Mathematics

Planning Guide

Grade 5 *Fractions*

Number

Specific Outcome 7

This Planning Guide can be accessed online at:
http://www.learnalberta.ca/content/mepg5/html/pg5_fractions/index.html

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Planning Guide: *Grade 5 Fractions*

Strand: Number
Specific Outcome: 7

This *Planning Guide* addresses the following outcomes from the Program of Studies:

Strand: Number	
Specific Outcome:	7. Demonstrate an understanding of fractions by using concrete, pictorial and symbolic representations to: • create sets of equivalent fractions • compare fractions with like and unlike denominators.

Curriculum Focus

The changes to the curriculum targeted by this sample include:

- The general outcome focuses on number sense; whereas the previous math curriculum specified demonstrating a number sense for whole numbers 0 to 100 000 and exploring proper fractions and decimals.
- The specific outcome includes creating equivalent fractions and comparing fractions; whereas the previous math curriculum included describing equivalent fractions and ordering proper fractions and decimals. Both curriculums include the connections among the concrete, pictorial and symbolic representations for equivalent fractions.

What Is a Planning Guide

Planning Guides are a tool for teachers to use in designing instruction and assessment that focuses on developing and deepening students' understanding of mathematical concepts. This tool is based on the process outlined in *Understanding by Design* by Grant Wiggins and Jay McTighe.

Planning Steps

The following steps will help you through the Planning Guide:

Step 1: Identify Outcomes to Address

Guiding Questions

- What do I want my students to learn?
- What can my students currently understand and do?
- What do I want my students to understand and be able to do based on the Big Ideas and specific outcomes in the program of studies?

Big Ideas

Students develop and demonstrate understanding of equivalent fractions and the comparison of fractions by connecting the concrete, pictorial and symbolic representations. They should be encouraged to use a variety of strategies to make sense out of fractions relating to parts of a whole region or whole set. For example, students can use pattern blocks, buttons, fraction strips, fraction circles or geoboards to concretely represent fractions and then draw appropriate diagrams together with the corresponding symbolic representations.

By observing students, asking questions and listening to their explanations, the teacher probes for deeper understanding. The *Professional Standards for School Mathematics* states that it is beneficial for students to create their own models and allow for some confusion to reveal what students do and do not understand (NCTM 1991, p. 163). Van de Walle and Lovin (2006) expand upon this idea when they state that to help "students create an understanding of equivalent fractions is to have them use models to find different names for a fraction" (p. 81).

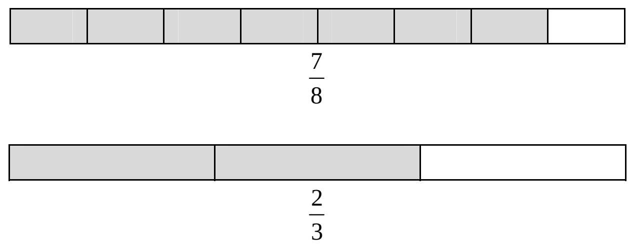
Van de Walle and Lovin (2006) provide the following big idea about equivalent fractions:

> "Two equivalent fractions are two ways of describing the same amount by using different-sized fractional parts. For example, in the fraction $\frac{6}{8}$, if the eighths are taken in twos, then each pair of eighths is a fourth. The six-eighths then can be seen to be three-fourths" (p. 66).

Problem solving plays a major role in developing mathematical power with fractions. As students create and solve a wide variety of problems related to fractions, they must be engaged in "making conjectures and constructing arguments, validating solutions and evaluating the reasonableness of mathematical claims" (NCTM 1991, p. 21).

Assessment of problem solving requires that the teacher "look beyond the answer to the reasoning behind the solution. This evidence can be found in written and oral explanations, drawings and models" (NCTM 2000, p. 187). As teachers reflect on this assessment data, they are able to plan future instruction that best suits the students' needs in developing understanding of fractions or any other mathematical concept. Van de Walle and Lovin (2006) elaborate further: "In a problem-based classroom, students can develop an understanding of equivalent fractions and also develop from that understanding a conceptually based algorithm" (p. 81). They go on to caution that an instructional error is to formulate the rule too quickly. It is important that students construct their own meaning and use intuitive methods first.

Using visual images of fractions such as fractions strips should help many students to think flexibly in comparing fractions (NCTM 2000, p. 216). *Principles and Standards for School Mathematics* provides the following example to illustrate how a student might communicate with understanding that $\frac{7}{8}$ is greater than $\frac{2}{3}$.



The $\frac{7}{8}$ portion is one piece less than a whole, and so is $\frac{2}{3}$. But the missing piece for $\frac{7}{8}$ is smaller than the missing piece for $\frac{2}{3}$. So $\frac{7}{8}$ is bigger than $\frac{2}{3}$.

As students develop understanding of equivalent fractions, they are able to use them to compare fractions with different denominators. By finding common denominators, the fractions are then compared by looking at which numerator is greater; i.e., which fraction has more of the same-sized parts (Van de Walle and Lovin 2006, p. 76).

Sequence of Outcomes from Program of Studies

See http://education.alberta.ca/teachers/core/math/programs.aspx for the complete program of studies.

Grade 4	Grade 5	Grade 6
Specific Outcomes 8. Demonstrate an understanding of fractions less than or equal to one by using concrete, pictorial and symbolic representations to: • name and record fractions for the parts of a whole or a set • compare and order fractions • model and explain that for different wholes, two identical fractions may not represent the same quantity • provide examples of where fractions are used.	**Specific Outcomes** 7. Demonstrate an understanding of fractions by using concrete, pictorial and symbolic representations to: • create sets of equivalent fractions • compare fractions with like and unlike denominators.	**Specific Outcomes** 4. Relate improper fractions to mixed numbers and mixed numbers to improper fractions.

Step 2: Determine Evidence of Student Learning

Guiding Questions

- What evidence will I look for to know that learning has occurred?
- What should students demonstrate to show their understanding of the mathematical concepts, skills and Big Ideas?

Using Achievement Indicators

As you begin planning lessons and learning activities, keep in mind ongoing ways to monitor and assess student learning. One starting point for this planning is to consider the achievement indicators listed in the program of studies. You may also generate your own indicators and use them to guide your observation of the students.

The following indicators may be used to determine whether or not students have met this specific outcome. Can students:

- create a set of equivalent fractions and explain, using concrete materials, why there are many equivalent fractions for any given fraction?
- model and explain that equivalent fractions represent the same quantity?
- determine if two given fractions are equivalent, using concrete materials or pictorial representations?
- formulate and verify a rule for developing a set of equivalent fractions?
- identify equivalent fractions for a given fraction?
- compare two given fractions with unlike denominators by creating equivalent fractions?
- position a given set of fractions with like and unlike denominators on a number line, and explain strategies used to determine the order?
- connect the concrete, pictorial and symbolic representations of fractions?
- solve problems involving equivalent fractions and the comparison of fractions and clearly explain the process?

Sample behaviours to look for related to these indicators are suggested for some of the activities listed in **Step 3, Section C: Choosing Learning Activities** (p. 12).

Step 3: Plan for Instruction

Guiding Questions

- What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should I use?
- How will I meet the diverse learning needs of my students?

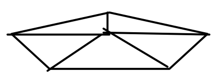
A. Assessing Prior Knowledge and Skills

Before introducing new material, consider ways to assess and build on students' knowledge and skills related to fractions. For example:

Provide counters and fractions bars for the students to use as needed.

