Mathematics

Planning Guide

Grade 5

Multiplying and Dividing Whole Numbers

Number

Specific Outcomes 5 and 6

This Planning Guide can be accessed online at:
http://www.learnalberta.ca/content/mepg5/html/pg5_multiplyingdividingwholenumbers/index.html

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Planning Guide: *Grade* 5 *Multiplying and Dividing Whole Numbers*

Strand: Number
Specific Outcomes: 5, 6

This *Planning Guide* addresses the following outcomes from the Program of Studies:

Strand: Number	
Specific Outcomes:	5. Demonstrate, with and without concrete materials, an understanding of multiplication (2-digit by 2-digit) to solve problems. 6. Demonstrate, with and without concrete materials, an understanding of division (3-digit by 1-digit), and interpret remainders to solve problems.

Curriculum Focus

The changes to the curriculum targeted by this sample include:

- The general outcome focuses on number sense.
- The specific outcome includes multiplying (2-digit by 2-digit) with or without concrete materials to solve problems; whereas the previous math curriculum included estimating, mentally calculating, computing or verifying the product (3-digit by 2-digit) of whole numbers.

What Is a Planning Guide?

Planning Guides are a tool for teachers to use in designing instruction and assessment that focuses on developing and deepening students' understanding of mathematical concepts. This tool is based on the process outlined in *Understanding by Design* by Grant Wiggins and Jay McTighe.

Planning Steps

The following steps will help you through the Planning Guide:

Step 1: Identify Outcomes to Address

Guiding Questions

- What do I want my students to learn?
- What can my students currently understand and do?
- What do I want my students to understand and be able to do based on the Big Ideas and specific outcomes in the program of studies?

Big Ideas

To successfully solve problems using operations, students must understand what each number in the problem represents and their relation to the answer. Then students require computational fluency to carry out the appropriate operation.

Principles and Standards for School Mathematics describes the development of computational fluency:

> "Computational fluency refers to having efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate flexibility in the computational methods they choose, understand and can explain these methods, and produce accurate answers efficiently" (NCTM 2000, p. 152).

Van de Walle and Lovin (2006) present the following big ideas related to personal, flexible, invented strategies in computation:

1. Flexible methods of computation involve taking apart and combining numbers in a wide variety of ways.
2. Invented strategies are flexible methods of computing that vary with the numbers and the situation. . . . [These strategies] must be constructed by the student.
3. Flexible methods for computation require a good understanding of the operations and properties of the operations, especially the commutative property and the distributive property for multiplication. How the operations are related – addition to subtraction, addition to multiplication, and multiplication to division – is also an important ingredient.

Students should solve a variety of problems involving multiplication and division so they have abundant experience in realizing that numbers have different meanings, depending on the context. Numbers can represent the number of groups, the number of items in each group or the total number of items. As students make sense of the problem, they will be able to communicate the meaning of each number, perform the appropriate calculation and correctly answer the question asked in the problem. *Principles and Standards for School Mathematics* makes the following suggestions related to problem solving:

- Modelling multiplication problems with pictures, diagrams, or concrete materials helps students learn what the factors and their product represent in various contexts (NCTM 2000, p. 151).
- [Students] should learn the meaning of a remainder by modeling division problems and exploring the size of remainders given a particular divisor (NCTM 2000, p. 151).
- As students develop methods to solve multidigit computation problems, they should be encouraged to record and share their methods. As they do so, they can learn from one another, analyze the efficiency and generalizability of various approaches, and try one another's methods (NCTM 2000, p. 153).

The role of the teacher is as a guide at the side, probing students' thinking with thought provoking questions, providing meaningful problem contexts and differentiating instruction to accommodate students' needs; i.e., encouraging the use of personal strategies with the aid of manipulatives, diagrams, and/or mental calculation. The goal is to develop student success in problem solving with operations as they invent strategies with understanding. Teachers must provide guidance to ensure that the strategy is:

- efficient enough to be used regularly
- mathematically valid
- generalizable.

Sequence of Outcomes from Program of Studies

See http://education.alberta.ca/teachers/core/math/programs.aspx for the complete program of studies.

Grade 4	Grade 5	Grade 6
Specific Outcomes	**Specific Outcomes**	**Specific Outcomes**
6. Demonstrate an understanding of multiplication (2- or 3-digit by 1-digit) to solve problems by: • using personal strategies for multiplication with and without concrete materials • using arrays to represent multiplication • connecting concrete representations to symbolic representations • estimating products • applying the distributive property. 7. Demonstrate an understanding of division (1-digit divisor and up to 2-digit dividend) to solve problems by : • using personal strategies for dividing with and without concrete materials • estimating quotients • relating division to multiplication.	5. Demonstrate, with and without concrete materials, an understanding of multiplication (2-digit by 2-digit) to solve problems. 6. Demonstrate, with and without concrete materials, an understanding of division (3-digit by 1-digit), and interpret remainders to solve problems.	2. Solve problems involving whole numbers and decimal numbers. 6. Demonstrate an understanding of multiplication and division of decimals (1-digit whole number multipliers and 1-digit natural number divisors). 7. Explain and apply the order of operations, excluding exponents, with and without technology (limited to whole numbers).

Step 2: Determine Evidence of Student Learning

Guiding Questions

- What evidence will I look for to know that learning has occurred?
- What should students demonstrate to show their understanding of the mathematical concepts, skills and Big Ideas?

