Mathematics

Planning Guide

Grade 4 *Multiplication and Division Part A*

Number

Specific Outcomes 4, 5

This Planning Guide can be accessed online at:
http://www.learnalberta.ca/content/mepg4/html/pg4_multiplicationanddivisiona/index.html

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Planning Guide: *Grade 4 Multiplication and Division Part A*

Strand: Number
Specific Outcomes: 4, 5

This *Planning Guide* addresses the following outcomes from the Program of Studies:

Strand: Number	
Specific Outcomes:	4. Apply the properties of 0 and 1 for multiplication and the property of 1 for division. 5. Describe and apply mental mathematics strategies, such as: • skip counting from a known fact • using doubling or halving • using doubling or halving and adding or subtracting one more group • using patterns in the 9s facts • using repeated doubling to determine basic multiplication facts to 9×9 and related division facts.

Curriculum Focus

This sample targets the following changes in the curriculum:

- The General Outcome focuses on number sense, whereas the previous mathematics curriculum specified applying arithmetic operations on whole numbers and illustrating their use in creating and solving problems.
- The Specific Outcome focuses on listing mental strategies that may be used in determining number facts, whereas the previous mathematics curriculum specified mental strategies as one choice in a list of methods for multiplication and division.

What Is a Planning Guide?

Planning Guides are a tool for teachers to use in designing instruction and assessment that focuses on developing and deepening students' understanding of mathematical concepts. This tool is based on the process outlined in *Understanding by Design* by Grant Wiggins and Jay McTighe.

Planning Steps

The following steps will help you through the Planning Guide:

Step 1: Identify Outcomes to Address

Guiding Questions

- What do I want my students to learn?
- What can my students currently understand and do?
- What do I want my students to understand and be able to do based on the Big Ideas and specific outcomes in the program of studies?

Big Ideas

Developing basic multiplication facts to 9 × 9 and related division facts requires that the students have a strong foundation in patterns, number relationships, place value, and the meaning, relationships and properties of operations as described below.

- Many patterns are used in developing mental strategies, such as skip counting from a known fact and using the constant sum of the digits in products with the 9s facts.
- Number relationships are evident when using the properties of operations or other strategies, such as repeated doubling; e.g., $4 \times 6 = (6 \times 2) \times 2 = 24$.
- Place value is used extensively in various strategies, such as doubling and adding one more group; e.g., $3 \times 7 = 2 \times 7 + 7 = 14 + 7 = 21$.
- The meaning of multiplication and division is crucial as the students develop understanding of multiplication and division facts. Word problems are the key in developing this understanding.
- The relation between multiplication and division is the foundation on which the students learn the division facts. Word problems are the key to develop this connection (Van de Walle 2001, p. 144).
- The properties of multiplication and division used in developing mental strategies include:
  - commutative property of multiplication; e.g., $6 \times 2 = 2 \times 6$
  - associative property of multiplication; e.g., $(2 \times 2) \times 6 = 2 \times (2 \times 6)$, used in repeated doubling when finding the product of 4 and 6
  - distributive property; e.g., $7 \times 7 = (7 \times 5) + (7 \times 2) = 35 + 14 = 49$
  - identity element for multiplication is 1; i.e., any number multiplied by 1 remains unchanged. Similarly, any number divided by 1 remains unchanged
  - zero property; i.e., any number multiplied by zero is zero.

The students construct meaning of these properties by solving appropriate story problems and follow-up discussion.

Van de Walle lists the following as Big Ideas in helping children master number facts:

1. Number relationships can be used to help remember basic facts.
2. For subtraction facts, the concept "think addition" is the most important idea. (Similarly, for division facts, the concept "think multiplication" is the most important idea.)

3. There are patterns and relationships in basic facts. You can figure out new or unknown facts from the ones you already know.
4. All the facts can be learned with the help of efficient strategies.

To clarify the second point, he states:

> "… mastery of multiplication facts and connections between multiplication and division are the key elements of division fact mastery. Word problems continue to be a key vehicle to create this connection" (Van de Walle 2001, p. 144).

He goes on to say that "an efficient strategy is one that can be done mentally and quickly" (Van de Walle 2001, p. 129). There is one rule for teaching number facts: "Do not subject any student to fact drills unless the student has developed efficient strategies for the facts being practiced" (Van de Walle 2001, p. 144).

Efficient strategies for number facts result when the students develop personal mental strategies that are built on their understanding of numbers, compare these strategies to those used by others and choose strategies that suit their strengths and the numbers being multiplied or divided (Willis et al. 2006, p. 152). Fact strategy instruction includes two important facets:

- learning how to use a strategy
- learning how to select an appropriate strategy when it is needed (Van de Walle 2001, p. 130).

Sequence of Outcomes from the Program of Studies

See http://education.alberta.ca/teachers/core/math/programs.aspx for the complete program of studies.

