample Activity 7: Identify Missing Numbers in a Given Hundred Chart 21
  - Sample Activity 8: Identify Errors in a Given Hundred Chart 23
  - Sample Activity 9: Order a Given Set of Numbers in Ascending or Descending Order, and Verify the Result, Using a Hundred Chart, Number Line, Ten Frames or by Making References to Place Value 24
  - Sample Activity 10: Estimate the Number of Groups of Ten in a Given Quantity, Using 10 as a Referent 26
  - Sample Activity 11: Select Between Two Possible Estimates for a Given Quantity, and Explain the Choice 27

Step 4: Assess Student Learning 28
A. Whole Class/Group Assessment 28
B. One-on-one Assessment 34
C. Applied Learning 34

Step 5: Follow-up on Assessment 34
A. Addressing Gaps in Learning 34
B. Reinforcing and Extending Learning 35

Bibliography 36

Planning Guide: *Grade 2 Place Value to 100*

Strand: Number
Specific Outcomes: 5, 6, 7

This *Planning Guide* addresses the following outcomes from the Program of Studies:

Strand: Number	
Specific Outcomes:	5. Compare and order numbers up to 100. 6. Estimate quantities to 100, using referents. 7. Illustrate, concretely and pictorially, the meaning of place value for numerals to 100.

Curriculum Focus

The focus of these three specific outcomes is the development of number sense through experiences with numbers to 100. "Number sense" can be described as a "good intuition about numbers and their relationships. It develops gradually as a result of exploring numbers, visualizing them in a variety of contexts, and relating them in ways that are not limited by traditional algorithms" (Howden 1989, p. 11). Lessons designed for students in meeting these specific learner outcomes develop an understanding of place value, relationships between quantities represented by numerals to 100 and the concept of estimation. Students become more adept at comparing and estimating quantities as they accumulate benchmarks or referents and use groupings to efficiently compare and calculate. Parallel to the development of place value concepts is the development of their communication skills with the language of math in words, symbols and illustrations.

This sample targets the following changes to the curriculum:

- The General Outcome focuses on number sense; whereas the previous mathematics curriculum divided the "Number" strand under the subheadings of "Number Concepts" and "Number Operations." The underlying change is in the greater emphasis now given to making sense of the number system as a way of describing the environment and solving problems. In recognition of the need for students to each construct his or her mathematical concepts, more emphasis is given to students' learning experiences, metacognition and sharing strategies to solve problems or represent mathematically things in their environment. Flexibility and proficiency are expected while allowing for personal preferences. More emphasis is on students actively participating in mathematics to develop understanding, then explaining their thinking to demonstrate their level of development. Correct answers and procedures are recognized as an insufficient foundation for numeracy. This informs the instruction that follows. Assessment is through observation during lessons, student work, and interviews and from performance tasks more often than in the past. Less emphasis is on what marks to assign the student for their performance. More emphasis is on what can we be sure students understand and how this informs our decisions about what instruction should follow for each student or group of students. Are our students through their experiences learning that the number system and mathematics make sense?

What Is a Planning Guide?

Planning Guides are a tool for teachers to use in designing instruction and assessment that focuses on developing and deepening students' understanding of mathematical concepts. This tool is based on the process outlined in *Understanding by Design* by Grant Wiggins and Jay McTighe.

Planning Steps

The following steps will help you through the Planning Guide:

- **Step 1: Identify Outcomes to Address** (p. 5)
- **Step 2: Determine Evidence of Student Learning** (p. 7)
- **Step 3: Plan for Instruction** (p. 8)
- **Step 4: Assess Student Learning** (p. 28)
- **Step 5: Follow-up on Assessment** (p. 36)

Step 1: Identify Outcomes to Address

Guiding Questions

- What do I want my students to learn?
- What can my students currently understand and do?
- What do I want my students to understand and be able to do based on the Big Ideas and specific outcomes in the program of studies?

Big Ideas

- Sets of ten can be regarded as single entities. Quantities can be counted in groups of ten and combined with the units to describe a quantity. For example, 4 sets of ten and 2 units/singles/ones is another way to express 42 individual objects. This is fundamental to our base ten number system.
- The position of the digits within a numeral determines the quantity a digit represents. This is the basis of place value.
- There are patterns in the way numerals are formed so that each decade has a 0 through 9 sequence. There is a similar sequence in the progression through the decades. These and many other patterns are easily observed on a hundred chart. If students know the counting sequence, they do not necessarily know place value.
- A number or value may be expressed in a multitude of ways. For example, 83 can be 8 tens and 3 ones or 7 tens and 13 units (part–whole relationship).
- A referent or known quantity is useful as a benchmark or anchor in making estimates or in making comparisons.
- It is not always necessary to have an accurate count. Sometimes knowing an approximate value is sufficient.
- The size of the groups being counted can change. For example, when counting tens and ones, it is important to be able to count by tens and switch over to counting by ones. Likewise, when counting money, it is helpful to be able to change from counting by 25s, to 10s, then 5s and 1s, as needed.

Sequence of Outcomes from the Program of Studies

See http://education.alberta.ca/teachers/core/math/programs.aspx for the complete program of studies.

Grade 1	Grade 2	Grade 3
Specific Outcomes	Specific Outcomes	Specific Outcomes
5. Compare sets containing up to 20 elements, using: • referents • one-to-one correspondence to solve problems.	5. Compare and order numbers up to 100.	3. Compare and order numbers to 1000.
6. Estimate quantities to 20 by using referents.	6. Estimate quantities to 100, using referents.	4. Estimate quantities less than 1000, using referents.
4. Represent and describe numbers to 20, concretely, pictorially and symbolically.	7. Illustrate, concretely and pictorially, the meaning of place value for numerals to 100.	5. Illustrate, concretely and pictorially, the meaning of place value for numerals to 1000.

Step 2: Determine Evidence of Student Learning

Guiding Questions

- What evidence will I look for to know that learning has occurred?
- What should students demonstrate to show their understanding of the mathematical concepts, skills and Big Ideas?

