Mathematics

Planning Guide

Grade 4 *Multiplication and Division Part B*

Number

Specific Outcomes 6, 7

This Planning Guide can be accessed online at:
http://www.learnalberta.ca/content/mepg4/html/pg4_multiplicationanddivisionb/index.html

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Planning Guide: *Grade 4 Multiplication and Division Part B*

Strand: Number
Specific Outcomes: 6, 7

This *Planning Guide* addresses the following outcomes from the Program of Studies:

Strand: Number

Specific Outcomes:

6. Demonstrate an understanding of multiplication (2- or 3-digit by 1-digit) to solve problems by:
   - using personal strategies for multiplication with and without concrete materials
   - using arrays to represent multiplication
   - connecting concrete representations to symbolic representations
   - estimating products
   - applying the distributive property.
7. Demonstrate an understanding of division (1-digit divisor and up to 2-digit dividend) to solve problems by:
   - using personal strategies for dividing with and without concrete materials
   - estimating quotients
   - relating division to multiplication.

Curriculum Focus

- The General Outcome focuses on number sense, whereas the previous mathematics curriculum specified applying arithmetic operations on whole numbers and illustrating their use in creating and solving problems.
- The Specific Outcome for multiplication focuses on using personal strategies, arrays, estimating, connecting concrete and symbolic representations, and the distributive property, whereas the previous mathematics curriculum was limited to connecting manipulatives, diagrams and symbols.
- The Specific Outcome for division includes using personal strategies with and without concrete materials, estimating, and relating division to multiplication, whereas the previous mathematics curriculum focused on connecting manipulatives, diagrams and symbols as well as the connections between multiplication and division.

What Is a Planning Guide?

Planning Guides are a tool for teachers to use in designing instruction and assessment that focuses on developing and deepening students' understanding of mathematical concepts. This tool is based on the process outlined in *Understanding by Design* by Grant Wiggins and Jay McTighe.

Planning Steps

The following steps will help you through the Planning Guide:

Step 1: Identify Outcomes to Address

Guiding Questions

- What do I want my students to learn?
- What can my students currently understand and do?
- What do I want my students to understand and be able to do based on the Big Ideas and specific outcomes in the program of studies?

Big Ideas

Principles and Standards for School Mathematics states that "developing [computational] fluency requires a balance and connection between conceptual understanding and computational proficiency" (NCTM 2000, p. 35). Conceptual understanding requires flexibility in thinking about the structure of numbers (base ten system), the relationship among numbers and the connections between multiplication and division. The ability to generate equivalent representations of the same number provides a foundation for using personal strategies to multiply and divide, recognizing that for some problems either operation may be used. Computational proficiency includes both efficiency and accuracy. Personal strategies must be compared and evaluated to derive methods that are efficient as well as accurate.

Understanding the operations of multiplication and division and the connections between them is crucial. John Van de Walle states, "Multiplication involves counting groups of like size and determining how many are in all (multiplicative thinking). … Division names a missing factor in terms of the known factor and the product" (2001, p. 107). He goes on to say, "Division and multiplication problems presented in pairs give you an opportunity to focus on the relationships between these operations" (2001, p. 119). For example, one problem may repeat equal quantities while another problem partitions a quantity into equal parts (Willis 2006).

Problems showing addition and subtraction are similar to problems showing multiplication and division because in the former the focus is on identifying the part or the whole while in the latter, the focus is on the number of groups, quantity in each group or the whole.

The connections between addition and multiplication can be clarified by writing an addition sentence as well as a multiplication sentence for early multiplication activities. Similarly, the connections between subtraction and division can be clarified by writing a subtraction sentence (repeated subtraction) as well as a multiplication sentence for early division activities.

By using a variety of problems, the students will construct their own meaning for the inverse relationship between multiplication and division and for the following properties:

- commutative property of multiplication (numbers can be multiplied in any order)
- identity element for multiplication (any number multiplied by one remains unchanged)
- distributive property; e.g., $4 \times 65 = (4 \times 60) + (4 \times 5) = 240 + 20 = 260$.

Van de Walle and Lovin (2006) identify four different classes of multiplicative structures as follows:

- Equal-Group Problems that can be subdivided into three groups:
  - Whole Unknown (Multiplication with Groups including Rates)
  - Size of Group Unknown or Equal Grouping (Partition Division)
  - Number of Groups Unknown or Equal Sharing (Measurement Division)
- Multiplicative Comparison Problems that can be subdivided into three groups:
  - Product Unknown (Multiplication)
  - Set Size Unknown (Partition Division)
  - Multiplier Unknown (Measurement Division)
- Combination Problems (Cartesian Products) that can be subdivided into two groups:
  - Product Unknown
  - Size of Set Unknown
- Product of Measures Problems (as in finding area).

By solving problems in contexts that relate to their own lives, the students use their prior knowledge to make sense out of the problem, estimate the answer and use computational strategies that they are able to explain and justify. The students' understanding of multiplication and division is enhanced as they develop their own methods and share them with one another, explaining why their strategies work and are efficient to use (NCTM 2000, p. 220).

The use of manipulatives or models helps the students to understand the structure of the story problem and also connects the meaning of the problem to the number sentence (Van de Walle 2001, p. 108). To develop understanding of the meaning of operations, the students connect the story problem to the manipulatives, create a number sentence and then use personal strategies to solve the problem. Van de Walle states:

> "It is useful to think of models, word problems, and symbolic equations as three separate languages. Each language can be used to express the relationships involved in one of the operations. Given these three languages, a powerful approach to helping children develop operation meaning is to have them make translations from one language to another" (2001, p. 109).

Sequence of Outcomes from the Program of Studies

Grade 3

Specific Outcomes

11. Demonstrate an understanding of multiplication to 5×5 by:
    - representing and explaining multiplication using equal grouping and arrays
    - creating and solving problems in context that involve multiplication
    - modelling multiplication using concrete and visual representations, and recording the process symbolically
    - relating multiplication to repeated addition
    - relating multiplication to division.
12. Demonstrate an understanding of division (limited to division related to multiplication facts up to 5×5) by:
    - representing and explaining division using equal sharing and equal

Grade 4

Specific Outcomes

8. Demonstrate an understanding of multiplication (2- or 3-digit by 1-digit) to solve problems by:
    - using personal strategies for multiplication with and without concrete materials
    - using arrays to represent multiplication
    - connecting concrete representations to symbolic representations
    - estimating products
    - applying the distributive property.
9. Demonstrate an understanding of division (1-digit divisor and up to 2-digit dividend) to solve problems by:
    - using personal strategies for dividing with and without concrete materials
    - estimating quotients
    - relating division to multiplication.

