Mathematics

Planning Guide

Grade 4 *Addition and Subtraction*

Number

Specific Outcome 3

This Planning Guide can be accessed online at:
http://www.learnalberta.ca/content/mepg4/html/pg4_additionandsubtraction/index.html

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Planning Guide: *Grade 4 Addition and Subtraction*

Strand: Strand
Specific Outcome: 3

This *Planning Guide* addresses the following outcomes from the Program of Studies:

Strand: Number	
Specific Outcome:	3. Demonstrate an understanding of addition of numbers with answers to 10 000 and their corresponding subtractions (limited to 3- and 4-digit numerals) by: • using personal strategies for adding and subtracting • estimating sums and differences solving problems involving addition and subtraction.

Curriculum Focus

This sample targets the following changes in the curriculum:

- The General Outcome focuses on number sense, whereas the previous mathematics curriculum specified applying arithmetic operations on whole numbers and illustrating their use in creating and solving problems.
- The Specific Outcome focuses on using personal strategies, whereas the previous mathematics curriculum specified connecting manipulatives, diagrams and symbols.
- The Specific Outcome includes estimating sums and differences, whereas the previous mathematics curriculum did not include estimating.

What Is a Planning Guide?

Planning Guides are a tool for teachers to use in designing instruction and assessment that focuses on developing and deepening students' understanding of mathematical concepts. This tool is based on the process outlined in *Understanding by Design* by Grant Wiggins and Jay McTighe.

Planning Steps

The following steps will help you through the Planning Guide:

Step 1: Identify Outcomes to Address

Guiding Questions

- What do I want my students to learn?
- What can my students currently understand and do?
- What do I want my students to understand and be able to do based on the Big Ideas and specific outcomes in the program of studies?

Big Ideas

Principles and Standards for School Mathematics states that computational fluency is a balance between conceptual understanding and computational proficiency (NCTM 2000, p. 35). Conceptual understanding requires flexibility in thinking about the structure of numbers (base ten system), the relationship among numbers and the connections between addition and subtraction. The ability to generate equivalent representations of the same number provides a foundation for using personal strategies to add and subtract, recognizing that for some problems either operation may be used. Computational proficiency includes both efficiency and accuracy. Personal strategies must be compared and evaluated to derive methods that are efficient as well as accurate.

Understanding the operations of addition and subtraction and the connections between them is crucial. John Van de Walle states, "Addition names the whole in terms of the parts, and subtraction names a missing part" (2001, p. 107). Only addition is used in finding the whole when given the parts; however, either addition or subtraction may be used in finding a missing part when given the whole and the other part(s). He goes on to say that addition and subtraction problems include three main types:

- problems involving change—changing a number by adding to it or taking from it
- part–part–whole problems—considering two static quantities either separately or combined
- comparison problems—determining how much two numbers differ in size.

Recognizing which numbers in a problem refer to a part or to a whole helps the students see the inverse relationship between addition and subtraction and understand their properties (Willis et al. 2006, p. 30). By using a variety of problems, the students will construct their own meaning for the inverse relationship between addition and subtraction and for the following properties:

- commutative property of addition—numbers can be added in any order
- identity element for addition—any number added to zero remains unchanged.

By solving problems in contexts that relate to their own lives, the students use their prior knowledge to make sense out of the problem, estimate the answer and use computational strategies that they are able to explain and justify. The students should always be encouraged to estimate prior to calculating the answer. The students' understanding of addition and subtraction

is enhanced as they develop their own methods and share them with one another, explaining why their strategies work and are efficient to use (NCTM 2000, p. 220).

The use of manipulatives or models helps the students understand the structure of the story problem and also connects the meaning of the problem to number sentence (Van de Walle 2001, p. 108). To develop understanding of the meaning of operations, the students connect the story problem to the manipulatives, to the number sentence and then use personal strategies to solve the problem. Van de Walle states:

> "It is useful to think of models, word problems, and symbolic equations as three separate languages. Each language can be used to express the relationships involved in one of the operations. Given these three languages, a powerful approach to helping children develop operation meaning is to have them make translations from one language to another" (2001, p. 109).

Sequence of Outcomes from the Program of Studies

See http://education.alberta.ca/teachers/core/math/programs.aspx for the complete program of studies.

Grade 3

Specific Outcomes

8. Apply estimation strategies to predict sums and differences of two 2-digit numerals in a problem-solving context.
9. Demonstrate an understanding of addition and subtraction of numbers with answers to 1000 (limited to 1-, 2- and 3-digit numerals), concretely, pictorially and symbolically, by:
   - using personal strategies for adding and subtracting with and without the support of manipulatives
   - creating and solving problems in context that involve addition and subtraction of numbers.

Grade 4

Specific Outcomes

4. Demonstrate an understanding of addition of numbers with answers to 10 000 and their corresponding subtractions (limited to 3- and 4-digit numerals) by:
   - using personal strategies for adding and subtracting
   - estimating sums and differences
   - solving problems involving addition and subtraction.

Grade 5

Specific Outcomes

2. Use estimation strategies, including:
   - front-end rounding
   - compensation
   - compatible numbers

   in problem-solving contexts.

Step 2: Determine Evidence of Student Learning

Guiding Questions

- What evidence will I look for to know that learning has occurred?
- What should students demonstrate to show their understanding of the mathematical concepts, skills and Big Ideas?