Using Achievement Indicators

As you begin planning lessons and learning activities, keep in mind ongoing ways to monitor and assess student learning. One starting point for this planning is to consider the achievement indicators listed in the *Mathematics Kindergarten to Grade 9 Program of Studies with Achievement Indicators*. You may also generate your own indicators and use them to guide your observation of the students.

The following indicators may be used to determine whether or not students have met specific outcomes 5, 6 and 7. Can students:

- order a given set of numbers in ascending or descending order, and verify your results using a hundred chart, number line, ten frames or by making references to place value?
- identify and explain errors in a given ordered sequence?
- identify missing numbers in a given hundred chart?
- identify errors in a given hundred chart?
- estimate a given quantity by comparing it to a referent (known quantity)?
- estimate the number of groups of ten in a given quantity, using 10 as a referent?
- select between two possible estimates for a given quantity, and explain the choice?
- explain and show with counters the meaning of each digit for a given 2-digit numeral with both digits the same; e.g., for the numeral 22, the first digit represents two tens (twenty counters) and the second digit represents two ones (two counters)?
- count the number of objects in a given set, using groups of 10s and 1s, and record the result as a 2-digit numeral under the headings 10s and 1s?
- describe a given 2-digit numeral in at least two ways; e.g., 24 as two 10s and four 1s, twenty and four, two groups of ten and four left over, and twenty-four ones?
- illustrate using ten frames and diagrams that a given numeral consists of a certain number of groups of ten and a certain number of ones?
- illustrate, using base ten materials, that a given numeral consists of a certain number of tens and a certain number of ones?
- explain why the value of a digit depends on its placement within a numeral.
- order the digits shown on two dice reliably to make either the smallest or largest numeral possible?

Sample behaviours to look for related to these indicators are suggested for some of the activities listed in **Step 3, Section C: Choosing Learning Activities** (p. 11).

Step 3: Plan for Instruction

Guiding Questions

- What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should I use?
- How will I meet the diverse learning needs of my students?

A. Assessing Prior Knowledge and Skills

Before introducing new material, consider ways to assess and build on students' knowledge and skills related to counting. For example:

- Have students illustrate numbers to 20 with various counters. Ask them if they can show 21, 22, 23, … 29. How would they show 30?
- Ask where they have experienced numbers larger than 20? The calendar is likely to have been a daily experience in kindergarten and Grade 1. Other experiences may range from the number of kids in the class to amount of money in their allowance.
- Check which manipulatives they have experienced. Are they familiar with ten frames and place-value mats, with or without labels or number flips (from 0-9)? Have they counted with beans and portion cups? Have they used Unifix or multilink cubes? Have they used the hundred chart? If so, how?
- Ask students to describe their experiences with estimating in Grade 1. Do they recall the term "estimate" and understand that it means to make a guess as to what amount would be as close to the answer as they can predict. Did they have few or many experiences with estimating and were the numbers limited to 20 and under?
- Have students compare numbers up to 20 and explain how they know which numbers are greater or less than the others in comparisons of 2 numerals, then 3 numerals. Do they use terms "greater than" and "less than" or "bigger than" and "smaller than" or simply "more/bigger" and "less/smaller"? Can students order numerals from largest to smallest, not just smallest to largest?

If a student appears to have difficulty with these tasks, consider further individual assessment, such as a structured interview, to determine the student's level of skill and understanding. See **Sample Structured Interview: Assessing Prior Knowledge and Skills** (p. 9).

Sample Structured Interviews: Assessing Prior Knowledge and Skills

Number Sense, Specific Outcome 7: Place Value

Directions	Date: **Not Quite There**	**Ready to Apply**
Give the student a bag or pile of items such as 49 counters, and ask the student to count them. Ask the student to write the numeral for the amount.	Counts these randomly by ones. Numeral may be written backwards, such as 94 for 49, or not at all.	Organizes counters into groups of ten and the left over ones and can easily recount or check the count. Writes numeral for number with correct tens and ones digits.
Ask the student: What number is ten more than this number? What number is ten less than this number?	Counts on fingers or with counters or refers to the hundred chart or other aids in the room such as a number line.	Readily responds with the correct number for ten more and ten less based upon place value knowledge and can demonstrate by adding a ten stick, not ten units.
Ask the student to make 4 ten sticks. Then set out 3 more cubes. Ask how many cubes are there now.	If the response is 7, you know the student is not ready to see groups of cubes as both one and ten things simultaneously (Van de Walle and Lovin 2006, p. 125).	Replies 43 directly or counts: 10, 20, 30, 40, 41, 42, 43.
On a work space, show 45 tiles lined up in tens and ones. Ask the student, "How many tiles are there?" Then move all the tiles into a pile. Ask the student, "How many tiles are there now?"	Although the student may respond without counting by ones that there are 45 tiles, when presented with the second question he or she begins to recount the tiles by ones or twos. The student may not understand *conservation of number* yet and definitely does not understand the notion of equivalencies.	Responds without counting by ones that there are 45 tiles and recognizes without counting that after the rearranging of the tiles, there are still 45. This notion of equivalencies is essential for many computational strategies (Van de Walle and Lovin 2006, p. 125).

Kathy Richardson in *Assessing Math Concepts: Grouping Tens* (2002) provides a complex interview using three categories: needs instruction, needs practice or ready to apply. She includes more questions than those in the previous chart.

B. Choosing Instructional Strategies

Consider the following guidelines for teaching:

- Provide varied manipulatives, enough for all students to use during lessons.
- Insure that students are familiar with the various manipulatives, allowing time to explore the manipulatives if students have not used them before.
- Teach the same concept or skill with more than one manipulative, so students can generalize that the mathematics involved is applicable in more than the case of a single manipulative.
- Students do not have to be able to spell all the words correctly that they need to communicate their work and thinking to you and others.
- Establish a safe community for risk-taking and for expressing differences. Assure students that making errors gives them opportunities to learn.
- Have manipulative materials available for students to use as needed, but do not force students to use them all of the time. If students develop strategies to solve problems, it is not necessary for them to spend time setting out manipulative representations for every number or problem they encounter. If students make mistakes, ask them to show you the number or problem with their manipulatives. The student is likely to notice and correct the error in doing so.
- Provide time for students to share their answers and reasoning or strategies.
- Be sure that students have the opportunity to work with base ten units that are groupable before they convert to pregrouped ten sticks, single rods that are the length of ten joined centicubes. Likewise, students who are going to be using beans that have been glued onto Popsicle sticks as ten sticks would be better off to begin using beans and portion cups.
- Challenge the students to make and do similar problems with or without models to clarify their explanations.