Grade 3
Specific Outcomes

10. Apply mental mathematics strategies and number properties, such as:
    - using doubles
    - making 10
    - using the commutative property
    - using the property of zero
    - thinking addition for subtraction

    for basic addition facts and related subtraction facts to 18.
11. Demonstrate an understanding of multiplication to 5×5 by:
    - representing and explaining multiplication using equal grouping and arrays
    - creating and solving problems in context that involve multiplication
    - modelling multiplication using concrete and visual representations, and recording

Grade 4
Specific Outcomes

4. Apply the properties of 0 and 1 for multiplication and the property of 1 for division.
5. Describe and apply mental mathematics strategies, such as:
    - skip counting from a known fact
    - using doubling or halving
    - using doubling or halving and adding or subtracting one more group
    - using patterns in the 9s facts
    - using repeated doubling

    to determine basic multiplication facts to 9×9 and related division facts.

Grade 5
Specific Outcomes

3. Apply mental mathematics strategies and number properties, such as:
    - skip counting from a known fact
    - using doubling or halving
    - using patterns in the 9s facts
    - using repeated doubling or halving

    to determine, with fluency, answers for basic multiplication facts to 81 and related division facts.
4. Apply mental mathematics strategies for multiplication, such as:
    - annexing then adding zero
    - halving and doubling
    - using the distributive property.

the process symbolically

- relating multiplication to repeated addition
- relating multiplication to division.

12. Demonstrate an understanding of division (limited to division related to multiplication facts up to 5×5) by:
    - representing and explaining division using equal sharing and equal grouping
    - creating and solving problems in context that involve equal sharing and equal grouping
    - modelling equal sharing and equal grouping using concrete and visual representations, and recording the process symbolically
    - relating division to repeated subtraction
    - relating division to multiplication.

Step 2: Determine Evidence of Student Learning

Guiding Questions

- What evidence will I look for to know that learning has occurred?
- What should students demonstrate to show their understanding of the mathematical concepts, skills and Big Ideas?

Using Achievement Indicators

As you begin planning lessons and learning activities, keep in mind ongoing ways to monitor and assess student learning. One starting point for this planning is to consider the achievement indicators listed in the *Mathematics Kindergarten to Grade 9 Program of Studies with Achievement Indicators*. You may also generate your own indicators and use them to guide your observation of the students.

The following achievement indicators may be used to determine whether the students have met this specific outcome.

- Determine the answer to a given question involving the multiplication of a number by 1 and explain the answer.
- Determine the answer to a given question involving the multiplication of a number by 0 and explain the answer.
- Determine the answer to a given question involving the division of a number by 1 and explain the answer.
- Recognize whether the numbers in the problem represent the whole, the number of groups or the quantity in each group.
- Recognize whether the unknown in the problem represents the whole, the number of groups or the quantity in each group.
- Solve word problems to demonstrate properties of multiplication and division and other mental strategies.
- Use concrete and visual representations, including arrays, to illustrate various mental strategies.
- Provide examples for applying mental mathematics strategies:
  - doubling; e.g., for 4×3, think $2 \times 3 = 6$, and $4 \times 3 = 6 + 6$
  - doubling and adding one more group; e.g., for 3×7, think $2 \times 7 = 14$, and $14 + 7 = 21$
  - using ten facts when multiplying by 9; e.g., for 9×6, think $10 \times 6 = 60$, and $60 - 6 = 54$; for 7×9, think $7 \times 10 = 70$, and $70 - 7 = 63$
  - using patterns in the 9s facts; e.g., sum of the digits in each product is 9, the first digit in the product is one less than the number multiplying 9
  - halving; e.g., if 4×6 is equal to 24, then 2×6 is equal to 12
  - relating division to multiplication; e.g., for $64 \div 8$, think $8 \times \square = 64$
  - using the commutative property for multiplication; e.g., $5 \times 4 = 4 \times 5$

  - using the associative property of multiplication; e.g., $4 \times 6 = (2 \times 2) \times 6 = 2 \times (2 \times 6) = 2 \times 12 = 24$
  - using the distributive property; e.g., $7 \times 7 = (7 \times 5) + (7 \times 2) = 35 + 14 = 49$
  - using skip counting from a known fact; e.g., $6 \times 5 = 5 \times 5 + 5 = 25 + 5 = 35$
- Explain the use of place value in various strategies, such as doubling and adding one more group; e.g., $3 \times 7 = 2 \times 7 + 7 = 14 + 7 = 21$.
- Create a mental strategy that makes sense based on previous understanding.
- Create various mental strategies that can be used for a given number fact.
- Critique various mental strategies and choose one that is the most efficient for a given situation.
- Sort mental strategies into categories based on which strategies apply to various groups of number facts.

Some sample behaviours to look for in relation to these indicators are suggested for many of the instructional activities in **Step 3, Section C, Choosing Learning Activities** (p. 13).

Step 3: Plan for Instruction

Guiding Questions

- What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should I use?
- How will I meet the diverse learning needs of my students?

A. Assessing Prior Knowledge and Skills

Before introducing new material, consider ways to assess and build on the students' knowledge and skills related to multiplying and dividing. For example:

- Describe a rule for adding and subtracting zero. Illustrate this rule in a problem situation.
- You have 15 candies and give 7 of them to your friend. How many candies do you have left? Explain how you get your answer by using a mental strategy.
- Explain how to use the "using doubles" strategy to add 6 and 7.
- Explain how you would use the "thinking of addition" strategy in solving the following problem:

  Your mother has 17 apples. She uses 8 apples to make an apple pie. How many apples does she have left?

