Mathematics

Planning Guide

Grade 4 *Fractions and Decimals*

Number
Specific Outcomes 8, 9, 10

This Planning Guide can be accessed online at:
http://www.learnalberta.ca/content/mepg4/html/pg4_fractionsanddecimals/index.html

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Planning Guide: *Grade 4 Fractions and Decimals*

Strand: Number
Specific Outcomes: 8, 9, 10

This *Planning Guide* addresses the following outcomes from the Program of Studies:

Strand: Number	
Specific Outcomes:	8. Demonstrate an understanding of fractions less than or equal to one by using concrete, pictorial and symbolic representations to: • name and record fractions for the parts of a whole or a set • compare and order fractions • model and explain that for different wholes, two identical fractions may not represent the same quantity • provide examples of where fractions are used. 9. Represent and describe decimals (tenths and hundredths), concretely, pictorially and symbolically. 10. Relate decimals to fractions and fractions to decimals (to hundredths).

Curriculum Focus

This sample targets the following changes in the curriculum:

- The General Outcome focuses on number sense, whereas the previous mathematics curriculum specified demonstrating number sense for whole numbers and exploring proper fractions.
- The Specific Outcome includes comparing fractions, whereas the previous mathematics curriculum included comparing fractions in Grade 5.
- The Specific Outcome includes decimals with tenths and hundredths, whereas the previous mathematics curriculum focused on hundredths because tenths were introduced in Grade 3.
- The Specific Outcome focuses on fractional parts of whole or a set, whereas the previous mathematics curriculum focused on fractional parts of a region or a set.

What Is a Planning Guide?

Planning Guides are a tool for teachers to use in designing instruction and assessment that focuses on developing and deepening students' understanding of mathematical concepts. This tool is based on the process outlined in *Understanding by Design* by Grant Wiggins and Jay McTighe.

Planning Steps

The following steps will help you through the Planning Guide:

Step 1: Identify Outcomes to Address

Guiding Questions

- What do I want my students to learn?
- What can my students currently understand and do?
- What do I want my students to understand and be able to do based on the Big Ideas and specific outcomes in the program of studies?

Big Ideas

Fractions form a number system that includes the whole numbers and also an infinite set of numbers between every whole number. Fractions are numbers that can be written in the form $\frac{a}{b}$, where *a* and *b* are whole numbers, $b \neq 0$. The density property of fractions states that between any two fractions there is another fraction. Both the whole numbers and the fractions are infinite but the set of fractions are dense and the whole numbers are not. Also, the set of fractions is closed under addition, multiplication and division, whereas the whole numbers are closed under addition and multiplication; e.g., the sum of any two whole numbers is another whole number.

Number sense with fractions and decimals requires that the students develop a conceptual understanding of fractions and decimals as numbers. To work effectively with fractions and decimals, the students should demonstrate the ability to:

1. Represent numbers using words, models, diagrams and symbols and make connections among various representations.
2. Give other names for numbers and justify the procedures used to generate the equivalent forms.
3. Describe the relative magnitude of numbers by comparing them to common benchmarks, given simple estimates, ordering a set of number, and finding a number between two numbers.

Adapted with permission from James Vance, "Rational Number Sense: Development and Assessment," *delta-K* 28, 2 (1990), p. 24. *delta-K* is published by The Alberta Teachers' Association.

The students construct a firm foundation for fraction concepts by experiencing and discussing activities that promote the following understandings:

1. Fractional parts are equal shares or equal-sized portions of a whole or unit. A unit can be an object or a collection of things. More abstractly, the unit is counted as 1. On the number line, the distance from 0 to 1 is the unit.
2. Fractional parts have special names that tell how many parts of that size are needed to make the whole. For example, *thirds* require three parts to make a whole.
3. The more fractional parts used to make a whole, the smaller the parts. For example, eighths are smaller than fifths.
4. The denominator of a fraction indicates by what number the whole has been divided in order to produce the type of part under consideration. Thus, the denominator is a divisor. In

> practical terms, the denominator names the kind of fractional part that is under consideration. The numerator of a fraction counts or tells how many of the fraction parts (or the type indicated by the denominator) are under consideration. Therefore, the numerator is a multiplier—it indicates a multiple of the given fractional part.

Adapted from Van de Walle, John A., LouAnn H. Lovin, *Teaching Student-Centered Mathematics, Grades K–3* (p. 251). Published by Allyn and Bacon, Boston, MA. Copyright © 2006 by Pearson Education. Adapted by permission of the publisher.

The iterative nature of a fraction describes the bottom number (denominator) as telling what is being counted and the top number (numerator) telling what the count is (Van de Walle and Lovin 2006, p. 259).

The three models for fractions include:

- the region or area model, such as two-thirds of a pizza. Note: The equal parts must be the same size (area) but not necessarily the same shape (congruence) (Van de Walle 2001, p. 211)
- the length or measurement model, such as two-thirds of a paper strip or two-thirds of the distance from 0 to 1 on a number line
- set model, such as two-thirds of a set of 3 kittens (Van de Walle and Lovin 2006).

Conceptual understanding of comparing fractions is developed when the students use a variety of ways to relate fractions including the following:

- *more of the same size* in which the denominators of the fractions are the same; e.g., five-eighths is greater than three-eighths
- *same number of parts but parts of different sizes* in which the numerators of the fractions are the same; e.g., three-quarters is greater than three-fifths
- *more or less than one-half or one whole* in which numerator of the fraction is compared to the denominator in deciding its relation to a given benchmark; e.g., three-eighths is less than one-half because three is less than half of eight
- *distance from one-half or one whole* in which the fraction is one fractional part away from a whole or one-half; e.g., four-fifths is greater than three-quarters because they are each one fractional part away from a whole. Fifths are smaller than quarters and therefore four-fifths is closer to a whole than three-quarters.

Adapted from Van de Walle, John A., LouAnn H. Lovin, *Teaching Student-Centered Mathematics, Grades K–3* (p. 265). Published by Allyn and Bacon, Boston, MA. Copyright © 2006 by Pearson Education. Adapted with permission of the publisher.

Conceptual understanding of decimals requires that the students connect decimals to whole numbers and to fractions. Decimals are shown as an extension of the whole number system by introducing a new place value, the tenth's place, to the right of the one's place. The tenth's place follows the pattern of the base ten number system by iterating one-tenth ten times to make one whole or a unit (Wheatley and Abshire 2002, p. 152). Similarly, the hundredth's place to the right of the tenth's place iterates one-hundredth ten times to make one-tenth.

The connection between decimals and fractions is developed conceptually when the students read decimals as fractions and represent them using the same visuals. For example, 0.8 is read as eight-tenths and can be represented using fraction strips or decimal strips (Wheatley and Abshire 2002).

Sequence of Outcomes from the Program of Studies

See http://education.alberta.ca/teachers/core/math/programs.aspx for the complete program of studies.

Grade 3
Specific Outcomes

13. Demonstrate an understanding of fractions by:
    - explaining that a fraction represents a part of a whole
    - describing situations in which fractions are used
    - comparing fractions of the same whole that have like denominators.

Grade 4
Specific Outcomes

8. Demonstrate an understanding of fractions less than or equal to one by using concrete, pictorial and symbolic representations to:
    - name and record fractions for the parts of a whole or a set
    - compare and order fractions
    - model and explain that for different wholes, two identical fractions may not represent the same quantity
    - provide examples of where fractions are used.
9. Represent and describe decimals (tenths and hundredths), concretely, pictorially and symbolically.
10. Relate decimals to fractions and fractions to decimals (to hundredths).

Grade 5
Specific Outcomes

7. Demonstrate an understanding of fractions by using concrete, pictorial and symbolic representations to:
    - create sets of equivalent fractions
    - compare fractions with like and unlike denominators.
8. Describe and represent decimals (tenths, hundredths, thousandths), concretely, pictorially and symbolically.
9. Relate decimals to fractions and fractions to decimals (to thousandths).
10. Compare and order decimals (to thousandths) by using:
    - benchmarks
    - place value
    - equivalent decimals.

Step 2: Determine Evidence of Student Learning

Guiding Questions

- What evidence will I look for to know that learning has occurred?
- What should students demonstrate to show their understanding of the mathematical concepts, skills and Big Ideas?

