Planning Guide: *Grade 4 Solving Equations*

Strand: Patterns and Relations (Variables and Equations)
Outcomes: 5, 6

Curriculum Highlights

This sample targets the following changes in the curriculum:

- The General Outcome focuses on representing algebraic expressions in multiple ways, whereas the previous mathematics curriculum focused on communicating rules for and predicting from numerical and non-numerical patterns, including those found in the community.
- The Specific Outcome focuses on expressing a given problem as an equation in which a symbol is used to represent an unknown and then solving the one-step equation, whereas the previous mathematics curriculum focused on identifying and explaining mathematical relationships and patterns using tables, graphs, objects, Venn/Carroll/tree diagrams and technology.

Step 1: Identify Outcomes to Address

Guiding Questions

- What do I want my students to learn?
- What can my students currently understand and do?
- What do I want my students to understand and be able to do, based on the Big Ideas and specific outcomes in the program of studies?

Big Ideas

Mathematics is often referred to as the science of patterns. Patterns permeate every mathematical concept and are found in the everyday contexts. The various representations of patterns, including symbols and variables, provide valuable tools in making generalizations of mathematical relationships. Some characteristics of patterns include the following:

- There are different types of patterns that can be modelled in a variety of ways.
- Patterns using concrete and pictorial representations of a problem can be translated into patterns using numbers and symbols in equations.
- Patterns are used to generalize relationships; e.g., numbers may be added or multiplied in any order without changing the sum or product.
- Equality is used to express relationships. The symbols used on either side of the equal sign represent a quantity. The equal sign is "a symbol of equivalence and balance" (NCTM 2000, p. 39).
- Patterns using symbols and variables provide a means of describing change mathematically; e.g., 5 more than, 3 less than.
- Variables and equations are used to mathematically model everyday problems that describe quantitative relationships.

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Variables and symbols stand for any one of a set of numbers or objects and have multiple meanings. They are used as:

- representations of specific unknown values; e.g., 3 + □ = 15
- representations of quantities that vary or change; e.g., □ + △ = 15
- placeholders in a generalized expression or formula; e.g., □ = 10△, where □ represents the number of centimetres and △ represents the number of metres.

Adapted from Van de Walle, John A., LouAnn H. Lovin, *Teaching Student-Centered Mathematics, Grades 3–5* (pp. 290, 307). Published by Allyn and Bacon, Boston, MA. Copyright © by Pearson Education. Adapted by permission of the publisher.

Mathematician's rules or conventions dictate the use of variables for quantities that change or are dynamic.

- If the same variable or symbol is used repeatedly in the same equation, then there is only one possible solution for that variable or symbol; e.g., □ + □ = 20; the unique solution is to place 10 in each of the squares.
- If different variables or symbols are used in the same equation, then there are many different possible solutions, one of which may be the same number replacing each variable or symbol; e.g., □ + △ = 20, some solutions include 0 + 20, 10 + 10, ½ + 19 ½, and 25 – 5.

Adapted from Alberta Education, *Patterns and Pre-Algebra, Grades 4–6* (unpublished workshop handout) (Edmonton, AB: Alberta Education, 2007), p. 67.

The majority of the work done by young students in using variables and symbols is focused on specific unknowns. They "use symbolic representations and drawings to help them solve problems and communicate their ideas" (Van de Walle and Lovin 2006, p. 303).

Algebraic reasoning is directly related to patterns because this reasoning focuses on making generalizations based on mathematical experiences and recording these generalizations using symbols or variables (Van de Walle and Lovin 2006).

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Sequence of Outcomes from the Program of Studies

See (Web address) for the complete program of studies.

Grade 3	**Grade 4**	**Grade 5**
Specific Outcomes	Specific Outcomes	Specific Outcomes
4. Solve one-step addition and subtraction equations involving a symbol to represent an unknown number.	5. Express a given problem as an equation in which a symbol is used to represent an unknown number. 6. Solve one-step equations involving a symbol to represent an unknown number.	2. Express a given problem as an equation in which a letter variable is used to represent an unknown number (limited to whole numbers). 3. Solve problems involving single-variable, one-step equations with whole number coefficients and whole number solutions.

Step 2: Determine Evidence of Student Learning

Guiding Questions

- What evidence will I look for to know that learning has occurred?
- What should the students demonstrate to show their understanding of the mathematical concepts, skills and big ideas?

As you begin planning lessons and learning activities, keep in mind ongoing ways to monitor and assess student learning. One starting point for this planning is to consider the achievement indicators listed in the *The Alberta K–9 Mathematics Program of Studies with Achievement Indicators* (Alberta Education 2007). You may also generate your own indicators, and use these to guide your observation of the students.

The following achievement indicators may be used to determine whether the students have met this specific outcome.

- Explain the purpose of the symbol, such as a triangle or circle, in a given addition, subtraction, multiplication or division equation with one unknown; e.g., $36 \div \square = 6$.
- Express a given pictorial or concrete representation of an equation in symbolic form.
- Identify the unknown in a problem, represent the problem with an equation and solve the problem concretely, pictorially or symbolically.
- Create a problem in context for a given equation with one unknown.
- Represent and solve a given one-step equation concretely, pictorially or symbolically.
- Solve a given one-step equation using guess and test.

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- Describe orally, the meaning of a given one-step equation with one unknown.
- Solve a given equation when the unknown is on the left or right side of the equation.
- Represent and solve a given addition or subtract problem involving a "part–part–whole" or comparison context using a symbol to represent the unknown.
- Represent and solve a given multiplication or division problem involving equal grouping or partitioning (equal sharing) using symbols to represent the unknown.

Some sample behaviours to look for in relation to these indicators are suggested for many of the instructional activities in Step 3, Section C, Choosing Learning Activities.

Step 3: Plan for Instruction

Guiding Questions

- What learning opportunities and experiences should I provide to promote learning of the outcomes and permit the students to demonstrate their learning?
- What teaching strategies and resources should I use?
- How will I meet the diverse learning needs of my students?

A. Assessing Prior Knowledge and Skills

Before introducing new material, consider ways to assess and build on the students' knowledge and skills related to solving one-step addition and subtraction equations involving symbols representing an unknown number. For example:

- Explain the purpose of the box in the following equation: $15 - \square = 8$.

- Write the following equation using another symbol for the unknown instead of a box.
  $50 = 20 + \square$

- You have 24 marbles and your friend gives you some more marbles. Now you have 32 marbles in all. How many marbles did your friend give you?
  a. Write an equation to show what is happening in this problem.

  b. Solve the problem. Explain your thinking.

