Mathematics

Planning Guide

Grade 4 *Area*

Shape and Space (Measurement)

Specific Outcome 3

This Planning Guide can be accessed online at:
http://www.learnalberta.ca/content/mepg4/html/pg4_area/index.html

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Planning Guide: *Grade 4 Area*

Strand: Shape and Space (Measurement)
Specific Outcome: 3

This *Planning Guide* addresses the following outcomes from the Program of Studies:

Strand: Shape and Space (Measurement)	
Specific Outcome:	3. Demonstrate an understanding of area of regular and irregular 2-D shapes by: • recognizing that area is measured in square units • selecting and justifying referents for the units cm^2 or m^2 • estimating area, using referents for cm^2 or m^2 • determining and recording area (cm^2 or m^2) • constructing different rectangles for a given area (cm^2 or m^2) in order to demonstrate that many different rectangles may have the same area.

Curriculum Focus

This sample targets the following changes in the curriculum:

- The General Outcome focuses on using direct or indirect measurement to solve problems, whereas the previous mathematics curriculum focused on estimating, measuring and comparing using decimal numbers and standard units of measure.
- The Specific Outcome focuses on introducing area, whereas in the previous mathematics curriculum, area was introduced in earlier grades.
- The Specific Outcome includes referents for the units cm^2 or m^2, whereas the previous mathematics curriculum does not.

The previous mathematics curriculum included specific outcomes that focused on proportionality (relating the size of the unit to the number of units measured), selecting the most appropriate standard unit, and comparing and ordering areas, whereas the new mathematics curriculum does not.

What Is a Planning Guide?

Planning Guides are a tool for teachers to use in designing instruction and assessment that focuses on developing and deepening students' understanding of mathematical concepts. This tool is based on the process outlined in *Understanding by Design* by Grant Wiggins and Jay McTighe.

Planning Steps

The following steps will help you through the Planning Guide:

- **Step 1: Identify Outcomes to Address** (p. 4)
- **Step 2: Determine Evidence of Student Learning** (p. 6)
- **Step 3: Plan for Instruction** (p. 8)
- **Step 4: Assess Student Learning** (p. 27)
- **Step 5: Follow-up on Assessment** (p. 33)

Step 1: Identify Outcomes to Address

Guiding Questions

- What do I want my students to learn?
- What can my students currently understand and do?
- What do I want my students to understand and be able to do based on the Big Ideas and specific outcomes in the program of studies?

Big Ideas

Van de Walle and Lovin define area as "a measure of the space inside a region or how much it takes to cover a region" (2006, p. 234).

In using any type of measurement, such as length, area or volume, it is important to discuss the similarities in developing understanding of the different measures. First identify the attribute to be measured, then choose an appropriate unit and finally compare that unit to the object being measured (NCTM 2000, p. 171). As with other attributes, it is important to understand the attribute of area before measuring.

Key ideas in understanding the attribute of area include the following:

- conservation—an object retains its size when the orientation is changed or it is rearranged by subdividing it in any way
- iteration—the repetitive use of a identical nonstandard or standard units of area to entirely cover the entire surface of the region
- tiling—the units used to measure the area of a region must not overlap and must completely cover the region, leaving no gaps
- additivity—add the measures of the area for each part of a region to obtain the measure of the entire region
- proportionality—there is an inverse relationship between the size of the unit used to measure area and the number of units needed to measure the area of a given region; i.e., the smaller the unit, the more you need to measure the area of a given region
- congruence—comparison of the area of two regions can be done by superimposing one region on the other region, subdividing and rearrangement as necessary
- transitivity—when direct comparison of two areas is not possible, use a third item that allows comparison; e.g., to compare the area of two windows, find the area of one window using nonstandard or standard units and compare that measure with the area of the other window; i.e., if A = B and B = C, then A = C
- standardization—using standard units for measuring area such as cm^2 and m^2 facilitates communication of measures globally
- unit/unit-attribute relations—units used for measuring area must relate to area; e.g., cm^2 must be used to measure area and not cm or mL.

Adapted from Alberta Education, *Teaching Measurement Concepts, Grades 4–6* (unpublished workshop handout) (Edmonton, AB: Alberta Education, 2006), Research section, pp. 2–4.

Sequence of Outcomes from the Program of Studies

See http://education.alberta.ca/teachers/core/math/programs.aspx for the complete program of studies.

Grade 3
Specific Outcomes

5. Demonstrate an understanding of perimeter of regular and irregular shapes by:
   - estimating perimeter, using referents for cm or m
   - measuring and recording perimeter (cm, m)
   - constructing different shapes for a given perimeter (cm, m) to demonstrate that many shapes are possible for a perimeter.

Grade 4
Specific Outcomes

4. Demonstrate an understanding of area of regular and irregular 2-D shapes by:
   - recognizing that area is measured in square units
   - selecting and justifying referents for the units cm^2 or m^2
   - estimating area, using referents for cm^2 or m^2
   - determining and recording area (cm^2 or m^2)
   - constructing different rectangles for a given area (cm^2 or m^2) in order to demonstrate that many different rectangles may have the same area.

Grade 5
Specific Outcomes

2. Design and construct different rectangles, given either perimeter or area, or both (whole numbers), and make generalizations.

Step 2: Determine Evidence of Student Learning

Guiding Questions

- What evidence will I look for to know that learning has occurred?
- What should students demonstrate to show their understanding of the mathematical concepts, skills and Big Ideas?

Using Achievement Indicators

As you begin planning lessons and learning activities, keep in mind ongoing ways to monitor and assess student learning. One starting point for this planning is to consider the achievement indicators listed in the *Mathematics Kindergarten to Grade 9 Program of Studies with Achievement Indicators*. You may also generate your own indicators and use them to guide your observation of the students.

The following achievement indicators may be used to determine whether the students have met this specific outcome.

- Describe area as the measure of surface recorded in square units.
- Identify and explain why the square is the most efficient unit for measuring area.
- Provide a referent for a square centimetre and explain the choice.
- Provide a referent for a square metre and explain the choice.
- Determine which standard square unit is represented by a given referent.
- Estimate the area of a given 2-D shape using personal referents.
- Determine the area of a regular 2-D shape and explain the strategy.
- Determine the area of an irregular 2-D shape and explain the strategy.
- Construct a rectangle for a given area.
- Demonstrate that many rectangles are possible for a given area by drawing at least two different rectangles for the same given area.
- Explain that the area of a given region remains the same when the orientation changes or it is subdivided differently (conservation of area).
- Explain the inverse relationship between the size of the unit used to measure the area of a region and number of units required.
- Use direct and indirect measurement (apply the transitive property) to measure regions.
- Explain the need for standard units in measuring area.
- Identify which units of measure are appropriate to measure area; e.g., cm^2, m^2.
- Identify which units of area are appropriate to measure various regions; e.g., smaller regions require smaller units of measure.
- Explain that all measurements for area are approximate with a margin for error.
- Recognize that the smaller the unit used to measure area the more precise the measurement will be.
- Compare the areas of two or more regions and explain the process used.