  Solve this problem another way by using the strategy of 'thinking of addition.'
- How would you change the following subtraction number sentence into an addition number sentence with the same answer? $16 - 9 = \square$
- How would you complete the following number sentence to make it a true statement? $6 + \square = 8 + 6$
- Explain how you know whether the solution to the following problem makes sense or not:

  You have $6 and earn $7 more. How much money do you have now?

  Solution: $6 + 7 = 6 + 4 + 3 = 10 + 3 = 13$

  I have $13 now.

- Solve the following problem using any mental strategy and explain your thinking:

  You read for 15 minutes. You read 6 minutes more than Wei reads and 3 minutes less than Beth reads. How many minutes do you each read? How many minutes do you read altogether?

If a student appears to have difficulty with these tasks, consider further individual assessment, such as a structured interview, to determine the student's level of skill and understanding. See **Sample Structured Interview: Assessing Prior Knowledge and Skills** (p. 11).

Sample Structured Interview: Assessing Prior Knowledge and Skills

Directions	Date:	
	Not Quite There	**Ready to Apply**
"You have 15 candies and give 7 of them to your friend. How many candies do you have left? Explain how you get your answer by using a mental strategy."	• Is unsuccessful in solving the problem. • Solves the problem but has difficulty explaining the mental strategy used. • Solves the problem using an inefficient strategy, such using a diagram.	• Successfully solves the problem and clearly explains an efficient mental strategy used in solving the problem.
"Explain how to use the 'using doubles' strategy to add 6 and 7."	• Adds 6 and 7 but is not able to use the 'using doubles' strategy. • Adds 6 and 6 but forgets to add 1 to complete the sum.	• Clearly explains how to use the 'using doubles' strategy to add 6 and 7; e.g., take one away from the 7, add 6 and 6, then add the 1 to make 13.
"Explain how you would use the 'thinking of addition' strategy in solving the following problem: Your mother has 17 apples. She uses 8 apples to make an apple pie. How many apples does she have left?" Solve this problem another way by using the strategy of 'thinking of addition.'	• Adds 17 and 8 to get the answer to the problem. • Uses a different mental strategy to solve the problem but is unsuccessful. • Uses an inefficient mental strategy such as drawing a diagram to solve the problem. • Uses addition to solve the problem by adding 9 to 8 but has difficulty explaining why it must be 9 that is added to 8 to make 17. • Can only explain one way to use the 'thinking addition' strategy.	• Uses the mental strategy, 'thinking of addition,' and explains the process clearly; e.g., 8 plus 2 is 10, 10 plus 7 is 17, 2 plus 7 is 9 so she has 9 apples left. • Provides another way to use addition to solve the problem using the 'thinking of addition' strategy; e.g., 8 plus 8 is 16, 16 + 1 is 17, 8 plus 1 is 9 so she has 9 apples left.

"Explain how you know whether the solution to the following problem makes sense or not: You have $6 and earn $7 more. How much money do you have now? Solution: 6 + 7 = 6 + 4 + 3 = 10 + 3 = 13 I have $13 now."	• Knows the answer is correct but cannot explain any solution to get the answer. • Solves the problem using another mental strategy, such as doubles plus one. • Does not indicate that the given solution makes sense. • Needs guidance to see that 7 is shown as 4 plus 3 but can then explain the solution.	• Clearly explains why the given solution makes sense; e.g., to make 10, take 4 from 7 and add it to the 6, then add 10 and the remaining 3 to get 13. • Another solution might be as follows: to make 10, take 3 away from 6 and add it to 7, then add 10 and the remaining 3 to get 13.

B. Choosing Instructional Strategies

Consider the following guidelines for teaching mental mathematics strategies to determine basic multiplication facts to 9 × 9 and related division facts:

- access the prior knowledge needed in learning number facts and encourage the development of a variety of mental strategies to suit the developmental level of each student
- use story problems that will help the students develop mental strategies as they solve the problems. For example:
  - use word problems to show the commutative property for multiplication by creating one problem in which 3 is the number of groups and another word problem in which 3 is the size of each group
  - use word problems to develop the more difficult number facts, such as putting 8 apples into each of 7 bags
  - use word problems to provide clues for a specific strategy, such as doubling by putting 6 pencils in each of 4 boxes and then putting 6 pencils in each of 8 boxes ($4 \times 6 = 24$, $8 \times 6 = 2 \times (4 \times 6) = 2 \times 24 = 48$)
- use story problems that will help the students develop the connection between multiplication and division
- provide the students with concrete and visual materials, including arrays, as thinking tools in developing mental strategies
- develop a strong foundation of number relationships and operations (multiplication and division) by connecting the concrete to the symbolic representation
- have the students create, compare and critique various mental strategies and decide which strategy is best to use with which number facts
- expect that the students understand strategies presented by other students but do not require them to adopt these strategies unless they choose to do so
- develop efficient strategies for knowing the number facts, recognizing that different strategies work for different students and provide practice in using and selecting mental strategies
- practise only those number facts for which the students have learned efficient strategies (Van de Walle 2001).

C. Choosing Learning Activities

The following learning activities are examples of activities that could be used to develop student understanding of the concepts identified in Step 1.