1. For each question, explain why it shows fifths or why it does not show fifths.

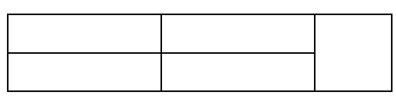
   a.

   b.

   c.

2. a. Draw a diagram for $\frac{3}{4}$ as part of a region.

   b. Draw a diagram for $\frac{3}{4}$ as part of a set.

3. For each of the following pairs of fractions, circle the larger fraction. Explain how you know it is larger.

   a. $\frac{2}{5}$ and $\frac{2}{3}$

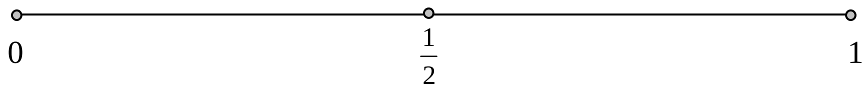
   b. $\frac{5}{8}$ and $\frac{7}{8}$

   c. $\frac{5}{6}$ and $\frac{7}{8}$

   d. $\frac{2}{3}$ and $\frac{3}{8}$

4. Place the following fractions on the number line below. Explain your thinking.

$$\frac{9}{10}, \frac{3}{5}, \frac{3}{4}, \frac{2}{5}$$

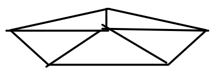
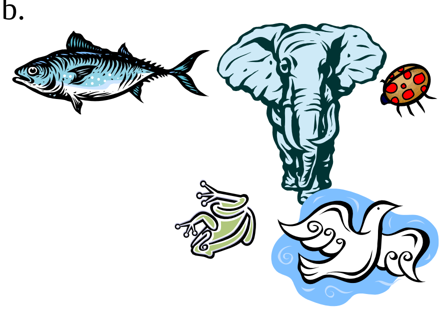


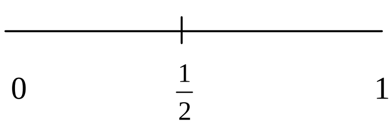
5. You have 6 rocks and paint 5 of them. What fraction of the rocks did you paint? Explain your thinking. Write the fraction name and draw a diagram to represent the fraction.

6. Gracie ate $\frac{5}{8}$ of her pizza and Joshua ate $\frac{5}{8}$ of his pizza. Joshua said that he ate more pizza than Gracie. Explain how Joshua could be right by using diagrams and words.

If a student appears to have difficulty with these tasks, consider further individual assessment, such as a structured interview, to determine the students' level of skill and understanding. See **Sample Structured Interview: Assessing Prior Knowledge and Skills** (p. 9).

Sample Structured Interview: Assessing Prior Knowledge and Skills

Directions	Date:	
	Not Quite There	**Ready to Apply**
Place the following diagrams before the student and say, **"For each question, explain why it shows fifths or why it does not show fifths."** a.  b. 	Does not answer the questions correctly. Answers one or more questions correctly but is unable to explain why it shows fifths or why it does not show fifths.	Answers the questions correctly and explains clearly why it shows fifths or why it does not show fifths.
Instruct the student, **a. "Draw a diagram for $\frac{3}{4}$ as part of a region.** **b. Draw a diagram for $\frac{3}{4}$ as part of a set."**	Does not answer either question correctly. Does not understand the difference between part of a region and part of a set. Draws a diagram correctly for only one of the questions.	Draws a diagram correctly for part a, showing that quarters of a region must have the same size but not necessarily the same shape. Draws a diagram correctly for part b, showing that the four parts of a set must have the same number of elements in each part but the elements can be of different sizes.

Place the following pairs of fractions before the student and say, **"For each of the following pairs of fractions, circle the larger fraction. Explain how you know it is larger. "** a. $\frac{2}{5}$ and $\frac{2}{3}$ b. $\frac{5}{8}$ and $\frac{7}{8}$ c. $\frac{5}{6}$ and $\frac{7}{8}$ d. $\frac{2}{3}$ and $\frac{3}{8}$	Guesses which fraction is larger and is not able to explain why it is larger. Correctly answers some of the questions with a vague explanation but incorrectly answers others; e.g., answers correctly only when the denominators of the fractions are the same or the numerators of the fractions are the same. Correctly answers each question but is unable to explain why.	Correctly answers each question and explains clearly why one fraction is larger than the other by using sound mathematical reasoning; e.g., a. Since the numerators are the same in both fractions, the larger fraction is the one with the least denominator because the smaller the denominator the larger each piece in the whole. d. Since 2 is more than half of 3, and 3 is less than half of 8, then $\frac{2}{3}$ is greater than $\frac{1}{2}$ while $\frac{3}{8}$ is less than $\frac{1}{2}$, resulting in $\frac{2}{3}$ being the larger fraction.
Place the following fractions and the number line before the student and say, **"Place these fractions on the number line below. Explain your thinking."** $\frac{9}{10}, \frac{3}{5}, \frac{3}{4}, \frac{2}{5}$ 	Does not place any of the fractions correctly on the number line and is unable to explain his or her thinking. Places some of the fractions on the number line correctly but is unable to explain his or her thinking. Places all of the fractions on the number line correctly but is unable to explain his or her thinking or gives vague explanations.	Places all of the fractions correctly on the number line and explains his or her thinking clearly with sound mathematical reasoning; e.g., Since 3 is a little more than half of 5, it is placed a little to the right of $\frac{1}{2}$. Since 2 is a little less than half of 5, it is placed a little to the left of $\frac{1}{2}$. Since tenths are smaller than quarters, then $\frac{9}{10}$ is greater than $\frac{3}{4}$ because $\frac{9}{10}$ is only $\frac{1}{10}$ away from 1 while $\frac{3}{4}$ is $\frac{1}{4}$ away from 1.

Place the following problem before the student and read it orally, **"You have 6 rocks and paint 5 of them. What fraction of the rocks did you paint? Explain your thinking. Write the fraction name and draw a diagram to represent the fraction."**	Does not write the correct fraction name nor draw a diagram to correctly represent the fraction. Writes the correct fraction name or draws an appropriate diagram but not both and is unable to explain his or her thinking. Writes the correct fraction name and draws an appropriate diagram but is unable to explain his or her thinking or the explanation is vague.	Writes the correct fraction name, draws an appropriate diagram and explains his or her thinking clearly.
Place the following problem before the student and read it orally, **"Gracie ate $\frac{5}{8}$ of her pizza and Joshua ate $\frac{5}{8}$ of his pizza. Joshua said that he ate more pizza than Gracie. Explain how Joshua could be right by using diagrams and words."**	Does not explain why Joshua could be right. Explains why Joshua could be right by using either diagrams or words but not both.	Explains clearly why Joshua could be right by using diagrams and words.

B. Choosing Instructional Strategies

Consider the following guidelines for teaching equivalent fractions and ordering fractions:

- Access students' prior knowledge of fractions and build on this understanding.
- To develop understanding, include everyday problem-solving contexts for equivalent fractions and the comparison of fractions, then use concrete representations and connect them to pictorial and symbolic representations.
- To demonstrate understanding, have the students represent the symbolic equivalent fractions and the comparison of fractions concretely and pictorially.
- Provide many examples of the three models for equivalent fractions and comparison of fractions: part of a region, part of a length or measurement and part of a set.
- By using examples and non-examples, have the students construct the meaning that equivalent fractions represent the same part of a given whole.
- Guide the students in constructing their own rules that are mathematically sound for developing a set of equivalent fractions.
- Emphasize that fractions can be compared only if they all refer to parts of the same whole.
- Reinforce the relationship between the symbolic and pictorial modes (two symbolic equivalent fraction names and two corresponding pictorial equivalent fractions) by assigning problems in which three of these are provided and the student determines the fourth by using their models (Van de Walle and Lovin 2006).
- Emphasize the meaning of a fraction as the various ways to compare fractions are explored, communicated and evaluated for mathematically sound reasoning.
- Encourage flexibility in thinking as students compare fractions in a problem-solving context.