Some sample behaviours to look for in relation to these indicators are suggested for many of the instructional activities in **Step 3, Section C, Choosing Learning Activities** (p. 14).

Step 3: Plan for Instruction

Guiding Questions

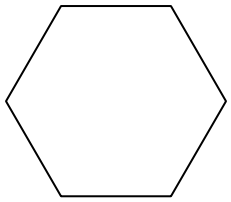
- What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should I use?
- How will I meet the diverse learning needs of my students?

A. Assessing Prior Knowledge and Skills

Before introducing new material, consider ways to assess and build on the students' knowledge and skills related to measurement. For example:

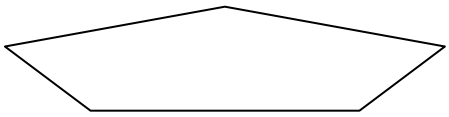
- Measure and record the perimeter of the following shapes. Explain your thinking.

  a.

  What is the least number of sides that you would have to measure to find the perimeter of this figure?

  b.

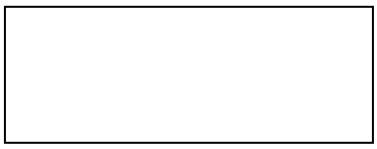
  

  What is the least number of sides that you would have to measure to find the perimeter of this figure?

- Draw a shape on centimetre grid paper that has a perimeter of 18 cm. Explain how you know your answer is correct.
- Draw two rectangles on centimetre grid paper, each with a perimeter of 24 cm.
  a. Explain how you know your answer is correct.

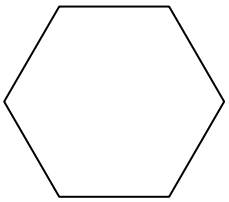
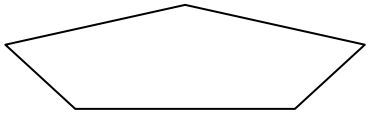
  b. How many different rectangles could you draw with a perimeter of 24 cm where the sides are measured in whole centimetres? Explain how you know.

- Estimate the perimeter of the following shape using personal referents. Explain your thinking.



If a student appears to have difficulty with these tasks, consider further individual assessment, such as a structured interview, to determine the student's level of skill and understanding. See **Sample Structured Interview: Assessing Prior Knowledge and Skills** (p. 10).

Sample Structured Interview: Assessing Prior Knowledge and Skills

Directions	Date:	
	Not Quite There	**Ready to Apply**
Place the following figures, one at a time, before the student. Provide a 30 cm ruler. a. "Measure and record the perimeter of the following shape. Explain your thinking."  "What is the least number of sides that you would have to measure to find the perimeter of this figure?" b. "Measure and record the perimeter of the following shape. Explain your thinking."  "What is the least number of sides that you would have to measure to find the perimeter of this figure?"	• Doesn't understand what to do to find the perimeter. • Is unsuccessful in using a ruler to measure a side of the 2-D shape. • Needs assistance in placing the ruler on the line segment and counting the number of centimetres in the measure. • Measures all the sides of the regular hexagon rather than measuring one side and multiplying it by six. • Measures one side of the irregular pentagon and multiplies the measure by five, not realizing that the shape is irregular. • Neglects to measure one or more of the required sides of the pentagon. • Measures the sides of the pentagon but adds them up incorrectly. • Does not respond correctly to the least number of sides needed to measure one or both perimeters. • Does not explain the process used clearly.	• Measures one side of the regular hexagon correctly and multiplies it by six to get the correct perimeter in centimetres. • Explains the process clearly. • Measures the required sides of the irregular pentagon and calculates correctly to get the correct perimeter in centimetres (doubles the measure of appropriate sides rather than measuring each side). • Explains the process clearly.

Provide the student with centimetre grid paper and a 30 cm ruler. "Draw a shape on centimetre grid paper that has a perimeter of 18 cm. Explain how you know your answer is correct."	• Draws a shape but the perimeter is more or less than 18 cm. • Does not indicate how the shape was measured to check for accuracy. • Has difficulty justifying the answer.	• Draws a shape with a perimeter of 18 cm. • Indicates how the shape was measured to check for accuracy. • Justifies the process used.
Provide the student with centimetre grid paper and a 30 cm ruler. "Draw two rectangles on centimetre grid paper, each with a perimeter of 24 cm. c. Explain how you know your answer is correct. d. How many different rectangles could you draw with a perimeter of 24 cm where the sides are measured in whole centimetres? Explain how you know."	• Draws shapes other than rectangles with or without a perimeter of 24 cm. • Fails to draw either rectangle with a perimeter of 24 cm. • Draws one rectangle with a perimeter of 24 cm but is unable to draw another rectangle with the same perimeter. • Does not justify the process used. • Unable to answer part (b), answers it incorrectly and/or cannot explain the process used.	• Draws two rectangles, each with a perimeter of 24 cm. • Explains clearly why the drawings are correct. • Uses patterns or some other appropriate strategy to explain how many different rectangles with a perimeter of 24 cm can be drawn; e.g., Length Width 1 cm 11 cm 2 cm 10 cm 3 cm 9 cm 4 cm 8 cm 5 cm 7 cm 6 cm 6 cm

Place the following shape before the student. Ensure that no rulers are available for the student to see or to use. "Estimate the perimeter of the following shape using personal referents. Explain your thinking." 	• Estimates the perimeter but the estimate is not close to the actual measurement. • Does not use a personal referent in estimating the perimeter. • Is unable to explain the process used in finding the estimated perimeter.	Uses a personal referent to estimate the perimeter and explains the process clearly. • Provides a close estimate for the perimeter of the rectangle.