C. Choosing Learning Activities

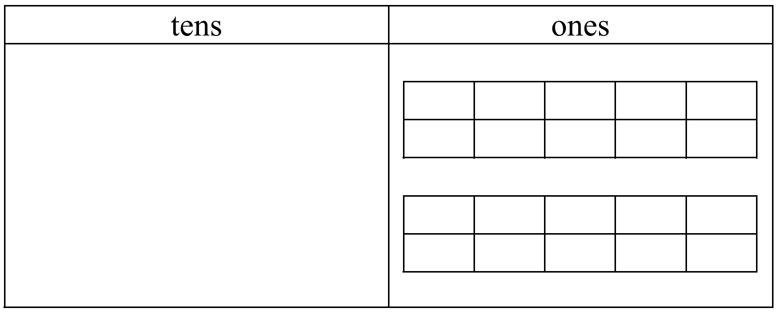
The following learning activities are examples that could be used to develop student understanding of the concepts identified in Step 1.

Sample Activities:

1. **Explaining and Demonstrating with Counters the Meaning of 2-digit Numbers, Including Those with Identical Numerals in Both the Tens and Ones Places** (p. 12)
2. **Count the Number of Objects in a Given Set, Using Groups of 10s and 1s, and Record the Result as a 2-digit Numeral Under the Headings 10s and 1s** (p. 15)
3. **Describe a Given 2-digit Numeral in at Least Two Ways** (p. 16)
4. **Illustrate, Using Ten Frames and Diagrams, That a Given Numeral Consists of a Certain Number of Groups of Ten and a Certain Number of Ones** (p. 17)
5. **Illustrate, Using Base 10 Materials, That a Given Numeral Consists of a Certain Number of Tens and a Certain Number of Ones** (p. 19)
6. **Identify and Explain Errors in a Given Ordered Sequence** (p. 20)
7. **Identify Missing Numbers in a Given Hundred Chart** (p. 21)
8. **Identify Errors in a Given Hundred Chart** (p. 23)
9. **Order a Given Set of Numbers in Ascending or Descending Order, and Verify the Result, Using a Hundred Chart, Number Line, Ten Frames or by Making References to Place Value** (p. 24)
10. **Estimate the Number of Groups of Ten in a Given Quantity, Using 10 as a Referent** (p. 26)
11. **Select Between Two Possible Estimates for a Given Quantity, and Explain the Choice** (p. 27)

Sample Activity 1: Explaining and Demonstrating with Counters the Meaning of 2-digit Numbers, Including Those with Identical Numerals in Both the Tens and Ones Places

1. Provide students with place-value mats that have columns under the headings "tens" and "ones," with 2 tens frames in the ones column, as shown below.



Provide students with counters such as Unifix cubes to make numbers as directed. For the first lesson students will need up to 50 counters and in future lessons up to 100.

You will need to insure that students are familiar with ten frames and know how to place the numbers on them systematically beginning with one in the left box of the upper row. As additional counters are placed on the frame, they are added in the next empty box from left to right until the top row is filled. To make 6, the new counter is placed in the bottom box on the left and additional counters are added in the first empty box on the left until the bottom row is complete and there are 10 counters shown. By using this system, students can develop visualization skills that enable them to identify numbers to 10 without counting by ones. As students use the second ten frame in addition to the first, they will be repeating the work they did in Grade 1 to count to 20. Later, they will be using up to 10 individual ten frames to show quantities to 100; however, on the place-value mat, they will need to have 2 ten frames so that they may use them when adding two quantities. They also will need the space to place counters from a group of ten they needed to dismantle when subtracting a larger ones place number from a smaller number.

Look For …

Do students:

- ☐ know where to place the counters on the ten frame?
- ☐ calculate the number of counters without counting the units by ones?
- ☐ count by tens and then the ones?
- ☐ verify their counts with explanations that include the number of tens and ones?
- ☐ explain why the same digit in the ones place has a different value when it is in the tens place?

It is helpful for students if the teacher uses an overhead projector to demonstrate as the lesson progresses. When using manipulatives such as Unifix, there may not be enough room on the transparency or overhead to show large numbers. You may use the units from base ten blocks on the overhead as a substitute for student Unifix cubes.

Ask the students to show these numbers: 1, 2, 5, 6, 9 and 10. Are students placing the counters on the upper frame in the ones column in the correct sequence? Ask students how they would make 11, 13, 15, 16, 18 and 20. Check to see if they are placing the additional counters on the second frame correctly. Ask students if there is another way to show 20 on the place-value mat. Someone may suggest taking ten and lining them up under the tens column. They may even have you make 2 tens sticks and move them to the tens column.

Now ask students how they would show you 21, 25, 26, 29 and then 30. Students may show you 2 tens sticks and ten counters in a ten frame or may have progressed to adding the third ten stick onto the tens side of the mat.

Continue asking students to make various numbers to 50. Ask them to explain how they know their counters add up to specified number.

2. In the following days' lessons, ask the students to demonstrate numbers up to 100. The latter may be shown as 9 tens and ten units in the ten frame or discussion may ensue about making 10 ten sticks and placing them in the tens column. A student may predict how the pattern in the system will continue by suggesting that 10 ten sticks should make a 100 in another column to the left. Be sure to vary the manipulatives used in these additional lessons so the students see that the mathematical concepts are related to any materials, not just Unifix cubes. Some alternative manipulatives are beans and portion cups (available from paper and restaurant supply distributors), beans and bean sticks (10 beans glued to a Popsicle stick), base ten rods and centicubes, plastic chain links or paper clips; pennies and portion cups for sets of ten. Note that experience has shown that the students who benefit most from using bean sticks are those involved in making them. Also, should you use pennies, note that using dimes for the tens at this point is not helpful to the students who are still in the process of learning that one item can have the value of ten items.