Using Achievement Indicators

As you begin planning lessons and learning activities, keep in mind ongoing ways to monitor and assess student learning. One starting point for this planning is to consider the achievement indicators listed in the *Mathematics Kindergarten to Grade 9 Program of Studies with Achievement Indicators*. You may also generate your own indicators and use them to guide your observation of the students.

The following achievement indicators may be used to determine whether the students have met this specific outcome.

- Explain how to keep track of digits that have the same place value when adding numbers (limited to 3- and 4-digit numerals).
- Explain how to keep track of digits that have the same place value when subtracting numbers (limited to 3- and 4-digit numerals).
- Describe a situation in which an estimate rather than an exact answer is sufficient.
- Estimate sums and differences using different strategies; e.g., front-end estimation and compensation.
- Solve problems that involve addition and subtraction (limited to 3- and 4-digit numerals) and decide on the reasonableness of the answer by connecting it to the estimated sum or difference.
- Recognize related problems that can help with the problem.
- Know what each number in the problem means in relation to a part or a whole.
- Solve problems that involve addition and subtraction in more than one way, limited to 3- and 4-digit numerals. For example, 385 + ☐ = 500 or 500 – 385 = ☐ .
- Use the relationship among operations to improve proficiency in problem solving.
- Know and draw on number facts and other number relationships.
- Use the structure of the base ten number system to calculate sums and differences; e.g., adds 300 to 3689 efficiently and explains the process using place value.
- Explain how a personal strategy for adding and subtracting works and apply it to another similar problem (limited to 3- and 4-digit numerals).
- Create a different personal strategy for adding and subtracting and decide which strategy is most efficient in solving problems.
- Create a problem given a number sentence for addition or subtraction.
- Analyze a personal strategy created by another person and decide if it makes sense in solving an addition or a subtraction problem.
- Solve problems that involve addition and subtraction of more than 2 numbers.

Some sample behaviours to look for in relation to these indicators are suggested for many of the instructional activities in **Step 3, Section C, Choosing Learning Activities** (p. 12).

Step 3: Plan for Instruction

Guiding Questions

- What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should I use?
- How will I meet the diverse learning needs of my students?

A. Assessing Prior Knowledge and Skills

Before introducing new material, consider ways to assess and build on the students' knowledge and skills related to counting. For example:

- Model the addition of 127 and 48 using concrete or visual representations and record the process symbolically.
- Subtract 48 from 73 by modelling the subtraction using concrete or visual representations and record the process symbolically.
- Create an addition or subtraction story problem for the number sentence:
  33 – 18 = ☐ or 18 + ☐ = 33.
- Determine the sum of 185 and 25 using a personal strategy and explain how the strategy works.
- Subtract 39 from 278 using a personal strategy and explain how the strategy works.
- Solve the following problems in more than one way using personal strategies and explaining how the strategies work:
  - After receiving 16 hockey cards from a friend, Nicholas has a total of 135 hockey cards. How many hockey cards did he have before receiving some from his friend?
  - Jimmy has 57 cents and Mary has 85 cents. Mary has how many more cents than Jimmy?

If a student appears to have difficulty with these tasks, consider further individual assessment, such a structured interview, to determine the student's level of skill and understanding. See **Sample Structured Interview: Assessing Prior Knowledge and Skills** (p. 9).

Sample Structured Interview: Assessing Prior Knowledge and Skills

Directions	Date:	
	Not Quite There	**Ready to Apply**
"Add 127 and 48 using the base ten blocks or drawing diagrams. Write a number sentence to show what you have done."	• Has difficulty representing the numbers with the blocks or diagrams. • Represents the numbers with the blocks or diagrams but does not regroup to show 1 hundred, 7 tens and 5 ones. • Represents the numbers with blocks or diagrams and regroups but does not write a number sentence to record the process symbolically.	• Represents the numbers with the blocks, regroups correctly and records the process symbolically.
"Subtract 48 from 73 using the base ten blocks or diagrams. Write a number sentence to show what you have done."	• Has difficulty representing the numbers with the blocks or diagrams. • Represents the numbers with the blocks or diagrams but does not regroup to show 73 to show 6 tens and 13 ones. • Represents the numbers with blocks or diagrams and regroups but does not write a number sentence to record the process symbolically.	• Represents the numbers with the blocks, regroups correctly and records the process symbolically.
"Create a story problem for the number sentence: 18 + ☐ = 33."	• Creates a story problem using the numbers provided but the meaning of the story is not represented by the number sentence. • If the story requires the addition of 18 and 33, understanding of the number sentence is not there. If the story requires the subtraction of 18 from 33 using a 'take away' situation, understanding of the number sentence is not there, even though the answer in each case is the same.	• Creates a story problem that is represented by the number sentence. For example: I have 18 cents. My mother gives me some more money and now I have 33 cents. How much money did my mother give to me?

"Solve the following problem in more than one way, using strategies that make sense to you and explain how the strategies work: After Nicholas received 16 hockey cards from a friend, he had a total of 135 hockey cards. How many hockey cards did he have before receiving some from his friend?" Provide base ten blocks for the student to use, if necessary. "Solve the following problem in more than one way, using strategies that make sense to you and explain how the strategies work: Jimmy has 57 cents and Mary has 85 cents. Mary has how many more cents than Jimmy?"	• Has difficulty representing the numbers with the blocks or diagrams. • Represents the numbers with the blocks or diagrams but cannot translate the concrete representation into a personal strategy using symbols. • Represents the numbers with blocks or diagrams and records the process symbolically using a personal strategy but does not explain how the strategy works. • Uses a personal strategy to solve the problem and explains why this strategy leads to a correct answer but is unable to solve the problem a different way.	• Uses a personal strategy to solve the problem and explains why this strategy leads to a correct answer. • Recognizes that the first problem can be an addition problem by adding on to 16 to get 135 and it can also be a subtraction problem by taking 16 away from 135. • Recognizes that the second problem can be a subtraction problem by subtracting 57 from 85 and it can also be an addition problem by adding on to 57 to get 85.