B. Choosing Instructional Strategies

Consider the following general strategies for teaching (Van de Walle 2001):

- access prior knowledge on using perimeter in the real world
- introduce area by drawing on familiar and accessible contexts to illustrate uses of area (NCTM 2000)
- review the process used in developing understanding of perimeter and use a similar process in developing understanding of area stressing that the attribute changes but the process is similar:
  - explain that the attribute to be measured is area
  - check for conservation of area; e.g., rearrange a given shape and determine if the student realizes that the area of the shape remains unchanged
  - always estimate prior to comparing or measuring areas
  - make direct comparisons; e.g., compare the areas of two shapes by superimposing one shape on the other—subdividing one shape may be necessary
  - estimate the area of the shape using nonstandard units of measure; e.g., tiles or lima beans. Use various techniques for estimating area:
    - referents—use a referent for the single unit of measure and iterate this unit mentally to obtain the estimate; e.g., use the size of the fingernail on your smallest finger as a referent for 1 cm^2
    - chunking—estimate the area of a smaller portion of a shape initially and use this estimate to estimate the entire area of the shape; e.g., estimate the area of a smaller section of the floor and then multiply that answer by the number of these sections in the entire floor
  - have the students share their strategies for estimating area
  - accept a range of estimates—within 10% or 20% of the actual measure is reasonable (Van de Walle 2001, p. 295)
  - encourage the students to measure the area after each estimate so that they develop a better sense of area
  - use nonstandard units of measure that have the same attribute as the item being measured; e.g., use tiles or lima beans to measure a given shape
  - make indirect comparisons using a nonstandard unit of measure that has the same attribute as the item being measured; e.g., use tiles to measure a desk top and compare this measure to the number of tiles needed to measure another desk top
  - measure the area of the shape using larger then smaller nonstandard units of measure to establish that the smaller the unit of measure the more you need to measure the area of a given shape; e.g., more small tiles are needed than large tiles to measure the area of a given shape
  - explain the need to use standard units to measure area to facilitate communicating various areas globally
  - measure the area of a given shape using an appropriate instrument with standard units of measure; e.g., use transparencies with centimetre grid paper to lay over given shapes to find the areas in square centimeters

  - make a simple measuring instrument using familiar unit models; e.g., cut large squares, maybe 30 cm on a side, to measure larger areas and cut small squares, maybe 5 cm to 10 cm on a side, to measure smaller areas. Explain the inverse relationship between the size of the unit used to measure the area of a region and number of units required.

- integrate the strands by:
  - using patterns to develop understanding of area
  - connecting area to arrays used in relating multiplication and division of whole numbers
  - connecting the area concepts to fractions of a region (the denominator of a fraction indicates the number of equal parts into which the region is divided; these equal parts have the same area but not necessarily the same shape).

C. Choosing Learning Activities

The following learning activities are examples of activities that could be used to develop student understanding of the concepts identified in Step 1.

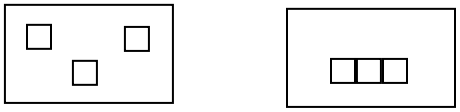
Sample Activities:

1. **Teaching Conservation of Area** (p. 15)
2. **Teaching That Area Is Measured in Square Units** (p. 17)
3. **Teaching the Use of Referents for cm^2 or m^2 in Estimating Area** (p. 20)
4. **Teaching How to Determine and Record Area (cm^2 or m^2)** (p. 21)
5. **Teaching That Many Different Rectangles Can Be Constructed for a Given Area (cm^2 or m^2)** (p. 25)

Sample Activity 1: Teaching Conservation of Area

1. Quilts
   a. Present the following scenario to the students:
   Grandma is making a quilt with three patches of blue cloth on a white background. She needs help deciding where to put the patches. If grandma wants the patches to cover the same amount of space on the white background, should she scatter them about or put them together in a group? Explain your answer.

   b. Provide the students with three pieces of blue paper to represent the patches of blue cloth. Ask them to move the papers around on their desk, which can be the white background for the quilt, and decide which arrangement would be best for the quilt. If necessary, provide scaffolding by moving three pieces of paper on the overhead projector into different positions. Example:



   c. Through discussion, have the students verbalize that grandma can put her three patches of blue cloth anywhere on the white background and they will always cover the same amount of surface.

Look For …

Do students:

- ☐ recognize that rearranging shapes results in shapes with the same area?
- ☐ explain why the areas remain the same when shapes are rearranged?
- ☐ apply the conservation of area to everyday contexts?

2. Changing Shapes
   a. Provide each student with at least four congruent squares each of a different colour. Have the students compare the squares by superimposing one on the other to show that they are congruent—same size and shape. Reaffirm that these squares each cover the same amount of surface or have the same area.

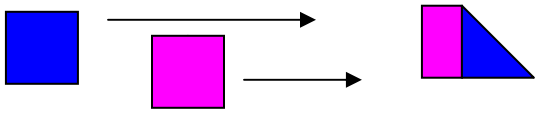
   b. Have the students cut one of their squares along the diagonal to make two triangles. Instruct them to rearrange the two triangles to make as many different shapes as possible with two sides aligned.

   Possible shapes:

   c. Encourage the students to make other designs by cutting a square in different ways and rearranging the pieces. Reinforce that the pieces of one square must be rearranged to make one design—the pieces are the same colour. The students may glue their designs on newspaper and place them into groups on the floor, justifying the categories.

Ask the students to describe the differences and similarities among the designs. Guide the discussion to generalize that all the designs cover the same amount of surface or have the same area because each design was cut from a congruent square—same size and shape.

d. Focus the students' attention on the group of designs that were made by rearranging two halves of a square. Ask the students what fraction is shown by one part of each design. Reinforce that when a whole is divided into halves, each half has the same size (area) but not necessarily the same shape. Ask the students the following question:
If you take one-half of one square and made a design with one-half of a different square, will the design still cover the same surface (or have the same area) as the other designs? Explain your answer.



Guide the discussion so that the students communicate that halves of the same-sized wholes are equal (same area), therefore, two halves will cover the same surface (or have the same area) as any of the designs created.

Adapted from Alberta Education, *Diagnostic Mathematics Program, Elementary: Measurement, Division II* (Edmonton, AB: Alberta Education, 1990), pp. 140–141.

Sample Activity 2: Teaching That Area Is Measured in Square Units

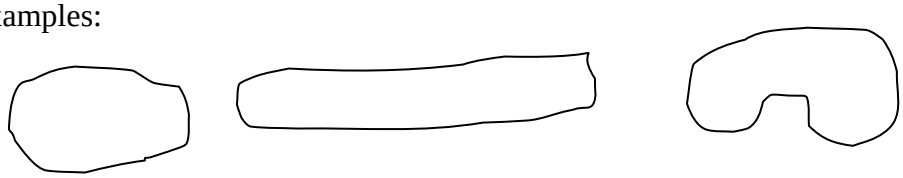
1. Concept of Area
   a. Have the students share some everyday contexts in which they need to know the amount of surface covered, such as painting a wall and tiling a floor.
   b. Draw on prior knowledge of perimeter to share ideas of where perimeter is used everyday. Have the students review what is used to measure the perimeter of 2-D shapes. Ask if these units would be useful in measuring the amount of surface covered or the area of 2-D shapes. Have the students suggest something that could be used to completely cover a surface, such as a piece of paper. Have lima beans, tiles and other suitable objects available for the students to see and, hopefully, suggest as useful in measuring area.