Sample Activities:

1. **Teaching Mental Strategies to Determine Multiplication Facts to 9 × 9 and Related Division Facts** (p. 14)

Sample Activity 1: Teaching Mental Strategies to Determine Multiplication Facts to 9 × 9 and Related Division Facts

1. Develop Mental Strategies through Problem Solving

   Various problems can be used for different purposes in developing mental strategies.

   a. To develop understanding of the commutative property of multiplication as a tool in solving facts to 9 × 9, present the following problem to the students:
   Lauren puts 4 candies in each of 9 bags. Brandon puts 9 candies in each of 4 bags. How many candies does Lauren put into bags? How many candies does Brandon put into bags?

   Look For …
   Do students:
   - ☐ show understanding of the problems?
   - ☐ respond positively to the guiding questions?
   - ☐ verbalize the mental strategies used and apply them successfully to other situations?

   Suggested guiding questions:

   - How are the two situations the same? How are they different?
   - How might you represent these two situations using arrays? (Note: Guide the students to see that the same array can be used to represent both situations.)
   - Do you know how the number of candies for the two children compare before calculating the answers? How do you know?
   - Is it easier for you to find the number of Lauren's candies or Brandon's candies? Why?
   - Which products might it be easier to find by reversing the order of the numbers before multiplying? Sample answer: It would be easier to find 7 groups of 5 rather than 5 groups of 7 because skip counting by 5s is easier than skip counting by 7s.
   - Write a number sentence to show that the order in which you multiply numbers does not matter. Sample answer: $5 \times 8 = 8 \times 5$.

   b. To prompt thinking about specific mental strategies such as doubling, present the following problem to the students:

   Keri puts 6 pencils in each of 4 boxes. Sue puts 6 pencils in each of 8 boxes. How many pencils does each girl have?

   Suggested guiding questions:

   - Explain how you could use the answer for the number of pencils Keri has to find the number of pencils Sue has.
   - Explain how the following number sentences could be used in solving this problem: $4 \times 6 =$ ☐ $2 \times 4 \times 6 =$ ☐
   - For what other number facts would this strategy be useful? Examples might include: $6 \times 7 = 2 \times 3 \times 7 = 2 \times 21 = 42$.

- Apply this strategy to solving the following problem.
  Marcy has four pages with 7 pictures on each page. How many pictures does she have on these pages?

  Suggested solution: $4 \times 7 = 2 \times 2 \times 7 = 2 \times 14 = 28$.

  Marcy has 28 pictures on these pages.

c. To develop mental strategies for the more difficult multiplication facts, present the following problem to the students:

   Ted puts 7 pieces of pepperoni on each of 7 pizzas. How many pieces of pepperoni does he use for the pizzas?

   To solve this problem mentally, the students might break up the 7 pizzas into two groups: 5 pizzas in one group and 2 pizzas in the other group. Then they find the number of pieces of pepperoni for each group and add these two numbers together. Symbolically, this would be represented as: $7 \times 7 = 7 \times 5 + 7 \times 2 = 35 + 14 = 49$. This is an application of the distributive property. Other mental strategies could be used as well to show the thinking done in finding the answer to this problem.

d. To develop understanding of one and zero in solving facts to 9×9, present the students with the following problem:

   You place one balloon in each of 8 party bags. How many balloons do you put into the bags? Explain your thinking.

   Suggested guiding questions:

   - If you placed one balloon in each of 9 party bags, how would the answer to the problem change?
   - What if you placed one balloon in each of 30 party bags, how would the answer to the problem change?
   - Write a number sentence for each situation. How are the number sentences the same?
   - What general statement can you make about multiplying any number by one?
   - Create a problem in which you are dividing a number by one.
   - What general statement can you make about dividing any number by one?
   - Create a problem in which you are multiplying a number by zero.
   - What general statement can you make about multiplying any number by zero?

2. Relate Division to Multiplication and Develop Mental Strategies by Using Arrays

Present the students with a variety of multiplication and division problems and have them solve the problems by using arrays.

Look For …

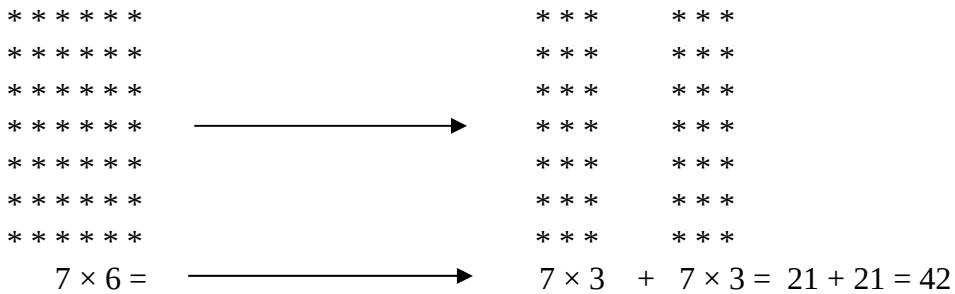
Do students:

- ☐ explain the connection between division and multiplication by using arrays in the given problem?
- ☐ create arrays for other problems and use them in multiplication and division number facts?
- ☐ verbalize the mental strategies used and represented in arrays?
- ☐ apply the mental strategies developed to other appropriate number facts?