Using Achievement Indicators

As you begin planning lessons and learning activities, keep in mind ongoing ways to monitor and assess student learning. One starting point for this planning is to consider the achievement indicators listed in the *Mathematics Kindergarten to Grade 9 Program of Studies with Achievement Indicators*. You may also generate your own indicators and use them to guide your observation of the students.

The following achievement indicators may be used to determine whether the students have met this specific outcome.

- Represent a given fraction using a region, object or set.
- Explain that the equal parts of a region must be the same size (area) but not necessarily the same shape (congruence).
- Identify a fraction from its given concrete representation.
- Name and record the shaded and nonshaded parts of a given set.
- Name and record the shaded and nonshaded parts of a given whole region, object or set.
- Represent a given fraction pictorially by shading parts of a given set.
- Represent a given fraction pictorially by shading parts of a given whole region, object or set.
- Explain how denominators can be used to compare two given unit fractions with numerator 1.
- Order a given set of fractions that have the same numerator and explain the ordering.
- Order a given set of fractions that have the same denominator and explain the ordering.
- Identify which of the benchmarks 0, $\frac{1}{2}$, or 1 is closer to a given fraction.
- Name fractions between two given benchmarks on a number line.
- Order a given set of fractions by placing them on a number line with given benchmarks.
- Provide examples of when two identical fractions may not represent the same quantity; e.g., half of a large apple is not equivalent to half of a small apple; half of ten Saskatoon berries is not equivalent to half of sixteen Saskatoon berries.
- Provide an example of a fraction that represents part of a set and a fraction that represents part of a region from everyday contexts.
- Write the decimal for a given concrete or pictorial representation of part of a set, part of a region or part of a unit of measure.
- Represent a given decimal using concrete materials or a pictorial representation.
- Explain the meaning of each digit in a given decimal with all digits the same.
- Represent a given decimal using money values (dimes and pennies).

- Record a given money value using decimals.
- Provide examples of everyday contexts in which tenths and hundredths are used.
- Model, using manipulatives or pictures, that a given tenth can be expressed as hundredths; e.g., 0.9 is equivalent to 0.90 or 9 dimes are equivalent to 90 pennies.
- Express orally and in written form a given fraction with a denominator of 10 or 100 as a decimal.
- Read decimals as fractions; e.g., 0.5 is zero and five tenths.
- Express orally and in written form a given decimal in fractional form.
- Express a given pictorial or concrete representation as a fraction or decimal; e.g., 15 shaded squares on a hundred grid can be expressed as 0.15 or $\frac{15}{100}$.
- Express orally and in written form the decimal equivalent for a given fraction; e.g., $\frac{50}{100}$ can be expressed as 0.50.
- Connect decimals to fractions by reading decimals as fractions and representing them using the same visuals such as fraction bars and decimal bars (Wheatley and Abshire 2002).

Some sample behaviours to look for in relation to these indicators are suggested for many of the instructional activities in **Step 3, Section C, Choosing Learning Activities** (p. 12).

Step 3: Plan for Instruction

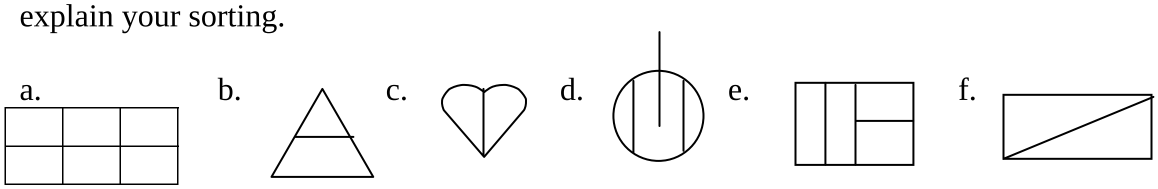
Guiding Questions

- What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should I use?
- How will I meet the diverse learning needs of my students?

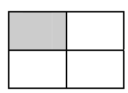
A. Assessing Prior Knowledge and Skills

Before introducing new material, consider ways to assess and build on the students' knowledge and skills related to fractions. For example:

- Describe two everyday examples of where you use fractions.
- Sort the following diagrams into those that represent equal parts and those that do not, and explain your sorting.



- Draw a diagram to represent the fraction $\frac{3}{4}$.
- Given the following diagram:

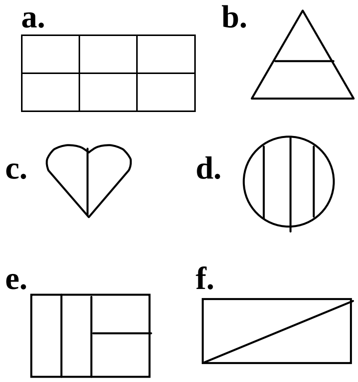


  a. Write the fraction shown by the shaded part of the diagram.
  b. Write the fraction shown by the unshaded part of the diagram.

- Explain the meaning of the numerator and denominator in the fraction $\frac{2}{3}$ and use a model to illustrate what you mean.

If a student appears to have difficulty with these tasks, consider further individual assessment, such as a structured interview to determine the student's level of skill and understanding. See **Sample Structured Interview: Assessing Prior Knowledge and Skills** (p. 10).

Sample Structured Interview: Assessing Prior Knowledge and Skills

Directions	Date:	
	Not Quite There	**Ready to Apply**
"Sort the following diagrams into those that represent equal parts and those that do not, and explain your sorting." (Show the student the following diagrams.) 	• Thinks that a and c show equal parts but not the other diagrams but cannot explain why. • Thinks that only a and c show equal parts because you could fold them to show it. • Thinks that only a, c and f show equal parts because the parts are the same size and shape.	• Thinks that a, c, e and f show equal parts because they all show parts that are the same size. The parts in e are the same size because you cut the rectangle in half and then cut each of the halves in half to get the quarters.
"Draw a diagram to represent the fraction $\frac{3}{4}$." (Show the student the fraction symbol.)	• Draws a diagram of a region divided into 4 equal parts but does not shade in 3 of them. • Draws a diagram of a region divided into 4 unequal parts and shades 3 of the parts.	• Draws a diagram of a region divided into 4 equal parts (all the same size but not necessarily the same shape) and shades in 3 of the parts.

"Given the following diagram: (Show the student the diagram.)  a. Write the fraction shown by the shaded part of the diagram. b. Write the fraction shown by the unshaded part of the diagram."	• Mixes up the shaded and unshaded parts of the diagram. • Writes a fraction with the incorrect numerator and/or denominator.	• Writes $\frac{1}{4}$ for the shaded part and $\frac{3}{4}$ for the unshaded part.
"Explain the meaning of the numerator and denominator in the fraction $\frac{2}{3}$ and use a model to illustrate what you mean." (Show the student the fraction symbol.)	• Draws a diagram of a region and shades 2 of 3 unequal parts with no explanation. • Draws a diagram of a region and shades 2 of 3 equal parts with no explanation. • Draws a diagram of a region and shades 2 of 3 equal parts and explains that the whole is divided into 3 parts but does not specify equal parts, or if equal parts are mentioned then the meaning of equal is unclear.	• Draws a diagram of a region and shades 2 or 3 equal parts with a clear explanation of the meaning of the numerator and the denominator; e.g., the denominator is 3 and divides the whole into 3 equal parts that are all the same size and the numerator is 2 which counts 2 of these equal parts.

B. Choosing Instructional Strategies

Consider the following guidelines for teaching fraction and decimals:

- Access the students' prior knowledge of fractions and decimals and build on this understanding.
- Relate fractions to whole numbers. Both represent numbers or a quantity. As whole numbers increase in size so does the quantity that they represent. With fractions, there is an inverse relationship between the number of parts in a whole and the size of the parts; i.e., the greater the denominator the smaller each part of the whole.
- To develop understanding, include everyday contexts for fractions and decimals, then use concrete representations and connect them to pictorial and symbolic representations.
- To demonstrate understanding, have the students represent the symbolic fractions and decimals concretely and pictorially.
- Provide many examples of the three models for fractions: part of a region, part of a length or measurement and part of a set.
- By using examples and nonexamples, have the students construct the meaning that the denominator is the number of equal parts of a whole, all of which are the same size. The equal parts of a region must be the same size but not necessarily the same shape whereas the equal parts of a set have the same number of elements in each part.
- Emphasize the iterative nature of a fraction by describing the bottom number (denominator) as telling what is being counted and the top number as telling what the count is (Van de Walle and Lovin 2006, p. 259). For example, focus on thirds and quarters, not on one-third and one-quarter.
- Reinforce the relationship between the symbolic and pictorial modes (symbolic fraction name, pictorial parts, pictorial whole) by posing problems in which two of these are provided and the student determines the third by using their models (Van de Walle and Lovin 2006).
- Emphasize the meaning of a fraction as the various ways to compare fractions are explored. Encourage flexibility in thinking as the students compare fractions.
- Develop understanding of decimals by relating them to whole numbers and to fractions.
- Use everyday contexts such as money or units of measurement to facilitate understanding of decimals.