Look For …

Do students:

- ☐ estimate area using nonstandard units?
- ☐ use, repeatedly, the same size unit to find the area of a shape?
- ☐ show with examples that more units are needed to measure a given area when the units are smaller?

2. Using Nonstandard Units to Measure Area
   a. Provide each group of students with a copy of three shapes that could represent different garden plots. Have lima beans, tiles, buttons, pattern blocks and other nonstandard units for area available for the students to use. Present the following problem:
   Mr. McGregor wants the largest possible garden plot to plant his carrots. He knows that he has to share some of them with the rabbits. Which garden plot should he choose? Estimate first, then find the area of each garden plot. Explain your thinking.
   Examples:

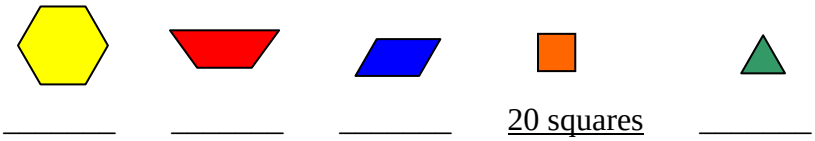


   b. Have the students explore different ways to find the area of the garden plots, such as covering the surface with objects such as lima beans or tiles, folding the paper to make a grid and counting the squares, and drawing a grid on the paper. Ask the students to share the different ways that they might use to find the area of the garden plots. Then have them choose a method and use to solve the problem.
   c. Have the student share their answers to the problem and discuss which method they think is the most accurate in finding the areas. Guide the discussion to make the following generalizations about finding the area of a surface:
      - use, repeatedly, the same size unit
      - cover all the surface, leaving no gaps
      - use a variety of ways to find area, remembering that the smaller the unit used the more of them will be needed to cover a given surface.

Adapted from Alberta Education, *Teaching Measurement Concepts, Grades 4–6* (unpublished workshop handout) (Edmonton, AB: Alberta Education, 2006), pp. 44–46.

Extensions:

- Iteration with Pattern Blocks
  Provide the students with rectangular papers that each measure 10 cm by 13 cm. Have them estimate how many copies of each shape of pattern block it would take to cover the rectangle. Then have the students measure the area using each of the shapes in turn.

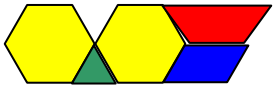


_____ _____ _____ 20 squares _____

Adapted from W. George Cathcart, Yvonne M. Pothier and James H. Vance, *Learning Mathematics in Elementary and Middle Schools* (2nd ed.) (Scarborough, ON: Prentice-Hall Canada, 1997), p. 211. Adapted with permission from Pearson Education Canada.

- Area of Designs with Noncongruent Parts
  Provide the students with pattern blocks and review the relationship among the different blocks.
  Present the students with the following design. Ask them to find the area by using:
  - triangles—green pattern blocks
  - rhombuses—blue pattern blocks
  - trapezoids—red pattern blocks
  - hexagons—yellow pattern blocks.

  Have them explain their thinking.



Reinforce the idea that the same unit for area must be repeated in finding the area of any surface. Explain that the square unit is the most common unit used to measure area but hexagons, trapezoids, rhombuses and triangles can also be used.

- Using Circles, Squares and Triangles to Measure Area
  Cut out congruent shapes of circles, squares and equilateral triangles. Provide the students with packages of each shape as well as a rectangular mat to measure the area. Have the students use the different shapes to measure the area of the mat and discuss the advantages and/or disadvantages of using each shape.

Adapted from Alberta Education, *Diagnostic Mathematics Program, Elementary: Measurement, Division II* (Edmonton, AB: Alberta Education, 1990), p. 142.

3. Focusing on the Square Unit

Look For …

Do students:

- ☐ know the essential characteristics of a square?
- ☐ explain clearly why the square unit is the preferred unit for measuring area?

a. Have the students make a nonstandard unit for area that they could use to measure their desktop. Remind them that the desk must be completely covered with the unit they choose and that they must repeat their unit at least four times to measure the area of their desk. They should be ready to justify their choice of unit. Provide paper and scissors.

b. Have the students compare their answers and share the units that they created to measure area. Provide triangular, rectangular and square units if the students do not include these as their created units. Show a variety of different shaped triangles and rectangles.
Guide the discussion to include the following ideas:
   - circles are not useful units in measuring area because they leave gaps
   - triangles and rectangles do not leave gaps but there are many different shapes for triangles and rectangles so saying a unit is triangular is not descriptive enough
   - squares do not leave gaps and there is only one shape for a square so everyone knows the shape of a square unit.

c. Provide the students with square units to measure their desks or measure congruent mats. Give some groups large squares and other groups small squares. Compare the answers for the areas found. Generalize: to compare areas, the same size unit of measure must be used; i.e., either small squares or large squares. Review the fact that the smaller the unit used to measure area the more of these units are needed.

Adapted from Alberta Education, *Diagnostic Mathematics Program, Elementary: Measurement, Division II* (Edmonton, AB: Alberta Education, 1990), pp. 142–143.

Sample Activity 3: Teaching the Use of Referents for cm^2 or m^2 in Estimating Area

Look For …

Do students:

- ☐ compare the standard units for perimeter with the standard units for area?
- ☐ use a personal referent for estimating area that relates well to the standard units?
- ☐ explain clearly how to estimate the area of a given shape?

1. Standard Units for Area

   a. Review the linear units (centimetre and metre) used to find the perimeter of 2-D shapes. Explain that these standard units of measure were used to find perimeter so that the perimeters of shapes could be compared and communicated clearly. Connect the need for standard units in finding perimeter to the need for standard units in finding area.

   b. Review the discussion on using a square unit for area in the previous activity and ask the students to suggest standard units for measuring area. Through discussion, conclude that the standard units used for area are cm^2 and m^2. Write the symbols and explain that 1 cm^2 is read as "one square centimetre," **not** "one centimetre squared."

      Note: One square centimetre is a measure for the area of a variety of shapes, whereas one centimetre squared is a square that is 1 cm on each side.

   c. Provide the students with centicubes or the units of the base ten materials and explain that the area of one face is 1 cm^2. Also, provide them with centimetre grid paper on which they may place the centicubes or base ten units. Have the students use their centimetre rulers to measure the side of each square on the centimetre grid paper to verify that each square is 1 cm^2.