- Solve the following equation and use a diagram to explain the process.
  $4 + 5 = \square + 2$

- Solve the following equation and explain your thinking.
  $- 13 = 20$

- Lori said that the box in the following equation stands for more than one number. Is Lori correct? Why or why not?
  $6 + 8 = \square + 4$

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If a student appears to have difficulty with these tasks, consider further individual assessment, such as a structured interview, to determine the student's level of skill and understanding (see Sample Structured Interview: Assessing Prior Knowledge and Skills).

B. Choosing Instructional Strategies

Consider the following general strategies for teaching the use of equations involving a symbol to represent an unknown number.

- Build on the students' knowledge from the previous grade in using equations to write addition and subtraction equations.
- Connect the concrete, pictorial and symbolic representations consistently as the students develop and demonstrate understanding of equations.
- Use everyday contexts for problems that the students can relate to so that they can translate the meaning of the problem into an appropriate equation using a symbol to represent the unknown number.
- Provide a variety of problems for the students to express as equations. Include problems that illustrate the various types of addition and subtraction (e.g., change, part–part–whole and comparison) as well as multiplication and division (e.g., of equal sharing, equal grouping, comparison problems and combination).
- Review the relationship between addition and subtraction number sentences as well as the relationship between multiplication and division number sentences.
- Have the students create problems for a variety of number sentences illustrating addition and subtraction, including examples of change, part–part–whole and comparison problems.
- Have the students create problems for a variety of number sentences illustrating multiplication and division, including examples of equal sharing, equal grouping, comparison problems and combination problems.
- Encourage the students to write equations in various ways to represent the meaning of a given problem.
- Include examples of equations in which the unknown is on the left or the right side of the equation.
- Emphasize that the equal sign is a symbol of equivalence or balance of the two quantities on either side of the equation.

C. Choosing Learning Activities

The following learning activities are examples of activities that could be used to develop student understanding of the concepts identified in Step 1.

Sample Activities for Teaching Students to Express a Given Problem as an Equation Using a Symbol to Represent an Unknown Quantity

1. Story Translations for Addition and Subtraction Problems
   Provide the students with part–part–whole and comparison word problems and have them explore the idea of a symbol representing a specific, unknown quantity as they translate the problems into written equations. Review the meaning of the equal sign as a symbol of equivalence or balance of the two quantities on either side of the equation.

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Examples of problems:

<table>
<tr><td rowspan="2">Part–Part–Whole</td><td colspan="3">Whole Unknown</td><td colspan="3">Part Unknown</td></tr>
<tr><td colspan="3">Connie has 15 red marbles and 28 blue marbles. How many marbles does she have?</td><td colspan="3">Connie has 43 marbles. 15 are red and the rest are blue. How many blue marbles does Connie have?</td></tr>
<tr><td rowspan="2">Comparison</td><td colspan="2">Difference Unknown</td><td colspan="2">Unknown Big Quantity</td><td colspan="2">Unknown Small Quantity</td></tr>
<tr><td colspan="2">Connie has 15 red marbles and 28 blue marbles. How many more blue marbles than red marbles does Connie have?
(Compare)</td><td colspan="2">Connie has 15 red marbles and some blue marbles. She has 13 more blue marbles than red ones. How many blue marbles does Connie have?</td><td colspan="2">Connie has 28 blue marbles. She has 13 more blue marbles than red ones. How many red marbles does Connie have?</td></tr>
</table>

a. Part–Part–Whole Problem
Cut strips of cash-register tape cut into the following lengths:
- 46 cm and 54 cm
- 48 cm and 52 cm
- 52 cm and 48 cm
- 56 cm and 44 cm.

Provide each student with one strip of the pre-cut cash register tape. Have them measure their strips and record the measurements on sticky notes they stick on the front of their shirts. Their challenge is to find a partner who has a strip that, when added to theirs, makes a longer strip that is exactly 1 m long.

Encourage the students to use mental mathematics strategies to predict how long their partner's strip will have to be. Once they find their partner, have them verify their solution by placing the strips together and comparing the combined length to a metre stick.

Look For …
Do students:
- ☐ apply previous knowledge in understanding the four operations?
- ☐ draw appropriate diagrams to represent the types of problems for the four operations?
- ☐ relate addition to subtraction and multiplication to division in writing equations and solving problems?
- ☐ show flexibility in using a variety of symbols to represent an unknown number in an equation?

Ask the student to draw a diagram to represent the situation, perhaps using a solid line to represent their string and a dotted line to represent their partner's string.

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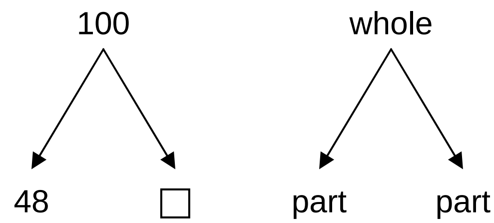
Model this part–part–whole situation:



Through discussion, have the students verbalize various ways that equations can be written to represent the situation. Encourage the students to use a variety of symbols. Ensure that the students include equations in which the symbol to represent the unknown quantity is on the left side and other equations in which the unknown quantity is on the right side.

48 + □ = 100	⬭ + 48 = 100	100 = ⬡ + 48	100 = 48 +
100 – 48 = ⬠	◇ = 100 – 48	100 – ▢ = 48	48 = 100 – ◺

Provide the students other part–part–whole problems to solve, with either the whole or one of the parts as the unknown. Have the students solve these problems using manipulatives, then draw appropriate diagrams and write equations with a symbol for the unknown.

b. Comparison Situations
Provide the students with interlocking cubes (white and two other colours). Pose the following problem:

> Connie has 15 red marbles and 24 blue marbles. How many more blue marbles than red marbles does Connie have?

Have the students model this situation by building two columns with the cubes, one representing the red marbles and the other representing the blue marbles. To find the difference between the two columns, white cubes are added to the red column to represent the difference between the two quantities. Have the students draw a diagram to represent the situation.

Encourage the students to write as many different equations as they can, using symbols to represent this situation.
Ask the students to share their equations and ensure that a wide variety of equations are included.