C. Choosing Learning Activities

The following learning activities are examples of activities that could be used to develop student understanding of the concepts identified in Step 1.

Sample Activities:

1. **Teaching the Naming and Recording Fractions for the Parts of a Region or Set** (p. 13)
2. **Teaching That for Different Wholes, Identical Fractions May Not Represent the Same Quantity** (p. 20)
3. **Comparing and Ordering Fractions** (p. 21)
4. **Teaching Decimals by Showing Connections to Fractions** (p. 24)

Sample Activity 1: Teaching the Naming and Recording Fractions for the Parts of a Region or Set

1. Parts of a Region: Same Size but Not Necessarily the Same Shape

a. Provide the students with rectangular pieces of paper. Have them fold the paper to make quarters in as many different ways as possible. Ask the students to record each design by drawing the rectangle on centimetre grid paper or dot paper.

b. Provide each group of students with a transparency of centimetre grid paper to copy one of their designs to share with the class on the overhead projector.
Bring the examples of rectangles with quarters that are not congruent to the attention of the entire class. The students presenting the designs justify why they are divided into quarters. The purpose of this activity is to have the students construct meaning through exploration that the denominator of a fraction divides the whole region into equal parts that are all the same size (area) but not necessarily the same shape (Curcio and Bezuk 1994).

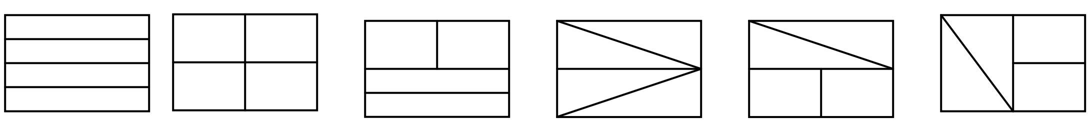
Ongoing Assessment

Look For …

Do students:

- ☐ create a variety of designs showing quarters that all have the same size but not necessarily the same shape for a given whole?
- ☐ explain why a given design shows quarters by focusing on the size or the area of each of the equal parts?
- ☐ transfer the learning about quarters to other denominators?

Examples of possible designs made by the students:



Extensions:

- Have the students sort the designs into groups in different ways and justify the categories. For example, designs showing quarters and designs not showing quarters; or designs showing quarters with congruent parts, designs showing quarters with noncongruent parts, and designs not showing quarters.
- Use a variety of shapes such as circles, trapezoids and hexagons divided into parts. Some examples will show equal parts, others will not. Some will have a different number of parts. The students sort the shapes and justify their categories.
- Have the students show quarters on 5 × 5 geoboards and record their designs on geopaper. This activity foreshadows work done on equivalence in Grade 5.
- Lead a discussion as to whether one-quarter of one design is equal to one-quarter of another design. Why or why not? This foreshadows work done on comparing fractions.
- Have the students draw designs so that $\frac{1}{2}$ is coloured red, $\frac{1}{4}$ is coloured blue, $\frac{1}{8}$ is coloured green and the remaining part is left white.

Pattern Block Designs

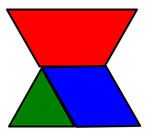
Provide the students with pattern blocks and have them create various designs to make regions (no spaces between the blocks). Have the students write the fraction of the whole region made up by various shapes, such as hexagons, trapezoids, rhombuses or triangles. Encourage the students to include the various shapes as part of their design so that they need to use logical thinking to find the needed fractions of the whole.

Look For …

Do students:

- ☐ create a variety of designs using different pattern blocks?
- ☐ explain why the fraction symbol represents the fraction of the design for a given pattern block?
- ☐ transfer the learning from one design to another design?

Example:

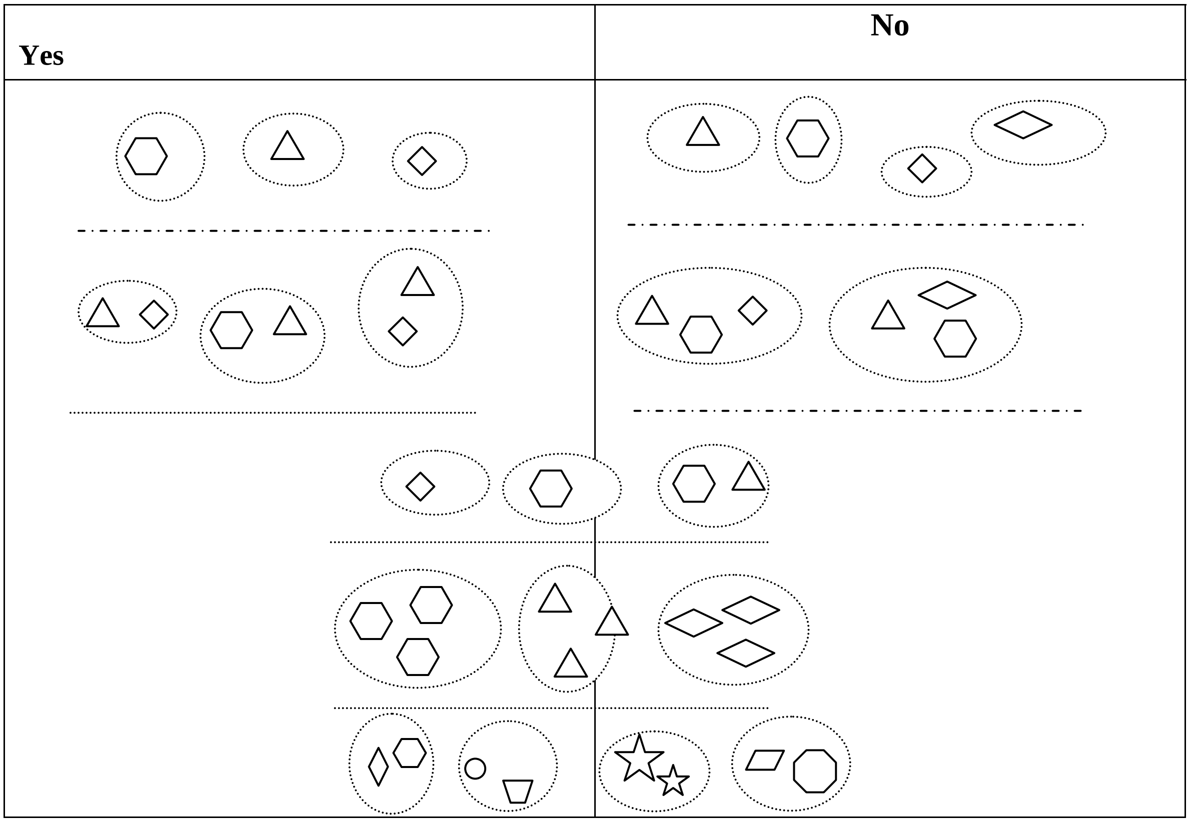


Name of Pattern Block	Fraction of the Whole
Trapezoid	$\frac{1}{2}$
Rhombus	$\frac{1}{3}$
Triangle	$\frac{1}{6}$

2. Part of a Set: Same Number of Elements in Each Part

Use concept attainment to stimulate the students' thinking about what visual representations show a fractional part of a set, such as thirds.

a. Use a Yes/No chart as shown below on the overhead projector or drawn on the board. Place an example of a set divided into thirds in the "Yes" column and a nonexample of thirds in the "No" column. Example:



Have the students explain the difference between the two.

b. Use the students' responses to decide the next example and nonexample to use. For example, if you place a set divided into thirds in the "Yes" column and a set divided into quarters in the "No" column, the students might say that one has 3 parts and the other has 4 parts. Since you want to focus on the equal parts, show the next example where there are 6 pattern blocks in each column but the grouping is different. Have the students explain how the two examples in the "Yes" column are the same and how they are different from the two examples in the "No" column.

c. Place the next example on the line between the "Yes" and the "No" columns and have the students decide where they think the example should go and justify their response. After discussion, place the example in the correct column to continue attaining the concept of thirds of different sets. Continue with examples and nonexamples until the class discussion summarizes that:
   - the examples in the "Yes" column all show thirds of sets of objects

Look For …

Do students:

- ☐ explain how the examples in the "Yes" column are the same by focusing on the characteristics they have in common but are different from the examples in the "No" column?
- ☐ decide into which column to place a given set and justify their choice?
- ☐ transfer the learning about fractions of sets to other everyday contexts?
- ☐ connect the fraction symbols to the concrete or pictorial representation of sets?