2. Referents for Area and Estimating Area

   a. Review the referents used for centimetre (the width of the pinky finger) and metre (the distance from the teacher's finger tip to his or her opposite shoulder). Ask the students to suggest a suitable referent for 1 cm^2 and explain why they think it would work.

   b. Have the students use their referent for 1 cm^2 to estimate the area of a book cover in square centimetres. Then, have them check their estimate by finding the area of the book cover by overlaying a transparency of a centimetre grid.

   c. Similarly, discuss possible referents for 1 m^2 after the students make a square on the floor that is one square metre, using masking tape or other suitable materials. Have the students use their referents and estimate the area of a large tabletop or a section of the classroom floor. Use a square piece of paper that is 1 m by 1m to measure the area and check the estimates.

Sample Activity 4: Teaching How to Determine and Record Area (cm^2 or m^2)

1. Estimate and Compare Areas
   a. Provide the students with pairs of paper rectangles, such as the following:
   First pair: 2 cm by 9 cm, 3 cm by 6 cm
   Second pair: 1 cm by 10 cm, 3 cm by 5 cm

   Have the students estimate which rectangle has the greater area and explain their thinking.

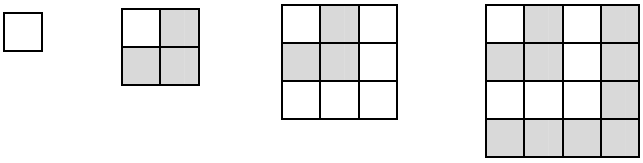
Look For …
Do students:
- ☐ rearrange the rectangles and superimpose one on the other to compare the areas?
- ☐ use more than one way to compare areas?

Adapted from Van de Walle, John A., LouAnn H. Lovin, *Teaching Student-Centered Mathematics Grades K–3* (p. 235). Published by Allyn and Bacon, Boston, MA. Copyright © 2006 by Pearson Education. Adapted by permission of the publisher.

   b. Provide the students with scissors, centimetre grid paper overlays, squares that are each 1 cm^2 and centimetre rulers.

   Have the students decide which rectangle in each pair has the greater area and explain their thinking. Encourage the students to share their ideas and critique which strategy for finding area works best for them.

2. Patterns in Area: Growing Squares
   Provide the students with a diagram showing the following pattern:



Look For …
Do students:
- ☐ apply their knowledge of patterns to area?
- ☐ discover a rule to describe the relationship between the number of rows in the square and the area of the square?

Suggested investigations:
- Describe the patterns you see in the "growing squares." Example: 1, 1 + 3 = 4, 1 + 3 + 5 = 9 and so on.
- Draw the fifth square. Explain your thinking.
- Predict the area of the tenth square. Explain your thinking.
- What is the area of the 20^{th} square? How do you know?
- Create a different pattern using area of rectangles and share it with others.

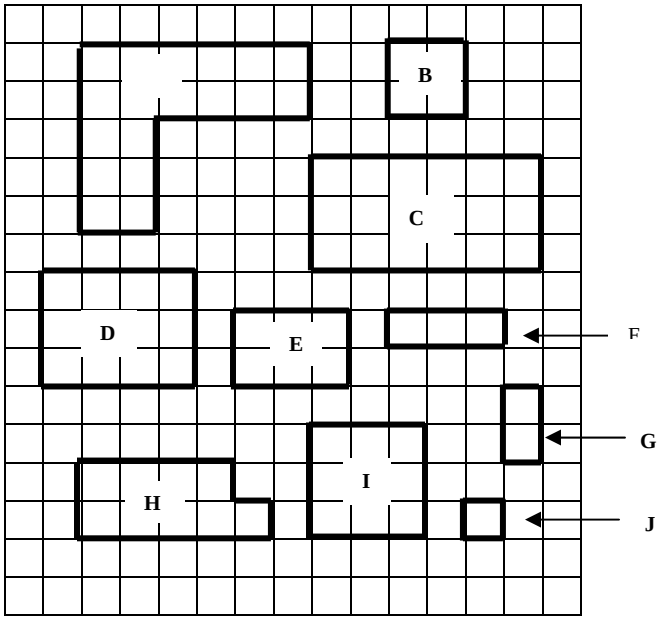
Adapted from the National Council of Teachers of Mathematics, *Principles and Standards for School Mathematics* (Reston, VA: The National Council of Teachers of Mathematics, 2000), pp. 159, 160. Adapted with permission of the National Council of Teachers of Mathematics.

3. Lake and Island Board

The Lake and Island Board uses one square centimetre as the basic unit for measuring area.

a. Provide the student with copies of the Lake and Island Board shown below without the centimetre grid. Make a transparency of the board without the centimetre grid to use for discussion with the whole class.

Lake and Island Board



b. Present the students with a set of problems, such as the following:
   - Suppose you and your family are moving to Lake and Island Country. You wish to purchase the largest island available so that there is plenty of room for the family. Which island would you purchase? Estimate first and then find the areas of the islands to determine your answer. Do you have a choice? If so, what would be the advantages and disadvantages of either choice?
   - Some friends of yours, the Jones, also wish to purchase an island. They want one that is only one-third as large as yours. Which island would you suggest that they buy? Do they have a choice? If so, what is it?
   - As time goes by, members of your family grow up. Two of the grown children, Lauren and Kevin, each wish to buy an island the same size. Which two islands would you suggest they purchase? Estimate first and then find the areas to determine your answer.
   - A plant disease epidemic breaks out and all the islands have to be sprayed. What is the total area of all the islands?
   - The Jones' family wants to double the amount of land they now own. Which set of islands could they buy to satisfy this need?
   - Name a pair of islands in which the area of one island is three times the area of the other island.
   - Name a pair of islands in which the area of one island is double the area of the other island.
   - Draw as many different rectangular islands with whole number dimensions that are the same area as Island D. Explain your thinking.

Look For …

Do students:

- ☐ use a referent in estimating the areas of the islands?
- ☐ use an appropriate strategy to find the areas of various islands?
- ☐ restate the problems in their own words to show understanding?
- ☐ draw different rectangles with the same area as a given rectangle?

4. Frayer Model for Area
   Provide the students with a template for the Frayer Model and have them fill in the sections individually or as a group to consolidate their understanding of area.
   A sample of a Frayer Model is provided below.

Frayer Model for Area

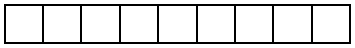
Definition
Area is the measure of the space inside a region.

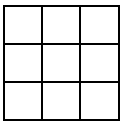
Characteristics

- the area of a region remains the same when the region is rearranged
- area can be measured in nonstandard or standard units for area
- the smaller the unit of measure the greater the number of units needed to measure a given area
- when comparing areas, the same units must be used
- standard units for area include cm^2 and m^2
- for a given area, there are usually many different rectangles than can be drawn.