a. Emphasize the connection between multiplication and division as well as the commutative property of multiplication.
Present the students with the following problem and have them represent the problem using an array:
Gary sets up 7 rows of chairs in the gym with 8 chairs in each row. How many chairs did he set up in the gym?
When the students make the arrays using tiles or drawing them on grid paper, have them write the different multiplication and division number sentences that are represented by this array: $7 \times 8 = 56$, $8 \times 7 = 56$, $56 \div 8 = 7$, $56 \div 7 = 8$.
Have them create problems that would be appropriate for each number sentence written.
Through discussion, have the students come to the following generalizations:

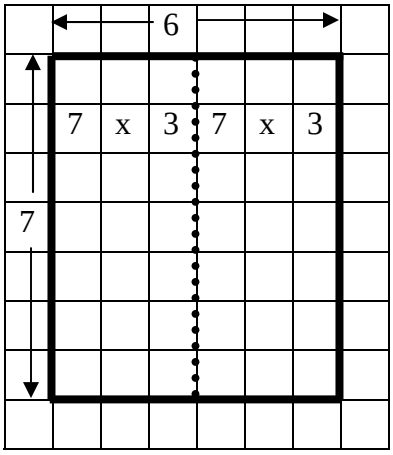
- division problems can be solved by relating them to appropriate multiplication sentences
- numbers can be multiplied in any order and the answer is still the same. This is not true for the division of numbers
- learning one multiplication fact can be used in learning other related multiplication and division facts.

b. Show how various mental strategies can be visually represented using arrays.
Present the students with the following problem and have them represent the problem using an array:
You decorate a page by pasting 7 rows of stars with 6 stars in each row. How many stars do you use to decorate the page?
When the students make the arrays using tiles or drawing them on grid paper, guide them by adjusting the array to show two parts as shown below:



Another way to think of 7×6 is doubling 7×3 or $7 \times 3 \times 2$.
Answer: I use 42 stars to decorate the page.

Encourage the students to transfer the arrays made with counters to centimetre grid paper as shown below:



Have the students label the array as shown and write a number sentence to represent what is shown by the array:

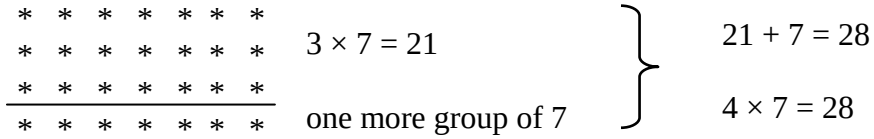
$7 \times 6 = 7 \times 3 \times 2 = 21 \times 2 = 42.$

Discuss other ways to write the number sentences and guide discussion to show the connection to the distributive property:
$7 \times 6 = 7 \times 3 + 7 \times 3 = 21 + 21 = 42.$
Have the students apply this strategy to other appropriate problems.

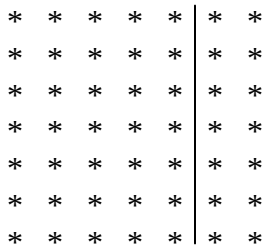
Through discussion, have the students verbalize that this array shows visually the mental strategy for finding products called *doubling* (Willis 2006).

Other mental strategies that apply the *distributive property* and build on the students' understanding of multiplication facts to 5×5 can also be shown with arrays.

- Show the mental strategy for *one more group* as follows:
  Example: for 4×7, think 3×7 is 21 and then add one more group of 7 to make 28.



- Show the mental strategy, *facts-of-5*, which is used with larger facts where at least one of the factors is greater than 5.
  Example: for 7×7, think 7×5 is 35 and 7×2 is 14. Then add 35 and 14 to make 49.



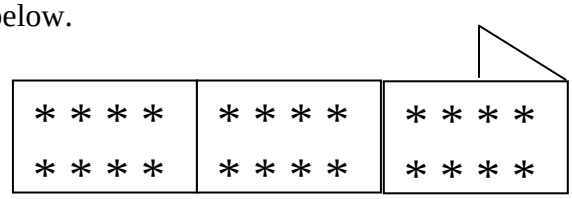
$7 \times 5 = 35$ $7 \times 2 = 14$ $35 + 14 = 49$ Therefore, $7 \times 7 = 49$

Provide reinforcement by asking the students to take a given multiplication fact, such as 8×6, and break it apart in as many ways as possible using the distributive property

represented by arrays drawn on centimetre grid paper. For each example, have the students write an appropriate number sentence such as $8 \times 6 = (8 \times 5) + (8 \times 1) = 40 + 8 = 48$.

3. Folded Cards

To help the students understand the mental strategy of *adding one more group*, prepare some folded cards similar to the one shown below.



When folded as shown, $3 \times 8 = 24$.
Unfold the other section that shows another group of 8 to represent $4 \times 8 = 32$.

Have the students verbalize that 4×8 is 3 groups of 8 plus one more group of 8, making 32.
This activity could be adapted to visually represent the *doubling* strategy; e.g., 2 eights are 16; therefore, 4 eights would be twice as much or 32.

Look For …

Do students:

- ☐ verbalize the strategies shown on the folded cards?
- ☐ apply these strategies to appropriate number facts?
- ☐ make other folded cards to represent these strategies apply to other number facts?
- ☐ represent, symbolically, the mental strategies shown on the folded cards?