Grade 5

Specific Outcomes

1. Use estimation strategies, including:
    - front-end rounding
    - compensation
    - compatible numbers

    in problem-solving contexts.
5. Demonstrate, with and without concrete materials, an understanding of multiplication (2-digit by 2-digit) to solve problems.
6. Demonstrate, with and without concrete materials, an understanding of division (3-digit by 1-digit), and interpret remainders to solve problems.

grouping
- creating and solving problems in context that involve equal sharing and equal grouping
- modelling equal sharing and equal grouping using concrete and visual representations, and recording the process symbolically
- relating division to repeated subtraction
- relating division to multiplication.

Step 2: Determine Evidence of Student Learning

Guiding Questions

- What evidence will I look for to know that learning has occurred?
- What should students demonstrate to show their understanding of the mathematical concepts, skills and Big Ideas?

Using Achievement Indicators

As you begin planning lessons and learning activities, keep in mind ongoing ways to monitor and assess student learning. One starting point for this planning is to consider the achievement indicators listed in the *Mathematics Kindergarten to Grade 9 Program of Studies with Achievement Indicators*. You may also generate your own indicators and use them to guide your observation of the students.

The following achievement indicators may be used to determine whether the students have met this specific outcome.

- Model a given multiplication problem using the distributive property;
  e.g., $8 \times 365 = (8 \times 300) + (8 \times 60) + (8 \times 5)$.
- Use concrete materials, such as base ten block or their pictorial representations, to represent multiplication and record the process symbolically.
- Create and solve a multiplication problem that is limited to 2- or 3-digits by 1-digit and record the process.
- Create, classify and solve problems that involve equal grouping (repeated addition), comparison and combinations.
- Estimate a product using a personal strategy; e.g., 2×243 is close to or a little more than 2×200, or close to or a little less than 2×250.
- Model and solve a given multiplication problem using an array and record the process.
- Solve a given multiplication problem and record the process.
- Solve a given division problem with a remainder using arrays or base ten materials and connect it to the symbolic representation.
- Solve a given division problem using a personal strategy and record the process.
- Create and solve a division problem involving a 1- or 2-digit dividend and record the process.
- Create, classify and solve problems that involve equal sharing and equal grouping.
- Estimate a quotient using a personal strategy; e.g., $86 \div 4$ is close to $80 \div 4$ or close to $80 \div 5$.
- Relate division to multiplication in solving problems.
- Identify what is given in a multiplication problem and what is the unknown, using the following terms: number of groups, quantity in each group or whole.

Some sample behaviours to look for in relation to these indicators are suggested for many of the instructional activities in **Step 3, Section C, Choosing Learning Activities** (p. 13).

Step 3: Plan for Instruction

Guiding Questions

- What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should I use?
- How will I meet the diverse learning needs of my students?

A. Assessing Prior Knowledge and Skills

Before introducing new material, consider ways to assess and build on the students' knowledge and skills related to counting. For example:

- Identify events from experience that can be described as multiplication, equal sharing or equal grouping; e.g., explain what you have done that could be called equal sharing.
- Illustrate, with counters or a diagram, a given story problem (orally, shared reading, written) involving repeated addition, equal sharing or equal grouping, and record a number sentence. Then solve the problem. Examples:
  - How many legs are on 4 horses? Show how you know using counters or a diagram. Write a number sentence. Solve the problem.
  - Susie picks 20 apples. She puts them into bags containing 4 apples each. How many bags did she use?
- Write $5 \times 4 = \square$ using repeated addition.
- Write $5 + 5 + 5 = \square$ using a multiplication sentence.
- Create a story problem for the number sentence, $3 \times 4 = \square$, and use counters or a diagram to show what it means in two different ways, equal groups and arrays.
- Show that $3 \times 2 = 2 \times 3$ using counters in an array.
- Create an array and write the multiplication and division number sentences that are shown by this array.
- Create a story problem for the number sentence, $9 \div 3 = \square$, and use counters to show what it means.
- Write $12 \div 4 = \square$ using repeated subtraction.
- Write $10 - 5 - 5$ using a division expression.
- Solve the following problems. Show your work.
  - You have 15 cookies to share equally among 3 children. How many cookies will each child receive?
  - How many pennies are the same as 5 nickels?

If a student appears to have difficulty with these tasks, consider further individual assessment, such as a structured interview, to determine the student's level of skill and understanding. See **Sample Structured Interview: Assessing Prior Knowledge and Skills** (p. 10).

Sample Structured Interview: Assessing Prior Knowledge and Skills

Directions	Date:	
	Not Quite There	**Ready to Apply**
"How many legs are on 4 horses? Show how you know using counters or a diagram. Write a number sentence. Solve the problem."	• Has difficulty representing the problem with the counters or diagrams. • Represents the problem with the counters or diagrams but does not write a number sentence. • Represents the problem with counters or diagrams and writes a number sentence but is not able to solve the problem.	• Represents the problem with counters or play horses, writes an appropriate number sentence and solves the problem.
"Create a story problem for the number sentence, $3 \times 4 = \square$. Use counters or a diagram to show what it means in two different ways, equal groups and arrays."	• Has difficulty creating a story problem for the number sentence. • Creates a story problem but has difficulty using counters or a diagram to show what it means. • Creates a story problem and uses counters or a diagram to show what it means only one way, either equal groups or arrays but not both.	• Creates a story problem and uses counters or a diagram correctly to show what it means in two ways: equal groups and arrays.
"Create a story problem for the number sentence, $9 \div 3 = \square$, and use counters to show what it means."	• Creates a story problem using the numbers provided but the meaning of the story is not represented by the number sentence. • If the story requires the multiplication of 9 and 3, understanding of the number sentence is not there.	• Creates a story problem, using equal sharing or equal grouping, showing the meaning of the number sentence. For example: • I share 9 pencils equally with 3 friends. How many pencils does each friend receive?

"Solve the following problems. Use counters or a diagram and a number sentence. 1. You have 15 cookies to share equally among 3 children. How many cookies will each child receive? 2. How many pennies are the same as 5 nickels? 3. You have 20 apples. You put them into bags containing 4 apples each. How many bags do you need?"	• Has difficulty representing the problems with counters. • Represents the problems with counters but has difficulty completing the problem. • Has difficulty writing a number sentence to represent the problem. • Solves one or two problems correctly but has difficulty solving the other problem(s). • Confuses multiplication with division. • Does not distinguish between the equal sharing process in problems 1 and the equal grouping process used in problem 2.	• Solves both problems correctly with the work showing that the process of multiplication and division is understood. • The student uses counters or diagrams and a number sentence to represent the problem and explains how they related to the numbers provided in the problem.

K What facts do I **KNOW** from the information in the problem?	**N** Which information do I **NOT** need?	**W** **WHAT** does the problem ask me to find?	**S** What **STRATEGY** will I use to solve the problem?

Adapted from Mary Lee Barton and Clare Heidema, *Teaching Reading in Mathematics: A Supplement to Teaching Reading in the Content Areas Teacher's Manual* (Aurora, CO: McREL [Mid-continent Research for Education and Learning], 2002), p. 113. Adapted with permission from Mid-continent Research for Education and Learning.