C. Choosing Learning Activities

The following learning activities are examples of activities that could be used to develop student understanding of the concepts identified in Step 1.

Sample Activities:

Sample Activity 1: Fractions on Geoboards (Meaning of a Fraction and Equivalent Fractions)

The purpose of this activity is to link an investigation of fractional parts (including equivalent fractions) with area on the geoboard. The students should understand that each of the fractional parts should have the same area but do not necessarily have the same shape. The focus is on "developing visual, geometric representations of fractions and building students' knowledge of common equivalents such as $\frac{1}{4}$ and $\frac{2}{8}$" (Burns 1992, p. 219).

Have the students work in groups using the geoboards and then have them draw their designs on dot paper or grid paper and write the appropriate math symbols for the pictorial representations.

Materials: 5 × 5 geoboards, dot paper or grid paper to illustrate the 5 × 5 geoboards

A blackline master for 1-cm square dot paper can be downloaded from www.ablongman.com/vandewalleseries. Located in Volume 3, the blackline master is labelled as Blackline Master 10.

Look For …

Do students:

- ☐ create quarters to show that each quarter of the whole region must have the same size but not necessarily the same shape?
- ☐ show flexibility in thinking as they create the quarters?
- ☐ apply their understanding of fractions to solving a problem using equivalent fractions?
- ☐ connect the concrete, pictorial and symbolic representations?
- ☐ demonstrate understanding by solving similar problems?

Problem:
You are to make a design by dividing a square with an area of 16 square units into quarters.

a. What are some possible designs?

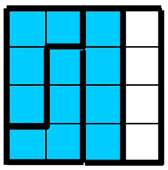
b. If $\frac{3}{4}$ of the design is to be coloured blue, what is the area of the blue part? Shade $\frac{3}{4}$ of one design. Draw a diagram and write an appropriate number sentence.

For part a, encourage the students to make designs in which each quarter is the same size but not the same shape. To differentiate instruction, make the problem simpler by using halves instead of quarters.

Sample solution for part b:

Pictorial Representation

Symbolic Representation



Each quarter has an area of 4 square units. Therefore, three of the quarters have an area of 12 square units. This can be written as equivalent fractions:

$\frac{3}{4} = \frac{?}{16}$ $\frac{3}{4} = \frac{12}{16}$

The area of the blue part is 12 square units.

Have the students demonstrate their understanding of equivalent fractions by solving the following problem using math symbols and verifying that their answers are correct by drawing a diagram or using a geoboard.

Problem:

A square cake is divided into 16 equal pieces. You eat $\frac{3}{8}$ of the cake. How many pieces did you eat? Use a number sentence to solve the problem and draw a diagram to show that your answer is correct.

Sample Activity 2: Fraction Strips (Equivalent Fractions)

Use fraction strips to help the students understand the concept of equivalent fractions. Ideas for using fraction strips complete with blackline masters are found on pages 286–297 of the *Diagnostic Mathematics Program, Division II, Numeration*. A blank copy of the fraction strips is provided at the end of this plan.

Provide each group of students with a set of fraction strips and have them solve the following problem.

Problem:
A rectangular cake is divided into 12 equal pieces. Hungry Harry eats $\frac{2}{3}$ of the cake. How many pieces did he eat? Explain your thinking by using diagrams and symbols.

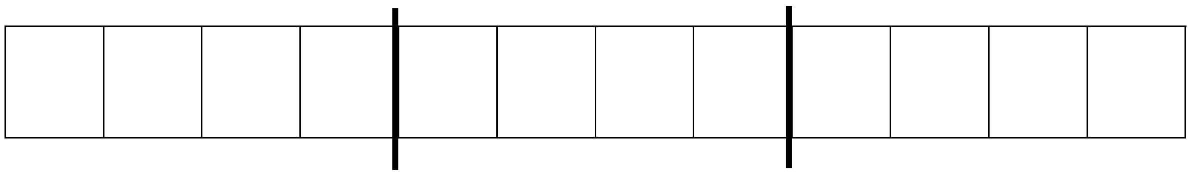
Look For …

Do students:

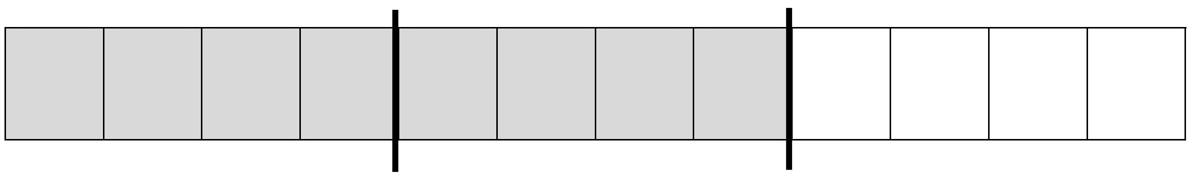
- ☐ use the appropriate fraction strips to show equivalent fractions?
- ☐ apply their understanding of fractions to solving a problem using equivalent fractions; i.e., divide the fraction strip showing twelfths into three equal parts and shading two groups of three equal parts?
- ☐ connect the concrete, pictorial and symbolic representations?
- ☐ demonstrate understanding by solving similar problems?

Example of a Solution

Use the fraction strip that is divided into 12 equal parts. Show thirds by dividing the strip into 3 equal parts as shown.



Since Harry ate $\frac{2}{3}$ of the cake, shade 2 of the 3 equal parts to show how many pieces he ate.



Using symbolic representation showing equivalent fractions:

$\frac{2}{3} = \frac{?}{12}$ $\qquad$ $\frac{2}{3} = \frac{\mathbf{8}}{12}$

Harry ate 8 pieces of the cake.

Have the students demonstrate their understanding of equivalent fractions by solving the following problem using math symbols and verifying that their answers are correct by drawing a diagram. Provide fractions strips or other manipulatives as needed.

Problem:
A rectangular cake is divided into 10 equal pieces. You eat $\frac{3}{5}$ of the cake. How many pieces did you eat? Use a number sentence to solve the problem and draw a diagram to show that your answer is correct.

Look For …

Do students:

- ☐ use the fraction circles to generate a set of equivalent fractions?
- ☐ formulate a mathematically sound rule in their own words for developing a set of equivalent fractions?
- ☐ show flexibility in thinking as they create the fifths?
- ☐ apply their understanding of fractions to solving a problem using equivalent fractions?
- ☐ connect the concrete, pictorial and symbolic representations?
- ☐ use other manipulatives to generate sets of equivalent fractions?

Sample Activity 3: Fraction Circles (Generalizing a Rule for Developing a Set of Equivalent Fractions)

Use fraction circles to help the students understand the concept of equivalent fractions and generalize a rule for developing a set of equivalent fractions. Ideas for using fraction circles complete with blackline masters are found on pages 298–314 of the *Diagnostic Mathematics Program, Division II, Numeration*.

Provide each group of students with a copy of a circle with part of it shaded; e.g., $\frac{1}{2}$ shaded. Have the students explore how many different names can be written for $\frac{1}{2}$ by drawing line segments on the shaded circle.