Real-life Problem and Visual Representation

Cindy wants a rectangular flowerbed that is 9 m^2. What are the different flowerbeds that she could have if the lengths and widths are whole numbers?





Cindy could have 2 different flowerbeds—one that is 1 m by 9 m and the other one that is 3 m by 3 m.

Area

Examples

Area is used in the following:

- tiling floors
- seeding lawns
- painting walls
- buying windows
- covering counter tops.

Nonexamples

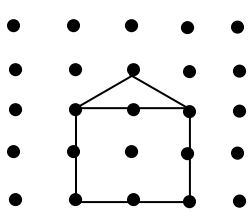
Area is not used in the following:

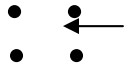
- fencing around a garden
- lace around a tablecloth
- liquid in a glass
- weight of a person
- cans of pop in a box.

Format adapted from D. A. Frayer, W. C. Frederick and H. J. Klausmeier, *A Schema for Testing the Level of Concept Mastery* (Working Paper No. 16/Technical Report No. 16) (Madison, WI: Research and Development Center for Cognitive Learning, University of Wisconsin, 1969). Adapted with permission from the Wisconsin Center for Education Research, University of Wisconsin-Madison.

Sample Activity: 5: Teaching That Many Different Rectangles Can Be Constructed for a Given Area (cm^2 or m^2)

1. Area with Geoboards
   a. Provide the students with geoboards. Explain that the nonstandard unit of measure is the smallest square on the geoboard. Use a transparent geoboard on the overhead to display some shapes, such as the following:





One square unit

   Have the students find the area of the shapes, stating the number of square units; e.g., the area of the entire shape is 5 square units. The students should explain their thinking.

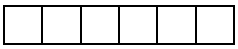
   b. Reverse the procedure and provide the students with a given area, such as 6 square units. The students are then asked to make shapes on their geoboards that have the given area, justifying their answers. Have the students compare their shapes to show that many different shapes can be created for a given area.

Look For …

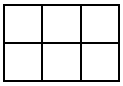
Do students:

- ☐ think flexibly and create various shapes for a given area?
- ☐ explain patterns that they discover in the different arrays constructed to represent various areas?
- ☐ translate learning from one problem to another?

2. Arrays with Tiles
   Provide the students with tiles and centimetre grid paper. Give them the following instructions:
   a. For each of the areas from 1 cm^2 to 20 cm^2, find all the possible rectangular arrays using whole numbers. For example, the possible arrays for an area of 6 cm^2 would be as follows:



1 row of 6 square units



2 rows of 3 square units

   b. Draw the arrays for each area on the centimetre grid paper.
      - Describe any patterns in the different rectangular arrays for the given areas.
      - Predict how many different arrays can be made to represent 36 cm^2. Draw the arrays to check your prediction.

   Make a class display of the various groups of arrays by having the students cut the appropriate arrays from the grid paper and hang them under the number showing the area of the arrays. For example, the two arrays shown above would be hung under the number 6 cm^2. Use the class display to facilitate class discussion and the discovery of patterns.

Examples of patterns:

- all the even numbered areas can be displayed using a double row but not the odd numbers
- all the areas with square numbers such as 4 cm^2 and 9 cm^2 can be displayed as a square as well as one long rectangle
- some of the areas can be represented by only one array; e.g., the prime numbers, such as 3 cm^2 and 7 cm^2
- some of the areas can be represented by more than one array; e.g., the composite numbers, such as 4 cm^2 and 6 cm^2
- all the areas can be represented using one row; i.e., every number is divisible by 1.

3. Area Problems

Present the following problem to the students:

> Farmer Jones is making a rectangular garden plot with an area of 24 m^2. Draw all the possible rectangular garden plots that he could make, using only whole numbers for the lengths and widths. How do you know that you have included all the possible garden plots?
> Draw all the rectangles on centimetre grid paper, where one square centimetre represents one square metre.

Encourage the students to use patterns in deciding whether or not they have included all the possible rectangular shapes for a given area. For example, an area of 24 m^2 could be represented by arrays that have: 1 row of 24 square metres
2 rows of 12 square metres
3 rows of 8 square metres
4 rows of 6 square metres.

Apply the process in solving other problems with other areas.

Look For …

Do students:

- ☐ restate the problem in their own words to show understanding?
- ☐ apply their understanding of patterns to solve the problem?
- ☐ apply the problem solving strategy to other similar problems?

4. Area Investigator

Present the following problem to the students:

> Tracy made this generalization when she was creating different rectangles for a given area, "the greater the area, the more rectangles that can be made to represent the area." Decide whether Tracy's statement is true or false. Why do you think so?

Step 4: Assess Student Learning

Guiding Questions

- Look back at what you determined as acceptable evidence in Step 2.
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Sample Assessment Tasks

In addition to ongoing assessment throughout the lessons, consider the following sample activities to evaluate students' learning at key milestones. Suggestions are given for assessing all students as a class or in groups, individual students in need of further evaluation, and individual or groups of students in a variety of contexts.

A. Whole Class/Group Assessment

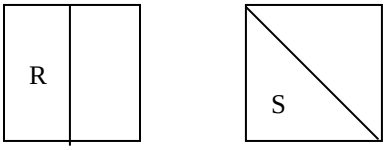
Note: Performance-based assessment tasks are under development.

Have the students complete Part A (Estimating Area) of the assessment and hand it in. Then provide them with Part B (Finding Area) along with centimetre rulers, centimetre grid paper, pattern blocks and other manipulatives that they may wish to use.

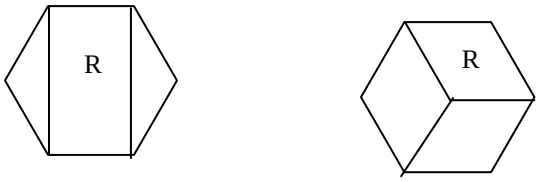
Part A: Estimating Area

1. For each of the following pairs of congruent shapes, decide if Part R has the same area as Part S. Explain your thinking.

   a.