3. There are several additional manipulatives to be added in the lessons that assist students in connecting the quantities/number to the numerals and the number words. The first is number flips. A description of number flips can be found in *Mathematics Their Way* (page 363) and in the blackline masters for that book. The *Numeration* book of the *Alberta Diagnostic Mathematics Program, Division I* provides a blackline master (page 58) for such number cards.

   A further step in preparing students for reading and writing number words is to give them number word cards to place below their mats to correspond to the numbers shown. Students can be given cards with the words one through nine in one colour, eleven through nineteen in another colour and twenty, thirty, forty … ninety in yet another colour. When given hyphens to use in number words from twenty-one through ninety-nine with the exception of the decade numbers, students can make the number words to go with the quantities shown on their place-value charts. These cards can be housed in envelopes and/or plastic bags. This exercise may also be included in an activity centre where students have assigned quantities to show or may throw two ten-sided dice and then make the corresponding quantity and show its number word. A master sheet with the number words to 100 would allow students to self-correct. The manual practice of placing the hyphen between tens and ones from twenty-one through ninety-nine may help students remember to include it when learning to write these number words in Grade 3. Students will also be working to improve their reading of number words.

4. As the place-value lessons are repeated and become more challenging, you will need to ask students to explain what each digit in the 2-digit numeral means or stands for. In the numeral 34, for example, students explain that the 3 represents 3 tens (or 30 counters) and the 4 represents 4 ones (or 4 counters). After a few numerals in which the digits vary, ask the students to make a number in which both digits of the numeral are identical and explain the meaning of each digit (22, 33, 44, 55, 66, 77, 88 or 99).

Look For …

Do students:

- ☐ automatically take the number of tens groups as the tens digit in the number?
- ☐ explain that the tens digit means *that many* tens and can tell you the correct value it represents, such as in 43, the 4 means forty or 4 tens?
- ☐ do they reach for the correct number of units as shown in the ones place?

Sample Activity 2: Count the Number of Objects in a Given Set, Using Groups of 10s and 1s, and Record the Result as a 2-digit Numeral Under the Headings 10s and 1s (Specific Outcome 7, Achievement Indicator b)

1. This is easiest to cover in an activity centre since having bags of concrete items pre-counted for each student requires a great deal of time and many counters. If you are preparing individual bags for students to do a few samples together or as an introduction, older students in the school may willingly volunteer to prepare them. At the activity centre, have place-value mats and any necessary items such as portion cups. The prepared bags of counted items will need to be placed there, along with recording sheets that identify the counters and have columns prepared and labeled for tens and ones. It is advisable that each bag holds a different type of counter so that students do not inadvertently change the counts in each bag. The counters can be those already mentioned in the lessons above or could be others such as buttons, coins, rings, tiles, bread tags, stickers, used stamps, paper clips, crayons, pattern blocks, and various commercial counters such as teddy bears. Remember to supply containers that can be used to organize the groups of ten; e.g., plastic margarine containers or egg carton cups.

2. Another way to have students meet this criterion is to use stickers to make a set on a sheet of card stock, which can be placed under leftover laminating paper. Students can use non-permanent overhead marking pens to circle sets of ten to do their count of tens and ones. The recording sheet might be as follows:

a.

There are

10s	1s

butterflies.

b.

There are

10s	1s

ladybugs.

c.

There are

10s	1s

fish.

Look For …

Do students:

- ☐ draw circles accurately around groups of ten?
- ☐ count by tens and then add the ones to find the total?
- ☐ record the correct digits in the tens and ones places?
- ☐ read these numerals accurately when asked to read their completed sheets?
- ☐ explain how they know they have the right number recorded?

Sample Activity 3: Describe a Given 2-digit Numeral in at Least Two Ways (Specific Outcome 7, Achievement Indicator c)

1. This outcome could have been integrated into many of the previous lessons. As you made numbers, such as 24, you could explain that since it means 2 tens and 4 ones and that is 20 + 4, these are other ways to describe twenty-four. Some students will say, "Twenty-four is 2 groups of ten with 4 left over." As you model recording these numbers, you have the opportunity to label them as "standard form" (or the way we usually write 24) and the expanded forms, those that are stretched out like a rubber band that is extended. The expanded forms are: 20 + 4 or 2 tens and 4 ones. Learning these expanded forms helps students to prepare for their work in operations and to understand equalities by finding many other names for a number. A challenge for early finishers or to the class would be to collect other names for a number. It might be a different number each night and sharing the students' ideas is part of the following day's math routine. In the case of 24, their responses might include some of the following:

5 + 5 + 5 + 5 + 4	10 + 10 + 4
10 +10 + 2 +2	10 + 5 + 5 + 1 + 1 + 1 + 1 +1
26 – 2	10 + 5 + 5 + 1 + 1 + 1 + 2
30 – 6	100 – 76

2. You can check student knowledge of the standard and expanded forms by asking students for these forms in class day after day as a minor review or mini-lesson. Notes can be kept as to who has given you these forms and added to their portfolio or your assessment data. Some teachers find it helpful to have date-stamped sticky notes at hand to take notes about individuals during the school day that can then be filed for reference in the students' assessment files. Another method for saving this data is to prepare a class list with the skill in a column and simply put in the date beside students' names when they have successfully given you the expanded forms during class.

3. Students can do a board relay of "Do You Know Me?" in which selected students are shown a numeral and challenged to be the first to write on the board the two expanded forms of this numeral. They can also be shown the expanded forms and asked to write the standard form. When the game is first played, all students earn a point for their teams when they write the two forms.

4. For a paper and pencil assessment, students may be asked to complete a form such as:

   Complete this chart.