15 + □ = 24	⬭ + 15 = 24	24 = 15 + ⬡	24 = △ + 15
24 – ⬠ = 15	24 – 15 = ◇	15 = 24 – ▢	◺ = 24 – 15

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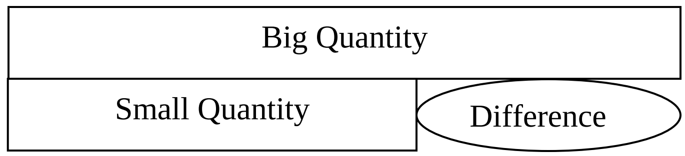
Summarize the comparison problems by showing the students a general model that they can use to think about other comparison problems.



Note: Model suggested by Karen Fuson in "Meaning of Numerical Operations through Word Problem Solving: Access to All through Student Situational Drawings within an Algebraic Approach." Presentation at the Annual Conference for the National Council of Supervisors of Mathematics, St. Louis, USA, April 26, 2006.

Have the students solve other comparison problems, with either the big quantity, small quantity or difference as the unknown. Encourage the students to draw diagrams and write equations to represent the situations.

Note: In the primary grades, the students have worked with symbols as specific unknowns in join and separate problems. Samples of these problems can also be provided as review.

Examples of problems:

	Result Unknown	**Change Unknown**	**Start Unknown**
Join	Connie had 15 marbles. Juan gave her 28 more marbles. How many marbles does Connie have altogether?	Connie has 15 marbles. How many more marbles does she need to have 43 marbles altogether?	Connie had some marbles. Juan gave her 15 more marbles. Now she has 43 marbles. How many marbles did Connie have to start with?
Separate	Connie had 43 marbles. She gave 15 to Juan. How many marbles does Connie have left?	Connie had 43 marbles. She gave some to Juan. Now she has 15 marbles left. How many marbles did Connie give to Juan?	Connie had some marbles. She gave 15 to Juan. Now she has 28 marbles left. How many marbles did Connie have to start with?

Emphasize that the power of patterns and relations in solving problems is that the same equation can be used to represent a variety of different addition and subtraction problems.

Adapted from Alberta Education, *Patterns and Pre-Algebra, Grades 4–6* (unpublished workshop handout) (Edmonton, AB: Alberta Education, 2007), pp. 142–144.

2. Story Translations for Multiplication and Division Problems
   Provide the students with equal grouping, equal sharing, comparison, area or array and combination word problems and have them explore the idea of a symbol representing a specific, unknown quantity as they translate the problems into written equations. Review the meaning of the equal sign as a symbol of equivalence or balance of the two quantities on either side of the equation.

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Examples of problems:

Problem Type	**Multiplication** **(givens: number of groups and number of objects in each group)**	**Measurement Division** **(givens: total number of objects and the number of objects in each group)**	**Partitive Division** **(givens: total number of objects and the number of groups)**
Grouping/ Partitioning **Equal Groups**	Gene has 4 tomato plants. There are 6 tomatoes on each plant. How many tomatoes are there altogether?	Gene has some tomato plants. There are 6 tomatoes on each plant. Altogether there are 24 tomatoes. How many tomato plants does Gene have?	Gene has 4 tomato plants. There is the same number of tomatoes on each plant. Altogether there are 24 tomatoes. How many tomatoes are on each tomato plant?
Multiplicative Comparison	The giraffe in the zoo is 3 times as tall as the kangaroo. The kangaroo is 6 feet tall. How tall is the giraffe? The giraffe is 18 feet tall. The kangaroo is 6 feet tall. The giraffe is how many times taller than the kangaroo? The giraffe is 18 feet tall. She is 3 times as tall as the kangaroo. How tall is the kangaroo?		
Area and Array	A farmer plants a rectangular vegetable garden that measures 2 m along one side and 5 m along an adjacent side. How many m^2 of garden did the farmer plant? A baker has a pan of fudge that measures 8 inches on one side and 9 inches on another side. If the fudge is cut into square pieces 1 inch on a side, how many pieces of fudge does the pan hold? A farmer plans to plant a rectangular vegetable garden. She has enough room to make the garden 5 m on one side. How long does she have to make the adjacent side in order to have 10 m^2 of garden?		
Combinations	The Friendly Old Ice Cream Shop has 2 types of cones (waffle and plain). They have 5 flavours of ice cream (chocolate, vanilla, strawberry, rainbow and tiger). How many one-scoop combinations of an ice cream flavour and cone type can you get at the Friendly Old Ice Cream Shop?		

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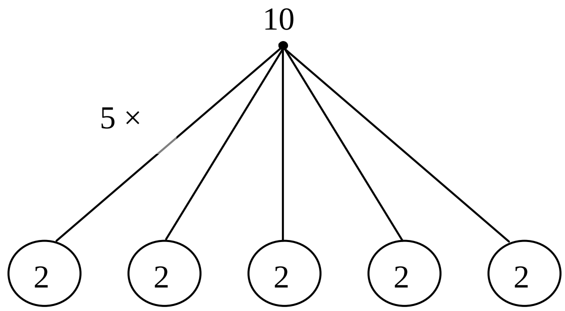
Models

Note: Models suggested by Karen Fuson in "Meaning of Numerical Operations through Word Problem Solving: Access to All through Student Situational Drawings within an Algebraic Approach." Presentation at the Annual Conference for the National Council of Supervisors of Mathematics, St. Louis, USA, April 26, 2006.

Equal Grouping and Equal Sharing

1. Amy has 5 cousins. She is making 2 puppets for each cousin. How many puppets will Amy need to make?
2. Amy made 10 puppets to divide equally among her 5 cousins. How many puppets will each cousin get?
3. Amy made 10 puppets for her cousins. Each cousin will get 2 puppets. How many cousins does Amy have?



Have the students write equations to represent the situations, using as many different ways as possible. The equations for problem 2 could include the following:

10 ÷ 5 = □	△ = 10 ÷ 5	10 ÷ ◺ = 5	5 = 10 ÷ ⬡
5 × ⬠ = 10	10 = 5 × ⬭	◇ × 5 = 10	10 = ⯃ × 5

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Comparisons

1. Bill has 2 apples. Kim has **5 times** as many apples as Bill. How many apples does Kim have?
2. Kim has 10 apples. Bill has $\frac{1}{5}$ as many apples as Kim. How many apples does Bill have?
3. Bill has 2 apples. Kim has 10 apples. Kim has how many times the number of apples as Bill?