B. Choosing Instructional Strategies

Consider the following guidelines for teaching multiplication and division:

- Teach in a problem-solving context. Research shows that by solving problems using multiplication and division, the students create personal strategies for computing and develop understanding about the relationship between the operations and their properties (NCTM 2000).
- Choose problems that relate to the children's own lives (Van de Walle 2001).
- Provide a variety of problems representing the different multiplication and division situations with varying degrees of difficulty to differentiate instruction.
- Work with the whole group initially and have the students paraphrase the problem to enhance understanding (Willis et al. 2006) and to recognize whether the numbers in the problem refer to the whole, the number of groups or the quantity in each group. Discuss whether the unknown refers to the whole, the number of groups or the quantity in each group, thereby facilitating thinking about which operation to use in solving the problem.
- Have the students estimate the answer to the problem before calculating so that they are better able to determine the reasonableness of their answers.
- Have base ten materials available for the students to use as needed.
- Provide time for the students to create their personal strategies to solve the problem and share these strategies with members of their group or with the entire class.
- Guide the discussion by asking questions to encourage thinking about number relationships, the connection among the operations, and their personal strategies.
- Have the students compare their answers to the estimates that they made before solving the problems.
- Challenge the students to solve the problem another way, do a similar problem without models or clarify the explanation of their personal strategies.
- Have the students critique their personal strategies as well as those of their classmates to decide which strategy works best for them and why.
- Have the students create problems for a variety of number sentences illustrating multiplication and division, including examples of equal sharing, equal grouping, comparison problems and combination problems.

C. Choosing Learning Activities

The following learning activities are examples of activities that could be used to develop student understanding of the concepts identified in Step 1.

Sample Activities:

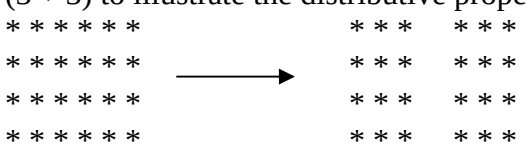
1. **Teaching Personal Strategies for Multiplying and Dividing** (p. 14)
2. **Teaching Estimating Products and Quotients** (p. 21)
3. **Teaching Solving Problems Involving Multiplication and Division** (p. 25)

Sample Activity 1: Teaching Personal Strategies for Multiplying and Dividing

1. Multiplying 2- and 3-digit Numbers by 1-digit Numbers Using Personal Strategies, Concrete Materials Connected to Symbolic Representations, Arrays, Distributive Property and Connections to Division

   a. Draw on prior knowledge by reviewing some personal strategies used by the students in solving problems by multiplying 1-digit numbers. Have the students share their ideas by multiplying a variety of 1-digit numbers.

   Example:
   Problem—Four bags each contain 6 apples. How many apples are there in the four bags?
   Show 4 groups of 6 as an array with 4 rows of 6 counters. Then separate each of the rows of 6 counters into two groups
   (3 + 3) to illustrate the distributive property.

   Connect the action to the number sentence:
   $4 \times 6 = (4 \times 3) + (4 \times 3) = 12 + 12 = 24$.

   The 4 represents the number of groups, the 6 represents the quantity in each group and the 24 represents the whole.

   Discuss related problems and other number sentences using the following array:
   $6 \times 4 = 24$, $24 \div 4 = 6$, and $24 \div 6 = 4$.

   b. Connect this process to multiplying a 2-digit number by a 1-digit number. Use counters at first and then base ten materials.

   Present the following problem to the students:
   John jogs for 48 minutes each day. How many minutes does he jog in 3 days?

   Guide discussion as to whether the numbers in the problem and the unknown refer to the whole, the number of groups or the quantity in each group. Discuss which operation would be used to solve the problem and what would be a good estimate for the answer. Have the students justify all their answers.

Look For …

Do students:

- ☐ know and draw on number facts and other number relationships?
- ☐ solve the problem with counters or base ten materials but need guidance in using symbols to show what they did?
- ☐ show understanding of place value and number relationships so that their strategy makes sense?
- ☐ explain why the steps that they use work?
- ☐ relate multiplication to division?

Encourage the students to represent the problem using counters and then base ten materials by arranging them in an array.

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Provide time for the students to solve the problem by writing a number sentence and their personal strategies to show calculations of the product and then compare it to their estimate. Have the students explain how their strategies work by relating to place value and have them decide on which strategy is most efficient for them to use in calculating other products. Guide the students to use the distributive property in calculating products:
$48 = 40 + 8$ Therefore, $3 \times 48 = (3 \times 40) + (3 \times 8) = 120 + 24 = 144$

Answer: John jogs for 144 minutes in 3 days.

c. Have the students apply their personal strategy to solve a similar problem with different numbers, such as jogging 85 minutes each day for 4 days.
Have the students discuss whether the problem could be represented by the number sentence, $48 \times 3 = 144$. Why or why not?

d. Have the students create a division problem using the same context. For example:
John jogs for a total of 144 minutes on 3 days. If he jogs the same length of time each day, how many minutes does he jog each day? Extend to include 3-digit numbers in the multiplicand, using different problem contexts.

Sample personal strategies for other problems:

First sample (multiply a 2-digit number by a 1-digit number):

$4 \times 24 = \square$

********** ********** * * * *
********** ********** * * * *
********** ********** * * * *
********** ********** * * * *

Group ten ones to make one ten.

$4 \times 24 = (4 \times 20) + (4 \times 4) = 80 + 16 = 96$

OR
$$\begin{array}{r} 24 \\ \times\ 4 \\ \hline 16 \\ 80 \\ \hline 96 \end{array}$$

OR
$$\begin{array}{r} {\scriptstyle 1} \\ 24 \\ \times\ 4 \\ \hline 96 \end{array}$$

OR
$$\begin{array}{r} 24 \\ \times\ 4 \\ \hline 80 \\ 16 \\ \hline 96 \end{array}$$

Second sample (multiply a 3-digit number by a 1-digit number):

$3 \times 126 =$

☐ ********** ********** * * * * * *
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☐ ********** ********** * * * * * *

Group ten ones to make one ten.

$3 \times 126 = (3 \times 100) + (3 \times 20) + (3 \times 6)$
$= 300 + 60 + 18$
$= 378$

OR
$$\begin{array}{r} 126 \\ \times\ 3 \\ \hline 18 \\ 60 \\ 300 \\ \hline 378 \end{array}$$

OR
$$\begin{array}{r} {\scriptstyle 1} \\ 126 \\ \times\ 3 \\ \hline 378 \end{array}$$

OR
$$\begin{array}{r} 126 \\ \times\ 3 \\ \hline 300 \\ 60 \\ 18 \\ \hline 378 \end{array}$$

2. Multiplying 2- and 3-digit Numbers by 1-digit Numbers Using Personal Strategies without Concrete Materials

Look For …

Do students:

- ☐ know and draw on number facts and other number relationships?
- ☐ solve the problem with base ten materials but need guidance in using symbols to show what they did?
- ☐ use personal strategies that work for smaller products but are not efficient for calculating larger products?
- ☐ show understanding of place value and number relationships so that their strategy makes sense?
- ☐ explain why the steps that they use work?

a. Draw on prior knowledge by reviewing some personal strategies used by the students in solving problems by multiplying 1-digit numbers. Have the students share their ideas by multiplying a variety of 1-digit numbers.
Present the following problem to the students:
You saved $26 last year. This year you saved 4 times as much money as last year. How much money did you save this year?