K What facts do I **KNOW** from the information in the problem?	**N** Which information do I **NOT** need?	**W** **WHAT** does the problem ask me to find?	**S** What **STRATEGY** will I use to solve the problem?

Adapted from Mary Lee Barton and Clare Heidema, *Teaching Reading in Mathematics: A Supplement to Teaching Reading in the Content Areas Teacher's Manual* (2nd ed.) (Aurora, CO: McREL [Mid-continent Research for Education and Learning], 2002), p. 113. Adapted with permission of McREL.

B. Choosing Instructional Strategies

Consider the following guidelines for teaching addition and subtraction:

- Teach in a problem-solving context. Research shows that by solving problems using addition and subtraction, the students create personal strategies for computing and develop understanding about the relationship between the operations and their properties (NCTM 2000, p. 153).
- Choose problems that relate to the children's own lives (Van de Walle 2001).
- Provide a variety of problems representing the different addition and subtraction situations with varying degrees of difficulty to differentiate instruction.
- Work with the whole group initially and have the students paraphrase the problem to enhance understanding (Willis et al. 2006) and to recognize which numbers in a problem refer to a part or to a whole.
- Have the students estimate the answer to the problem before calculating so that they are better able to determine the reasonableness of their answers.
- Have base ten materials available for the students to use as needed.
- Provide time for the students to create their personal strategies to solve the problem and share these strategies with members of their group or with the entire class.
- Guide the discussion by asking questions to encourage thinking about number relationships, the connection between addition and subtraction, and their personal strategies.
- Have the students compare their answers to the estimates that they made before solving the problems.
- Challenge the students to solve the problem another way, do a similar problem without models or clarify the explanation of their personal strategies.
- Have the students critique their personal strategies as well as those of their classmates to decide which strategy works best for them and why.
- Have the students create problems for a variety of number sentences illustrating addition and subtraction.

C. Choosing Learning Activities

The following learning activities are examples of activities that could be used to develop student understanding of the concepts identified in Step 1.

Sample Activities:

1. **Teaching Personal Strategies for Adding and Subtracting** (p. 13)
2. **Teaching Estimating Sums and Differences** (p. 18)
3. **Teaching Solving Problems Involving Addition and Subtraction** (p. 22)

Sample Activity 1: Teaching Personal Strategies for Adding and Subtracting

1. Examining Number Sentences for Addition and Subtraction

   Encourage the students to use base ten materials to help them understand this process and to explain their thinking.

   a. Provide the students with an addition number sentence such as the following:
   328 + 462 = 330 + 460

   b. Have them decide if the number sentence is true or false and why they think so. Encourage the students to think of the equal sign as "the same as" so that they are deciding if the two sides balance each other. In the example above, the statement is true because 2 is taken away from 462 and added to 328, thereby keeping the sum constant.

   c. Similarly, provide the students with a subtraction number sentence such as the following:
   482 – 348 = 484 – 350

   In this subtraction example, the statement is true because 2 is added to each number on the left of the number sentence, thereby keeping the difference constant.

   d. Provide reinforcement by giving the students a list of addition and subtraction number sentences similar to the examples above. Some are true and some are false. Have the students decide which number sentences are true or false and explain how they know.

   e. Have the students add and subtract 3-digit numbers and use the constant sum or constant difference in their personal strategies.

Look For …

Do students:

- ☐ know and draw on number facts and other number relationships?
- ☐ explain their reasoning using mathematical language?
- ☐ use the base-ten materials to show that their reasoning is correct?
- ☐ verbalize the difference between making a constant sum and making a constant difference?
- ☐ apply the constant sum and constant difference in their personal strategies for adding and subtracting?

2. Adding Three-digit Numbers

   a. Draw on prior knowledge by reviewing some personal strategies used by the students in adding 2-digit numbers. Have the students share their ideas by adding a variety of 2-digit numbers.

   b. Present the following problem to the students: Michael reads 152 pages one day and 178 pages the next day. How many pages did he read in the two days?

   Guide discussion as to whether the numbers in the problem refer to parts or the whole, which operation would be used to solve the problem and what would be a good **estimate** for the answer.

   c. Provide time for the students to solve the problem by writing a number sentence and recording their personal strategies to show how they calculated the sum.

   Have base ten materials and number lines available for the students to use as needed.

Look For …

Do students:

- ☐ know and draw on number facts and other number relationships?
- ☐ solve the problem with base-ten materials but need guidance in using symbols to show what they did?
- ☐ add the hundreds easily but have difficulty adding the tens and ones?
- ☐ show understanding of place value and number relationships so that their strategy makes sense?
- ☐ explain why the steps that they use work?

   d. Have the students explain how their strategies work by relating to place value and have them decide on which strategy is most efficient for them to use in calculating other problems.

   e. Have the students apply their personal strategy to solve a similar problem with different numbers, such as 85 pages on the first day and 258 pages on the second day.

   f. Extend to include 3- and 4-digit numbers using different problem contexts.