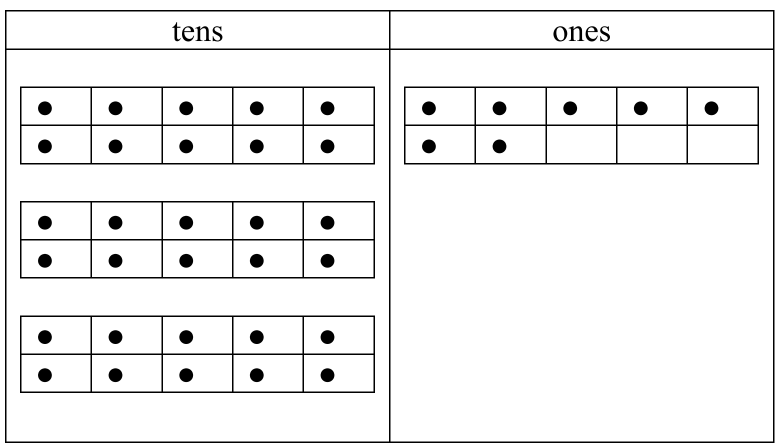
standard form	___tens___ones	_____ + _____
31		
	5 tens 8 ones	
		40 + 0
76		
		20 +3
	6 tens 5 ones	

   If you finish early, write as many other ways to name any of the numbers on this page as you can.

Sample Activity 4: Illustrate, Using Ten Frames and Diagrams, That a Given Numeral Consists of a Certain Number of Groups of Ten and a Certain Number of Ones
(Specific Outcome 7, Achievement Indicator d)

1. Students are given ten individual ten frames and 100 or more counters. They are asked to use them to make the numbers listed on the board, the overhead, a chart or at the top of their recording page. Explain to students that since you can't get around fast enough to see everyone's work, you would like them to record their work on the page. If this is the first time students have had to record ten frames, you may need to take a few minutes to show the students how it can be done as 2 columns with 5 dots down or as 2 rows with 5 dots across, all lined up on the tens side of the place-value mat. The assignment may be as follows:

   Make the numbers 37, 64, 92 and 85 using ten frames. The first example is done for them either on their sheets as below or on the board so that students understand what is expected of their drawings.



   To integrate this with criterion f (explaining why the value of the digit depends upon its placement within a numeral), begin to ask students how they knew what number of complete ten frames they would need and how many dots to place in the incomplete ten frame. It may be helpful to use pennies as counters for this activity, if you are asking the students to illustrate a counter by a dot. It may be easier and faster for them to make a dot than a square representing a cube or a tile. A bingo dabber could help them make a coloured circle faster and be motivating, but it will require their ten frame diagrams to be quite large.

   On-going Assessment:
   In interviews, ask each student, or any whom you are concerned about, how many of these ten frames the child would need to show the number 23, then 65, then 93? If the student answers 2 for 23, ask how many full frames will you have? Will there be any counters left over that would not fit on those frames? So how many counters would you need altogether? The student may have perceived that you only wished to know the number of full frames; however, if the student responded that you would need 5 frames, you can see that it is likely

that the student's place value understanding is not being applied to these two-digit numbers or is not yet developed. Check their answers to the next two numbers just to be sure.

Sample Activity 5: Illustrate, Using Base 10 Materials, That a Given Numeral Consists of a Certain Number of Tens and a Certain Number of Ones (Specific Outcome 7, Achievement Indicators e, f)

If students have not handled base ten blocks in your class previously, take time to develop their understanding of how these materials are related. Ideally, all students should begin with centicubes that will interlock to make ten sticks. Students need to see that ten individual centicubes can be joined together to create a ten stick, sometimes referred to as a "rod." Since these ten sticks take time to make and break apart easily while being moved, it is handy to substitute sticks the same length that are single pieces of plastic. When it is necessary to break up a ten stick, students can substitute for the unitary ten stick. In taking time to make this connection, you help prevent student confusion over why one long rod is called ten. Students gradually come to understand that "ten" is not a digit, but a label for a grouping of nine plus one more. These sets or groups may be made and dismantled, as needed.

Once students are ready to use the centicubes and rods with some understanding of their relative values, ask the students to represent the following numbers on their place-value mats: 35, 53, 18, 81 and 50.

- After the students have displayed the base ten blocks required to make each number, ask the students how they knew how many rods and centicubes to place on their mats.
- After they have made the first two numbers and explained how they knew how many blocks to display, ask them if 35 is the same as 53.
- Then ask them why not, as both numerals are made up of a 5 and a 3?
- Let students verbalize that the position of each digit in the numeral determines its value.

Sample Activity 6: Identify and Explain Errors in a Given Ordered Sequence (Specific Outcome 5, Achievement Indicator b)

The level of difficulty in this task lies in the size of the numbers used and the complexity of the sequences. Early in the year, students should be ready to identify and explain errors in a sequence that is counting by ones, fives and tens to 100 forward or by ones backward from 20 to 0, as these are skills from the Grade 1 curriculum. They should also be able to count by 2s from 0 to 20.

1. Sample sequences to begin the year (ask students to identify and explain errors):
   a. 4, 6, 8, 10, 12, 16, 18, 20
      Do they notice a number is missing and the sequence jumps four between 12 and 16?
   b. 0, 2, 4, 6, 9, 11, 13, 15, 17, 19
      Do they notice that the sequence added 3 to 6; then returns to counting by 2s?
   c. 51, 52, 53, 54, 55, 56, 58, 59, 60
   d. 35, 30, 35, 50, 60, 65, 70, 75, 80
   e. 10, 20, 30, 40, 50, 60, 80, 90, 100

   As the year progresses, this activity should be done repeatedly with more complex patterns. Integrate this with their developing knowledge of sequences as in Number Sense Specific Outcome 1, in which they count forward and backward from 0 to 100 by 2s, 5s and 10s using varying starting points. They also learn to count by 10s using varying starting points from 1 through 9 and to count by 2s using odd numbers. This also integrates with the students' work on increasing patterns in Patterns and Relations.

2. An ideal way to integrate this outcome regularly is to have a hundred chart that is made from clear strips of acetate over canvas stitched into a hundred pockets (10 X 10), into which fit number cards 1 to 100. Each day, place a sequence in the hundred pocket chart that has an error and ask students to spend a few minutes finding and preparing to explain the error. When class begins, have them share their answers.