- the thirds for any given whole set always have the same number of objects in each third; e.g., in the first example, each third has one object; in the second example, each third has two objects
- the objects in each third for a given whole set do not have to be the same size, just the number of objects in each of thirds has to be the same
- the number of objects in each third depends on the size of the set.

Extension:

- Use concept attainment to develop understanding that the denominator of a fraction shows equal parts of a region and these parts must be the same size (area) but not necessarily the same shape.
- Use concept attainment to develop understanding of the similarities and differences between fractions of a set and fractions of a region.
- Provide the students with counters, pattern blocks or other concrete materials as well as paper plates. Have the students use a given number of objects such as 8, 12, 16 or 20 and show quarters by dividing the set of objects into four equal groups, placing each group on a separate paper plate. Have the students draw diagrams to match each concrete representation. The students could shade their diagrams to show $\frac{3}{4}$ in each case.
- Have the students use 20 objects to show the fraction $\frac{2}{5}$.
- Have the students exchange diagrams of sets with equal part(s) shaded for other students to write the appropriate fraction for the shaded or unshaded part.
- Have the students discuss whether half of a set of 20 objects is the same, more or less than half a set of 10 objects. Why? This foreshadows work done on comparing fractions.

3. Similarities and Differences

After the students understand halves, thirds and quarters of a region and of a set, have them draw diagrams to show fifths using a region and then using a set. Have the students discuss the similarities and differences between the two representations and write their answers in a chart as follows:

Similarities	Differences

Look For …

Do students:

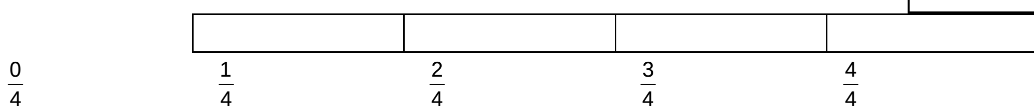
- ☐ apply their knowledge of regions and sets by drawing diagrams to show fifths in each case?
- ☐ explain the similarities between fractions of a region and fractions of a set by focusing on the meaning of the numerator and denominator?
- ☐ explain the differences between regions and sets by focusing on the meaning of equal parts?

Extensions:

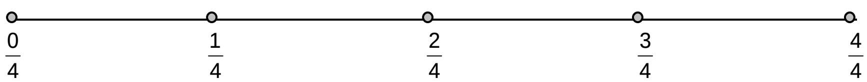
- Have the students use objects or diagrams to show three-quarters of a region and three-quarters of a set, discussing similarities and differences.
- Have the students discuss the similarities and differences between:
  - the numerator of a fraction and the denominator
  - fractions and whole numbers
  - fractions and decimals.

4. Fraction Bars to Number Line
   Provide each student with a strip of paper at least 25 cm long. Have them measure and cut off a strip that is 20 cm long. By folding or by measuring, have the students divide the strip into 4 equal parts and mark the parts to show $\frac{0}{4}$, $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$, and $\frac{4}{4}$.

Look For …
Do students:
- ☐ measure equal parts on the number line and use the fraction symbols appropriately?
- ☐ transfer the knowledge about quarters on a number line to other fractions on a number line?



Then have them draw a line segment 20 cm long and mark the same fractions on the line segment.



This activity provides a foundation for work done later in comparing fractions by using benchmarks on a number line.
Have the students show halves, fifths and tenths on other strips and transfer the fractions to a number line (Alberta Education 1990).

Frayer Model

Have the students summarize their understanding of proper fractions by completing a Frayer Model such as the following example. If the students are not familiar with using the Frayer Model, then this strategy for consolidating understanding of concepts should be modelled first (*I do*) and then done with the students (*We do*) before having them complete one on their own (*You do*).

Look For …
Do students:
- ☐ apply their knowledge of fractions and write a definition and the characteristics in their own words that is mathematically correct?
- ☐ create and justify examples and nonexamples of regions and sets for a given fraction?

Frayer Model for a Fraction

Definition

A proper fraction is a number that represents part of a whole region, set or length.

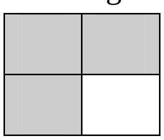
Characteristics

- part of a region, set or length
- the numerator counts
- the denominator shows what is counted
- the denominator divides the whole into equal parts
- equal parts of a region have the same size but not necessarily the same shape
- equal parts of a set have the same number of objects

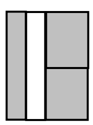
Fraction

Examples of $\frac{3}{4}$ shaded:

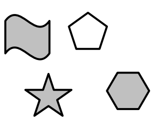
First Region



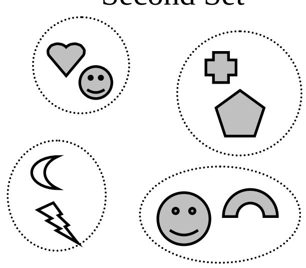
Second Region



First Set

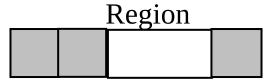


Second Set

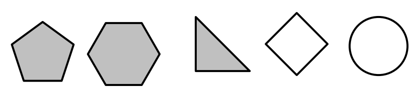


Nonexamples of $\frac{3}{4}$ shaded:

Region



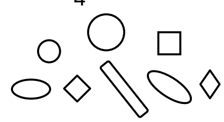
Set



Format adapted from D. A. Frayer, W. C. Frederick and H. J. Klausmeier, *A Schema for Testing the Level of Concept Mastery* (Working Paper No. 16/Technical Report No. 16) (Madison, WI: Research and Development Center for Cognitive Learning, University of Wisconsin, 1969). Adapted with permission from the Wisconsin Center for Education Research, University of Wisconsin-Madison.

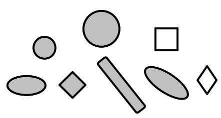
5. Problems Using Two out of Three
   Review the connections between the whole, the part and the fraction symbol. Provide various problems that give two of these—whole, part and fraction—and guide the students to use their models to determine the third. Provide problems using part of a region and part of a set. Problems with part of a set are provided as examples below:

   a. Given the whole and fraction, find the part.
      You have 8 candies and eat one-quarter ($\frac{1}{4}$) of them. How many candies did you eat? Explain how you know.

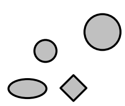


Whole Set of Candies

   b. Given the whole and the part, find the fraction.
      You have 8 candies and eat 6 of them. What fraction of the candies do you eat? Explain how you know.
      The shaded candies show the candies that are eaten.



   c. Given the part and the fraction, find the whole.
      You eat 4 candies that make up one-half ($\frac{1}{2}$) of the candies in your pocket. How many candies did you have in your pocket at the beginning? How do you know?



Eaten Candies

Look For …

Do students:

- ☐ explain their thinking by using the diagrams and fraction symbols appropriately?
- ☐ recognize the different problem types for fractions by verbalizing the unknown in the problem and what the numbers in the problem represent?
- ☐ ask questions to clarify the meaning of problems?
- ☐ transfer their understanding to other similar problems?

Adapted from Van de Walle, John A., LouAnn H. Lovin, *Teaching Student-Centered Mathematics, Grades K–3* (pp. 261–262). Published by Allyn and Bacon, Boston, MA. Copyright © 2006 by Pearson Education. Adapted with permission of the publisher.

Sample Activity 2: Teaching That for Different Wholes, Identical Fractions May Not Represent the Same Quantity

1. Licorice Fallacy

   By providing everyday contexts in which the whole region varies in size, you stimulate the students' thinking to generalize that when comparing fractions, the whole must be the same size for each fraction. For example, half of a large watermelon is much more than half of a small watermelon.

   Present the following situation to the students:
   Tara ate one-half of her licorice and Ben ate one-half of his licorice. Tara said that she ate more licorice than Ben. Explain how Tara could be right by using diagrams and words.