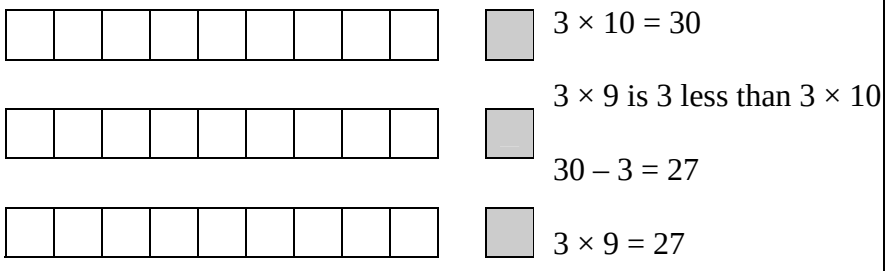
Adapted from W. George Cathcart, Yvonne M. Pothier and James H. Vance, *Learning Mathematics in Elementary and Middle Schools* (2nd ed.) (Scarborough, ON: Prentice-Hall Canada Inc., 1977), p. 122. Adapted with permission from Pearson Education Canada.

4. Develop Patterns in the 9s Facts

Write the 9s facts vertically and have the students describe any patterns that they see.
Through discussion, have the students verbalize the following two patterns among others:

a. the sum of the digits in each product is 9
b. the tens digit in the product is always one less than the number that is multiplying 9.

Encourage the students to explain why these patterns exist for the 9s facts but not the other number facts. During the discussion, suggest that the students use the ten facts along with number relationships to help them discover the connection; e.g., for 3×9, use $9 = 10 - 1$ and think $3 \times 10 = 30$ and $30 - 3 = 27$. This can be shown with tiles or on grid paper as follows:



Look For …

Do students:

- ☐ describe the patterns in the 9s facts?
- ☐ connect the patterns in the 9s facts to place value?
- ☐ verbalize the mental strategies used in applying patterns to the 9s facts using a variety of situations?
- ☐ represent, visually, another example using patterns in the 9s facts such as 6×9?

Adapted from John A. Van de Walle, *Elementary and Middle School Mathematics: Teaching Developmentally*, 4/e (pp. 141, 142). Published by

After the students understand the patterns in the 9s facts and can explain how they relate to place value, show them the finger patterns that relate to the patterns in the 9s facts.
Example: To multiply 3 × 9, hold up 10 fingers facing away from you, then bend the third finger from the pinky on the left hand so that you have 2 fingers extended on one side of the bent finger and a total of 7 fingers extended on the other side, showing 27 (Willis 2006).

2 7

5. Create and Share Mental Strategies for a Collection of Number Facts
Provide the students with a number fact and have them develop mental strategies that work for this number fact. Ask them which strategies are more efficient and which strategies could be used for other number facts. Have them indicate to which number facts the strategies could be applied.
Example: Given the number fact 6 × 8, possible strategies could include:

Look For …
Do students:
- ☐ create more than one mental strategy for solving a number fact?
- ☐ apply a mental strategy successfully to another number fact that they choose?
- ☐ verbalize the mental strategies used and apply them successfully to other situations?

- use nice numbers and adjusting: think 5 × 8 = 40 and add 8 to this product to make 48. Another way to write this is 5 × 8 + 1 × 8 = 48 using the distributive property.
- use halving and doubling: think half of 6 is 3, 3 × 8 = 24, double 24 to get 48
- use the commutative property of multiplication and repeated doubling: think 6 × 8 = 8 × 6, 2 × 6 = 12, double 12 to get 24, double 24 to get 48
- use the distributive property: think 6 × 8 = (6 × 6) + (6 × 2) = 36 + 12 = 48.

Have the students share their strategies and suggest other number facts that could be solved using any given strategy.

Through discussion, have the students verbalize the following:

- The halving and doubling strategy could be applied to number facts that have at least one even number.
- The commutative property works for all multiplication facts but not division facts. It is efficient to use the commutative property when it is easier to multiply by the second number than by the first number given in the number fact.
- The distributive property works for all multiplication facts but is efficient only if the number parts chosen are easier to multiply than the original number.

Provide time to practise, using the strategies that were created.

6. Sort Mental Strategies and Number Facts
   Provide the students with mental strategies, each written on a separate piece of paper, as well as a variety of multiplication and division number facts on separate pieces of paper. Have the students sort the mental strategies and group them according to which number fact(s) they apply. Clarify that a given mental strategy can apply to more than one number fact and many different mental strategies can be used to solve a number fact. Have them explain their grouping and then have them regroup to show another way to sort the strategies and the number facts.

   Look For …
   Do students:
   - ☐ explain why a certain group of mental strategies applies to specific number facts?
   - ☐ group the mental strategies another way and explain how this group applies to specific number facts?
   - ☐ explain why a certain group of number facts can be solved by a specific mental strategy?

   Adaptations:
   a. Instruct the students to put the number facts that can be solved by a mental strategy into one group, the number facts that can be solved by a different mental strategy in another group, and so on. Remind the students that a number fact may be solved by more than one mental strategy so the groupings could be represented by an overlapping Venn diagram. Have them justify their groupings.

   b. Have the mental strategies displayed on a poster in the classroom. Provide the students with number facts with each fact on a separate piece of paper. The students group the number facts according to which mental strategy or strategies that they would use to get the answers efficiently.