Sample personal strategies:

<table>
<tr>
<td>First sample (add ones, add tens, add hundreds, then combine):

152 + 178 = ☐
152
<u>178</u>
10 8 + 2 = 10
120 50 + 70 = 120
<u>200</u> 100 + 100 = 200
330 the total of the partial sums = 330

Michael read 330 pages in the two days.</td>
<td>Second sample (add the hundreds, then add the 2-digit number by moving some to make tens):
152 + 178 = ☐
152 = 100 + 52 178 = 100 + 78
100 + 100 = 200
52 + 75 = ? (review from previous grade)
Use nice numbers and compensate.
52 – 2 = 50
78 + 2 = 80
50 + 80 = 130
Since 2 was added to one number and subtracted from another number, the sum remains constant.
The sum is 200 + 130 = 330.
Michael read 330 pages in the two days.</td>
</tr>
<tr>
<td colspan="2">Third sample (nearest ten and constant sum):
152 + 178 = ☐
Think 152 + 178 = 150 + 180. Since 2 was added to one number and subtracted from another number, the sum remains constant.
150 + 180 = 330
Michael read 330 pages in the two days.</td>
</tr>
</table>

3. Subtracting Three-digit Numbers (Regrouping the Tens)

   a. Draw on prior knowledge by reviewing some personal strategies used by the students in subtracting 2-digit numbers. Have the students share their ideas by subtracting a variety of 2-digit numbers.

   b. Present the following problem to the students:
   Vanlee has 253 jellybeans and gives 145 to her friends. How many jellybeans does she have left?

   c. Guide discussion as to whether the numbers in the problem refer to parts or the whole, which operation would be used to solve the problem and what would be a good **estimate** for the answer.

   d. Provide time for the students to solve the problem by writing a number sentence and recording their personal strategies to show take-away or counting up.
   Have base ten materials and number lines available for the students to use as needed.

Look For …

Do students:

- ☐ know and draw on number facts and other number relationships?
- ☐ solve the problem with base-ten materials but need guidance in using symbols to show what they did?
- ☐ subtract the hundreds easily but have difficulty subtracting the tens and ones?
- ☐ show understanding of place value and number relationships so that their strategy makes sense?
- ☐ explain why the steps that they use work?

e. Have the students explain how their strategies work by relating to place value and have them decide on which strategy is most efficient for them to use in calculating other problems.

f. Have the students apply their personal strategy to solve a similar problem with different numbers, such as 392 jellybeans at the beginning and 75 jellybeans given away. Then extend to include 3- and 4-digit numbers using different problem contexts.

Sample personal strategies:

First sample (take away tens then ones):	Second sample (counting up):
253 – 145 = ☐ 253 = 200 + 53 145 = 100 + 45 53 – 45 = 53 – 40 = 13 Take away 5 more: 13 – 5 = 8 200 – 100 = 100 100 + 8 = 108 Vanlee has 108 jellybeans left.	145 + ☐ = 253 145 + 100 = 245 245 + 8 = 253 Answer: 100 + 8 = 108. Vanlee has 108 jellybeans left.

Third sample (look at the numbers and decide if regrouping is necessary, symbolic representation of work with base ten materials):

253 – 145 = ☐

Regrouping the tens and the ones is necessary because 5 ones cannot be taken away from 3 ones so you must regroup one ten into ten ones.

Regroup 253 to show 2 hundreds, 4 tens and 13 ones.

```
   4 13
2  5  3
1  4  5   Subtract the ones, tens and hundreds to complete the calculation.
1  0  8
```

Vanlee has 108 jellybeans left.

4. Subtracting Three-digit Numbers (Regrouping the Hundreds and Tens)

   a. Present the following problem to the students:
   Your family drives to grandma's house, which is 376 km away. After driving for 196 km, your family stops for a bite to eat. How much farther does your family have to travel to reach grandma's house?

   b. Guide discussion as to whether the numbers in the problem refer to parts or the whole, which operation would be used to solve the problem and what would be a good **estimate** for the answer.

   c. Provide time for the students to solve the problem by writing a number sentence (☐ + 196 = 376 or 376 – 196 = ☐) and their personal strategies to show take-away or counting up. Have base ten materials and number lines available for the students to use as needed.

Look For …

Do students:

- ☐ know and draw on number facts and other number relationships?
- ☐ solve the problem with base-ten materials but need guidance in using symbols to show what they did?
- ☐ subtract the hundreds easily but add the 20 instead of subtracting it?
- ☐ show understanding of place value and number relationships so that their strategy makes sense?
- ☐ explain why the steps that they use work?

d. Have the students explain how their strategies work by relating to place value and have them decide on which strategy is most efficient for them to use in calculating other problems.

e. Have the students apply their personal strategy to solve a similar problem with different numbers, such as 429 km to grandma's house and 159 km travel before a stop.

f. Extend to include 3- and 4-digit numbers using different problem contexts.