Problem:
Find as many different names as you can for fractions that are equivalent to (or represent the same part of the whole as) $\frac{1}{2}$. Explain your thinking by using diagrams and symbols.

Make a transparency of the shaded circle as well as transparencies of congruent unshaded circles divided into quarters, eighths and sixteenths. As students share their ideas, superimpose the appropriate unshaded circles onto the shaded circle showing $\frac{1}{2}$.

Have the students write the corresponding symbolic representation for each pictorial representation; i.e.,

$$\frac{1}{2}=\frac{2}{4}=\frac{4}{8}=\frac{8}{16}$$

Encourage the students to continue the pattern without drawing diagrams or using transparency overlays. Ask them how many fractions are equivalent to $\frac{1}{2}$. As they study the pattern, they should generalize that there are an infinite number of equivalent fractions for $\frac{1}{2}$ or for any fraction.

Use a similar procedure with a circle that is $\frac{2}{3}$ shaded.

Through discussion, have the students study the pattern and generalize a rule for finding equivalent fractions.

Rule

If you multiply (or divide) the top (numerator) and the bottom (denominator) of a fraction by the same number, you make another fraction that names the same amount as the first (original fraction).

Note: Other manipulatives besides fraction circles can be used to guide students in formulating a rule for developing a set of equivalent fractions. For example, fraction strips can be used. The strip is folded in half with half shaded, then folded in quarters, followed by eighths and so on. See pages 320–322 of the *Diagnostic Mathematics Program, Division II, Numeration* for detailed instructions and visuals.

Sample Activity 4: Pattern Blocks (Equivalent Fractions including Basic Fractions)

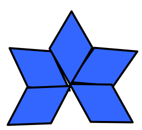
Use pattern blocks to help the students understand the concept of equivalent fractions and generalize a rule for developing a set of equivalent fractions and for finding the basic fraction. Ideas for using pattern blocks complete with blackline masters are found on pages 315–319 of the *Diagnostic Mathematics Program, Division II, Numeration*.

Encourage students to use a variety of manipulatives as they develop their understanding of equivalent fractions. Pattern blocks are used in the following problems but other manipulatives could be used if desired.

Provide the students with appropriate pattern blocks and triangular dot paper and present them with the following problem.

Problem:

Hungry Hannah eats $\frac{2}{5}$ of the cake shown below. If the cake is cut into 10 equal pieces, how many pieces does she eat? Explain your thinking by using diagrams and symbols.



Look For …

Do students:

- ☐ explain that two tenths of a whole is the same as one fifth of that whole using pattern blocks (triangles and parallelograms)?
- ☐ apply their understanding of fractions to solving a problem using equivalent fractions?
- ☐ connect the concrete, pictorial and symbolic representations?
- ☐ explain the use of equivalent fractions in creating basic or simplified fractions?

Solution:

To scaffold student thinking, suggest that they draw the shape on triangular dot paper and shade $\frac{2}{5}$ of the cake to show the part eaten by Hannah. Then encourage them to cover the entire pattern block shape with the green triangles to show that the whole can be written as $\frac{10}{10}$ with $\frac{4}{10}$ of the shape shaded. Similarly, have them draw line segments in their diagrams to convert each rhombus into two triangles.

Have them write an appropriate number sentence; i.e.,

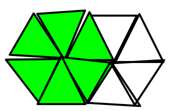
$$\frac{2}{5} = \frac{?}{10} \qquad \frac{2 \times 2}{5 \times 2} = \frac{\mathbf{4}}{10}$$

Hannah ate 4 pieces of the cake.

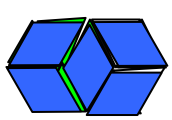
Then present the students with the following problem that applies the concept of equivalent fractions to finding a basic fraction. Provide the students with pattern blocks or other manipulatives as needed. Also, provide the students with triangular or isometric dot paper for drawing appropriate diagrams related to the concrete representation.

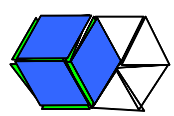
Problem:

Nicholas and Brooklyn bought the same amount of pizza. Nicholas ate $\frac{6}{10}$ of his pizza, as shown by the shading in the diagram below. Brooklyn ate the same amount of her pizza. She ate 3 equal pieces. Brooklyn's pizza was divided into how many equal pieces? Explain your thinking by using diagrams and symbols.



Solution:
Encourage the students to construct Nicholas's pizza by using the green triangle pattern blocks. Then have them explore which other pattern block used repeatedly would perfectly cover the diagram. They will discover that the blue parallelogram pattern blocks can be used to perfectly cover the diagram and that three of these blocks cover the shaded part representing $\frac{6}{10}$ of the whole.





The whole is $\frac{5}{5}$. The shaded part is $\frac{3}{5}$.

Have the students draw appropriate diagrams on their triangular dot paper and write an appropriate number sentence:

$$\frac{6}{10} = \frac{3}{?} \qquad \frac{6 \div 2}{10 \div 2} = \frac{3}{\mathbf{5}}$$

Brooklyn's pizza was divided into 5 equal pieces.

Look For …

Do students:

- ☐ create quarters to show that each quarter of the whole set must have the same number of elements in each equal part, but that the elements do not have to be the same size?
- ☐ show flexibility in thinking as they create the quarters?
- ☐ apply their understanding of fractions to solving a problem using equivalent fractions?
- ☐ connect the concrete, pictorial and symbolic representations?
- ☐ demonstrate their understanding by solving similar problems?
- ☐ explain the use of equivalent fractions in creating basic or simplified fractions?

Sample Activity 5: Pattern Blocks (Part of a Set)

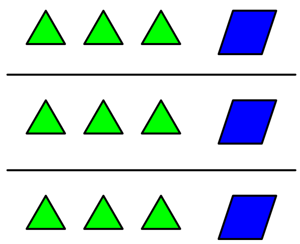
Pattern blocks can be used as part of a region or as part of a set. In this activity, the pattern blocks are used as part of a set.

Present the students with the following problem and provide pattern blocks for them to use to solve the problem.

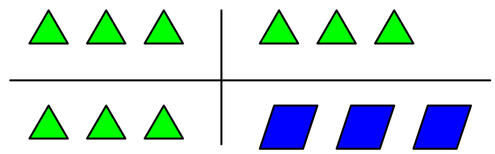
Problem:

You have a set of 12 pattern blocks and $\frac{3}{4}$ of them are triangles. How many are triangles? Use the pattern blocks and draw a diagram to show how you solved the problem. Write an appropriate number sentence.

Solution Samples:



or



Number sentence:

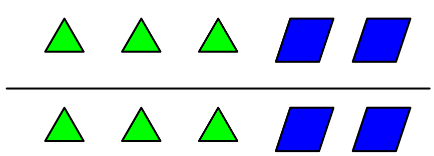
$\frac{3}{4} = \frac{?}{12}$ $\quad$ $\frac{3}{4} = \frac{\mathbf{9}}{12}$

Answer to the problem: There are 9 triangles.

Provide the students with other problems, such as the following, that require the use of equivalent fractions to make a basic fraction or write the fraction in simplest form.

Problem:
There are 6 triangles in a set of 10 pattern blocks. The triangles make up what fraction of the set? Write your answer in simplest form. Use the pattern blocks and draw a diagram to show how you solved the problem. Write an appropriate number sentence.