Guide discussion as to whether the numbers in the problem and the unknown refer to the whole, the number of groups or the quantity in each group. Discuss which operation would be used to solve the problem and what would be a good estimate for the answer. Have the students justify all their answers.

Provide time for the students to solve the problem by writing a number sentence and their personal strategies to show how to find the product.

To address different learning styles and abilities, have base ten materials available for the students who need the concrete or visual representation before using the symbolic personal strategies.

Have the students explain how their strategies work by relating to place value and have them choose the most efficient for them to use in calculating other problems.

b. Have the students apply their personal strategy to solve a similar problem with different numbers, such as saving $208 last year and saving 8 times as much this year.

c. Extend to include different problem contexts with different numbers.
Sample personal strategies:

First sample (repeated addition):	Second sample (distributive property):
26 + 26 = 52 26 + 26 = 52 52 + 52 = 104 You saved $104 this year.	$4 \times 26 = (4 \times 20) + (4 \times 6)$ $= 80 + 24$ $= 104$ You saved $104 this year.

Third sample (nice numbers and compensation):

26 – 1 = 25	OR	26 + 4 = 30
4 × 25 = 100		4 × 30 = 120
4 × 1 = 4		4 × 4 = 16
100 + 4 = 104		120 – 16 = 104
You saved $104 this year.		You saved $104 this year.

3. Dividing 1- and 2-digit Numbers by 1-digit Numbers with and without Remainders Using Personal Strategies, Concrete Materials Connected to Symbolic Representations, Arrays, and Connections to Multiplication

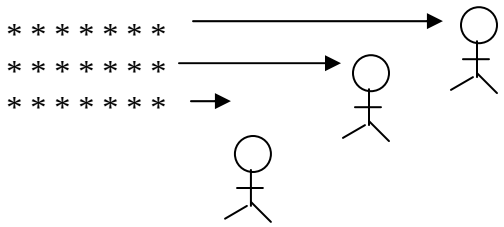
Look For …

Do students:

- ☐ know and draw on number facts and other number relationships?
- ☐ solve the problem with base ten materials but need guidance in using symbols to show what they did?
- ☐ show understanding of place value and number relationships so that their strategy makes sense?
- ☐ explain why the steps that they use work?
- ☐ interpret the remainder by using the context of the problem?
- ☐ relate division to multiplication?
- ☐ show, by using concrete materials, that repeated subtraction can be used to solve equal grouping problems but not equal sharing problems?

   a. Draw on prior knowledge by reviewing some personal strategies used by the students in solving problems by dividing numbers with dividends to 25. Have the students share their ideas by finding the quotients to equal sharing and equal grouping problems.

   Example:
   Problem—You have 24 apples to share equally among 3 friends. How many apples does each friend receive?
   Show the 24 apples as an array with 3 rows of 8 counters.
   Then show by using the sharing process (one for you and one for you and so on) that each friend receives 8 apples.



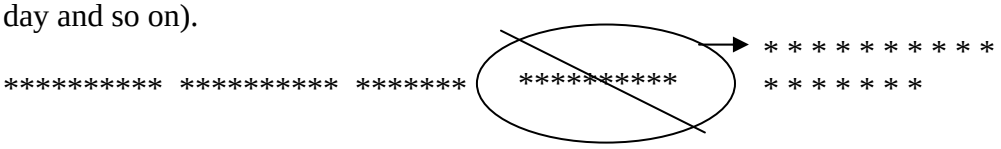
   Connect the action to the number sentence:
   24 ÷ 3 = 8 24 represents the whole, 3 represents the number of groups and the 8 represents the quantity in each group.

   Discuss related problems and other number sentences using this array, such as:
   24 ÷ 8 = 3, 3 × 8 = 24 and 8 × 3 = 24.

   b. Connect this process to dividing larger 2-digit numbers by a 1-digit number. Use counters at first and then base ten materials.
   Present the following problem to the students:
   Andre collected $57 in 3 days. If he collected the same amount of money each day, how much was his daily collection?

Guide discussion as to whether the numbers in the problem and the unknown refer to the whole, the number of groups or the quantity in each group. Discuss which operation would be used to solve the problem and what would be a good estimate for the answer. Have the students justify all their answers.
Encourage the students to represent the problem using counters and then base ten materials by arranging them in an array.

Show that 57 counters or base ten materials can be rearranged into an array by using the equal sharing process (one for the first day, one for the second day and one for the third day and so on).



Regroup one ten into ten ones to complete the sharing process as shown.

First day ********** * * * * * * * * * 1 ten and 9 ones or 19 dollars for each day
Second day ********** * * * * * * * * *
Third day ********** * * * * * * * * *

Sample personal strategies:

57 ÷ 3 = 19

Andre saved $19 each day for 3 days.

10 + 9 = 19

$$\begin{array}{r|l} 3 & 57 \\ \underline{30} & \\ 27 & \\ \underline{27} & \end{array}$$

OR

$$\begin{array}{r|l} 3 & 57 \\ \downarrow & \underline{3} \\ & 27 \\ & \underline{27} \end{array}$$

Provide time for the students to solve the problem by writing a number sentence and their personal strategies to show calculations of the quotient and then compare them to their estimate. **Emphasize the connections between the concrete representation and the symbolic personal strategies**.

c. Encourage the students to connect division to multiplication by using multiplication to check that their quotient is correct.

d. Have the students discuss whether the problem could be represented by the number sentence, $3 \times \square = 57$. Why or why not?
Have the students create a multiplication problem using the same context. For example: Andre collects $19 a day for 3 days. How much money does he collect in all?

e. Have the students explain how their strategies work by relating to place value and have them decide on which strategy is most efficient for them to use in calculating other quotients.

f. Have the students apply their personal strategy to solve a similar problem with different numbers, such as the following:
Sam saved $85 in 2 days. If he collected the same amount of money each day, how much was his daily collection?

Encourage discussion about what to do with the dollar left over as a remainder.

g. Have the students apply their personal strategy to solve a variety of division problems that include equal sharing and equal grouping with and without remainders. Note that the traditional division algorithm is built on the process involved with fair-share problems (Van de Walle 2001). To solve equal grouping problems, repeated subtraction can be used but is not efficient if the quotient is a fairly large number.

4. Dividing 1- and 2-digit Numbers by 1-digit Numbers Using Personal Strategies without Concrete Materials

a. Draw on prior knowledge by reviewing some personal strategies used by the students in solving problems by dividing 1- and 2-digit numbers, with dividends up to and including 25. Have the students share their ideas by dividing a variety of division problems, including both equal sharing and equal grouping.
Present the following problem to the students:
You have 98 beads to make 4 necklaces. If each necklace has the same number of beads, how many beads are on each necklace? Will any beads be left over? If so, how many?