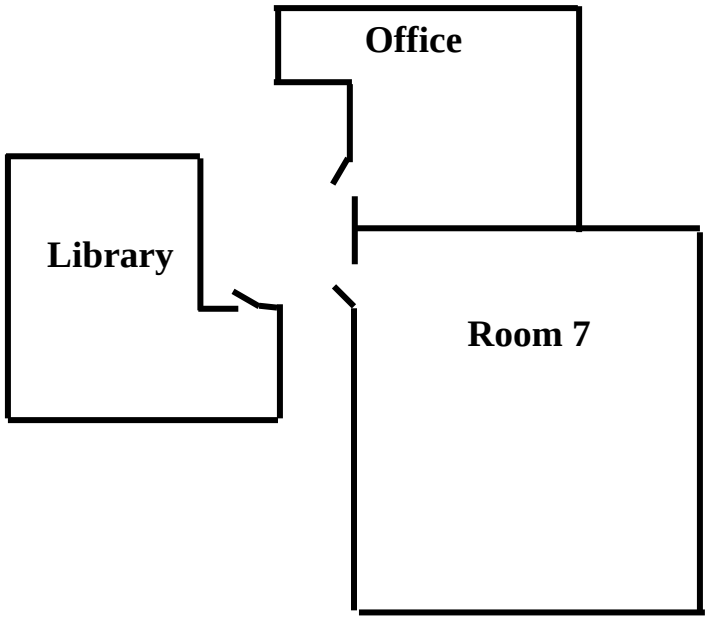


   b.



2. Estimate the area of the following rectangle. Explain the referent that you are using to make the estimate.

3. The following diagram shows the part of a floor plan for a school. If the area of Room 7 is about 90 m^2, estimate that area of the library. Explain your thinking.



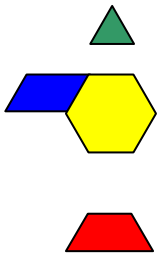
Part B: Finding Area

Provide the students with scissors, 30-cm rulers, tiles, pattern blocks, centimetre grid paper and other manipulatives as needed.

1. Compare the areas of two rectangles. One rectangle is 4 cm by 10 cm. The other rectangle is 5 cm by 8 cm.
   a. Draw diagrams on the centimetre grid paper provided and use words to explain your thinking.
   b. Explain how you could compare the areas of these two rectangles by rearranging one rectangle.

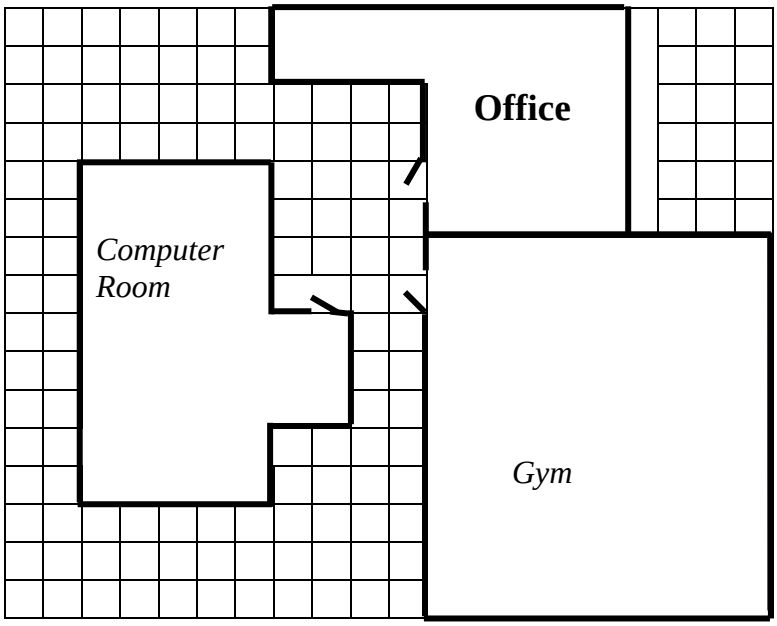
2. Explain why area is measured in square units.

3. Find the area of the following pattern block design by measuring with:
   a. triangles (green pattern blocks)
   b. rhombuses (blue pattern blocks).
   Explain your thinking.



4. Keri measured a pattern block design with the yellow hexagons and said that the measure of the area was 8 hexagons. Mark measured the same design with the green triangles and said that the area was 48 triangles. Could they both be correct? Why or why not?

5. Use the partial floor plan for a school as shown below to answer this question.

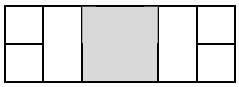


= 1 m^2

   a. Find the area of the computer room. Explain your thinking.

   b. Find the area of the office. Explain your thinking.

6. The area of the entire design below is 12 m^2.



Find the area of the shaded part. Explain your thinking.

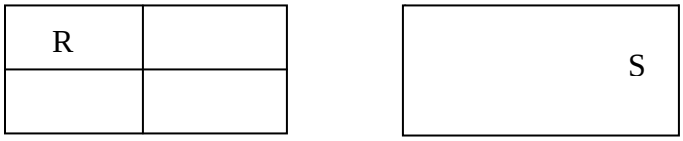
7. Sammy wants a dog pen for his dog. He wants the pen to have an area of 32 m^2.
   a. Draw **all** the possible rectangular pens that have the area of 32 m^2 on centimetre grid paper. The sides of the rectangles must be measured in whole numbers.

   b. Explain how you know that you have drawn all the possible rectangular pens.

Which pen you would advise Sammy to use? Explain why.

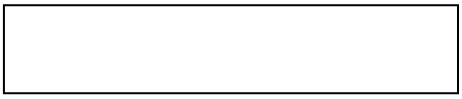
B. One-on-one Assessment

1. Place the following pair of rectangles before the student. Explain that the two rectangles are congruent (same size and shape). Ask the student to decide if Part R has the same area as Part S and explain his or her thinking.



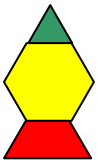
If the student has difficulty, provide the student with scissors and suggest that he or she cut out Part R and rearrange it to see if it can fit right over top of Part S. Another way to guide thinking is to ask the student what fraction of the whole is represented by Part R. Then ask what fraction of the whole is represented by Part S, reminding the student that the whole is the same size for both rectangles. Having the student think about an everyday context for the problem, such as comparing two pieces of cake from congruent cakes, might be useful.

2. Ask the student to estimate the area of the following rectangle. Have the student explain what referent he or she uses to estimate the area.



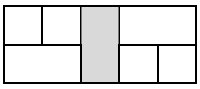
If difficulty arises, ask the student to draw a square about the size of a square centimetre. If the student is unable to do this, draw the square centimetre for him or her. Ask the student to think of some object that he or she has with him or her at all times that is about one square centimeter; e.g., smallest fingernail. Suggest that the student use this referent in deciding how many times it could be placed on the rectangle to completely cover it.

3. Place the following pattern block design in front of the student. Provide him or her with other pattern blocks as needed. Ask the student to find the area of the design by measuring with the green triangles. Have the student explain his or her thinking orally as the work is completed.



The student may not know that the unit for measure has to be the same unit repeated for the entire design. Suggest that the student use the pattern blocks and cover the design with the green triangles. Make a different design and have the student find the area using a given pattern block to see if he or she can transfer the learning.