Solution Samples:



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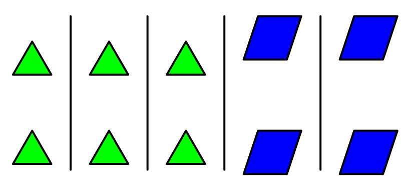


Number sentence:

$\frac{6}{10} = \frac{?}{?}$ $\quad$ $\frac{6 \div 2}{10 \div 2} = \frac{\mathbf{3}}{\mathbf{5}}$

The triangles make up $\frac{3}{5}$ of the set of pattern blocks.

Have the students demonstrate their understanding of equivalent fractions by solving the following problem using math symbols and verifying that their answers are correct by drawing a diagram or using manipulatives.

You have a set of 18 pattern blocks and $\frac{2}{3}$ of the set is made up of triangles. How many triangles are in the set?

Solution: $\frac{2}{3} = \frac{?}{18} \longrightarrow \frac{2 \times 6}{3 \times 6} = \frac{\mathbf{12}}{18}$ There are 12 triangles in the set.

Sample Activity 6: Buttons or Pasta (Part of a Set)

Provide each student with buttons or pasta of different sizes and colours. Solve similar problems to the ones given in Activity 1 – Pattern Blocks, but change the numbers and the context.

Sample Problem:

You have a set of 10 buttons and $\frac{2}{5}$ of them are black. How many black buttons are in the set? Explain using diagrams and symbols.

Sample Activity 7: Modified Frayer Model for Developing and Demonstrating Understanding of Equivalent Fractions

Have the students summarize their understanding of equivalent fractions by completing a Modified Frayer Model, such as the following example. If students are not familiar with using the Modified Frayer Model, then this strategy for consolidating understanding of concepts should be modelled first and then done with the students before having them complete one on their own.

Look For …

Do students:

- ☐ apply their knowledge of equivalent fractions and write a definition in their own words that is mathematically correct?
- ☐ create and justify examples and non-examples of equivalent fractions?
- ☐ draw diagrams and write appropriate symbols for equivalent fractions?
- ☐ create a problem that can be solved using equivalent fractions?

Modified Frayer for Equivalent Fractions

Equivalent Fractions

Definition

Two fractions are equivalent if they represent two ways of describing the same amount of a given whole by using different-sized fractional parts (Van de Walle and Lovin 2006, p. 66).

Examples

$\frac{3}{4} = \frac{9}{12}$

$\frac{10}{12} = \frac{5}{6}$

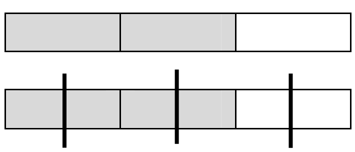
Non-examples

$\frac{3}{4} \neq \frac{11}{12}$

$\frac{10}{12} \neq \frac{4}{6}$

Visual & Numeric Representation

Part of a Region

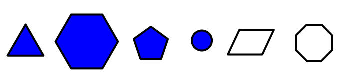


$\frac{2}{3}$ shaded

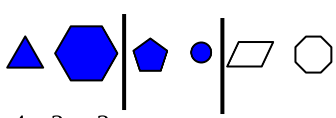
$\frac{4}{6}$ shaded

$\frac{2 \times 2}{3 \times 2} = \frac{4}{6}$

Part of a Set



$\frac{4}{6}$ shaded



$\frac{2}{3}$ shaded

$\frac{4 \div 2}{6 \div 2} = \frac{2}{3}$

Real Life Application

One board is divided into 12 equal parts and 8 parts are painted. Another board, the same size, is divided into 3 equal parts. How many parts of the second board must be painted to have the same amount painted as the first board?

Format adapted from D. A. Frayer, W. C. Frederick and H. J. Klausmeier, *A Schema for Testing the Level of Concept Mastery* (Working Paper/Technical Report No. 16) (Madison, WI: Research and Development Center for Cognitive Learning, University of Wisconsin, 1969). Adapted with permission from the Wisconsin Center for Education Research, University of Wisconsin-Madison.

Sample Activity 8: Real Cake and Paper Strips (Comparing Fractions)

To access students' prior knowledge about fractions, use a cake such as a puffed wheat cake as a manipulative. Provide the students with congruent strips of paper to represent the cake.

Problem:

Would you rather eat $\frac{1}{4}$ or $\frac{2}{8}$ of a cake? Explain using diagrams and symbols.

Have the students use their paper strips to show $\frac{1}{4}$ of the cake and also $\frac{2}{8}$ of the cake. Emphasize that the whole must be the same before comparing fractions. Divide the real cake into quarters and put icing on one of the quarters. Then divide the cake into eighths to show that $\frac{1}{4}$ of the cake is the same amount as $\frac{2}{8}$ of that cake.

Look For …

Do students:

- ☐ use equivalent fractions with a common denominator to decide if two fractions of a whole are equal or unequal?
- ☐ apply their understanding of fractions to solving a problem requiring the comparison of fractions?
- ☐ connect the concrete, pictorial and symbolic representations?

Have the students write the appropriate equivalent fractions.

Have the students demonstrate their understanding of comparing fractions by solving the following problem using math symbols and verifying that their answers are correct by drawing a diagram. Provide fractions strips or other manipulatives as needed.

Sample Activity 9: Comparing Fractions by Using Equivalent Fractions (Comparing Fractions)

Look For …
Do students:
- ☐ apply their understanding of equivalent fractions to solve a problem requiring the comparison of fractions?
- ☐ apply the rule for creating a set of equivalent fractions using the correct symbols?
- ☐ clearly explain the process used for comparing fractions?

Build on students' understanding of various ways to compare fractions such as using benchmarks or analyzing fractions with the same denominator but different numerators.

Review equivalent fractions and explain that equivalent fractions can be used to order fractions when the denominators of the fractions are different.

Present the students with the following problem.

Problem:
Explain whether $\frac{3}{5}$ or $\frac{5}{8}$ of a given pizza is larger. Explain your thinking.

Have the students explore various ways to solve the problem and then encourage them to share their ideas with the entire class. Capitalize on the discussion involving equivalent fractions. If no students use equivalent fractions to solve the problem, ask them to suggest how equivalent fractions could be used to compare these two fractions.

Guide the discussion to show that each fraction can be written as an equivalent fraction with a denominator of 40. When the denominators of each fraction are the same, then encourage the students to use their background knowledge to compare the fractions by looking at the numerators.

$\frac{3}{5} = \frac{24}{40}$ $\quad$ $\frac{5}{8} = \frac{25}{40}$

$\frac{24}{40} < \frac{25}{40}$ $\quad$ Therefore, $\frac{3}{5} < \frac{5}{8}$.

Answer to the problem:
$\frac{5}{8}$ of a pizza is greater than $\frac{3}{5}$ of the same pizza.

Provide the student with other problems that require the comparison of fractions.

Problem:
Johnny eats two sixths of a cake and Lori eats one third of the same cake. Who eats more cake or do they eat the same amount? Explain by using diagrams and symbols.

Problem:
Would you rather have $\frac{4}{10}$ of a cake or $\frac{3}{5}$ of the same cake? Explain by using diagrams and symbols.

Sample Activity 10: Number Line (Comparing Fractions)

Review the various ways to compare fractions, including the use of equivalent fractions, and also review the placement fractions on a number line.