Guide discussion as to whether the numbers in the problem and the unknown refer to the whole, the number of groups or the quantity in each group. Discuss which operation would be used to solve the problem and what would be a good estimate for the answer. Have the students justify all their answers. Provide time for the students to solve the problem by writing a number sentence and their personal strategies to show calculations for finding the quotient.

To address different learning styles and abilities, have base ten materials available for the students who still need the concrete or visual representation before using the symbolic personal strategies.
Have the students explain how their strategies work by relating to place value and have them decide on which strategy is most efficient for them to use in calculating other quotients.

b. Have the students apply their personal strategy to solve a similar problem with different numbers, such as having 84 beads to make 5 necklaces.

Look For …

Do students:

- ☐ know and draw on number facts and other number relationships?
- ☐ solve the problem with base ten materials but need guidance in using symbols to show what they did?
- ☐ use personal strategies that work for smaller quotients but are not efficient for calculating larger quotients?
- ☐ show understanding of place value and number relationships so that their strategy makes sense?
- ☐ explain why the steps that they use work?
- ☐ interpret the remainder by using the context of the problem?
- ☐ use the same number sentence for equal grouping and equal sharing problems but different personal strategies to suit the context of the problems?

Also include equal grouping problems by changing the problem as follows: You have 28 beads. Each necklace uses 8 beads. How many necklaces can you make? Will any beads be left over? If so, how many?

c. Extend to include different problem contexts with different numbers including answers with and without remainders.

Sample personal strategies:

<table>
<tr>
<td>First sample with equal sharing (multiplying and dividing):

You have 98 beads to make 4 necklaces. If each necklace has the same number of beads, how many beads are on each necklace? Will any beads be left over? If so, how many?

98 ÷ 4 = ☐ or 98 ÷ ☐ = 4

4 × 20 = 80
98 – 80 = 18
18 ÷ 4 = 4 with 2 remainder
20 + 4 = 24

Each necklace has 24 beads with 2 beads left over.</td>
<td>Second sample with equal sharing (multiplying and subtracting):

You have 98 beads to make 4 necklaces. If each necklace has the same number of beads, how many beads are on each necklace? Will any beads be left over? If so, how many?

98 ÷ 4 = ☐ or 98 ÷ ☐ = 4

20 + 4 = 24
4 ⟌ 9 8 | 20 OR 4 ⟌ 9 8
- 8 0 - 8 0
1 8 1 8
- 1 6 4 - 1 6
2 24 2
Answer: 24 with remainder 2
Each necklace has 24 beads with 2 beads left over.</td>
</tr>
<tr>
<td colspan="2">Third sample with equal grouping (repeated subtraction):

You have 28 beads. Each necklace uses 8 beads. How many necklaces can you make? Will any beads be left over? If so, how many?

28 ÷ 4 = ☐ 28 ÷ ☐ = 4

28 – 8 = 20 1 necklace
20 – 8 = 12 1 necklace
12 – 8 = 4 1 necklace
3 necklaces
I can make 3 necklaces with 4 beads left over.
Note: This is not an efficient strategy for larger quotients (Russell 2000).</td>
</tr>
</table>

Sample Activity 2: Teaching Estimating Products and Quotients

1. Estimating Products Using a Personal Strategy, such as the Nearest Multiple of Ten or a Hundred and Compensation

   a. Draw on prior knowledge by reviewing some estimation strategies used by the students in adding 2-digit numbers. Connect repeated addition to the multiplication of a 2-digit number by a 1-digit number and have the students transfer their strategies for adding 2-digit numbers to a multiplication problem. Have the students share their ideas by estimating the product of a 2-digit number by a 1-digit number in a problem-solving context.

   Present the students with the following problem:
   You have 3 pieces of licorice, each 27 cm long. **About** how many centimetres of licorice do you have?

   Have the students paraphrase the problem. Draw attention to the word **about**, which indicates that an estimated answer is needed and no calculation has to be done.

   Have the students represent the numbers using base ten materials to focus on the place values of the numbers. You might also represent the numbers using centimetre rulers. Model the thinking done in estimating by thinking aloud as you decide which operation to use and why. Use base ten materials along with mathematics symbols to show that 27 cm is close to 30 cm. Through discussion, have the students verbalize that 3×27 is close to or a little more than 3×20, or close to or a little less than 3×30; however, it is closest to the latter. Explain that you will use 30 to represent 27 because 30 is closer to 27 than 20. Adjustments or compensation will be made later to get a closer estimate.
   Use the number sentence, $3 \times 30 = 90$, to show the estimated product.

   Stimulate the students' thinking by asking whether 90 cm would be a good estimate for the answer. With the base ten blocks, the students should see readily that the blocks show a number less than 90. Therefore, a good estimate would be $90 - 10 = 80$ cm. Explain that the 10 is subtracted to compensate for the value that was added on when choosing the nearest multiple of ten and multiplying it by 3.

   Estimate: Three licorices would be a little less than 90 cm long or about 80 cm long.

   Have the students summarize how the nearest multiple of ten together with compensation provides a good estimate for products. Clarify that "compensation (adjusting) is a process used to add to or subtract from an initial estimate. It cuts across all estimation strategies" (Alberta Education 1990, p. 213).

Look For …

Do students:

- ☐ know and draw on number facts and other number relationships?
- ☐ explain how place value is used in the front-end strategy? The leading digit in a number together with its place value represents a good estimate for that number.
- ☐ explain how the front-end method relates to the nearest multiple of 10 or 100?
- ☐ explain how compensation provides a closer estimate than just using the front-end strategy?
- ☐ move away, gradually, from using the base ten materials and rely on their number sense for estimating products?

b. Have the students apply this estimation strategy and any other strategies that make sense to them in solving problems that require the multiplication of a 3-digit number by a 1-digit number. In these examples, the students would take the 3-digit number to the nearest multiple of 100, estimate the product and then compensate their answer appropriately.

The students should be encouraged to use various estimation strategies, such as the front-end strategy along with compensation.

2. Estimating Quotients Using a Personal Strategy, such as the Nearest Multiple of Ten or Compatible Numbers

Look For …

Do students:

- ☐ know and draw on number facts and other number relationships?
- ☐ explain how place value is used in finding the nearest multiple of ten?
- ☐ explain when to use which strategy to estimate and why?
- ☐ explain how compensation provides a closer estimate?
- ☐ gradually move away from using the base ten materials and rely on their number sense for estimating quotients?

a. Nearest Multiple of Ten

Draw on prior knowledge by reviewing some estimation strategies used by the students in subtracting 2-digit numbers. Connect repeated subtraction to the division of a 2-digit number by a 1-digit number and have the students transfer their strategies for subtracting 2-digit numbers to a division problem with equal grouping. Have the students share their ideas by estimating the quotient of a 2-digit number by a 1-digit number in a problem-solving context.