B	2				
K	2	2	2	2	2

Have the students write equations to represent the situations, using as many different ways as possible. The equations for problem 3 could include the following:

$10 \div 2 = \square$ $\qquad \triangle = 10 \div 2$ $\qquad 10 \div \text{◺} = 2$ $\qquad 2 = 10 \div \text{⬡}$

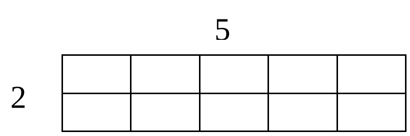
$2 \times \text{⬠} = 10$ $\qquad 10 = 2 \times \text{⬭}$ $\qquad \Diamond \times 2 = 10$ $\qquad 10 = \text{⯃} \times 2$

Array and Area

1 a. A garden has 2 rows and 5 columns of bean plants. How many plants are there in all?
b. The garden is 2 m wide and 5 m long. What is its area?

2 a. A garden has 10 bean plants in 2 equal rows. How many columns does it have?
b. The garden is 10 m^2 in area. It is 2 m wide. How long is it?

3 a. A garden has 10 bean plants in 5 equal columns. How many rows does it have?
b. The garden is 10 m^2 in area. It is 5 m long. How wide is it?



Have the students write equations to represent the situations, using as many different ways as possible. The equations for problem 2 could include the following:

$10 \div 2 = \square$ $\qquad \triangle = 10 \div 2$ $\qquad 10 \div \text{◺} = 2$ $\qquad 2 = 10 \div \text{⬡}$

$2 \times \text{⬠} = 10$ $\qquad 10 = 2 \times \text{⬭}$ $\qquad \Diamond \times 2 = 10$ $\qquad 10 = \text{⯃} \times 2$

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Combinations

1. Paco is making sandwiches on white bread and rye bread. The fillings are cheese, tuna, ham, peanut butter and egg salad. How many combinations can he make?
2. Paco made 10 different sandwiches. He used 5 kinds of fillings. How many kinds of bread did he use?
3. Paco made 10 different sandwiches. He used 2 kinds of bread. How many kinds of fillings did he use?

	C	T	H	P	E
W	WC	WT	WH	WP	WE
R	RC	RT	RH	RP	RE

Have the students write equations to represent the situations, using as many different ways as possible. The equations for problem 3 could include the following:

10 ÷ 2 = □   △ = 10 ÷ 52   10 ÷ ◺ = 2   2 = 10 ÷ ⬡

2 × ⬠ = 10   10 = 2 × ⬭   × 2 ◇ 10   10 = × 2 ⯃

Emphasize that the power of patterns and relations in solving problems is that the same equation can be used to represent a variety of different multiplication and division problems.

Adapted from Alberta Education, *Patterns and Pre-Algebra, Grades 4–6* (unpublished workshop handout) (Edmonton, AB: Alberta Education, 2007), pp. 145–149.

3. Creating Problems from Equations
   Provide the students with an equation using a symbol to represent the unknown; e.g.,
   48 ÷ ⬭ = 6. Discuss and write equations that are equivalent to the equation provided. Instruct the students to create a problem that could be represented by the given equation or one of the equivalent equations. Then tell the students to draw and label a diagram to represent the situation in the problem. Encourage the students to create more than one problem that could be represented by the given equation; e.g., equal grouping and equal sharing. Have the students share their problems and critique them for clarity and mathematical content.

Look For …

Do students:

- □ apply the four operations to everyday contexts?
- □ provide examples of equivalent equations?
- □ communicate clearly their examples of problems related to the four operations?

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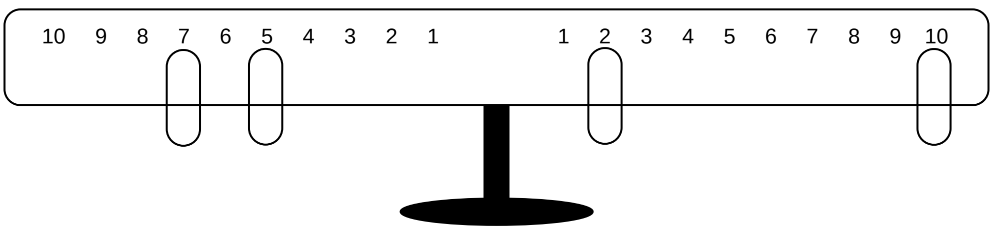
Sample Activities for Teaching Students to Solve One-step Equations Involving a Symbol to Represent an Unknown Quantity

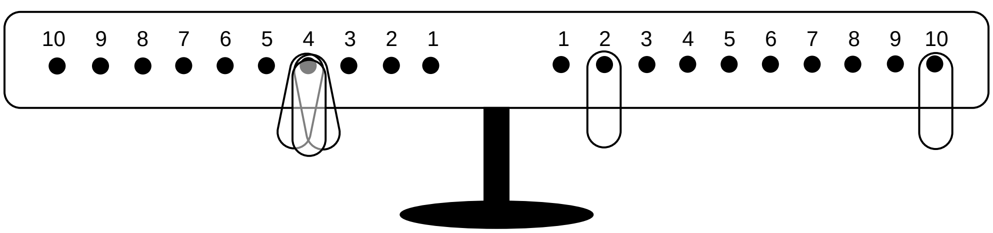
> **Look For …**
> Do students:
> - ☐ represent equations on a balance scale?
> - ☐ write equations that are represented on a balance scale?

1. Balance Scales and Number Sentences
   Provide the students with a pan balance and blocks (centicubes) or another type of balance scale to show equality. The students will find the value of specific unknown numbers as they explore the meaning of the equal sign in the context of both additive and multiplicative relationships.

   Have the students use the balance scale to find different ways to express the numbers 12, 18, 24 and 32 as either 2 addends or a multiplicative expression. Tell the students to write an appropriate equation for each representation on the balance scale.



$7 + 5 = 12$



$3 \times 4 = 12$

Adapted from Alberta Education, *Patterns and Pre-Algebra, Grades 4–6* (unpublished workshop handout) (Edmonton, AB: Alberta Education, 2007), p. 157.

2. Balance Scales and the Guess and Test Strategy for Solving Equations
   Provide the students with balance scales pictured in the previous activity or with pan balances and centicubes to use in solving one-step equations involving a symbol to represent an unknown number; e.g., 3 × ⬭ = 18.