Generalizations about Comparing Fractions (Van de Walle and Lovin 2006):

- *Same whole*: Comparisons with any model can only be made if both fractions are parts of the same whole.
- *Same number of parts but parts of different sizes*: As the denominator of a fraction increases with a constant numerator, the fraction decreases in size; e.g., $\frac{1}{3} > \frac{1}{4} > \frac{1}{5} > \frac{1}{6}$ and so on.
- *More of the same-size parts*: As the numerator of a fraction increases with a constant denominator, the fraction increases in size; e.g., $\frac{3}{8} < \frac{4}{8} < \frac{5}{8} < \frac{6}{8}$ and so on.
- *Distance from one whole*: As the denominator of a fraction increases with the numerator always one less than the denominator, the fraction increases in size; e.g., $\frac{2}{3} < \frac{3}{4} < \frac{4}{5} < \frac{5}{6}$ and so on.
- *More or less than a half*: Benchmarks such as $\frac{1}{2}$ are useful in comparing fractions; e.g., $\frac{3}{8} < \frac{2}{3}$ because 3 is less than half of 8 and 2 is more than half of 3.
- *Equivalent fractions and more of the same-size parts*: Using equivalent fractions creates a set of fractions with the same denominator so that the numerators can be compared in ordering the fractions; e.g., $\frac{3}{8} < \frac{2}{3}$ because $\frac{3}{8} = \frac{9}{24}$ and $\frac{2}{3} = \frac{16}{24}$.

Look For …

Do students:

- ☐ apply prior knowledge in ordering fractions?
- ☐ show flexibility in using a variety of appropriate strategies to order fractions?
- ☐ demonstrate understanding of placing fractions on a number line?

Ordering Fractions with Fraction Strips

Have the students use the fractions strips to order fractions that can easily be represented on these fraction strips such as $\frac{3}{5}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{7}{12}$ and $\frac{8}{10}$. Encourage the students to place the fractions on a number line after they have ordered them using the fraction strips. See the blackline master for fraction strips located at the end of this document.

Ordering Fractions without Using Fraction Strips
Present the students with the following problem. Encourage them to use strategies to order the fractions without using the fraction strips. If necessary, allow the students to use the fraction strips.

Problem:
Place the following fractions on the number line provided. Explain your thinking.
$\frac{5}{6}, \frac{3}{8}, \frac{1}{4}, \frac{1}{5}, \frac{7}{8}, \frac{4}{6}, \frac{7}{12}$

0 $\frac{1}{2}$ 1

Sample Solution:
Since $\frac{1}{4}$ and $\frac{1}{5}$ have the same numerators, then $\frac{1}{5}$ is less than $\frac{1}{4}$ because the denominator is greater, so the fraction is less.

Since $\frac{5}{6}$ and $\frac{7}{8}$ are fractions in which the numerator is one less than the denominator, then $\frac{5}{6}$ is less than $\frac{7}{8}$ because $\frac{5}{6}$ is one-sixth away from 1 while $\frac{7}{8}$ is only one-eighth away from 1.

Since 3 is less than half of 8, then $\frac{3}{8}$ is less than $\frac{1}{2}$.
$\frac{4}{6}$ and $\frac{7}{12}$ are both greater than $\frac{1}{2}$ because 4 is greater than half of 6, and 7 is greater than half of 12. To decide on the order of $\frac{4}{6}$ and $\frac{7}{12}$, change $\frac{4}{6}$ to an equivalent fraction with a denominator of 12; i.e., $\frac{4}{6} = \frac{8}{12}$. Since $\frac{7}{12} < \frac{8}{12}$, then $\frac{7}{12} < \frac{4}{6}$.

Answer to the problem:
The fractions ordered from least to greatest are placed on the number line below.

0 $\frac{1}{2}$ 1

$\frac{1}{5}$ $\frac{1}{4}$ $\frac{3}{8}$ $\frac{7}{12}$ $\frac{4}{6}$ $\frac{5}{6}$ $\frac{7}{8}$

$\frac{2}{3}$

Step 4: Assess Student Learning

Guiding Questions

- Look back at what you determined as acceptable evidence in Step 2.
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

In addition to ongoing assessment throughout the lessons, consider the following sample activities to evaluate students' learning at key milestones. Suggestions are given for assessing all students as a class or in groups, individual students in need of further evaluation, and individual or groups of students in a variety of contexts.

A. Whole Class/Group Assessment

An example of a performance based assessment task that could be used to evaluate student understanding of the outcome(s). Include a marking rubric to be used with the assessment task and, if possible, student exemplars.

Fractional Parts of Lasagnas
In this assessment task, students will solve problems to demonstrate their understanding of equivalent fractions as part of a whole (not as part of a set). They will explain the relationship between two equivalent fractions using drawings, symbols and words. Then they will apply a rule for creating another fraction equivalent to the first two fractions. Finally, students will apply this rule as they create equivalent fractions to compare two given fractions with unlike denominators.

Materials required: cm grid paper, manipulatives such as congruent strips of paper and fractions strips

Each student will:

- solve problems by creating equivalent fractions
- model and explain that equivalent fractions represent the same quantity
- create a set of equivalent fractions and explain why they are equivalent by using pictures, symbols and words
- apply a rule for developing a set of equivalent fractions
- compare two given fractions with unlike denominators by creating equivalent fractions.

Students should connect equivalent fractions to concepts of equal area regardless of the shape. They should be able to apply a rule for creating equivalent fractions and explain how the rule works. Also, students should show how to use equivalent fractions to determine if two fractions represent unequal or equal parts of the same whole.

Early finishers can write similar problems with equivalent fractions as part of sets.

Fractional Parts at a Picnic—Student Assessment Task

There were two lasagnas of the same size at the school picnic. Parents were careful to cut each pan of lasagna into equal portions. Tanya had 2 portions from one pan while Dave had 4 portions from the other pan. They both took the same amount of lasagna.

1. Draw diagrams to show how the two pans of lasagna were divided into portions so that Tanya's 2 portions are equal to Dave's 4 portions. Shade in the portion of lasagna that was eaten by each child.
   What fraction of one pan of lasagna did Tanya eat?
   What fraction of the other pan of lasagna did Dave eat?

2. How do you know these fractions are equivalent? Explain your thinking using pictures, symbols and words.
   Name another fraction that is equivalent to these two fractions. Explain how you know all three fractions are equivalent.

3. Parents also brought two identical cakes for dessert. If $\frac{2}{3}$ of the first cake was eaten and $\frac{3}{5}$ of the second cake was eaten, which cake had more eaten? Explain your thinking by creating equivalent fractions. Show all your work.

Student ___________________________________

SCORING GUIDE
Fractional Parts at a Picnic

Level / Criteria	4 Excellent	3 Proficient	2 Adequate	1 Limited *	Insufficient / Blank *
Represents equivalent fractions **Question #1**	The student draws very accurate representations of the two fractions using referent wholes of the same size. ☐	The student draws accurate but not perfect representations of the two fractions using referent wholes of the same size. ☐	The student draws representations of the two fractions but may not ensure that the wholes are the same size. ☐	The student draws inaccurate or confusing representations of the two fractions. ☐	No score is awarded because there is insufficient evidence of student performance based on the requirements of the assessment task. ☐
Proves that two fractions are equivalent and creates a third equivalent fraction **Question #2**	The student provides clear evidence of both conceptual and procedural understanding, such as reference to the relative size of the parts of the whole and finding common denominators. The student explains clearly how to apply the rule to create a third equivalent fraction. ☐	The student provides some evidence of conceptual understanding, such as "sixths are half of thirds," as well as procedural knowledge, such a finding common denominators. The student uses the rule accurately to create a third equivalent fraction. ☐	The student provides evidence of only procedural understanding, such as finding common denominators. The student creates a third equivalent fraction with no explanation of how it was done. ☐	The student provides little evidence of understanding the relationship between equivalent fractions. The student does not create a third equivalent fraction correctly. ☐	No score is awarded because there is insufficient evidence of student performance based on the requirements of the assessment task. ☐
Compares two fractions with unlike denominators by creating equivalent fractions **Question #3**	The student explains clearly how to create equivalent fractions to compare the two fractions with unlike denominators and correctly answers the problem. ☐	The student provides a limited explanation of how to create equivalent fractions to compare the two fractions with unlike denominators and correctly answers the problem. ☐	The student creates equivalent fractions to compare the two fractions with unlike denominators but provides no explanation. The student correctly answers the problem. ☐	The student does not create equivalent fractions to compare the two fractions with unlike denominators and does not correctly answer the problem. ☐	No score is awarded because there is insufficient evidence of student performance based on the requirements of the assessment task. ☐

* When work is judged to be limited or insufficient, the teacher makes decisions about appropriate intervention to help the student improve.