Sample personal strategies:

<table>
<tr>
<td>First sample (take away the hundreds, then the tens and ones):
376 – 196 = ☐
300 – 100 = 200 take away the hundreds
96 – 76 = 20 subtract the tens and ones
200 – 20 = 180 subtract the 20 because you must take 96 away from 76 to get the correct answer, even though 96 – 76 is used in the calculations.
The family has to travel 180 km more to reach grandma's house.</td>
<td>Second sample (counting up):

196 + ☐ = 376
196 + 4 = 200 add 4 to make a nice number
200 + 176 = 376 add 176 to get 376
176 + 4 = 180 add the two numbers that were added to 196 to count up to 376.
The family has to travel 180 km more to reach grandma's house.</td>
</tr>
<tr>
<td colspan="2">Third sample (look at the numbers and decide if regrouping is necessary, symbolic representation of work with base ten materials):

376 – 196 = ☐
Regrouping the hundreds and the tens is necessary because 9 tens cannot be taken away from 7 tens so you must regroup one hundred into ten tens.
Regroup 376 to show 2 hundreds, 17 tens and 6 ones.
2 17
3 7 6
1 9 6 Subtract the ones, the tens and the hundreds to complete the calculation.
1 8 0
The family has to travel 180 km more to reach grandma's house.</td>
</tr>
</table>

Sample Activity 2: Teaching Estimating Sums and Differences

1. Estimating Sums: Front-end Estimation and Compensation with Base Ten Materials

   a. Draw on prior knowledge by reviewing some estimation strategies used by the students in adding 2-digit numbers.

   b. Have the students share their ideas by estimating sums of 2-digit numbers in a problem-solving context.

   c. Present the students with the following problem:
   You drink 250 mL of milk on the first day, 375 mL of milk the second day and 450 mL of milk on the third day. **About** how many millilitres of milk did you drink during these three days?

   d. Have the students paraphrase the problem. Draw attention to the word **about**, which indicates that an estimated answer is needed and no calculation has to be done.

   e. Have the students represent the numbers using base ten materials to focus on the place values of the numbers. You might also represent the numbers using beakers.

   f. Model the thinking done in estimating by thinking aloud as you decide which operation to use and why. Explain that you will use the highest place value for each number to represent that number and then make some adjustments later.

   g. Use the base ten materials along with mathematics symbols to show $250 \rightarrow 200$ $375 \rightarrow 300$ $450 \rightarrow 400$ $200 + 300 + 400 = 900$.

   h. Remind the students that this method is called the front-end strategy.

   i. Stimulate the students' thinking by asking whether 900 mL would be a good estimate for the answer. With the base ten blocks, the students should see readily that the blocks remaining after the hundreds would together make almost two hundred. Therefore, a good estimate would be 900 + 200 = 1100 mL. Explain that this 200 is added on to compensate for the values that were dropped off when using the front-end strategy.

      Estimate: You drank a little less than 1100 mL in the three days.

   j. Have the students summarize how the front-end strategy together with compensation provides a good estimate for sums. Clarify that "compensation (adjusting) is a process used to add to or subtract from an initial estimate. It cuts across all estimation strategies" (Alberta Education

Look For …

Do students:

- ☐ know and draw on number facts and other number relationships?
- ☐ explain how place value is used in the front-end strategy? The leading digit in a number together with its place value represents a good estimate for that number.
- ☐ explain how compensation provides a closer estimate than just using the front-end strategy?
- ☐ move away, gradually, from using the base-ten materials and rely on their number sense for estimating sums?

1990, p. 213).

k. Apply this estimation strategy in solving other problems with 3- and 4-digit numbers with less and less use of the base ten materials.

The students should be encouraged to use various estimation strategies, such as rounding. In this case, rounding the numbers to find the estimated sum would not produce a very close estimate and compensation would still be needed.

2. Estimating Differences: Front-end Strategy and Compensation with Base Ten Materials

a. Draw on prior knowledge by reviewing some estimation strategies used by the students in subtracting 2-digit numbers.

b. Have the students share their ideas by estimating differences between 2-digit numbers in a problem-solving context.

c. Present the students with the following problem:
You drink 295 mL of milk one day and 415 mL of milk the next day. **About** how many more millilitres of milk did you drink on the second day than you drank on the first day?

d. Have the students paraphrase the problem. Draw attention to the word **about**, which indicates that an estimated answer is needed but no calculation has to be done.

e. Have the students represent the numbers using base ten materials to focus on the place values of the numbers. You might also represent the numbers using beakers.

Look For …

Do students:

- ☐ know and draw on number facts and other number relationships?
- ☐ explain how place value is used in the front-end strategy? The leading digit in a number together with its place value represents a good estimate for that number.
- ☐ explain how compensation provides a closer estimate than just using the front-end strategy?
- ☐ move away, gradually, from using the base-ten materials and rely on their number sense for estimating differences?

f. Model the thinking done in estimating by thinking aloud as you decide which operation to use and why. Explain that you will use the highest place value for each number to represent that number and then make some adjustments later.

g. Use the base ten materials along with mathematics symbols to show $295 \rightarrow 200$
$415 \rightarrow 400$ $400 - 200 = 200$.

h. Remind the students that this method is called the front-end strategy.

i. Stimulate the students' thinking by asking whether 200 mL would be a good estimate for the answer. With the base ten blocks, the students should see that the blocks remaining after the hundreds are compared and set aside, must also be compared for a closer estimate. Since you are comparing 15 with 95, then another 100 should be removed to

compensate for these remaining blocks. Clarify that "compensation (adjusting) is a process used to add to or subtract from an initial estimate. It cuts across all estimation strategies" (Alberta Education 1990, p. 213).

Therefore, a good estimate would be 200 – 100 = 100.

Estimate: You drink about 100 mL more milk on the second day than on the first day.

Note: If the numbers were different and the 15 was part of the smaller number while 95 was part of the larger number, then 100 would have to be added on to the estimate from the front-end strategy.

j. Have the students summarize how the front-end strategy together with compensation provides a good estimate for differences.

k. Apply this estimation strategy in solving other problems with 3- and 4-digit numbers with less and less use of the base ten materials.

The students should be encouraged to use various estimation strategies, such as rounding. In this case, rounding the numbers to find the estimated difference would work very well.