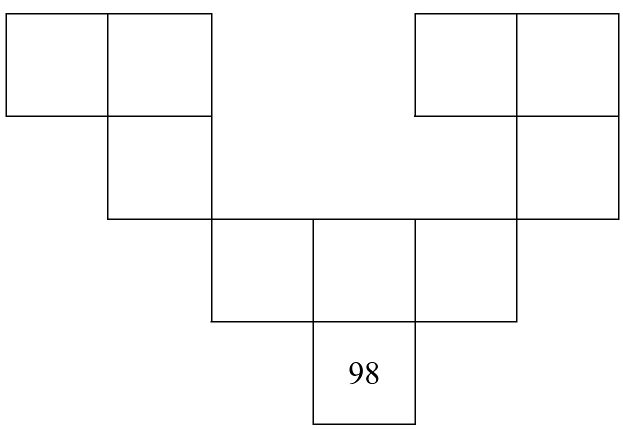
3. Students like to contribute and play teacher. Ask the students to think up number sequences that are familiar to the class and write them up with an error. Have them write the correct sequence and explain the error. Select from these student sequences for the day's challenge.

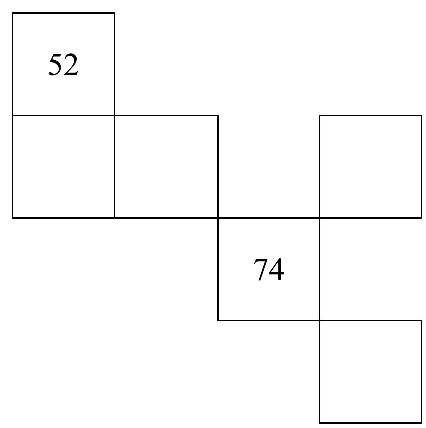
Sample Activity 7: Identify Missing Numbers in a Given Hundred Chart (Specific Outcome 5, Achievement Indicator c)

1. This can be done on a hundred chart as described above by just turning over the numbers you wish to be absent or placing a piece of coloured paper over them. Students tell you what the missing numbers are and you can pull out the coloured paper or reverse the number card so they can confirm their answers. Alternatively, you can have students write the missing numerals as you point to them.

   An overhead transparency of a hundred chart can be used instead of a pocket chart by obscuring numerals with opaque counters or pennies.

2. This skill can be practised on paper with just fragments of a blank hundred chart on which you have written in only a few numerals (Van de Walle and Lovin 2006). These could be made up as a station activity and students would print the missing numerals with a non-permanent marking pen on either acetate overlays or a transparency made from the original. They can also be configured with square sticky notes rather than grid paper. All squares must touch one another on either a side or at a vertex. Copies of these puzzle fragments could be available so that all students could do them at the same time or at various times, as either a mandatory or free-time activity. The following are examples of hundred chart puzzles:





3. Give the students a section of a blank hundred chart with just one number shown and ask them to find the neighbouring numerals.

		48	

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Sample Activity 8: Identify Errors in a Given Hundred Chart (Specific Outcome 5, Achievement Indicator d)

1. Again the pocket hundred chart can be used. Scramble the numbers and have the students find the errors. Explanations can be oral or students can record the errors and their explanations.

2. Create an activity centre with hundred charts that contain a single error or multiple errors. Indicate if the charts have more than one error or students may stop looking for errors after finding the first one. Note that it is difficult for students to make corrections to the chart if you leave out a complete row of numerals. Determine where you wish students to record their explanations of the errors, for example, under a personal copy of the chart or on another page.

Sample Activity 9: Order a Given Set of Numbers in Ascending or Descending Order, and Verify the Result, Using a Hundred Chart, Number Line, Ten Frames or by Making References to Place Value (Specific Outcome 5, Achievement Indicator a)

1. Ask the students to identify the smaller number in given pairs of numbers up to 100. Ask the students to explain how they know the number is smaller. This is a good opportunity to introduce or re-introduce the greater than (>) and less than (<) signs.

2. Have the students toss two ten-sided dice to make a numeral (or toss one twice), designating which throw is to be the tens digit and which the ones. Repeat to make a second numeral. Have the students make the numbers with base ten blocks or other manipulatives on place-value mats or with ten frames and compare the two numbers. To record their findings, they will use the numerals and the greater than or less than symbol. The recording sheets need boxes for recording the two numbers and a space between them for the symbol, as shown below:

10s	1s

10s	1s

   Ask the students to make two or three sets, compare them and record their results.

3. Have students in groups of three throw their dice to make a 2-digit numeral each and, using manipulatives, demonstrate the corresponding quantities. Ask the students to record their findings, arranging the numerals from smallest or least to greatest. It is important to use the word "least" in this work, so that when students are asked to arrange numbers from "least to greatest" or "greatest to least," they are clear on what is being asked. Challenge the students to find other ways to compare sets of numbers besides making them with manipulatives. Most students will be on the lookout for other ways at this point to avoid the labour involved in constructing the numbers with manipulatives each time. Ask the students to share how they knew which numbers were greatest and which were least. Students can be asked to work cooperatively to make a record of their game, including written explanations for one or two sequences of numbers.

Look For …
Do students:
- ☐ compare with base ten blocks or other manipulatives?
- ☐ verify their sequences by referring to base ten concepts?

4. Show the students a sequence of numerals on the board or an overhead. Ask the students to order them by ascending or descending value. Alternatively, to involve the students physically, print the numerals on sheets of paper and give them to the students, who are asked to line themselves up at the front of the room in order from least to greatest or from greatest to least. The numbers could be written on paper crowns that are placed on the students' heads and a student or group from the class could be asked to order the numerals by moving the students around. It is important to interchange between ascending and descending order so that students become accustomed to paying attention to the directions. Sometimes

students should be asked to record the list vertically, rather than horizontally, as some students find it difficult to interpret the order when the axis is altered.

Sample Activity 10: Estimate the Number of Groups of Ten in a Given Quantity, Using 10 as a Referent (Specific Outcome 6, Achievement Indicator b)

1. As the focus in math changes with different strands, adjust the estimation activities to match the strand. For example, when doing Measurement, students can estimate their height, then that of various adults in the school. They can estimate the number of steps to the gym or the time it will take them to tie their shoes. They can estimate the number of grams of spice in a particular container or the number of millilitres of liquid in a bottle. Another strategy for using referents to improve one's estimates is to know the quantity in a smaller container and then use that knowledge to predict the amount a larger container will hold. Have the students begin by estimating the number of groups of ten in a container, picture or measurement. If the container holds marbles, have the students place ten marbles in another container and use the information about the size of container that 10 marbles filled to make a better estimate. As students study mass, they can estimate how many groups of ten paperclips have an equivalent mass to a pattern block triangle. Then they can use that information to estimate how many groups of ten paperclips would be needed to have a mass equivalent to a pattern block square or trapezoid.