Present the students with the following problem:
You have ribbon that is 34 cm long. **About** how many pieces of ribbon, each 5 cm long, can be cut from this ribbon?

Have the students paraphrase the problem. Draw attention to the word **about**, which indicates that an estimated answer is needed and no calculation has to be done. Have the students represent the numbers using base ten materials to focus on the place values of the numbers. You might also represent the numbers using centimetre rulers.

Model the thinking done in estimating by thinking aloud as you decide which operation to use and why. Use base ten materials along with mathematics symbols to show that 34 cm is close to 30 cm. Through discussion, have the students verbalize that $34 \div 5$ is close to but a little more than $30 \div 5$, or close to but a little less than $40 \div 5$; however, it is closest to $30 \div 5$. Explain that you will use 30 to represent 34 because 30 is closer to 34 than 40. Adjustments or compensation will be made later to get a closer estimate, if necessary. Use the number sentence, $30 \div 5 = 6$, to show the estimated product.

Stimulate the students' thinking by asking whether 6 cm would be a good estimate for the answer. In this case, 6 pieces of ribbon would not only be a good estimate but also the calculated answer to the problem because you would not be able to have another 5 cm ribbon from the 4 cm remaining.
Estimate: About 6 pieces of ribbon can be cut from the 34 cm of ribbon.

Have the students summarize how the nearest multiple of ten provides a good estimate for quotients if the divisor divides evenly into the multiple of ten. Compensation is not needed when working with multiples of 10 but may be needed when working with multiples of 100, which will be used in Grade 5.

b. Compatible Numbers with Compensation
When the divisors in the problem do not divide evenly into multiples of ten, then compatible numbers is a strategy that the students might choose as their personal strategy to estimate quotients.

Present the students with the following problem:
You have 76 cm of ribbon to share equally among 9 people. About how much ribbon would each person receive?

Model the thinking done in estimating by thinking aloud as you decide which operation to use and why. Use base ten materials along with mathematics symbols to show that 75 is midway between 70 than 80 but neither 70 nor 80 is divisible by 9. Therefore, another strategy, called compatible numbers, will be used to estimate the quotient. Think, what number close to 75 is divisible by 9? We know that 72 is divisible by 9 so we will use 72 to estimate the quotient. Write the number sentence $72 \div 9 = 8$.

Through discussion, have the students verbalize that a good estimate would be a little more than 8 cm, because 75 cm is a little more than 72 cm.

Estimate: Each person would receive a little more than 8 cm of string.

Clarify that "compensation (adjusting) is a process used to add to or subtract from an initial estimate. It cuts across all estimation strategies" (Alberta Education 1990, p. 213).

Have the students apply these estimation strategies and any others that make sense to them in solving problems that require the division of a 1- or 2-digit number by a 1-digit number.

Look For …

Do students:

- ☐ explain the meaning of the problem and justify why only an estimate is needed or why a calculated answer is necessary as well?
- ☐ use compensation as well as other strategies to estimate the product or quotient?
- ☐ explain clearly the strategies that they used in estimating and why they work?

3. Estimating Sums and Differences: Classifying Problems

Draw on prior knowledge by reviewing some problem contexts that require only an estimate and not a calculation for the answer. Through discussion, have the students generalize that estimating is necessary for every problem involving multiplication or division because estimates must be made prior to any calculations so that the reasonableness of the calculated answer can be determined.

Present the students with problems and have them decide which problems can be answered with an estimate only and which problems require calculation as well as an estimate.

Examples of problems:

- You have $60. Do you have enough money to buy 3 CDs each worth $17?
- You travel 375 km each day for 3 days. Will you reach the cabin that is 1200 km away by the end of the third day?
- You set up 6 rows of chairs with 28 chairs in each row in the gym. Are there enough chairs to seat 180 people? How many chairs did you set up?
- Mary reads 96 pages and Tricia reads 8 pages. Mary reads how many times as many pages as Tricia?
- Serena has 50 kg of apples. If she puts 6 kg of apples in each basket, how many baskets can she fill?
- Tuan rode his bicycle every day for 8 days. He cycled 68 km. About how far did he ride each day?
- A toad jumps 135 cm on the first jump and twice as far on the second jump. About how far does it jump in all?
- You jog for 175 minutes each week. How many minutes do you jog in 28 days?

After classifying the problems as either needing only an estimate or needing both an estimate and a calculated answer, have the students answer the problems and share their solutions with one another.

Have the students create multiplication and division problems, some of which require only an estimate for an answer and others that require an estimate and a calculated answer.

Other strategies for estimating products and quotient are available in the *Diagnostic Mathematics Program, Elementary: Operations and Properties, Division II* (Alberta Education 1990, pp. 250–253).

Sample Activity 3: Teaching Solving Problems Involving Multiplication and Division

1. Four Corner Strategy

   Draw on prior knowledge by reviewing multiplication and division problems involving number facts to 5×5 or $25 \div 5$. Emphasize the connections among the story problems, the models/diagrams, the number sentences and the personal strategies used in calculations.
   Have the students divide their page into four sections to make graphic organizers and label them as follows:

Four Corner Strategy

Story Problem	Models/Diagrams
Number Sentence	Personal Strategy

Look For …

Do students:

- ☐ exhibit flexibility in writing number sentences in more than one way to show the relationship between multiplication and division?
- ☐ clearly explain their personal strategy and adjust it through discussion to make it more efficient?
- ☐ create problems to illustrate number sentences as well as write number sentences for problems?

Present the students with a problem or a number sentence involving multiplication or division, such as the following:

- You paid 95 cents for 5 apples. What is the cost of each apple?
- $5 \times \square = 95$

Have the students complete the graphic organizer by writing the story problem or the number sentence in one corner and filling in the other corners appropriately.

Adaptations:

- The students work in groups and fill in the graphic organizer on large chart paper that can be displayed and discussed with other students and the whole class.
- Use different labels for the four corners of the graphic organizer, such as word problem, number sentence, estimation and personal strategy.

2. Whole, the Number of Groups or the Quantity in Each Group?

   Draw on prior knowledge by reviewing multiplication and division problems involving number facts to 5×5 or $25 \div 5$.

   Guide discussion as to whether the numbers in the problems and the unknown refer to the whole, the number of groups or the quantity in each group. Have them solve the problems and explain their thinking.

   Problem examples:
   - How many different single-scoop ice cream cones can be made with 4 different kinds of cones and 28 different flavours of ice cream?
   - Star practises the piano each day for 75 minutes. How long does she practise the piano in a week?
   - You travel 84 km in three days. If you travel the same distance each day, how far do you travel each day?
   - You have 76 flowers to put into bouquets of 8 flowers each. How many bouquets can you make with these flowers?

Look For …

Do students:
- ☐ label, correctly, the numbers in each problem as the whole, the number of groups or the quantity in each group?
- ☐ identify whether the whole, the number of groups or the quantity in each group is not known in the problem?
- ☐ solve the problems and justify why the solution is correct?