3. Estimating Sums and Differences: Classifying Problems

a. Draw on prior knowledge by reviewing some problem contexts that require only an estimate and not a calculation for the answer. Through discussion, have the students generalize that estimating is necessary for every problem involving addition or subtraction because estimates must be made prior to any calculations so that the reasonableness of the calculated answer can be determined.

b. Present the students with problems and have them decide which problems can be answered with an estimate only and which problems require calculation as well as an estimate. Examples of problems:
   - Will a container that holds 2000 mL be large enough to hold 1350 mL of water from another container as well as 1015 mL of water from a different container?
   - You are travelling 1265 km to visit relatives. If you travel 568 km the first day, will you have to travel more or less than 700 km the next day to reach your destination in two days?
   - A book contains 458 pages and you have read 225 pages the first day and 125 pages the second day. After these two days, how many more pages do you have to read to finish this book?
   - Your three pet rocks weigh a total of 1625 g. If the first rock weighs 980 g and the second rock weighs 320 g, what is the weight of the third rock?
   - A toad jumps 135 cm on the first jump and 158 cm on the second jump. About how far does it jump in all?

Look For …

Do students:

- ☐ explain the meaning of the problem and justify why only an estimate is needed or why a calculated answer is necessary as well?
- ☐ use compensation as well as the front-end strategy or rounding to estimate the sum or difference?
- ☐ explain clearly the strategies that they used in estimating and why they work?

c. After classifying the problems as either needing only an estimate or needing both an estimate and a calculated answer, have the students answer the problems and share their solutions with one another.

d. Have the students create addition and subtraction problems, some of which require only an estimate for an answer and others that require an estimate and a calculated answer.

Other strategies estimating sums and differences are available in the *Diagnostic Mathematics Program, Elementary: Operations and Properties, Division II* (Alberta Education 1990, pp. 212–216).

Sample Activity 3: Teaching Solving Problems Involving Addition and Subtraction

1. Four Corner Strategy

   a. Draw on prior knowledge by reviewing addition and subtraction problems using 2- and 3-digit numbers.

   b. Emphasize the connections among the story problems, the models/diagrams, the number sentences and the personal strategies used in calculations.

   c. Have the students divide their page into four sections to make graphic organizers and label them as follows:

Four Corner Strategy

Story Problem	Models/Diagrams
Number Sentence	Personal Strategy

Look For …

Do students:

- ☐ exhibit flexibility in writing number sentences in more than on way to show the relationship between addition and subtraction?
- ☐ explain, clearly, their personal strategy and adjust it through discussion to make it more efficient?
- ☐ create problems to illustrate number sentences as well as write number sentences for problems?

   d. Present the students with a problem or a number sentence involving addition or subtraction, such as the following:
   - After your sister gives you 328 stickers from her sticker collection, you have a total of 445 stickers. How many stickers did you have in your collection before your sister gave you some?

   OR
   - ☐ + 328 = 445

   e. Have the students complete the graphic organizer by writing the story problem or the number sentence in one corner and filling in the other corners appropriately.

   Adaptations:
   - The students work in groups and fill in the graphic organizer on large chart paper that can be displayed and discussed with other students and the whole class.
   - Use different labels for the four corners of the graphic organizer such as word problem, number sentence, estimation and personal strategy.

2. Part or Whole?

   a. Draw on prior knowledge by reviewing addition and subtraction problems using 2- and 3-digit numbers.

   b. Have the students analyze a word problem to determine which numbers show the whole and the parts. Then have

Look For …

Do students:

- ☐ label, correctly, the numbers in each problem as the whole or the part?
- ☐ identify whether the whole or the part is not known in the problem?

them decide whether it is the whole or the part that is not known in the problem.
Problem examples:

- Lorne Cardinal made 2000 mL of punch by mixing raspberry juice and orange juice. He put 1344 mL of orange juice into the punch. How many millilitres of raspberry juice did he put into the punch?
- You are travelling 1235 km to visit relatives. If you travel 438 km the first day, how much farther do you have to travel? Will you have to travel more or less than 700 km the next day to reach your destination in two days?

3. Thumbs Up, Thumbs Down, Thumbs Sideways

   a. Present the students with a variety of addition and subtraction problems. Reading the problems orally and also having them displayed on the white board or the overhead projector addresses the different learning styles of the students.

   b. For each problem, ask the students to put their thumbs up if addition can be used to solve the problem, thumbs down if subtraction can be used and thumbs sideways if both addition and subtraction can be used. Have the students justify their choice, either in small groups or with the entire class.

   c. Finally, have the students write number sentences to support their choice. Emphasize the relationship between addition and subtraction as the students suggest different number sentences.

Look For …

Do students:

- ☐ classify, correctly, problems involving addition, subtraction or both operations?
- ☐ exhibit flexibility in writing number sentences in more than one way to show the relationship between addition and subtraction?
- ☐ justify their choice of operation(s) to solve a problem?

Adaptations:

- Include problems that require the addition or subtraction of more than 2 numbers.
- Provide the students with written copies of the problems and have them work in pairs or individually to classify the problems as addition, subtraction or both addition and subtraction. The students then write the appropriate number sentences for each problem.
- Have different groups of students take turns creating and classifying addition and subtraction problems.
- Have the students write equivalent number sentences for a given problem. Through discussion, have the students generalize that the semantic number sentence (the one that shows the meaning of the problem) is often rearranged to expedite calculation. For example, 234 + ☐ = 625 can be rewritten as 625 – 234 = ☐ .
  The first number sentence shows adding on, but a student might prefer to use subtraction to solve the problem and therefore rearrange the number sentence in the rewritten form.