2. Students can estimate the number of groups of ten in pictures. The students would circle only one group of 10 and then estimate how many groups of 10 the picture contains.

3. When students are working on measurement, they can estimate how many rods or decimetre strips they will need, and then how many centicubes, as they prepare to measure the width of items such as their desks, a door or a book.

Sample Activity 11: Select Between Two Possible Estimates for a Given Quantity, and Explain the Choice (Specific Outcome 6, Achievement Indicator c)

1. Ask the students which estimate they deem more accurate for a given pile of counters or items. The estimates need to be obviously distinct and one more clearly suitable than the other. Students can be shown a pile of counters and asked whether 30 or 90 counters is a better or closer estimate. If the students have handled those counters before, they should be able to determine which estimate is better.

2. While studying other strands such as Measurement, have the students select different units. For example:
   a. Would you measure the soccer field in centimetres or metres?
   b. Would you measure the time it takes you to tie your shoes in minutes or hours?
   c. Would you measure the distance to Vancouver in metres or kilometers?
   d. Would you measure your mass in tonnes or kilograms?
   e. Would you measure the mass of a spice container in grams or kilograms?

Step 4: Assess Student Learning

Guiding Questions

- Look back at what you determined as acceptable evidence in Step 2.
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

In addition to ongoing assessment throughout the lessons, consider the following sample activities to evaluate students' learning at key milestones. Suggestions are given for assessing all students as a class or in groups, individual students in need of further evaluation, and individual or groups of students in a variety of contexts.

A. Whole Class/Group Assessment

The following are two examples of performance-based assessment tasks that could be used to evaluate student understanding of two of the specific outcomes. Included are rubrics to be used with each assessment task.

Math 2 Number Sense: Place Value Performance Assessment

1. The children in Lakeside School wanted to find ways to recycle. They decided to recycle books they owned and would not read again. Hélène was in the Grade 2 classroom. Jamie was in the Grade 1 classroom. The Grade 2 class had brought 64 books to recycle. When Jamie came to meet Hélène to walk home at the end of the day, Jamie told Hélène that the Grade 1 class had brought in the same amount of books as the Grade 2 class, but when Hélène walked by Jamie's classroom door, a sign said they had recycled 46 books.

   Pretend you are Hélène and want to explain to Jamie whether the classes are tied or not. Use diagrams of two manipulatives that you would use to show Jamie how you know you are correct.

   Write what you would say to Jamie as you show Jamie these manipulatives.

   Why do you think Jamie thought that the two classes were tied?

Rubric for assessing student learning relative to Number Sense: Specific Outcome 7

	Insufficient or Blank	Not yet	Yes, but	Yes	Wow!
diagram	Lack of evidence of student learning from the diagram or diagram lacking.	No use of base ten groupings. Student may have drawn 46 and 64 books.	Drew one depiction of the two numerals, such as on ten frames or shows two sets of manipulative comparisons, but is inaccurate.	Represented both numerals accurately, using two different manipulatives with place-value mats.	Drew both a ten frame diagram and another base ten representation, knowing the frames would be most easily understood by a Grade 1 student. The base ten depiction is on a place-value mat with tens and ones labelled.
Written answers	Lack of evidence of student's learning based upon the written explanation or no explanation was written.	Explanation is based upon counting by ones and comparing quantities in total, not tens and ones. No generalization referring to digit position changing the value.	Reveals some comprehension of the variation in value due to digit position in the numeral, but gaps in the explanation make it difficult to understand and the generalization is absent or vague.	Description of tens and ones, some effort to compare quantities, and an attempted generalization communicates the idea that digit value is dependent upon position.	Clear description of the tens and ones and comparison of numbers. Also, a concise generalization about the order of digits impacting value.

Math 2 Number Sense: Specific Outcome 5: Comparing and Ordering Numbers Performance Assessment

1. The teacher needs help marking and recording marks. The teacher asked you to be her assistant. Your first job is to check the following students' hundred charts to see if they are filled in correctly. Please circle with a coloured pencil, marker or pen any numerals that are incorrect.

a.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	98	90
91	92	93	94	95	96	97	89	99	100

b.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	91	20
21	22	23	24	25	26	27	28	29	30
31	32	32	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
70	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

c.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	33	44	55	66	77	88	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	93	95	96	97	98	99	100

2. The second job you are to do for the teacher is to put the marks in order.

   These are the marks from the five tests that need to be put in order from the highest to the lowest mark: 75, 93, 86, 68, 79.

   ________, ________, ________, ________, ________

   These are the marks from the other five tests that need to be put in order from the least to the greatest mark: 85, 79, 67, 82, 97.

   ________, ________, ________, ________, ________

3. When the hundreds charts were given back to the students, the student with the first chart asked you to explain why you marked the numerals wrong. Print what you would tell that student on the lines below.

   ____

   ____

   ____

   ____

   ____

   ____

   ____

Rubric for Number Sense, Specific Outcome 5, Comparing and Ordering Numbers Performance Assessment

	Not Yet	More Practice Needed	Set to Go
Question 1: Spotting errors in the hundred chart	Missed three or more errors in hundred charts.	Missed one or two errors in hundred charts.	Found all errors in hundred charts.
Question 2: Ordering two sets of five numbers highest to lowest and least to greatest	One or more errors in sequencing the numbers.	Listed numbers in sequence, but not the correct sequence according to the directions.	All numbers in order according to the directions.
Question 3: Explaining an error in the hundred chart based upon place-value understanding	No answer or explanation makes no reference to place value. It may state the numerals don’t follow the pattern or the numerals are backwards.	Explanation includes that 98 is larger than 89 or that 89 is smaller than 98.	Explanation includes 98 means 90 + 8, but 89 means 80 + 9, making the latter less than the former or some expression of the generalization that position of the digits in the numeral changes its value.