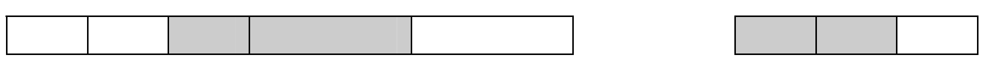
Look For …
Do students:
- ☐ provide everyday contexts to show that the same fraction may not represent the same quantity for different wholes?
- ☐ critique fallacies about fractions by drawing diagrams and using arguments that are mathematically sound?

Adapted from the National Council of Teachers of Mathematics, *Principles and Standards for School Mathematics* (Reston, VA: The National Council of Teachers of Mathematics, 2000), p. 199. Adapted with permission of the National Council of Teachers of Mathematics.

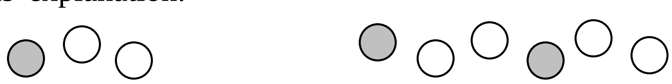
2. Critiquing Fallacies

   To reinforce the idea that the whole must be the same when comparing fractions, have the students examine misconceptions such as the ones described below:

   Lori and Francis believed that $\frac{2}{6}$ is greater than $\frac{1}{3}$.
   Lori's explanation:



   Francis' explanation:



Dogs Cats

   Through discussion, have the students verbalize that whether you are comparing fractions representing part of a region or part of a set you must remember that the whole must be same; e.g., you can't compare two fractions using different sized cakes or different sized sets of animals (NCTM 1991).

Sample Activity 3: Comparing and Ordering Fractions

1. More of the same-size parts

   Provide the students with copies of fraction bars showing various fractions such as thirds, quarters, fifths or tenths. For given fraction bars, such as the ones showing fifths, have the student name the shaded parts on the different bars; e.g., one-fifth, two-fifths, three-fifths, four-fifths and five-fifths. Ask them to explain which is larger, four-fifths or three-fifths of the fraction bar. Why do they think so? Similarly discuss why three-tenths is less than seven-tenths.

   Through discussion, have the students verbalize the following:
   Fractions with the same denominator have the whole divided into the same sized parts. Therefore, if you have more of these parts then the fraction is larger than if you have less of these parts. Example: two-thirds is greater than one-third because both fractions are thirds and you have more third in two- thirds than in one-third. Note: The wholes must be the same when comparing fractions.

   Have the students order a set of fractions having the same denominator with different numerators and then check their work by using the fraction bars.

Look For …

Do students:

- ☐ apply their understanding of numerators and denominators in comparing fractions?
- ☐ apply their understanding of number lines in using benchmarks to compare fractions?
- ☐ explain with clarity why one fraction is greater than another fraction by using different strategies depending on the relation between the fractions?

2. Same number of parts, but parts of different sizes
   With fractions, there is an inverse relationship between the number of parts in a whole and the size of the parts; i.e., the greater the denominator, the smaller each part of the whole. The following explorations guide the students to construct their own meaning of this inverse relationship.

   a. Provide the students with fraction bars for quarters and fifths. Have them compare three-quarters to three-fifths using the appropriate fraction bars and explain how they know. Guide discussion to generalize that the quarters are larger parts than the fifths because the whole is divided into fewer equal parts. Therefore, three-quarters must be greater than three-fifths because you are comparing three larger parts to three smaller parts. Similarly, have the students compare four-fifths and four-tenths and explain why one is greater than the other.

   b. Through discussion, have the students verbalize the following:
   Fractions with different denominators have the whole divided into the different sized parts. If you have same number of parts in two situations but the parts in one case are smaller than the parts in the other case, then the fraction with the smaller denominator (showing the larger parts) is greater. Example: four-fifths is greater than four-tenths because both fractions are focusing on four parts of the whole but the parts are different

sized. Fifths are greater than tenths, therefore, four-fifths is greater than four-tenths. Note: The wholes must be the same when comparing fractions.

c. Have the students order a set of fractions having the same numerator with different denominators and then check their work by using the fraction bars.

3. More or less than one-half (Using Benchmarks)

a. Provide the students with fraction bars showing halves and other fractions, such as thirds, quarters, fifths and tenths. Using the fraction bars, have the students order two fractions by comparing each fraction to one-half, such as one-quarter and two-thirds or three-fifths and eight-tenths. Through discussion, have the students generalize that some fractions can be ordered by deciding if they are greater than or less than one-half.

b. Ask the students to explain how they would know if a fraction was greater than or less than one-half without using the fraction bars. Guide the discussion to compare the numerator with the denominator of each fraction. If the numerator is less than half the denominator, then the fraction is less than one-half. Similarly, if the numerator is greater than half the denominator, then the fraction is greater than one-half. If the numerator is half the denominator, then the fraction shows another name for one-half.

c. Have the students order a pair of fractions having different numerators and denominators in which one fraction is less than one-half and the other fraction is greater than one-half.

4. Distance from zero or one whole

a. Provide the students a fraction bar showing a whole. Have them compare fraction bars showing four-fifths and nine-tenths with the whole and determine which fraction is greater. Guide discussion to focus on the fact that each fraction is one fractional part away from the whole. Since fifths are greater than tenths, then four-fifths is farther away from the whole than nine-tenths. Therefore, four-fifths is less than nine-tenths.

b. Have the students suggest other pairs of fractions that are close to one whole such as three-quarters and two-thirds and order them without using the fraction bars, explaining their thinking.

c. Similarly, have the students order fractions that are close to zero, such as one-eighth and one-tenth. Because tenths are smaller than eighths, one-tenth is closer to zero than one-eighth. Therefore, one-tenth is less than one-eighth.

d. Comparing fractions by using the distance of the fraction to one-half would require an understanding of equivalent fractions and this is part of the Grade 5 mathematics curriculum.

5. Fractions Marching in Order

Have the students draw a number line indicating 0, $\frac{1}{2}$ and 1.

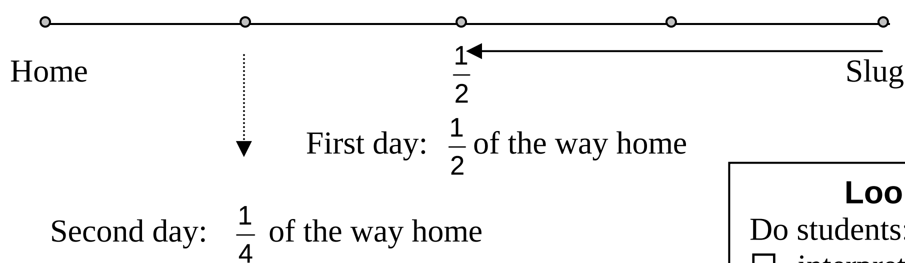
Provide the students with five or six fractions and have them place them in order on the number line, explaining how they know where each fraction should be placed. The students might wish to explain their thinking by referring to various strategies for comparing fractions, such as More of the Same Parts, Same Number of Parts, but Parts of Different Sizes, More or Less than One-half and Distance from Zero or One Whole.
Examples of fractions to place on the number line using the benchmarks might include:
$\frac{3}{4}, \frac{1}{5}, \frac{7}{8}, \frac{3}{6}, \frac{2}{5}, \frac{1}{3}$

Look For …
Do students:
- ☐ apply their knowledge of comparing two fractions to comparing more than two fractions?
- ☐ explain with clarity why one fraction is greater than another fraction by using different strategies depending on the relation between the fractions?

6. Homeward Bound
Relate the following scenario to the students.
A young slug named Slowpoke was frustrated because it was taking him so long to get home. A wise owl heard his complaints and gave this word of encouragement. "Think about travelling only half the distance to your home each day. In this way, the distance each day is shortened and you will eventually reach your home."

Ask the students if you agree with the wise owl. Why or why not?

Have the students predict the answer first and then test their prediction by using an appropriate strategy. Encourage the students to draw and number line and use fractions to illustrate the situation and come to a conclusion as to whether or not the slug will ever get home.



Guide discussion to conclude that the slug will never get home because another fraction can always be found between the given fraction and zero, which represents the slug's home. Adapt the scenario to reaching a finish line and include discussion about placing fractions between $\frac{9}{10}$ and 1 on the number line. Encourage the students to generalize that between any two fractions there is another fraction. This is the density property of fractions sets fractions apart from whole numbers.

Look For …
Do students:
- ☐ interpret the problem and apply the appropriate pattern?
- ☐ apply number facts (doubling) to create the different denominators for the fractions?
- ☐ explain with clarity why the slug will never reach home following the pattern suggested by the wise owl?