   Note: This sorting strategy is to be used **after** the students understand the mental strategies and have had practice using them in solving a variety of number facts.

7. Critique Strategies
   Provide the students with a multiplication or division problem and a solution to the problem.

   Example: You take 8 mL of cold medicine 8 times a day. How many millilitres of cold medicine do you take each day?

   Solution: 8×8, think $4 \times 4 = 16$, $2 \times 16 = 32$.

   I take 32 mL of cold medicine each day.

   Have the students critique the solution and make the correct changes.

   Look For …
   Do students:
   - ☐ explain why a solution is correct?
   - ☐ explain why a solution is incorrect and make the necessary changes to correct it?
   - ☐ apply this ability to critique mental strategies created to solve other problems in class discussions?

Step 4: Assess Student Learning

Guiding Questions

- Look back at what you determined as acceptable evidence in Step 2.
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Sample Assessment Tasks

In addition to ongoing assessment throughout the lessons, consider the following sample activities to evaluate students' learning at key milestones. Suggestions are given for assessing all students as a class or in groups, individual students in need of further evaluation, and individual or groups of students in a variety of contexts.

A. Whole Class/Group Assessment

Note: Performance-based assessment tasks are under development.

1. Explain the thinking you do as you use a mental strategy to solve each problem.
   a. You read for 7 hours during January and 3 times as long during February. How long do you read during February?

   b. You have 64 pictures to put on album pages that hold 8 pictures each. How many album pages do you need?

   c. What is the total weight of 4 turkeys each weighing 6 kg?

2. Write the multiplication and division number sentences shown by the following array.

   * * * * * * *
   * * * * * * *
   * * * * * * *
   * * * * * * *

3. Draw an array to show a solution for $42 \div 6 = \square$.

4. Describe a mental strategy for solving multiplication facts. Write three multiplication facts that can be solved using this mental strategy. Explain why this mental strategy works for the three multiplication facts.

5. Describe at least two mental strategies that can be used to solve the following problem:
   You are making 9 bouquets of flowers with 8 flowers in each bouquet. How many flowers do you use in making these bouquets?

6. How would you find the answer to the following division fact by relating it to multiplication?
   $30 \div 5 = \square$ Explain your thinking.

B. One-on-one Assessment

1. Write multiplication and division facts that can be represented by the following array.

   * * * * * * * *
   * * * * * * * *
   * * * * * * * *
   * * * * * * * *

   If the student has difficulty with the given array, use an array with fewer stars initially and then go back to the given array.

   Guiding questions to probe for understanding:

   - Pretend the stars in the array are chairs that you are setting up in the gym. How many rows of chairs are there?
   - How many chairs are in each row?
   - Write a multiplication sentence to show that there are a certain number of rows with a given number of chairs in each row.
   - Now, let's look at the up-and-down columns in the array. (Point to the columns.) How many columns are in the array?
   - How many chairs are in each column?
   - Write a multiplication sentence to show that there are a certain number of columns with a given number of chairs in each column.
   - Do you get the same answer when you change the order in which you multiply two numbers? Explain.
   - To make a division sentence, we need to know how many chairs there are in all and the number of chairs in each row. (Provide a general division sentence for the student if needed: number of chairs in all ÷ number of chairs in each row = number of rows.) Fill in the numbers to make a division fact.
   - Write another division fact using the array. (Provide the other general division sentence for the student if needed: number of chairs in all ÷ number of rows = number of chairs in each row.)

2. Use a mental strategy to solve the following problem. Explain the thinking you do as you use the mental strategy to solve the problem.
   You have 3 bags. You put 9 candies in each bag. How many candies do you need in all?

   If the student has difficulty understanding the problem and using a mental strategy to solve the problem, use the following guiding questions:

   - Use these tiles/counters to represent the action in the problem. (Or, have the student act out the problem by putting the candies into the bags.)
   - Explain which operation is used in this problem. How do you know?

- Write the number fact to show how this problem can be solved.
- How might you solve this problem without using the tiles/counters? If the student says drawing a diagram, encourage him or her to do so but continue probing and ask if the student can solve the problem mentally without objects or diagrams.
- (If the student struggles with any mental strategy, prompt him or her by suggesting that the 9 could be changed into a 10 to make the multiplication easier.) If you put 10 candies instead of 9 into each bag, explain how many candies would be in 3 groups of 10. (Suggest skip counting if necessary.)
- Explain how the answer to 3 groups of 10 could be adjusted to give the answer to 3 groups of 9. (If necessary, prompt by asking how many extra candies have been placed in the bags if 10 candies are in each bag instead of 9. The student should understand that each of the 3 bags have an extra candy so you must subtract 3.)

Provide a similar multiplication problem such as 4 bags with 9 candies in each bag and determine if the student can apply a mental strategy to this similar problem.

Depending on the student's understanding of number relationships, you may wish to prompt a mental strategy using doubling and adding one more group for 3×9, which can be written as $2 \times 9 + 9$. Or, you may wish to prompt repeated doubling for 4×9, which can be written as $2 \times (2 \times 9)$.