4. Choosing Number Sentences

   a. Present the students with a problem and have them choose which of the number sentences provided could be used to solve the problem. Ask why the number sentences chosen can be used to solve the problem.
   Example:
   Diego jogs for 459 minutes during one week while Dora jogs for 272 minutes that week. How much longer did Diego jog than Dora during that week?
   272 + 459 = ☐ ☐ = 459 – 272 272 + ☐ = 459
   459 – 272 = ☐ 459 + ☐ = 272

Look For …

Do students:
- ☐ choose, correctly, equivalent number sentences that relate to the problem?
- ☐ justify their choices by relating to the meaning of the problem and the relationship between addition and subtraction?

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5. Classifying Problems: Open and Closed Sorts

Look For …

Do students:
- ☐ create classifications for the problems based on the meaning of operations?
- ☐ exhibit flexibility in creating more than one way to sort the problems?
- ☐ justify why problems were placed in specific categories?

Open Sort

Present the students with a variety of addition and subtraction problems that are written on separate pieces of paper. Have the students work in groups to classify the problems into groups, label the groups and explain why the problems fit where they have been placed. Explain that some problems may fit in more than one group.

Challenge the students to:
- find another way to classify the problems
- create other problems and place them into the groups.

Some categories used by the students may include the following:
- addition, subtraction, both addition and subtraction
- part–part–whole, change, compare
- only estimation is needed, both estimation and a calculated answer are needed
- find the part, find the whole.

Closed Sort

Present the students with a variety of addition and subtraction problems that are written on separate pieces of paper and also provide them with the categories into which they are to group the problems. See the examples of categories given above.
The students sort the problems into the categories provided and justify their choices.

6. Similarities and Differences

a. Provide the students with two problems using the same numbers but different meanings for addition and subtraction, such as one showing change and one showing the part–part–whole relationship.

b. Ask the students to explain how the problems are the same and how they are different. They may wish to put their explanations in a graphic organizer, such as the one shown below:

Similarities	Differences

An example of a problem showing a change situation:

- You pour 355 mL of ginger ale into a pitcher and then add some raspberry juice to make a total of 808 mL of punch. How many millilitres of raspberry juice did you pour into the pitcher?
  An example of a problem showing a part–part–whole situation: Your class has collected 808 coins of which 355 are nickels. How many coins are not nickels?

Look For …

Do students:

- ☐ explain that in both problems the whole is known and one of the parts is unknown?
- ☐ explain that both problems can be solved using addition or subtraction?
- ☐ explain that both problems can be represented by the same number sentence?
- ☐ explain that both problems have the same numerical answer but different sentences to answer the question asked?
- ☐ explain that one problem shows action but not the other problem?
- ☐ explain that the units are different in the two problems?
- ☐ explain that the problems explain different real-life situations?

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Other strategies for estimating sums and differences, showing the connections between the operations and between the concrete and symbolic representations, are available in the *Diagnostic Mathematics Program, Elementary: Operations and Properties, Division II* (Alberta Education 1990, pp. 212–216).

Step 4: Assess Student Learning

Guiding Questions

- Look back at what you determined as acceptable evidence in Step 2.
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Sample Assessment Tasks

In addition to ongoing assessment throughout the lessons, consider the following sample activities to evaluate students' learning at key milestones. Suggestions are given for assessing all students as a class or in groups, individual students in need of further evaluation, and individual or groups of students in a variety of contexts.

A. Whole Class/Group Assessment

Note: Performance-based assessment tasks are under development.

Have counters and base ten materials for the student to use as needed.

1. Complete the chart below by drawing a diagram, writing a number sentence and using a personal strategy to solve the problem given. Write the answer to the problem in a complete sentence.

Story Problem You have 278 pennies in your coin collection. If you have 350 coins in all, how many coins are not pennies?	Models/Diagrams
Number Sentence	Personal Strategy

2. You make a punch by combining 1768 mL of orange juice with 355 mL of ginger ale. About how much punch did you make? Show the numbers you used for your estimate. Write your answer in a complete sentence.

3. Tai has 348 hockey cards. He gives 196 cards to Phu and 82 cards to Fung. How many hockey cards does he have left? Show your work and write your answer in a complete sentence.

4. Diego reads for 515 minutes during one week while Dora reads for 290 minutes that week.
   a. About how much longer does Diego read than Dora? Show the numbers you used for your estimate.

   b. Calculate how much longer Diego reads than Dora. Write your answer in a complete sentence.

5. One of your pet rocks weighs 1418 g. This rock and another pet rock weigh a total of 2196 g.
   a. Estimate the weight of the other pet rock. Show the numbers you used in your estimate.

   b. What does the other rock weigh? Show your work and write your answer in a complete sentence.

6. Your family is driving to grandpa's house that is 405 km away. Your dad drives 198 km and your mom drives the rest of the way. How many kilometres does your mom drive? Show your work and write your answer in a complete sentence.

7. You jog for a certain distance and then walk for 1350 m to cover a total distance of 2126 m. How far did you jog? Show your work and write your answer in a complete sentence.

8. Create a problem that can be represented by the number sentence: 350 + ☐ = 425. Explain how you know your problem matches the number sentence.

B. One-on-one Assessment

Assessment activities can be used with individual students, especially the students who may be having difficulty with the outcome.