Sample Activity 4: Teaching Decimals by Showing Connections to Fractions

1. Decimals with Money

   a. Provide the students with loonies, dimes and pennies. Review the relationship among the coins and focus on groups of ten. Relate these groups of ten to the base ten number system. Have the students write symbols for whole number amounts of money, such as \$15. Then focus on the necessity of writing values for money less than one loonie or one whole dollar. Explain that the whole number system is extended to accommodate the need to write numbers smaller than one by dividing the whole (dollar) into ten equal parts, called tenths (dimes). Have the students continue this pattern using their understanding of money; i.e., ten pennies make a dime and one hundred pennies make a dollar. Introduce the decimal symbol indicating that it separates the whole number from the fractional parts called tenths and hundredths. Have the students suggest how they might write 20 cents as a fraction of one dollar, using fractions and then decimals. Guide them to see that it can be written as $\$\frac{20}{100}$ but it is usually written as \$0.20, meaning that there are no dollars, but rather two-tenths of a dollar (two dimes) or twenty-hundredths of a dollar (20 cents).

   b. Provide opportunity for the students to write decimal symbols for various amounts of money. Also have them use money or drawings to show money amounts using decimals, such as \$0.32.

   c. Have the students use a place value mat showing hundreds, tens, ones, tenths and hundredths (Alberta Education 1990) to reinforce the connections between the concrete (money) and the symbolic representation for decimals.

2. Decimals Connected to Fractions (Tenths) Using Decimal Bars

   a. Provide the students with a set of decimal bars (congruent strips that form part of the set of fraction bars showing one whole, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 and 1.0 shaded) (Alberta Education 1990).

   Have the students relate the decimal bars to the fraction bars used previously to represent various fractions. Discuss the similarities and differences between the two sets of bars.

   Have the students write the fraction symbol for each of the decimal bars.

Look For …

Do students:

- ☐ connect money to the base ten number system?
- ☐ connect the concrete, pictorial and symbolic representations for decimals?
- ☐ use a variety of concrete representations for decimals?
- ☐ explain how decimals are an extension of the whole number system by using concrete, pictorial and symbolic representations?
- ☐ connect fractions (tenths and hundredths) to decimals, using symbols?

b. Review the base ten whole number system by having the students show hundreds, tens and ones using base ten materials. Explain that we often have measures or amounts that are less than one and are represented by fractions, such as four-tenths of a metre of string. Focus on the need to continue the pattern in our base ten number system, so that the unit or the whole is divided into ten equal parts or tenths and another place value is included to the right of the one's place, separated by a dot or decimal to show that it is a fractional part. Write four-tenths as a fraction and then as a decimal to show the connection between fractions and decimals as well as the connection between whole numbers and decimals; e.g., $\frac{4}{10} = 0.4$. Explain that we often write 0.4 m rather than using the fractional notation.

c. Have the students write the decimal notation beside the fractional notation for each of the decimal bars.

3. Base Ten Materials and Hundredths Grids

a. Provide the students with base ten materials and build on their prior knowledge of the whole number system with the unit representing the whole or one. Review an example with pattern blocks in which the whole can be the yellow hexagon with the green triangle representing one-sixth or the whole can be the blue rhombus with the green triangle representing one-half. Transfer this idea to the base ten materials. Say that the flat (previously known as the hundred's block) will now represent one whole. Through discussion, have the students verbalize that if the flat is one whole, then the long is one-tenth and the small cube is one-hundredth.

b. Provide practice in representing various decimals with the base ten materials and writing the appropriate decimal symbols. The overhead base ten materials are very useful as a means of showing various base ten representations to the whole class for them to discuss and critique.

Have the students use a place value mat showing hundreds, tens, ones, tenths and hundredths (Alberta Education 1990) to reinforce the connections between the concrete (base ten materials) and the symbolic representation for decimals.

c. Connect the work done with base ten materials to the pictorial representation by providing the students with a sheet of hundredths' grids. Have them shade in an amount to represent the decimal shown by the base ten materials.

d. Reinforce the connection between decimals and fractions by having the students write the fraction and the decimal for the shaded part. Conversely, provide the students with decimals or fractions (tenths and hundredths only) and have them shade the appropriate amounts on the hundredth grids. Encourage them to write the decimal and fraction for the unshaded part and compare the numbers they wrote for the shaded and unshaded parts. For example, if 0.32 is shaded then 0.68 is unshaded. The connection between these two decimals provides the foundation for adding and subtracting decimals later in Grade 5.

Other strategies for teaching fractions and decimals are available in the *Diagnostic Mathematics Program, Elementary: Numeration, Division II* (Alberta Education 1990, pp. 225–257).

Step 4: Assess Student Learning

Guiding Questions

- Look back at what you determined as acceptable evidence in Step 2.
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Sample Assessment Tasks

In addition to ongoing assessment throughout the lessons, consider the following sample activities to evaluate students' learning at key milestones. Suggestions are given for assessing all students as a class or in groups, individual students in need of further evaluation, and individual or groups of students in a variety of contexts.

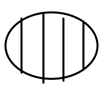
A. Whole Class/Group Assessment

Note: Performance-based assessment tasks are under development.

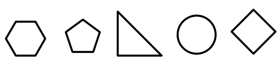
Have counters, base ten materials, play money and fraction bars for the students to use as needed.

1. For each diagram, explain why it shows fifths or why it does not show fifths.

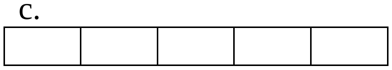
a.



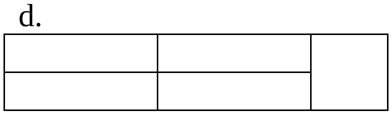
b.



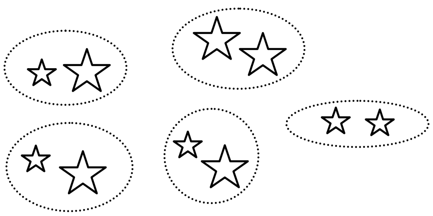
c.



d.



e.

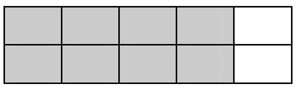


2. a. Draw a diagram for $\frac{2}{3}$ as part of a region.

   b. Draw a diagram for $\frac{2}{3}$ as part of a set.

3. Write a fraction and a decimal to show the shaded part of each of the following diagrams:

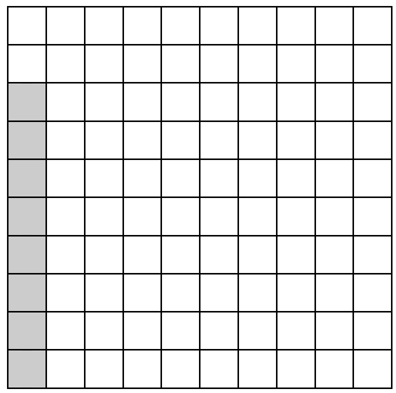
   a.



   Fraction: _________

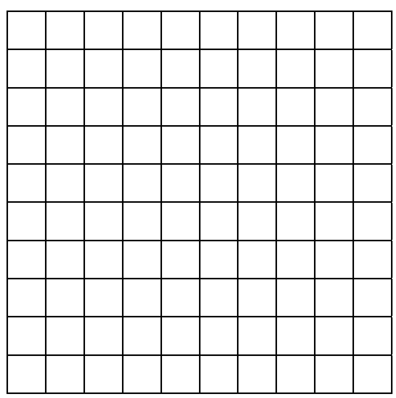
   Decimal: _________

   b.



   Fraction: _______ Decimal: ________

4.
   a. Shade the following diagram to show 0.45.



   b. Write the words you would say if you read this decimal.