B. One-on-one Assessment

To help the student solve problems related to the outcome, first review the meaning of fractions. Then review the meaning of equivalent fractions. Finally, apply the student's background knowledge to develop an understanding of comparing fractions. Use simple fractions such as $\frac{1}{2}$ or $\frac{3}{4}$ first and then include other fractions such as $\frac{3}{5}$ or $\frac{5}{8}$.

Review the Meaning of Fractions
If the student has difficulty representing fractions concretely, pictorially and symbolically, then time must be spent on developing understanding of the meaning of fractions.

Review the meaning of fractions by using a variety of manipulatives such as paper strips, fractions strips, fractions circles, pattern blocks and geoboards. Have the student write the fraction that is represented by the shaded or coloured part (concrete to symbolic). Then have the student draw a diagram to represent the fraction shown by the manipulatives (concrete to pictorial). For example, provide the student with a diagram that shows two out of three equal parts of a circle shaded and have the student write the appropriate fraction and explain his or her thinking.

After the students understand fractions by translating from the concrete to the pictorial to the symbolic representations, reverse the process and have the student demonstrate his or her understanding by translating from the symbolic to the pictorial or symbolic representations. For example, have the student use pattern blocks to show that $\frac{3}{5}$ of a set of blocks are triangles.

There is a structured interview on pages 27–28 of the *Diagnostic Mathematics Program, Division II, Numeration* that may be helpful to diagnose weaknesses in students who are having difficulty understanding the meaning of fractions, which is a necessary prerequisite for understanding equivalent fractions.

Equivalent Fractions
If the student has difficulty understanding equivalent fractions, then have the student spend more time solving problems by using concrete or pictorial representations first and then writing the appropriate number sentences. Students need to solve many different problems with a variety of manipulatives and have the time to ask questions and explore various solutions. Provide problems that have meaning and interest to the student.

Sample Problem:
You eat $\frac{2}{3}$ of the 6 chocolates in the box. How many chocolates did you eat?

Have the students take the correct number of counters to represent the 6 chocolates in the problem. Review the meaning of the 3 as the denominator in the fraction and have the student divide the counters into 3 equal groups. Then review the meaning of the 2 as the numerator in the fraction and have the student count the number of counters in 2 of the equal groups. Have the student draw a diagram to show what was done with the counters and write an appropriate number sentence showing the equivalent fractions: $\frac{2}{3} = \frac{4}{6}$.

Have the student demonstrate his or her understanding of equivalent fractions by solving other problems using the symbolic representation first and then proving that the answer is correct by drawing an appropriate diagram or using manipulatives.

Comparing Fractions
If the student has difficulty ordering fractions, review previous strategies for comparing fractions that the student has learned (see Activity 3—Number Line in Step 3 of this end-to-end plan).

Diagrams useful for reviewing strategies for comparing fractions are available in many resources such as on pages 312–313 of *Learning Mathematics in Elementary and Middle Schools* (1994).

Build on the student's background knowledge of comparing fractions and encourage him or her to use a variety of manipulatives and diagrams to show that one fraction is less than, equal to or greater than another fraction. Have the student apply his or her knowledge of equivalent fractions to compare fractions when the denominators are different and other strategies do not work.

Compare only two fractions initially and then move on to comparing more than two fractions.

Use concrete fractions strips to compare fractions and then connect these fraction strips to writing fractions on a number line.

C. Applied Learning

Provide opportunities for the students to use equivalent fractions and the comparison of fractions in a practical situation and notice whether or not the strategies transfer. For example, have the students order different lengths of ribbon by using only the measurements provided in proper fractions. Does the student:

- use an appropriate strategy to order the fractions based on the numbers provided?
- demonstrate flexibility by suggesting different strategies that could be used to order the numbers, including the use of equivalent fractions?
- explain his or her thinking as he or she applies the strategies to order the fractions?
- verify the order by cutting lengths of string to represent the various lengths of ribbon and aligning these lengths of strings along a line?
- transfer the learning to other everyday contexts that require the ordering of fractions?

Step 5: Follow-up on Assessment

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction?

A. Addressing Gaps in Learning

- Draw on the prior knowledge of students, spending time reviewing simple fractions as part of a region and part of a set. Review the meaning of fraction and how it relates to a part and to a whole.
- Provide everyday problem-solving contexts that students can relate to.
- Use a variety of concrete materials, such as folding paper, fractions strips, fraction circles, pattern blocks and geoboards. Connect the concrete to diagrams and symbols as students develop understanding of equivalent fractions and comparing fractions.
- Allow the students to use concrete materials as long as necessary to establish an understanding of the concepts.
- Emphasize the similarities and differences between part of a region and part of a set.
- Have the students demonstrate their understanding of equivalent fractions and comparing fractions by solving problems using symbols first and then verifying their solutions by using pictorial and/or concrete representations.
- Connect the number line to concrete fraction strips, recognizing that the number line is very abstract for many students.
- Ask guiding questions to direct the student's thinking. See the examples provided in Step 4, Part B: One-on-One Assessment.
- Provide time for the students to explore and construct their own meaning rather than being told.
- Encourage flexibility in thinking as students describe various ways to order fractions.
- Have the students share their thinking with others so that students having some difficulty hear how another person thinks about equivalent fractions and comparing fractions in kid-friendly language.

B. Reinforcing and Extending Learning

Students who have achieved or exceeded the outcomes will benefit from ongoing opportunities to apply and extend their learning. These activities should support students in developing a deeper understanding of the concept and should not progress to the outcomes in subsequent grades.

Consider strategies such as:

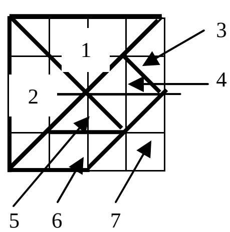
- Provide tips for parents on comparing fractions and decimals at home or in the community; e.g.,
  - Order different lengths of string using fractional measures. Order the symbolic fractions first and then verify the answer by using the strings and placing them side by side aligning them along a given line.
  - Use examples with fractions to compare two quantities, such as comparing $\frac{3}{8}$ of a pizza with $\frac{2}{5}$ of a pizza of the same size.
  - In comparing lengths or pizzas, use a variety of different fractions; e.g., same numerator and different denominators, same denominator and different numerators, fractions greater than $\frac{1}{2}$ and fractions less than $\frac{1}{2}$.
  - Ask for a length of rope that is between two measures such as $\frac{3}{4}$ m and $\frac{9}{10}$ m.
  - Determine how many pieces of pizza you ate if you ate $\frac{3}{4}$ of a pizza that is divided into 8 equal pieces.
- Provide the students with a fraction and ask them to find three fractions that are close in value to the fraction and explain why.
- Have the students write at least five fractions that are between $\frac{9}{10}$ and 1 and encourage them to explain their thinking.