4. Place the following design in front of the student and explain that the area of the entire design is 10^2 m.



Ask the student to find the area of the shaded part and to explain his or her thinking.

The student may not understand that all the units counted as a measure for area must be the same size. Suggest that the student make a grid on the diagram so that all the units are the same size. Then these units can be counted to determine the area. Provide the student with a ruler to draw the grid.

5. Place the following problem before the student.

> You are designing a pen for your rabbit. You want the pen to have an area of 30 m^2. Draw **all** the possible rectangular pens on centimetre grid paper. Use only whole numbers for the length and width of each rectangle.

Provide centimetre grid paper and a ruler. Ask the student to explain how he or she knows that all the possible rectangles have been drawn.

Have the student decide which rectangular pen he or she would use and why.

The student may have difficulty using such a large number. Replace 30 m^2 by 6 m^2 and ask the student to solve the simpler problem. Provide the student with tiles to represent the problem. If the student does not use an array, suggest that this be done. Have the student transfer this strategy used to solve the simpler problem to the original problem. Remind the student to draw the diagram for each array created with the 30 tiles. If the student works randomly and misses some rectangles, suggest that he or she uses a pattern to help solve the problem. For example, start with 1 row in the array and ask how many tiles it would have. With 2 rows in the array, how many tiles would be in each row?

C. Applied Learning

Provide opportunities for the students to use their understanding of area in a practical situation and notice whether or not this understanding transfers. For example, have the student decide whether or not cake is distributed equally when each of four students are given pieces that have the same area of the top surface but different shapes. Does the student:

- conserve area by recognizing that all pieces have the same area but different shapes?
- explain why the different shapes each have the same area?
- relate to fractions of regions by explaining that the cake is divided into quarters, meaning that each of the four parts has the same area regardless of whether each part has the same shape?
- apply this concept to other situations such as finding different sizes of paper that could be used to display 12 congruent photographs with no overlaps and completely covering the page?

Step 5: Follow-up on Assessment

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction?

A. Addressing Gaps in Learning

To improve estimating skills, have the students always estimate before measuring and then compare their measurements to the original estimate. As they become more familiar with the units used in area measurements, the students will have a better sense of estimating the area in the required units. Remind the student to use referents and/or chunking when estimating area. Have the students share their estimates and strategies for estimating. Begin by having the students estimate by comparing the areas of two different shapes. Then have them estimate the area of shapes. Accept a range of estimates and narrow the range as the students' estimating skills improve.

Conservation of area develops as the students mature. Continue to provide opportunities for the students to compare the areas of different shapes and have them communicate why they think as they do. Provide shapes that are easily rearranged so that one can be superimposed on the other. Encourage the students to cut the shapes and rearrange them as needed.

The students who have difficulty repeating the same unit when measuring area should have ample opportunity to explore using manipulatives such as tiles, pattern blocks and geoboards. Encourage the students to manipulate the concrete materials and draw on the diagrams to show the unit that is repeated in finding the area.

If the students have difficulty constructing different rectangles for a given area, use smaller numbers in the problem initially; i.e., do a simpler problem. Encourage the student to use tiles to represent the number as an array. See the prompts in the one-on-one assessment in the previous section.

B. Reinforcing and Extending Learning

The students who have achieved or exceeded the outcomes will benefit from ongoing opportunities to apply and extend their learning. These activities should support the students in developing a deeper understanding of the concept and should not progress to the outcomes in subsequent grades.

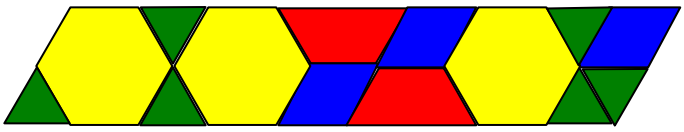
Consider strategies, such as the following.

- Provide tips for parents on helping their children to estimate and find areas of regular and irregular figures.
  - Ask the child to estimate the number of tiles needed to cover a specified floor area. Have the child explain how he or she estimated the area.
  - In board games, such as checkers, have the child estimate and then find the area of game board using a certain type of unit for measuring area. Have the child explain how he or she knows.
  - Ask the child to compare two picture frames and explain why the frames have the same or different areas.
- Provide the student with the following pair of rectangles: 3 cm by 8 cm and 4 cm by 5 cm. The rectangles should be blank except for the labels. Ask the student to compare the two rectangles and decide if they have the same area or if the area of one rectangle is greater than the area of the other rectangle. Encourage folding, cutting or other strategies to solve the problem. Have the student explain his or her thinking.

  Adapted from Van de Walle, John A., LouAnn H. Lovin, *Teaching Student-Centered Mathematics, Grades K–3*, (p. 235). Published by Allyn and Bacon, Boston: MA. Copyright © 2006 by Pearson Education. Adapted by permission of the publisher.

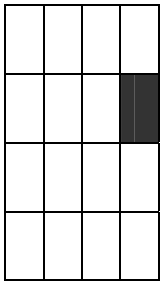
- Pose the following problem to the student:
  Prama said that if you double the lengths of a rectangle then you double the area. Do you agree with Prama? Why or why not?
- Provide the student with pattern blocks. Ask the student to find the area of challenging composite designs by using the following units to measure the area:
  - triangles—green pattern blocks
  - rhombuses—blue pattern blocks
  - trapezoids—red pattern blocks
  - hexagons—yellow pattern blocks.

  Example of a design:

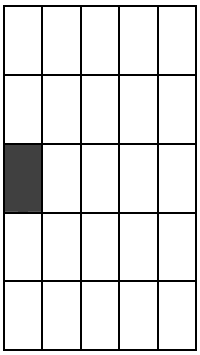


- Present the student with the following problem:
  A domino is a rectangular shape with an area of two square units and it is made up of two squares sharing a common side. A triomino is a shape with the area of three square units as shown in the diagram to the right.

  a. Use the L-shaped triominoes to tile the 4 by 4 checkerboard below. Use paper triomino models, and cover the entire checkerboard with the paper models except for the shaded square. Indicate your solution by labelling each triomino used—put an A in each of three squares for the first triomino used, put B in each of three squares for the second triomino used and so on.

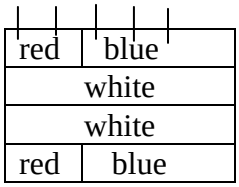


b. Use the paper triominoes to cover the 5 x 5 grid below except for the shaded square. Label your solution appropriately.



- Present the following problem to the student:

  If the total area of the flag shown below is 6 m^2, what is the area of each colour? Explain your thinking. (Hint: Divide the entire flag into 24 equal sized parts.)



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