5. Write $\frac{5}{100}$ as a decimal.

6. For each of the following pairs of fractions, circle the larger fraction. Explain how you know it is larger.

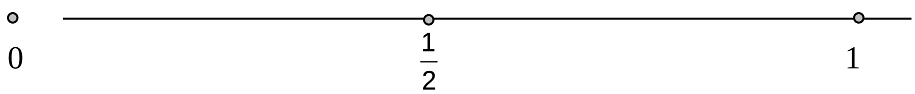
   a. $\frac{3}{5}$ and $\frac{3}{4}$

   b. $\frac{4}{6}$ and $\frac{5}{6}$

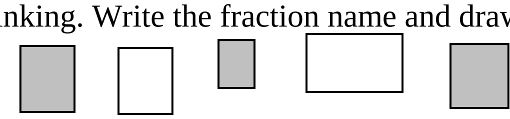
   c. $\frac{9}{10}$ and $\frac{7}{8}$

   d. $\frac{2}{3}$ and $\frac{3}{8}$

7. Place the following fractions on the number line below.
   $\frac{5}{6}$, $\frac{1}{4}$, $\frac{7}{8}$, $\frac{5}{8}$, $\frac{1}{10}$

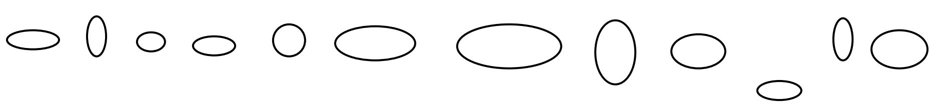


8. You have 5 books and read 3 of them. What fraction of your books did you read? Explain your thinking. Write the fraction name and draw a diagram to represent the fraction.



   The shaded books are the books you have read.

9. You have 12 pet rocks and paint $\frac{1}{4}$ of them. How many rocks do you paint? Explain your thinking. Complete the diagram to show quarters using your pet rocks.



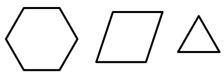
10. Explain the meaning of each digit in $3.33.

11. Tran ate $\frac{3}{4}$ of her pizza and Tung ate $\frac{3}{4}$ of his pizza. Tung said that he ate more pizza than Tran. Explain how Tung could be right by using diagrams and words.

B. One-on-one Assessment

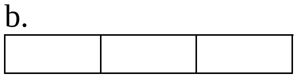
Have counters, base ten materials, play money and fraction bars for the students to use as needed.

1. Place the following diagrams in front of the student one at a time. Have the student explain orally why each diagram shows thirds or why it does not show thirds.

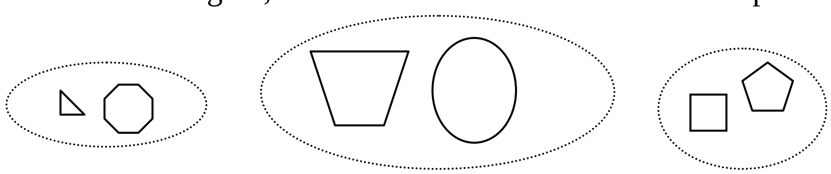
   a. 

   If the student says this represents thirds but cannot explain why, focus his or her attention on the number of pattern blocks shown. Provide the student with pattern blocks so he or she can see there are 3 separate blocks, making thirds. If the student says the diagram does not show thirds because the blocks are different sizes, transform the problem into one in which there are 3 children of different sizes. Ask if each child in a group of 3 children form one-third of the set?

   b.

   If the student has trouble explaining why this diagram represents thirds, ask him or her to count the parts and ask if each part is the same size. Ask how he or she knows. Encourage the student to trace the figure, cut it out and fold it to show that the parts are the same size.

   c. 

   If the student says that this diagram does not represent thirds or has trouble explaining why it does represent thirds, have him or her focus on the three dotted ovals and ask how many groups are shown. If the student says the diagram does not represent thirds because the groups are different sizes (one group has large figures, whereas the other two groups have smaller figures), ask him or her if there is the same number of shapes in each group. Prompt the student that for the parts of a set to be equal the number of items in each part must be the same.

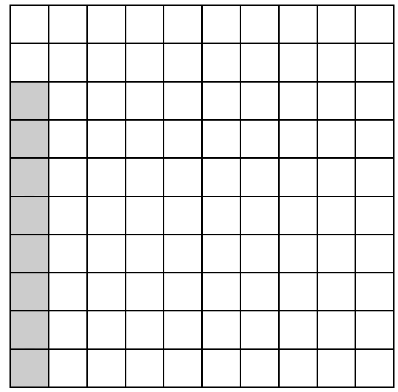
2. Place the following diagrams before the student one at a time. Have the student write a fraction and a decimal to show the shaded part of each of the diagrams. Have the student read the decimal that is written.

a.

Fraction: __________

Decimal: __________

b.



Fraction: ________ Decimal: _________

If the student has difficulty writing the fraction, have him or her count the total number of squares in the diagram and ask what part of the fraction that number represents. Then have him or her count the number of shaded squares and write that number as part of the fraction.

If the student has trouble writing the decimal, use the place value mat with the labels ones, tenths and hundredths. Have the student write the digits in the appropriate spaces on the mat, reminding him or her that number of shaded squares (tenths for the first diagram and hundredths for the second diagram) must be placed on the mat in the proper location to write the correct decimal.

If the student has difficulty reading the decimal, prompt by saying that the decimal is read the same way that the fraction is read.

3. Place the following pairs of fractions before the student, one at a time. Tell the student to circle the larger fraction and explain orally how he or she knows that the fraction is larger.

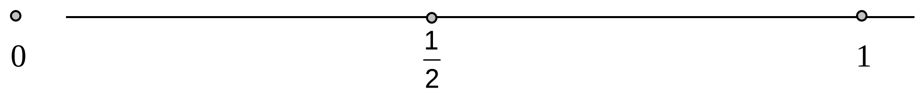
a. $\frac{3}{8}$ and $\frac{3}{5}$

If the student has difficulty ordering these fractions, ask him or her to explain what is the same about the two fractions and what is different. Then have him or her identify which fraction has three of the larger parts. Have the student use the fraction bars if he or she has difficulty telling you that fifths are larger than eighths. The fraction bars can also be used to highlight the fact that both fractions have 3 in the numerator.

b. $\frac{3}{4}$ and $\frac{2}{4}$

If the student has difficulty ordering these fractions, ask him or her to explain what is the same about the two fractions and what is different. Then have him or her identify which fraction has more quarters. Have the student choose the appropriate fraction bars to represent these two fractions and show that both bars show quarters but the bar showing three-quarters has one more quarter shaded than the bar showing two-quarters.

4. Have the following fractions printed on cards. Tell the student to place the fraction cards on the number line shown to show the order of the fractions.
$\frac{1}{5}$, $\frac{3}{5}$, $\frac{1}{3}$



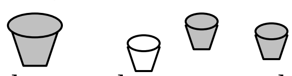
If the student has difficulty starting the task, provide him or her with only one fraction such as $\frac{1}{5}$ and ask him or her if the fraction is more or less than one-half, pointing to the one-half on the number line. Use an everyday context, such as comparing different lengths of licorice, so the students can relate better to the question. Remind the student to look at the denominator to see which fraction has bigger parts to make up the whole. Since both one-fifth and one-half have one for the top number, then you just have to order the fractions by looking at the bottom number. Halves are bigger than fifths (use the fraction bars if necessary to verify), therefore, one-fifth is less than one-half and is placed closer to zero.

Then have the student focus on one-third and check if he or she can apply a similar argument to one-third. Have the students use fraction bars if difficulty still exists.

Prompt the student to place the fraction, three-fifths, reminding him or her to decide if it is more or less than one half. Ask, if three more or less than half of five. Would you have more licorice if you had three-fifths or one-half?

5. Read the following problem orally to the student and also have it available for the student to read with you or reread as he or she solves the problem.

   You have 4 cupcakes and eat 3 of them. What fraction of the cupcakes did you eat? Explain your thinking. Write the fraction.

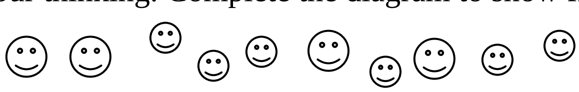
   

   The shaded cupcakes are the ones you have eaten.

   If difficulty arises, prompt the student to use the diagram to interpret the problem. Have the student explain orally what the numbers in the problems mean. Do they refer to the part or to the whole? Remind the student that the number of parts in the whole always goes on the bottom of the fraction.

6. Read the following problem orally to the student and also have it available for the student to read with you or reread as he or she solves the problem.

   You have 10 pets and $\frac{1}{5}$ of them are dogs. How many dogs do you have as pets? Explain your thinking. Complete the diagram to show fifths for your pets.

If difficulty arises, prompt the student to decide if the 10 represents all the pets or only part of the pets. Ask how many groups are needed. What does the one-fifth mean? Suggest that he or she divide the group of pets into fifths. Have the student focus on the meaning of one-fifth; i.e., the whole is divided into five equal parts with each part having the same number of items. Review the number facts to find the number of items in each group; i.e., 10 divided by 5 is 2. Have the students reread the problem and answer the question that is asked.