- Hexagon Challenge
  Have the students use triangular or isometric dot paper to draw a different regular hexagon for each of the following:
  - $\frac{1}{6}$ shaded
  - $\frac{1}{12}$ shaded
  - $\frac{1}{24}$ shaded

  The blackline master for isometric dot paper can be downloaded from www.ablongman.com/vandewalleseries. In Volume 3, the blackline master is labelled "BLM 11."

- Triangle Challenge
  Have the students use triangular or isometric dot paper to draw a different equilateral triangle for each of the following:
  - $\frac{1}{2}$ shaded
  - $\frac{1}{3}$ shaded
  - $\frac{1}{4}$ shaded
  - $\frac{1}{6}$ shaded
  - $\frac{1}{8}$ shaded
  - $\frac{1}{9}$ shaded
  - $\frac{1}{16}$ shaded

- Tangram Challenge
  A complete set of tangrams includes 7 pieces as shown below.



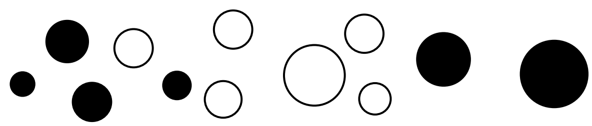
This diagram reproduced from Randy Crawford, "Make It," *Tangrams*, September 6, 2005, http://tangrams.ca/inner/makeset.htm (Accessed April 25, 2008).

Suppose you wish to make a tangram quilt as shown in the tangram diagram.

a. If you want $\frac{1}{4}$ of the quilt to be red, which piece or pieces could be red? List **all** the possibilities. Show and explain all your work.

- Marble Challenge

  How many black marbles should you remove so that $\frac{1}{4}$ of the remaining marbles are black?



This challenge adapted from Carole E. Greenes et al., *Techniques of Problem Solving* (kit) (Palo Alto, CA: Dale Seymour Publications, 1980), Card 148-D.

- Money Challenge

  Would you rather have $\frac{3}{4}$ of 20 nickels or $\frac{2}{3}$ of 15 dimes? Explain.

- Have the students place fractions such as $\frac{3}{12}$, $\frac{15}{20}$, $\frac{6}{18}$, $\frac{21}{24}$ and $\frac{8}{16}$ on a number line showing only fourths and convert each fraction into an equivalent basic fraction (Van de Walle and Lovin 2006, p. 115).

- Provide the students with a set of fractions in which each fraction has an equivalent fraction. Have the students pair each fraction with its equivalent fraction and explain why they make a pair (Van de Walle and Lovin 2006, p. 116).

- Challenge the students to order fractions such as $\frac{11}{20}$, $\frac{3}{5}$, $\frac{5}{12}$ and $\frac{7}{15}$ using common denominators. Encourage them to find the lowest common denominator by using prime factorization or another strategy that makes sense to them.

- Challenge the students to solve fraction riddles such as the following and to make up their own fraction riddles to share with others.

Circle the decimal that best fits the following clues. Explain your thinking.

- The denominator of the equivalent basic fraction is composite.
- It is not equal to $\frac{3}{4}$.
- It is between $\frac{1}{2}$ and 1.

$\frac{12}{16}$

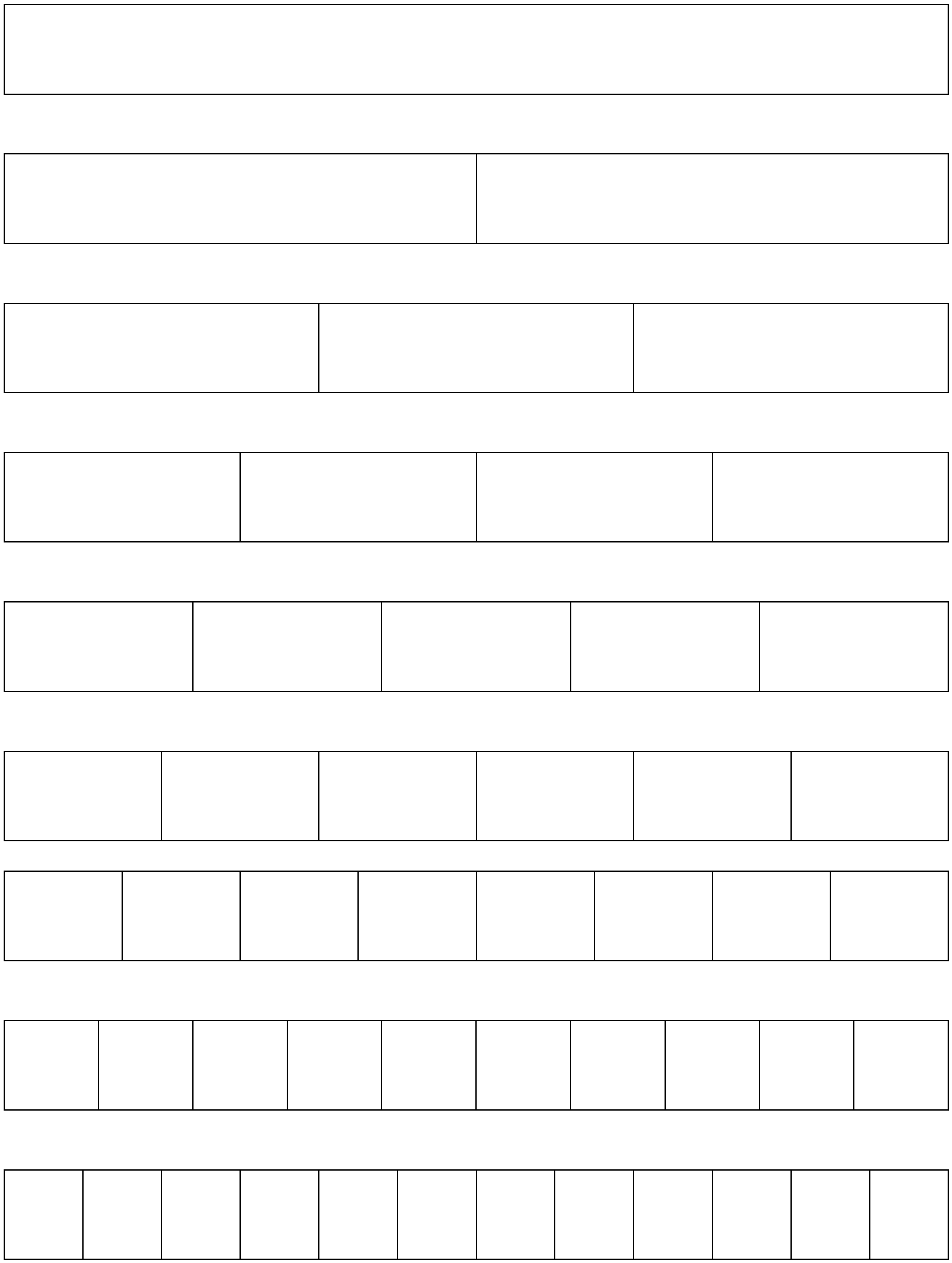
$\frac{6}{15}$

$\frac{15}{20}$

$\frac{10}{16}$

$\frac{8}{12}$

Fraction Strips



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