   Model the guess and test strategy as one strategy that may be used to solve the equation:

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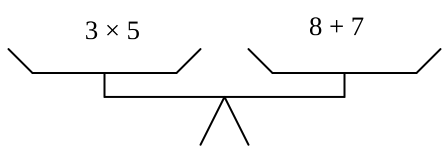
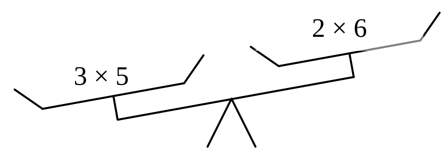
Think aloud that $3 \times \bigcirc = 18$ means three equal groups of something must balance 18. Say that you will try 3 groups of 4 on one side of the balance scale and 18 on the other side of the scale. Since the scale tips to show that 3 groups of 4 are not heavy enough, say that you must adjust the guessed answer to make it higher; e.g., 3 groups of 6. Show that 3 groups of 6 balances 18 on the scale and therefore this situation represents the equation and can be written as $3 \times 6 = 18$, $18 = 3 \times 6$, $18 \div 3 = 6$ or $18 \div 6 = 3$. Explain that for larger numbers, the students should keep track of their guesses in a table.

To provide further reinforcement, draw a simple 2-pan balance scale with a numeric expression in each pan. The students decide whether the pan will tilt or balance and write a corresponding equation to illustrate the situation. The students may test their hypothesis, using a real balance scale.

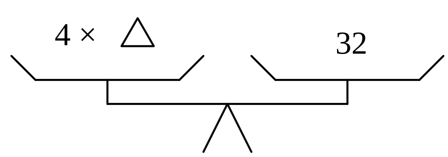


$3 \times 5 = 8 + 7$
Equation



$3 \times 5 > 2 \times 6$
Inequality

Then include a symbol on one side of the balance scale as shown below:



Remind the students that since the scale is balanced, an equation can be written to represent the situation illustrated. Have the students replicate the situation using blocks (centicubes) and a balance scale. Then have them write the equation and the solution. Encourage the students to write the equation in equivalent ways with the unknown on the left side of the equation in some examples and on the right side of the equation in other examples.

Include other examples using variables with varying levels of difficulty. Encourage the students to write the equations and then use the blocks and the balance scales to solve them. If necessary, model the use of guess and test as one strategy in finding the value for the unknown that will balance both sides of the equation. Keep track of the guesses made.

Look For …

Do students:

- ☐ apply their knowledge of the four operations in solving equations using guess and test?
- ☐ show that equality means the quantities on either side of the scale are balanced?
- ☐ use logical reasoning in applying the guess and check strategy; i.e., use the results of the previous guess to make the next guess?

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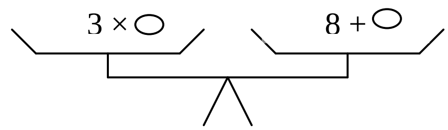
Example:



$$3 \times \bigcirc = 8 + \bigcirc$$
$$3 \times 4 = 8 + 4$$

Have the students create their own equation puzzles, using one unknown, and challenge their peers to solve the problem, using the balance scale to verify their equations.

Adapted from Alberta Education, *Patterns and Pre-Algebra, Grades 4–6* (unpublished workshop handout) (Edmonton, AB: Alberta Education, 2007), pp. 157–158.

3. Pan Balance Scales Using Known and Unknown Weights and Related Equations

Provide the students pan balance scales, weights and film containers filled with pinto beans. The film containers with pinto beans should all weigh the same and the weight depends on the equations that the students are to solve by using the balance scales. Reminder: 1 centicube weighs 1 g.

For example, if the students are to solve the equation $\square + 5 = 15$, then $\square$ represents the weight of the film container and pinto beans, which is 10 g.

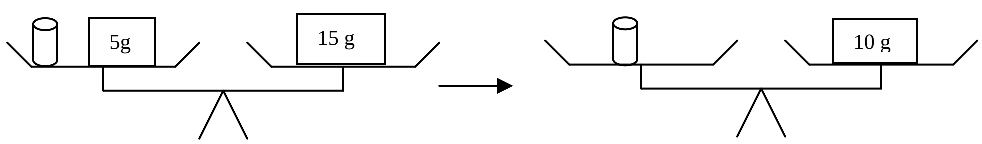
Present the students with a problem such as the following:

> If you add 5 g to the weight of an object (in this case the film container filled with pinto beans) you get a total weight of 15 g. What is the weight of the object? Use the balance scales to solve the problem. Draw a diagram and write equations to show your work.

Look For …

Do students:
- ☐ apply prior knowledge in using balance scales?
- ☐ use related number sentences to solve equations and maintain equality?
- ☐ translate easily among the concrete, pictorial and symbolic modes?

Sample student work:



$\square + 5 = 15$
$\square = 15 - 5$ using a related equation with subtraction
$\square = 10$

The weight of the object is 10 g.

Provide other problems that the students solve by using the balance scales and weights, writing the related equations and answering in a sentence with a numerical value and the proper units.

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4. Virtual Pan Balance—Shapes

Provide students with computers that are linked to the Internet and have them access the following Web site: http://illuminations.nctm.org/ActivityDetail.aspx?ID=33, and follow the instructions.

5. Virtual Pan Balance—Numbers

Provide students with computers that are linked to the Internet and have them access the following Web site: http://illuminations.nctm.org/ActivityDetail.aspx?ID=26, and follow the instructions.

6. Open Number Sentences

Use open number sentences as short warm-ups to start a mathematics class. They help the students to understand symbols as specific unknowns as they develop efficient mental mathematics strategies for computation.

Write an open number sentence on the board and ask the students how to make the statement true. Have the students justify their responses; e.g., using balance models, comparing distances on a number line.

23 + 15 = ⬭
38 = 23 + □
38 = △
33 + 25 = ⬠ + 23
23 + 15 = ◇ + 14

Look For …
Do students:
- ☐ apply the properties of operations in solving open number sentences?
- ☐ use compensation in solving open number sentences?
- ☐ show flexibility in using a variety of symbols to represent an unknown number in open number sentences?

By alternating the symbol used in these equations, the students understand that changing the symbol used does not change the equation.

23 + □ = 40
23 + △ = 40
23 = 40 – ⬡

Adapted from Alberta Education, *Patterns and Pre-Algebra, Grades 4–6* (unpublished workshop handout) (Edmonton, AB: Alberta Education, 2007), p. 165.

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Step 4: Assess Student Learning

Guiding Questions

- Look back at what you determined as acceptable evidence in Step 2.
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

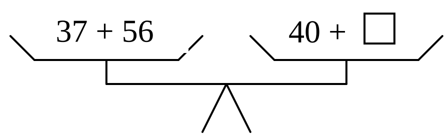
In addition to ongoing assessment throughout the lessons, consider the following sample activities to evaluate the students' learning at key milestones. Suggestions are given for assessing all students as a class or in groups, individual students in need of further evaluation, and individual or groups of students in a variety of contexts.