C. Applied Learning

Provide opportunities for the students to use their mental strategies for multiplication and division facts in a practical situation and notice whether or not the mental strategies transfer. For example, ask the student how many minutes he or she reads in a week if he or she reads for 9 minutes a day. Have the student explain how he or she finds the answer. Does the student:

- respond to the question without hesitation, showing that he or she is using a mental strategy efficiently?
- explain clearly his or her thinking in finding the product of 7 and 9?
- use another mental strategy for find the product of 7 and 9 when prompted to do so?

Step 5: Follow-up on Assessment

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction?

A. Addressing Gaps in Learning

If the student appears to be having difficulty with using mental strategies to determine basic multiplication facts to 9 x 9 and related division facts, provide guidance in understanding the operations by nesting the number facts in problem situations that relate to the student's experiences. Assessment by observing a student solving problems will provide valuable data to guide further instruction. By accommodating the individual learning styles, success will follow.

As the student solves a problem using a multiplication or division fact, provide guidance and prompts as follows:

- Diagnose the student's understanding of place value by having the student explain the meaning of a number, such as 25, and represent it visually. Provide activities to develop understanding of place value if necessary, such as adding 5 to 25 or 8 to 16, using base ten materials and their symbolic representations.
- Diagnose the student's understanding of number relationships by having the student write the number, such as 7 in a variety of ways; e.g., 1 + 6, 2 + 5 and 3 + 4. Provide activities to develop understanding of number and the various representations using counters and their symbolic representations.
- Build on prior knowledge by posing problems that use the number facts for multiplication to 5 × 5 and the related division facts. See the folded cards strategy described in the activities.
- Review the meaning of multiplication and division by having the student verbalize whether the numbers in the problem represent the part(s) or the whole and whether the unknown is the part or the whole.
- Review the commutative property for multiplication so that the students are successful in applying a mental strategy that works for a given fact, such as 3 × 4 to another fact in which the order is reversed; i.e., 4 × 3. Use arrays to show this relationship as well as the relationship between multiplication and division.
- Have counters and tiles available for the students to use in building arrays to represent the multiplication or division fact in the problem.
- Think aloud how you would use a mental strategy to solve a given problem. Use counters and/or diagrams to help the student understand what you are saying. Connect the visual display to the symbolic representation and explain why these symbols make sense.
- Focus on one type of mental strategy, such as doubling—choose a mental strategy that is developmentally appropriate for the student's background. Use a variety of problems with number facts that can be solved using this mental strategy; e.g., 4 × 3, 4 × 5, 6 × 7, 6 × 8 and so on. Use arrays to visually represent the strategy.

- After the student can explain clearly one mental strategy for a given number fact, encourage him or her to try another mental strategy that would work for that problem. For example, 4 × 9 can be solved using doubling or patterns in the 9s facts.
- Have the student decide which mental strategy is best for him or her in solving a given multiplication or division fact.
- Provide practice in using the mental strategies and selecting which strategy is best to use for a given multiplication or division fact.

B. Reinforcing and Extending Learning

The students who have achieved or exceeded the outcomes will benefit from ongoing opportunities to apply and extend their learning. These activities should support the students in developing a deeper understanding of the concept and should not progress to the outcomes in subsequent grades.

Consider strategies, such as the following.

- Provide tips for parents on practising multiplication and division facts at home or in the community.
  - Involve the children in finding how many candies are needed for 8 party bags if each bag has 6 candies. Other multiplication situations include finding the number of pencils in 6 packages of pencils each containing 4 pencils or finding how many books are read in a year if 5 books are read every two months. Division situations include finding how many cookies each of 3 children would receive if 18 cookies were distributed equally, or finding how many party bags you could fill with 36 items if 4 items were placed in each bag.
  - Have the children explain their thinking as they complete multiplication and division facts.
- Play games, such as the following. Use a deck of playing cards with the picture cards removed. The ace counts as one. Deal each player two cards. The remaining cards are placed upside down in the centre. Players take turns finding the product of the two numbers on the cards. The goal is to reach a certain number, say 100. With each turn, a player discards the two cards already used (place them in a pile at the side and shuffle them before reusing), takes two more cards from the pile, does the calculation and records his or her total to date. The first one to reach or exceed 100 is the winner. Adapt the game to use quotients by having the goal as the lowest total in a given time or a given number of plays. The students explore the concept of negative numbers as they 'go in the hole' when slipping below zero. Reminder, quotients are only used when they are whole numbers, otherwise the player must multiply the two numbers on the cards. To encourage the students to choose which operation and to avoid negative numbers, start at 50. The winner is the one who first reaches 100 or 0. Six- or eight-sided dice could be used instead of the cards.
- Have the students explore which numbers can be multiplied to make a given product, such as 36. Encourage the students to explain what these numbers have in common; e.g., all the numbers are divisible by 2 or 3. Have the students draw arrays to show the different numbers listed.

- Have the students create at least two mental strategies that could be used in solving a multiplication or division fact and explain which strategy is most efficient and why. This could be a journal entry.
- Have the students compare two mental strategies such as doubling and halving, and describe the similarities and differences between these two strategies.
- Have the students explain to other students how the patterns in the 9s facts relate to place value.

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