1. Ask the student to explain to you the connection between addition and subtraction by using counters or base ten materials. If necessary, coach the student by using the materials to show addition and ask how this situation could be changed into subtraction.

2. Present the following problem to the student and have him or her read it orally. Have base ten materials available to use as needed.
   - You pour 450 mL of milk out of a container, leaving 1325 mL in the container. How many millilitres of milk were in the container at the beginning?

   Pose the following questions to guide thinking, if necessary:
   - State the problem in your own words.
   - What do each of the numbers in the problem represent—a part or a whole?
   - What is the unknown in the problem—a part or a whole?
   - What number sentence could you write to show the meaning of the problem?
   - Does the problem use addition or subtraction or both? Explain.

- **About** how many millilitres of milk were in the container at the beginning? Explain your thinking.
- Use a strategy that makes sense to you to find the answer to the problem. Explain your thinking as you write the numbers.
- Explain how you know your answer makes sense and is reasonable.
- Would you solve the problem another way? Explain your thinking.

3. Use the same procedure as outlined in question 2 above with the following problem:
   - You give 263 of your 415 coins to a friend. How many coins do you have left?

Create a problem that can be shown by the number sentence, 220 + ☐ = 605.
Solve the problem you created by using a strategy that makes sense to you.

C. Applied Learning

Provide opportunities for the students to use addition and subtraction in a practical situation and notice whether or not the strategies transfer. For example, ask the student to compare height of the door to the width of the door by using centimetres. Does the student:

- use an estimate in doing the comparison?
- obtain the two measures and compare them in some way?
- use a personal strategy that makes sense in comparing the two measures?

Step 5: Follow-up on Assessment

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction?

A. Addressing Gaps in Learning

The students who have difficulty solving addition or subtraction problems by estimating and by using a personal strategy will enjoy more success if one-on-one time is provided. This time will allow for open communication to diagnose where the learning difficulties lie. Assessment by observing a student solving problems will provide valuable data to guide further instruction. Success in problem solving depends on a positive climate in which the students are confident in taking risks. By building on the understanding that each student already has and accommodating the individual learning styles, success will follow.

If the difficulty lies in understanding the problem, use the following strategies:

- provide problems that relate to the student's interest; use the student's name in the problem
- use smaller numbers in the problem initially
- have the student paraphrase the problem
- guide the student to determine if the numbers refer to a part or a whole
- ask the student if the unknown in the problem refers to a part of a whole
- provide base ten materials for the students to represent the problem as needed
- have the student decide which operation should be used and why
- ask guiding questions to show the connections between addition and subtraction and the possible option of using either operation
- provide a graphic organizer, such as the K–N–W–S chart (see Blackline Master).

If the difficulty lies in estimating sums and differences, use the following strategies:

- use the base ten materials to focus on the place values of the numbers and the relationship among the place values
- use smaller numbers initially and connect them to larger numbers; e.g., connect 30 to 300 to 3000
- convince the student of the need for estimating by citing many real-world examples of where estimating is needed
- review number facts and place value
- emphasize flexibility in estimating, capitalizing on the student's methods and fine tuning them for correctness and efficiency
- take small steps using the front-end strategy without compensation first, and then adding the compensation when the student sees the need for it in providing a better estimate.

If the difficulty lies in using personal strategies to solve addition and subtraction problems, use the following strategies:

- use smaller numbers in the problems initially
- review place value and number facts
- provide base ten materials as needed
- think aloud a personal strategy that you would use to solve the problem and explain why this strategy is more efficient than another one that you describe
- emphasize flexibility in choosing a personal strategy; a strategy that is efficient for one student may not be efficient for another student
- build on the student's understanding of place value and number facts to guide him or her in finding a strategy that works
- provide ample time for the students to think and ask questions to clarify thinking
- have the students work in groups so that they learn strategies from one another
- guide the students to critique various personal strategies to find one that can be used on a variety of problems efficiently
- have the students explain their personal strategies to the class so others can hear how they work in 'kid' language
- post various personal strategies in the classroom for the students to share and critique
- encourage the student to check the reasonableness of the answer using a given personal strategy by comparing the answer to the estimated answer provided earlier.

B. Reinforcing and Extending Learning

The students who have achieved or exceeded the outcomes will benefit from ongoing opportunities to apply and extend their learning. These activities should support the students in developing a deeper understanding of the concept and should not progress to the outcomes in subsequent grades.

Consider strategies, such as the following.

- Provide tips for parents on practising adding and subtracting at home or in the community. For example:
  - take the children shopping and have them estimate the total grocery bill prior to going through the check out
  - collect the cash register receipts, cover the total, and have the children estimate the total; or, tear off the totals and have the children match the receipts with the correct totals, using estimation
  - talk to your children about data in the newspaper and magazines, and encourage them to add and subtract mentally and explain how they are doing it.
- Have the students create problems showing the various types of addition and subtraction problems (change, part–part–whole, and comparison) and write appropriate number sentences for each one. These problems can be displayed in the chart on the bulletin board.
- Have the students create problems with different contexts but using the same numbers, such as 259 and 160. They could follow this up by having the class decide which of the problems could be solved using a given number sentence, such as $259 - 160 = \square$.

- Have the students convert single-step addition or subtraction problems into multistep problems and explain to other students how to solve them.
- Have the students critique other students' personal strategies and explain why they work or not. Which would be the most efficient and why?
- Have the students write an explanation for a personal strategy so that everyone in the class can understand it.
- Have the students compare the operations of addition and subtraction by discussing the commutative property (order property) and the zero property as well as other characteristics related to these two operations.

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