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3. Thumbs Up, Thumbs Down, Thumbs Sideways
   Present the students with a variety of multiplication and division problems. Reading the problems orally and also having them displayed on the white board or the overhead projector addresses the different learning styles of the students.
   For each problem, ask the students to put their thumbs up if multiplication can be used to solve the problem, thumbs down if division can be used and thumbs sideways if both multiplication and division can be used. Have the students justify their choice, either in small groups or with the entire class.

   Finally, have the students write number sentences to support their choices. Emphasize the relationship between multiplication and division as the students suggest different number sentences.

Look For …

Do students:
- ☐ classify, correctly, problems as involving multiplication, division or both operations?
- ☐ exhibit flexibility in writing number sentences in more than one way to show the relationship between multiplication and division?
- ☐ justify their choice of operation(s) to solve a problem?

Adaptations:

- Include problems that require the multiplication and division of more than two numbers. Examples:
  - Joey places five rows of 24 chairs in the gym. Tracy places twice as many chairs as Joey. How many chairs are placed in the gym?
  - Ninety-six apples are evenly distributed among 4 baskets. If one of the baskets is shared equally among 8 people, how many apples does each person receive?
  - Marcy and Tina buy three pieces of chocolate fudge that each cost 28 cents. If they share the cost of the fudge equally, how much does each person pay?
  - How many different outfits can Ragan make if she has 15 different shirts, 4 different pairs of pants and a 9 different pairs of socks?
- Provide the students with written copies of the problems and have them work in pairs or individually to classify the problems as multiplication, division or both multiplication and division. The students then write the appropriate number sentences for each problem.
- Have the different groups of students take turns creating and classifying multiplication and division problems.
- Have the students write equivalent number sentences for a given problem. Through discussion, have the students generalize that the semantic number sentence (the one that shows the meaning of the problem) is often rearranged to expedite calculation.
  For example, $85 \div 5 = \square$ can be written as $5 \times \square = 85$.
  The first number sentence shows dividing but a student might prefer to use multiplication to solve the problem and therefore rearrange the number sentence in the rewritten form.
- Have the students classify the problems according to whether the unknown is the whole, the number of groups or the quantity in each group.

4. Choosing Number Sentences
   Present the students with a problem and have them choose which of the number sentences provided could be used to solve the problem. Ask why the number sentences chosen can be used to solve the problem.

   Example:
   Diego saved $96 this month by doing odd jobs for the neighbours. Last month, he saved $8. How many times as much money did he save this month as last month?

   $96 \times 8 = \square$ $\quad \square = 8 \times 96$ $\quad 8 \times \square = 96$
   $96 \times \square = 8$ $\quad 96 \div 8 = \square$ $\quad 8 \div 96 = \square$
   $\square \div 8 = 96$ $\quad 96 \div \square = 8$ $\quad 8 \div \square = 96$

Look For …

Do students:

- ☐ choose, correctly, equivalent number sentences that relate to the problem?
- ☐ justify their choices by relating to the meaning of the problem and the relationship between multiplication and division?

5. Classifying Problems: Open and Closed Sorts

Open Sort

Present the students with a variety of multiplication and division problems that are written on separate pieces of paper. Have the students work in groups to classify the problems into groups, label the groups and explain why the problems fit where they have been placed. Explain that some problems may fit in more than one group. Challenge the students to:

- find another way to classify the problems
- create other problems and place them into the groups

Look For …

Do students:

- ☐ create classifications for the problems based on the meaning of operations?
- ☐ exhibit flexibility in creating more than one way to sort the problems?
- ☐ justify why problems were placed in specific categories?

Some categories used by the students may include the following:

- multiplication, division, both multiplication and division
- equal grouping, equal sharing (division problems)
- equal-group problems, comparison problems, combinations problems
- only estimates are needed, both estimates and a calculated answer are needed
- the unknown is the whole, the number of groups or the quantity in each group.

Closed Sort

Present the students with a variety of multiplication and division problems that are written on separate pieces of paper and also provide them with the categories into which they are to group the problems. See the examples of categories given above. The students sort the problems into the categories provided and justify their choices.

6. Similarities and Differences

Provide the students with two problems using the same numbers but different meanings for multiplication or division, such as one showing equal grouping and one showing the equal sharing. Ask the students to explain how the problems are the same and how they are different. They may wish to put their explanations in a graphic organizer, such as the one shown below:

Similarities	Differences

An example of a problem showing equal grouping:

You have 75 pictures put into albums. If each album page holds 4 pictures, how many pages do you need? Explain how you know.

An example of a problem showing equal sharing:

You have 75 marbles to share equally among 4 friends. How many marbles will each friend receive? Explain how you know.

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Other strategies for teaching multiplication and division problems, using arrays and showing the connections between the operations, are available in the *Diagnostic Mathematics Program, Elementary: Operations and Properties, Division II* (Alberta Education 1990, pp. 217–225).

Look For …

Do students:

- ☐ explain that in both problems the whole is known and one of the parts is unknown?
- ☐ explain that both problems can be represented by the same number sentence?
- ☐ explain that the concrete or visual representation in each problem is different?
- ☐ explain that the 4 represents the number in each group in the first problem but it represents the number of groups in the second problem?
- ☐ explain that the remainder in each problem is handled differently?
- ☐ explain that the units are different in the two problems?
- ☐ explain that the problems explain different real-life situations?

Step 4: Assess Student Learning

Guiding Questions

- Look back at what you determined as acceptable evidence in Step 2.
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Sample Assessment Tasks

In addition to ongoing assessment throughout the lessons, consider the following sample activities to evaluate students' learning at key milestones. Suggestions are given for assessing all students as a class or in groups, individual students in need of further evaluation, and individual or groups of students in a variety of contexts.

A. Whole Class/Group Assessment

Note: Performance-based assessment tasks are under development.

Have counters and base ten materials for the student to use as needed.

1. Complete the chart below by drawing a diagram, writing a number sentence and using a personal strategy to solve the problem given. Write the answer to the problem in a complete sentence.

Story Problem You have 4 cans of nuts that each weigh 350 g. What is the total weight of these cans of nuts?	Models/Diagrams
Number Sentence	Personal Strategy

2. You read 5 times as long this week as you read last week. If you read for 95 minutes this week, how long did you read last week?

3. For your lunch you have a sandwich, a dessert and a drink. How many different lunches could you make if you have 4 choices for the sandwich, 2 choices for dessert and 3 choices for a drink?

4. Write all the possible number sentences that are represented in the following array. Explain how each number sentence relates to the array.

   ********** ********** ********** * *
   ********** ********** ********** * *
   ********** ********** ********** * *

5. Sarah is putting 95 oranges into bags with 8 oranges in each bag. How many bags does she need? Explain your thinking.

6. For a school assembly, 9 rows of 38 chairs have been placed in the gym. Are there enough chairs for 370 students? Explain your thinking.