A. Whole Class/Group Assessment

Note: Performance-based assessment tasks are under development.

Have balance scales and centicubes available for the students to use as needed.

1.



   a. Write an equation to describe the quantities on the balance scale.

   b. Solve the equation. Explain your thinking.

2. Tran has 6 times as many stickers as Nicho. If Tran has 54 stickers, how many stickers does Nicho have?
   a. Write 2 different equivalent equations to represent the problem.

   b. Solve the equations. Show your work.

   c. Answer the question asked in the problem.

3. $\triangle + 23 = 48$
   a. Draw a diagram to represent the equation.

   b. Solve the equation.

   c. Write another equation that is equivalent to $\triangle + 23 = 48$.

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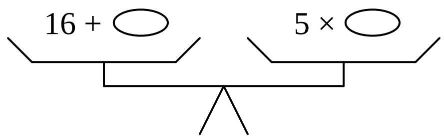
4. Pablo reads for 158 minutes and Juanita reads for 213 minutes. Juanita reads how many more minutes than Pablo?
   a. Write two different equivalent equations to represent the problem.

   b. Solve the equations.

   c. Answer the question asked in the problem.

5. 4 × ⬠ = 24
   a. Draw two different diagrams to represent the equation.

   b. Solve the equation.

6. 

   a. Write an equation to represent the situation shown on the balance scale.

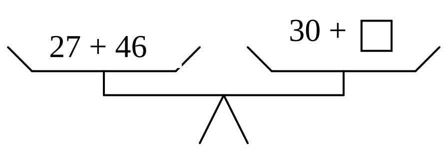
   b. Solve the equation. Explain your thinking.

7. Create a problem that can be represented by the following equation:
   8 = 96 ÷ ▭
   Solve the problem that you created. Show your work.

B. One-on-One Assessment

Have a balance scale and centicubes available for the student to use as needed.

1. Present the following labelled diagram of a balance scale to the student and ask him or her to answer the questions.



   a. Write an equation to describe the quantities on the balance scale.

   b. Solve the equation. Explain your thinking.

   The student may have difficult interpreting the diagram so suggest that he or she use a balance scale and centicubes to show what is in the diagram. Encourage the student to rearrange the centicubes to show 30 + 43 on the left side of the balance scale (take 3 away from 46 and add them to 27 keeping the total number of centicubes the same).

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2. Present the following problem to the student and have him or her answer the questions. Terry has 8 times as many marbles as Sammy. If Terry has 56 marbles, how many marbles does Sammy have?
   a. Write 2 different equivalent equations to represent the problem.

   b. Solve the equations. Show your work.

   c. Answer the question asked in the problem.

   If difficulty arises, replace the numbers in the problem with smaller numbers. This usually helps the students determine which operation to use and how to write the number sentence.

   Provide the student with base ten blocks or counters to use if he or she is struggling with number facts. Help the student snap together 56 blocks and have the student explore how these blocks can be separated into 8 equal groups. Encourage the student to draw an appropriate diagram.

   If difficulty still exists and the student writes $8 \times 56 = \square$, ask him or her who has more marbles, Terry or Sammy. Remind the student that multiplying a number by 8 makes the answer larger. Suggest that the student write 8 groups of some unknown number representing Sammy's marbles as 56. Remind the student that putting equal groups together can be written as a multiplication sentence.

   If the student has difficulty writing an equivalent equation, remind him or her that division and multiplication are related. Another option to consider is that the unknown can be written on either side of the equals sign to make an equivalent equation.

3. Present the following equation to the student and have him or her answer the questions.
   $\triangle - 23 = 48$
   a. Draw a diagram to represent the equation.

   b. Solve the equation.

   c. Write another equation that is equivalent to $\triangle - 23 = 48$.

   If the student has difficulty drawing a diagram, have him or her make up a real-world problem and then illustrate the problem with a diagram; e.g., 23 cm is cut off a ribbon and 48 cm remain. What was the length of the ribbon at the beginning?
   If the student has difficulty solving the equation, remind him or her that addition is related to subtraction so the student could write an equivalent addition statement. This would also help with part (c).

4. Present the following problem to the student and have him or her answer the questions. Emma jogs for 66 minutes on Monday and for 93 minutes on Tuesday. She jogs for how many more minutes on Tuesday than on Monday?
   a. Write two different equivalent equations to represent the problem.

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b. Solve the equations.
c. Answer the question asked in the problem.

If the student has difficulty deciding on which operation to use in writing equivalent equations, change the numbers to smaller numbers and have the student write an equation with smaller numbers. The larger numbers can then be substituted for the smaller numbers to complete the correct equations.

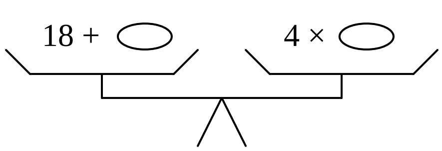
If the student writes a subtraction equation but is unable to write an equivalent equation, remind him or her that addition is related to subtraction.

If the student has difficulty solving the equation, provide base ten materials and have him or her represent 93 with the materials. Beneath these materials, have the student represent 66. Then have the student count to determine how many blocks must be added to 66 to obtain 93. Encourage the student to draw an appropriate diagram.

5. Present the following equation to the student and have him or her answer the questions.
   4 × ⬠ = 32
   a. Draw two different diagrams to represent the equation.
   b. Solve the equation.

   If the student has difficulty drawing the diagrams, suggest that he or she create a real-world problem that involves multiplication. If the student cannot create a problem, create contexts for him or her. Some examples include:
   - There are 4 equal groups of birds that make a total of 32 birds. How many birds are in each group? The student draws 4 groups and places an equal number of birds in each group until he or she obtains a total of 32 birds.
   - There are 4 rows of tiles with same number of tiles in each row making a total of 32 tiles. How many tiles are in each row? The student draws an array to show the tiles.
   - You have 3 times as long a ribbon as your friend. If your ribbon is 32 cm long, how long is your friend's ribbon? The student uses a ruler and draws a line segment 32 cm long and breaks it up into 4 equal lengths.

6. Present the following labelled diagram of a balance scale to the student and have him or her answer the questions.



   a. Write an equation to represent the situation shown on the balance scale.
   b. Solve the equation. Explain your thinking.

   If the student has difficulty writing the equation, remind him or her that the balance scale is balanced so it represents the same quantity on both sides so an equal sign can be used to connect the quantity in each pan.