B. One-on-one Assessment

One of the easiest ways to spot students having difficulty recognizing the relationship between the position of a digit and its place value is playing a dice game where students make the largest number from the digits shown on the die. Students who do not make the largest (or smallest, as directed) number reliably from the two dice tossed can be seen to need additional work before developing this concept. If a student has not performed satisfactorily on the first performance assessment above, a game with the teacher will show you if the student does know that the position of the digit influences its value or not. If the student reliably selects the larger number to be the tens digit when asked to make the largest number and the smaller number when asked to make the smallest number, you know the student understands the concept. What the student may be lacking is:

- understanding of the performance task
- concentration and stamina to complete the task
- language and organization necessary to describe his or her thinking
- fine motor skills to print the length of communication required
- experiences with manipulatives as a basis for the diagrams or
- health, energy and desire to perform the task on that particular occasion.

Based upon your knowledge of the student, you may be able to diagnosis which of these is most likely and remediate accordingly. If you think the student has fine motor issues, try asking the student to do a similar problem with the base ten materials in the room while explaining his or her thinking verbally.

There are additional structured interviews in the *Alberta Diagnostic Mathematics Program, Division I* book *Numeration* that may be helpful for students who do not seem ready for the Grade 2 curriculum.

C. Applied Learning

When students are counting over 20 items, provide opportunities for them to group the items into sets of ten and organize their counts and comparisons accordingly.

- For example, if students have been asked to estimate the number of buttons in a jar, when it comes time to find the winner, allow students to count the buttons by making groups of ten in portion cups and then count by tens before adding the ones leftover.
- In this way, several students can count tens and several others can check their counts.
- The whole class can finalize the check of the final number by counting the tens and then adding the ones.
- The students can explain how to write the numeral for this amount based upon the tens and ones counts.
- Have the students measure their heights using base ten rods and centicubes to encourage them to count tens and then ones.
- Counting pennies to find the total value is another job students can do both at home and at school.

- If there is a school penny drive or other collection in which there are many pennies collected, the students can count groups of ten into small portion cups until they have ten groups of ten and transfer these into a larger portion cup to be counted as a dollar.
- On occasion, items such as brochures or poppies arrive in the office and need to be divided into classroom quantities.
- Ask the office if your Grade 2 students can be the accountants.
- Have various teams make piles of tens and then the necessary ones to make each class's pile.
- Have the teams rotate so a second team of Grade 2 accountants audits their counting.

Step 5: Follow-up on Assessment

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction?

A. Addressing Gaps in Learning

If a student continues to have problems with place value, return to making various numbers to 99 with manipulatives and connecting them with the numerals they represent both with number flips and pictorially.

- Begin with numbers closer to 20 than 100 and gradually increase the quantities.
- Continue the activity and ask probing questions so that students can develop some meaning for the numbers and an understanding of the relationship between the numbers and their corresponding numerals.
- Begin with the easier questions such as, "What numbers are one more/less?" and move on to questions such as, "How much would we have if we added ten?" Check that the student's difficulty with an activity is not a result of some other learning challenge and not actually conceptual.
- If the student has difficulty with scribing, be sure to provide opportunities for the student to assess the number of items from a picture or diagram and show the numeral with sets of cards.
- The student can also ascertain the quantity on various place-value mats, rather than from diagrams. Van de Walle and Lovin (2006) point out that although physical models for base-ten concepts play a key role in helping students understand the idea of "a ten," students must construct the concept and impose it on the model; the model does not show the students the concept (p. 127).

If students have language or organizational problems that make it difficult for them to explain their actions and thoughts, provide question prompts that lead them from step to step, such as:

- What does the 3 in 37 mean?
- Which manipulatives should you use to represent the 3?
- How many do you need?
- Where will you place them on the place-value mat?
- What does the 7 stand for?
- Which manipulatives should you use for the 7?
- How many do you need?
- Where will you place them?
- Show the numeral with the number flips.
- How is it read? Repetition and gradually reducing prompts will help students be able to use the language of math to express themselves.

- If students are reversing digits in written numbers, for example printing 42 for 24, draw attention to the difference in value based upon the numeral's place.
- Ask them if their allowance were 75¢, would they be satisfied if their parents gave them 57¢ instead?
- If the student has not just made a careless error and really does not understand place value, he or she will need to be shown the difference between these quantities with pennies and perhaps even some examples of how much more could be bought with the greater value.
- In this case, return to more manipulative work with smaller quantities.

B. Reinforcing and Extending Learning

Students who have achieved or exceeded the outcomes will benefit from ongoing opportunities to apply and extend their learning. These activities should support students in developing a deeper understanding of the concept and should not progress to the outcomes in subsequent grades.

Consider strategies such as the following:

- Challenge students to make up as many combinations of tens and ones for a number as possible. Students might keep their own notebooks, beginning with 21 and add to this until they reach 99 or 100. For example, under 21 they would record 2 tens 1 one and 1 ten and 11 ones; under 32 they would record 3 tens + 2 ones, 2 tens + 12 ones and 1 ten and 22 ones. It may be helpful to the whole class if students contribute these examples to a large classroom chart.
- Encourage parents to give students opportunities to count quantities that range from 25 to 100, and provide means of organizing the items into tens and ones. For example, pennies, empty pop cans, pieces of pasta like macaroni, cereal, cards (Christmas or business), buttons, paper clips and small candies. Small bowls, margarine or yogurt containers, envelopes and plastic bags are all suitable for organizing groups of ten. Paper clips can be slipped together to make strings of ten.
- Encourage the students and their families to take part in preparing containers for estimation. If the family has a container that is transparent they can do without, have them fill it with small things to be counted and send it to school. Students and their families should do a count of the items they send beforehand and submit their counts. After the children have predicted the quantity in the container and the count has been verified by the students, the container should be refilled with at least one other set of items to be counted.
- Encourage parents to ask their children to estimate as they go through their household activities each week. For example, students could estimate how many litres of gasoline the car will take, how much the grocery or restaurant bill will be, how long it will take to play a game, and how tall each member of the family is in centimetres. With repeated opportunities for making estimates, the students should be able to give more and more accurate estimates. The second time students estimate how many litres of gasoline the family vehicle will take, they may confirm that it is as empty as last time and then make an estimate.

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