7. Jason rides 65 km in 4 days on his bicycle. About how many kilometres does he ride each day?

8. You save 6 times as much money this year as you saved last year. If you saved $125 last year, how much money did you save this year?

9. Explain why the following solution makes sense or not.
   $6 \times 128 = (6 \times 100) + (6 \times 20) + (6 \times 5) = 600 + 12 + 30 = 642$
   Answer: I saved $642 this year.

10. Create a problem that can be represented by the number sentence: $65 \div 4 = \square$.
    Explain how you know your problem matches the number sentence.

B. One-on-one Assessment

1. Ask the student to explain the connection between multiplication and division by using counters or base ten materials. If necessary, coach the student to make an array and show how the array shows both multiplication and division.

2. Present the following problem to the student and have him or her read it orally. Have base ten materials available to use as needed.

   A bottle contains 76 mL of medicine. Jerry takes 8 mL of medicine each hour. How many hours will pass before all the medicine is gone?

   Pose the following questions to guide thinking if necessary:
   - State the problem in your own words.
   - What do each of the numbers in the problem represent—whole, the number of groups or the quantity in each group?
   - What is the unknown in the problem—whole, the number of groups or the quantity in each group?
   - What number sentence could you write to show the meaning of the problem?
   - Does the problem use multiplication or division or both? Explain.

- **About** how many hours will pass before the medicine is all gone? Explain your thinking.
- Will any medicine be left over? Explain.
- Use a strategy that makes sense to you to find the answer to the problem. Explain your thinking as you write the numbers.
- Explain how you know your answer makes sense and is reasonable.
- Would you solve the problem another way? Explain your thinking.

3. Use a similar procedure as outlined in question 2 above with the following problem:
   You read for 255 minutes every week. How many minutes do you read in 3 weeks?

4. Create a problem that can be shown by the number sentence, $45 \div 6 = \square$.
   Solve the problem you created by using a strategy that makes sense to you.

C. Applied Learning

Provide opportunities for the students to use multiplication and division in a practical situation and notice whether or not the strategies transfer. For example, ask the students to estimate and then calculate the number of vans needed to transport 75 children if each van can hold 6 children.

Does the student:

- use an estimate to make the comparison?
- interpret the remainder correctly by including one more van for the remaining three children?
- use a personal strategy that makes sense in calculating the answer?

Step 5: Follow-up on Assessment

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction?

A. Addressing Gaps in Learning

The students who have difficulty solving problems by estimating and using a personal strategy for multiplication or division will enjoy more success if one-on-one time is provided in which there is open communication to diagnose where the learning difficulties lie. Assessment by observing a student solving problems will provide valuable data to guide further instruction. Success in problem solving depends on a positive climate in which the students are confident in taking risks. By building on the understanding that each student already has and accommodating the individual learning styles, success will follow.
If the difficulty lies in understanding the problem, use the following strategies:

- provide problems that relate to the student's interest; use the student's name in the problem
- use smaller numbers in the problem initially
- have the student paraphrase the problem
- guide the student to determine if the numbers refer to the whole, the number of groups or the quantity in each group
- ask the student if the unknown in the problem refers to the whole, the number of groups or the quantity in each group
- provide base ten materials for the students to represent the problem as needed
- have the student decide which operation should be used and why
- ask guiding questions to show the connections between multiplication and division and the possible option of using either operation in solving the problem
- provide a graphic organizer, such as the K–N–W–S chart (see Blackline Master).

If the difficulty lies in estimating products and quotients, use the following strategies:

- use the base ten materials to focus on the place values of the numbers and the relationship among the place values
- use smaller numbers initially and connect them to larger numbers; e.g., connect 30 to 300 to 3000
- convince the student of the need for estimating by citing many real-world examples of where estimating is needed
- review number facts and place value
- emphasize flexibility in estimating, capitalizing on the student's methods and fine tuning them for correctness and efficiency
- take small steps, using a range of acceptable estimates with closer estimates as the students become more comfortable with the process.

If the difficulty lies in using personal strategies to solve multiplication and division problems, use the following strategies:

- use smaller numbers in the problems initially
- review place value and number facts
- provide base ten materials as needed
- think aloud a personal strategy that you would use to solve the problem and explain why this strategy is more efficient than another one that you describe
- emphasize flexibility in choosing a personal strategy; a strategy that is efficient for one student may not be efficient for another student
- build on the student's understanding of place value and number facts to guide him or her in finding a strategy that works
- provide ample time for the students to think and ask questions to clarify thinking
- have the students work in groups so that they learn strategies from one another
- guide the students to critique various personal strategies to find one that can be used on a variety of problems efficiently
- have the students explain their personal strategies to the class so others can hear how they work in 'kid' language
- post various personal strategies in the classroom for the students to share and critique
- encourage the student to check the reasonableness of the answer using a given personal strategy by comparing the answer to the estimated answer provided earlier.

B. Reinforcing and Extending Learning

The students who have achieved or exceeded the outcomes will benefit from ongoing opportunities to apply and extend their learning. These activities should support the students in developing a deeper understanding of the concept and should not progress to the outcomes in subsequent grades.

Consider strategies, such as the following.

- Provide tips for parents on practising adding and subtracting at home or in the community. For example,
  - take the children shopping and have them estimate the total cost if you buy multiple copies of the same item
  - involve the children in real-life problem-solving activities, such as calculating the total time spent reading if 45 minutes are spent reading each day. Convert the problem into a division problem by providing the total number of minutes spent reading for a given number of days and asking for the number of minutes spent reading each day if the time is the same for each day.
  - talk to your children about data in the newspaper and magazines, and encourage them to estimate products and quotients and explain how they are doing it.

- Have the students create problems showing the various types of multiplication and division problems (equal group problems including equal grouping and equal sharing, comparison problems, combination problems) and write appropriate number sentences for each one. These problems can be displayed in chart on the bulletin board.

- Have the students create problems with different contexts but using the same numbers such as 78 and 8. They could follow this up by having the class decide which of the problems could be solved using a given number sentence such as $78 \div 8 = \square$.

- Have the students solve multi-step problems that involve the multiplication or division of more than two numbers. Examples:
  - Joey places five rows of 24 chairs in the gym. Tracy places twice as many chairs as Joey. How many chairs are placed in the gym?
  - Ninety-six apples are evenly distributed among 4 baskets. If one of the baskets is shared equally among 8 people, how many apples does each person receive?
  - Marcy and Tina buy three pieces of chocolate fudge that each cost 28 cents. If they share the cost of the fudge equally, how much does each person pay?
  - How many different outfits can Ragan make if she has 15 different shirts, 4 different pairs of pants and a 9 different pairs of socks?
  - You read 325 minutes a week. How many minutes do you read in 35 days?

- Have the students critique other students' personal strategies and explain why they work or not. Which would be the most efficient and why?

- Have the students write an explanation for a personal strategy so that everyone in the class can understand it.

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