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If the student has difficulty solving the equation, suggest that he or she use guess and test starting with a simple number like 2. Have the student try out the various guesses using the balance scale and the centicubes. Remind the student that whatever number he or she chooses, that number must be used in place of the unknown on both sides of the equation. Encourage the student to keep track of his or her guesses by recording them in a table.

7. Present the student with the following equation:
   $8 = 96 \div \boxed{}$
   Ask the student to create a problem that can be represented by this equation.
   Ask the student to solve the problem that he or she created and to show all his or her work.

   If the student has difficulty creating a problem, ask him or her to give examples of where division is used in everyday life. Also, start with smaller numbers and the student might be successful in creating a problem. Then, have the student substitute the larger numbers for the smaller numbers to complete the question. If the student has difficulty with division, remind him or her that division is related to multiplication so he or she could think of 8 equal groups of something totaling 96.

C. Applied Learning

Provide opportunities for the students to use equations in a practical situation and notice whether or not the understanding of equations transfers. For example, ask how many booklets can be made to create stories for primary children if there are 126 pages and each booklet has 9 pages. Does the student:

- recognize that the answer to the problem must be less than 126?
- need to use smaller numbers initially to determine what operation to use in solving the problem?
- represent the problem by counting the pages and removing groups of 9 until all the pages have been used?
- use a variety of division and multiplication equations to represent the problem when asked to do so?
- respond that more booklets can be made if asked to determine the effect of using 6 pages per booklet instead of 9?

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Step 5: Follow-up on Assessment

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction?

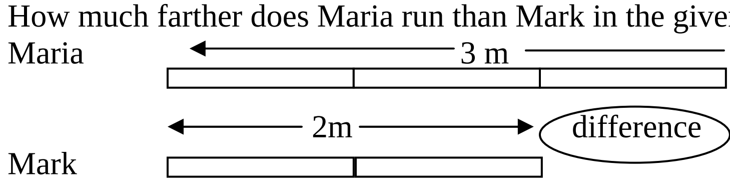
A. Addressing Gaps in Learning

If the student appears to be having difficulty **representing the meaning of a problem by writing an equation with a symbol** for the unknown problem, provide scaffolding as follows:

- Use problems with everyday contexts that are meaningful to the students so that they are motivated and are able to understand and explain the meaning of the problem.
- Use friendly numbers in the problems initially so that students can easily use manipulatives and/or draw diagrams to represent the problem and then connect them to the symbolic representation.
- Encourage the students to solve a simpler problem by substituting smaller whole numbers in the problem so that it is easier for them to represent it using manipulatives and/or diagrams and determine which operation to use when writing the equation and solving the problem. For example, given the following comparison problem:

    Maria runs 350 m in the same time that Mark runs 250 m. How much farther does Maria run than Mark in the given time?

        Change the problem to read, "Maria runs 3 m in the same time that Mark runs 2 m. How much farther does Maria run than Mark in the given time?"

        3 – 2 = ⬠ or 2 + ⬠ = 3 Therefore ⬠ = 1
        Maria runs 1 m farther than Mark in the given time.
        Using the original numbers in problem, 350 – 250 = △ or
        250 + △ = 350. Therefore, △ = 100.
        Maria runs 100 m farther than Mark in the given time.
- Use a variety of manipulatives, including balance scales, to represent the meaning of equality so that students are able to apply this understanding in writing equations with an unknown to represent different kinds of problems.
- Encourage the students to explain, orally, the meaning of the problem and justify why they choose to represent the meaning of the problem with a specific equation. Provide guidance as necessary to ensure that the equation does represent the meaning of the problem.
- Reinforce that a variety of symbols may be used to represent an unknown in a problem; e.g., 45 + □ = 350 or 45 + △ = 350. Clarify that the symbol used to represent the unknown can change without changing the relationship between the numbers in the problem.
- Clarify the mathematician's rule:

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  - an equation with two identical symbols has only one possible solution, that is, the same symbol used within one equation must represent the same number; e.g., $\square \times \square = 36$, $\square = 6$
  - an equation with two different symbols means that these symbols represent quantities that can vary rather than a specific unknown, $\square \times \triangle = 36$, $\square$ equals 6 and $\triangle$ also equals 6; in addition, these symbols represent other values whose product is 36 such as 4 and 9.
- Review the relationship between addition and subtraction problems and between multiplication and division problems by representing them with manipulatives and diagrams before writing the related equations.
- Provide the students with an equation and have them create a variety of problems that could be represented by that equation.

If the student appears to be having difficulty **solving a one-step equation with a symbol** for the unknown, provide scaffolding as follows:

- Build on students' understanding of number sentences from previous grades.
- Encourage students to use manipulatives to represent the equation and then draw a corresponding diagram.
- Have the students explain their thinking and provide guidance to overcome any misconceptions or misunderstandings.
- Translate the equation into a problem with an everyday context that the student understands and reinforce why the problem represents the equation.
- Have the students use manipulatives and diagrams to show the relationship between addition and subtraction, emphasizing that they are opposite operations, and then have the students write the related number sentences. Similarly, review the relationship between multiplication and division.
- Encourage the students to solve a simpler equation by substituting smaller whole numbers in the equation so that it is easier for them to represent it using manipulatives and/or diagrams and determine which operation to use when solving the equation.
- Review the role of symbols in equations, including the mathematician's rule, as described the problem-solving section located just prior to these statements.
- Reinforce understanding of equations by integrating them in every strand and emphasize the power of equations in mathematics.

B. Reinforcing and Extending Learning

The students who have achieved or exceeded the outcomes will benefit from ongoing opportunities to apply and extend their learning. These activities should support the students in developing a deeper understanding of the concept and should not progress to the outcomes in subsequent grades.

Consider strategies, such as the following.

- Provide tips for parents on practising writing and solving equations at home or in the community. For example, encourage the child to write equivalent number sentences for each of the following as they solve the problems:
  - ask what the weight of each bag of dog food is if the total weight of five bags is given

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- ask how many pieces of cutlery are needed in all for a dinner party with eight people if each person has five pieces of cutlery
- ask how much longer one child reads than another child, given the number of minutes each child reads
- include problems with money; e.g., if your child gets \$24, which is three times what his or her sibling receives, ask how much money the sibling would receive.

- Have the students create problems showing the various types of addition and subtraction problems (change, part–part–whole, comparison) and write appropriate equations for each one, using symbols to represent the unknowns. These problems can be displayed on the bulletin board.