C. Applied Learning

Provide opportunities for the students to use fractions and decimals in a practical situation and notice whether or not the strategies transfer. For example, ask the student if he or she would rather have three-fifths or three-quarters of a chocolate bar and explain why.

Does the student …

- indicate that he would rather have three-quarters of the chocolate bar because he would have more?
- explain that quarters are larger than fifths because there are fewer quarters than fifths in the whole?
- draw an appropriate diagram to explain his or her reasoning if asked to do so?
- apply his or her fraction sense to other situations when comparing fractions that have the same denominators and different numerators or are more or less than one-half?

Step 5: Follow-up on Assessment

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction?

A. Addressing Gaps in Learning

- Draw on the prior knowledge of the students, spending time reviewing simple fractions as part of a region before introducing fractions as part of a set. Review the meaning of fraction and how it relates to a part and to a whole.
- Provide everyday contexts for fractions and decimals that the students can relate to.
- Use concrete materials such as fractions bars, fraction blocks, base ten materials and money. Connect the concrete to diagrams and symbols.
- Allow the students to use concrete materials as long as necessary to establish an understanding of the concepts.
- Emphasize the similarities and differences between a fraction of a region and a fraction of a set.
- Connect the number line to concrete fraction bars, recognizing that the number line is very abstract for many students.
- Ask guiding questions to direct the student's thinking. See the examples provided on the one-on-one assessment.
- Provide time for the students to explore and construct their own meaning rather than being told.
- Encourage flexibility in thinking as the students describe various ways to order fractions.
- Have the students share their thinking with others so that the students having some difficulty hear how another person thinks about fractions and decimals in 'kid' language.

B. Reinforcing and Extending Learning

The students who have achieved or exceeded the outcomes will benefit from ongoing opportunities to apply and extend their learning. These activities should support the students in developing a deeper understanding of the concept and should not progress to the outcomes in subsequent grades.

Consider strategies, such as the following.

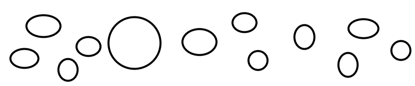
- Provide parents with suggestions of using fractions and decimals with their children, such as the following:
  - Have your child count money and write the decimal notation.
  - Tell your child that he or she can eat one-third of a given number of crackers or cookies, such as 6 or 9.

- Ask your child if he or she would fair if he or she had one-quarter of a small pizza and his or her brother had one-quarter of a large pizza. Explain why or why not.
- Have your child read recipes using fractions and then use the appropriate measuring cups to measure the required quantities.
- Have your child compare two fractions such as two-thirds of a cup of juice and three-quarters of a cup of juice and explain why one is larger than the other, then verify by using a measuring cup.

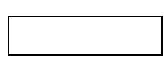
• Have the students make a set of cards that connect the diagram to the fraction to the decimal. For example, have the students draw a diagram on one card to show three-tenths, write the fraction $\frac{3}{10}$ on second card and write the decimal 0.3 on a third card. When the set of cards is completed, the students could use them to play memory (pick up a triplet rather than a pair), Go Fish, Rummy (focusing on three of a kind) or some other appropriate game that requires matching.

• Use everyday items such as books, stickers, pizza and cake to provide contexts for reinforcing the meaning of fractions and decimals as well as ordering fractions. For example, bring two cakes into the classroom, one smaller than the other. Tell the students that they will eat half of one cake. Ask them which cake they would rather have. Extend the problem by asking the students if they would rather have three-eighths or two-quarters of one of the cakes. Have them draw cakes, one showing eighths and one showing quarters, reminding them that the cakes must be the same size before fractional comparisons can be made. Challenge the students to draw one cake with pieces showing eighths and other pieces showing quarters.

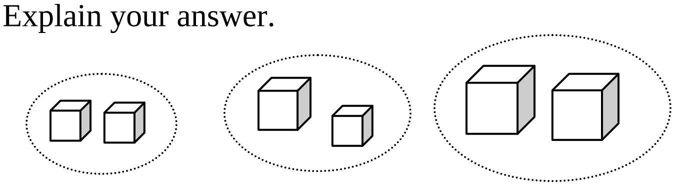
• Challenge the students by providing problems in which the part and the fraction are provided and the student must find the whole. Example:
  - If 12 cookies make up $\frac{3}{4}$ of a batch of cookies, how many cookies are in the entire batch? Explain your thinking.



  - If the following strip represents $\frac{2}{3}$ of a licorice, draw a diagram to show the length of the entire licorice. Explain your thinking.



  - If the following set of blocks is $\frac{3}{5}$ of the entire set, how many blocks are in the entire set? Explain your answer.



To make this problem more challenging, reword the problem as follows:

Six blocks make up three-fifths of the entire set. How many blocks are in the entire set? Draw a diagram to support your answer.

- Challenge the students to compare two fractions such as $\frac{5}{8}$ and $\frac{4}{6}$ by determining which fraction is closest to one-half. Note: The students need to understand equivalent fractions, a Grade 5 outcome, but one of the Grade 4 achievement indicators suggest that Grade 4s can determine which fraction is closer to one-half.

- Challenge the students to find another fraction between any two given fractions, such as $\frac{1}{2}$ and $\frac{5}{8}$. Have the students explain why the set of fractions is dense; i.e., between any two fractions there is always another fraction.

- Challenge the student to solve multistep problems, such as the following:

  – Three children raced to the flagpole. Chris got there in 24 seconds. Mary got there in half that time. Danny got there in half of Mary's time. How much longer did it take Chris to reach the flagpole than Danny? Explain your thinking.

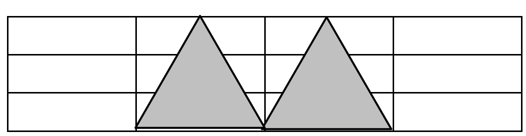
  – Three children raced to the flagpole. Jaden got there in half the time it took Tanya to get there. Maren got there in half of Jaden's time. If Maren reached the flagpole in 5 seconds, how long did it take Tanya to reach the flagpole? Explain your thinking.

  Adapted from Charles Allen et al. *Mathematics Assessment: A Practical Handbook for Grades 3–5, Classroom Assessment for School Mathematics K–12 Series* (Reston, VA: The National Council of Teachers of Mathematics, 2001), p. 106. Adapted with permission of the National Council of Teachers of Mathematics.

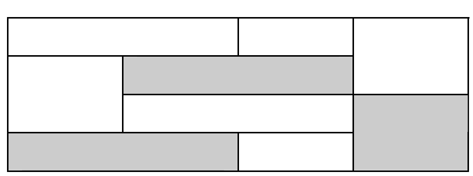
  – A loaf of bread is separated into several equal pieces. Geoff eats $\frac{1}{4}$ of the pieces. Bahati eats $\frac{1}{2}$ of the remaining pieces. There are 6 pieces left over. Into how many pieces was the original loaf divided? Explain your thinking.

- Challenge the students to find a fraction to represent the shaded portion of diagrams, such as the following:

a.

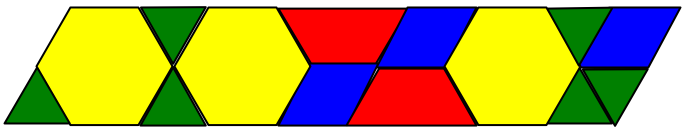


b.



- Ask the students to find what fraction of the whole design is the part shown by:
  - triangles—green pattern blocks
  - rhombuses—blue pattern blocks
  - trapezoids—red pattern blocks
  - hexagons—yellow pattern blocks.

  Provide pattern blocks for the students to use in solving this problem (Curcio and Bezuk 1994).



- Challenge the students to use triangular dot paper and draw a regular hexagon for each of the following:

  a. $\frac{1}{2}$ shaded b. $\frac{1}{12}$ shaded c. $\frac{1}{24}$ shaded

- Challenge the students to use triangular dot paper to draw a triangle with all sides equal for each of the following:

  a. $\frac{1}{2}$ shaded b. $\frac{1}{3}$ shaded c. $\frac{1}{4}$ shaded d. $\frac{1}{6}$ shaded

  e. $\frac{1}{8}$ shaded f. $\frac{1}{9}$ shaded g. $\frac{1}{16}$ shaded

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