- Have the students create problems showing the various types of multiplication and division problems (equal group problems including equal grouping and equal sharing, comparison problems, combination problems) and write appropriate equations for each one, using symbols to represent the unknowns. These problems can be displayed on the bulletin board.

- Have the students create problems with different contexts but using the same numbers, such as 72 and 8. They could follow this up by having the class decide which of the problems could be solved using a given number sentence, such as $72 \div 8 = \square$.

- Provide the students with a variety of story problems and have them draw diagrams and write equations to represent the situations. Encourage the students to write many equivalent equations to represent each situation. Examples of story problems:
  - You have 3 boxes of pencils with the same number of pencils in each box. There are 36 pencils in all.
  - You can make 24 different outfits by matching one shirt with one pair of pants. If you have 8 different shirts, how many different pairs of pants do you have?
  - A red ribbon is 36 cm long and a blue ribbon is 63 cm long. The blue ribbon is how much longer than the red ribbon?
  - You have 23 coins. Eight of the coins are quarters and the rest are dimes. How many dimes do you have?
  - You picked 48 apples. You picked 6 times as many apples as Tiny Tim. How many apples did Tiny Tim pick?
  - You have 78 cookies to share equally among 6 friends. How many cookies does each of your friends receive?

- Provide opportunities for the students to use the guess and test strategy together with appropriate diagrams and charts as they solve problems in which a symbol is used to represent an unknown number. Examples:

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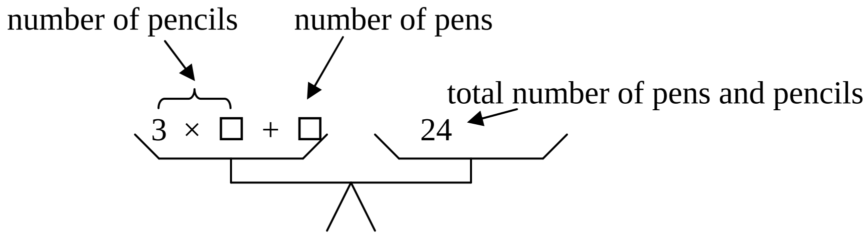
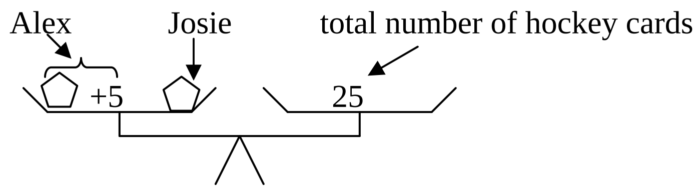
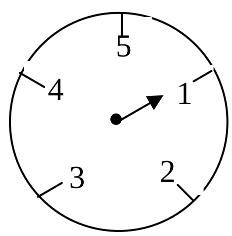
– You have three times as many pencils as pens. If you have 24 pens and pencils altogether, how many of each do you have?



– Two children have a collection of hockey cards. Alex has 5 more cards than Josie. If they have 25 hockey cards altogether, how many cards do they each have (Proulx 2006)?



- Use clock arithmetic to reinforce writing and solving equations in which a symbol is used to represent an unknown number. Clock arithmetic uses a clock with a given number of hours on it, such as the standard 12-hour clock or a 5-hour clock. The clock has only one hand.



Provide equations for the students to solve using the 5-hour clock. Examples:

– 4 + 3 = ▭ Answer: 2

– △ – 4 = 3 Answer: 2

– 3 × ⬡ = 2 Answer: 4

– 2 ÷ 3 = ⬭ Answer: 4 To divide, look for a related multiplication sentence; e.g., 3 × ⬭ = 2.

Have the students write equations for other students to solve. Change the clock to include a different number of hours, such as 6 hours, 8 hours or 12 hours (Cathcart 1997).

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- Sample Structured Interview: Assessing Prior Knowledge and Skills

<table>
<tr><td rowspan="2">Directions</td><td colspan="2">Date:</td></tr>
<tr><td>Not Quite There</td><td>Ready to Apply</td></tr>
<tr><td>Place the following equation before the student.
15 – ☐ = 8.
Say, "Explain the purpose of the box in this equation."</td><td>• Provides the answer to the equation but does not generalize the purpose of the box in the equation.</td><td>Says that the box represents a number that would make the equation true.</td></tr>
<tr><td>Place the following equation before the student:
50 = 20 + ☐
Say, "Write this equation using another symbol for the unknown instead of a box."</td><td>• Writes the answer to the equation but does not provide another symbol to represent the answer.</td><td>• Writes the equation replacing the box with a triangle or some other symbol to represent the number.</td></tr>
<tr><td>Present the following problem to the student and have him or her answer the part (a) before going on to part (b).
"You have 24 marbles and your friend gives you some more marbles. Now you have 32 marbles in all. How many marbles did your friend give to you?
a) Write an equation to show what is happening in this problem.
b) Solve the problem. Explain your thinking."</td><td>• Provides a procedure such as the subtraction algorithm but does not write an equation.
• Solves the problem but has difficulty explaining why the operation chosen makes sense.</td><td>• Writes an equation to show what is happening in the problem such as 24 + ☐ = 32.
• Solves the problem and explains the process by using a diagram and/or logical reasoning.</td></tr>
<tr><td>Place the following equation before the student.
4 + 5 = ☐ + 2

Say, "Solve this equation and use a diagram to explain the process."</td><td>• Solves the equation incorrectly by providing an answer such as 9.
• Can not use a diagram to explain the process.</td><td>• Uses a diagram appropriate to explain the process and solves that equation with the correct answer of 7.</td></tr>
</table>

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Place the following equation before the student: $\square - 13 = 20$ Say, "Solve this equation and explain your thinking."	• Solves the equation incorrectly. • Solves the equation correctly but is unable to explain why the process used makes sense.	• Solves the equation correctly and explains his or her thinking; e.g., the subtraction sentence is related to the addition sentence $20 + 13 = \square$; therefore, the answer is 33.
Place the following equation before the student: $6 + 8 = \square + 4$ Say, "Lori said that the box in this equation stands for more than one number. Is Lori correct? Why or why not?"	• Agrees with Lori that the box stands for more than one number; e.g., 14 or 18.	• Disagrees with Lori, stating that the box in the equation stands for only one number, 10, because 10 is the only number that makes the equation a true statement.

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Strand: Patterns and Relations (Variables and Equations)
Outcomes: 5, 6

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