Using Achievement Indicators

As you begin planning lessons and learning activities, keep in mind ongoing ways to monitor and assess student learning. One starting point for this planning is to consider the achievement indicators listed in the *Mathematics Kindergarten to Grade 9 Program of Studies with Achievement Indicators*. You may also generate your own indicators and use them to guide your observation of the students.

The following indicators may be used to determine whether or not students have met this specific outcome. Can students:

- illustrate partial products in expanded notation for both factors; e.g., 36 × 42, determine the partial products for (30 + 6) × (40 + 2)?
- represent both 2-digit factors in expanded notation to illustrate the distributive property; e.g., to determine the partial products of 36 × 42, (30 + 6) × (40 + 2) = 30 × 40 + 30 × 2 + 6 × 40 + 6 × 2 = 1200 + 60 + 240 + 12 = 1512?
- model the steps for multiplying 2-digit factors, using an array and base ten blocks, and record the process symbolically?
- describe a solution procedure for determining the product of two given 2-digit factors using a pictorial representation such as an area model?
- solve a given multiplication problem in context using personal strategies and record the process?
- refine personal strategies to increase their efficiency?
- create and solve a multiplication problem and record the process?
- model the division process as equal sharing using base ten blocks, and record it symbolically?
- explain that the interpretation of a remainder depends on the context:
  - ignore the remainder; e.g., making teams of 4 from 22 people
  - round up the quotient; e.g., the number of five passenger cars required to transport 13 people
  - express remainders as fractions; e.g., five apples shared by two people
  - express remainders as decimals; e.g., measurement and money?
- solve a given division problem in context, using personal strategies, and record the process?
- refine personal strategies to increase their efficiency?
- create and solve a division problem and record the process?

Sample behaviours to look for related to these indicators are suggested for some of the activities listed in **Step 3, Section C: Choosing Learning Activities** (p. 13).

Step 3: Plan for Instruction

Guiding Questions

- What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should I use?
- How will I meet the diverse learning needs of my students?

A. Assessing Prior Knowledge and Skills

Before introducing new material, consider ways to assess and build on students' knowledge and skills related to multiplication and division. For example:

- Write a number sentence and describe a personal strategy to solve the following problem. Write the answer to the problem in a complete sentence.
  You have 5 cans of nuts that each weigh 360 g. What is the total weight of these cans of nuts?
- You read 4 times as long this week as you read last week. If you read for 72 minutes this week, how long did you read last week? Show your work and write your answer in a complete sentence.
- Write all the possible number sentences that are represented in the following array. Explain how each number sentence relates to the array.

  ********** ********** * * *
  ********** ********** * * *
  ********** ********** * * *

- Sarah has 96 apples to put into bags and can put 6 apples in each bag. How many bags does she need? Explain your thinking. Write your answer in a complete sentence.
- You saved 3 times as much money this year as you saved last year. If you saved $128 last year, how much money did you save this year? Show your work and write your answer in a complete sentence. Explain why the following solution makes sense or not.
  $3 \times 128 = (3 \times 100) + (3 \times 20) + (3 \times 5) = 600 + 12 + 30 = 642$
  Answer: I saved $642 this year.
- Create a problem that can be represented by the following number sentence:
  $72 \div 3 = \square$. Explain how you know your problem matches the number sentence.

If a student appears to have difficulty with these tasks, consider further individual assessment, such as a structured interview, to determine the student's level of skill and understanding. See **Sample Structured Interview: Assessing Prior Knowledge and Skills** (p. 10).

Sample Structured Interview: Assessing Prior Knowledge and Skills

Directions	Date:	
	Not Quite There	**Ready to Apply**
Present the following problem to the student: You have 5 cans of nuts that each weigh 360 g. What is the total weight of these cans of nuts? Say, "**Write a number sentence and describe a personal strategy to solve the problem. Write the answer to the problem in a complete sentence."**	Writes an incorrect number sentence to represent the problem. Uses an incorrect operation, such as 5 + 360, to solve the problem Multiplies the two numbers correctly but is unable to describe with understanding the strategy used. Writes a sentence but the sentence does not answer the question asked.	Writes a correct number sentence to represent the problem; e.g., 5 × 360 = ? Solves the problem correctly using a personal strategy and describes this strategy clearly with understanding. Writes a complete sentence to answer the question.
Present the following problem to the student: **"You read 4 times as long this week as you read last week. If you read for 72 minutes this week, how long did you read last week? Show your work and write your answer in a complete sentence."**	Uses an incorrect operation to solve the problem; e.g., multiplies 72 by 4. Uses a correct operation to solve the problem but the work shown is vague and indicates limited understanding. Writes a sentence but the sentence does not answer the question asked.	Uses a correct operation to solve the problem and shows work that clearly indicates understanding of the process. Writes a sentence that answers the question asked.
Present the following problem to the student: **"Write all the possible number sentences that are represented by the following array. Explain how each number sentence relates to the array."** ********** ********** * * * ********** ********** * * * ********** ********** * * *	Writes some but not all of the possible number sentences represented by the array. Has difficulty showing the connection between multiplication and division. Explains vaguely or not at all how each number sentence relates to the array.	Writes all the possible number sentences represented by the array. Clearly demonstrates an understanding of the relationship between multiplication and division. Explains clearly how each number sentence relates to the array.

Present the following problem to the student: **"Sarah is putting 96 apples into bags with 6 apples in each bag. How many bags does she need? Explain your thinking. Write your answer in a complete sentence."**	Uses an incorrect operation to solve the problem; e.g., adds 96 and 6. Uses a correct operation to solve the problem, but the explanation is vague and indicates limited understanding. Writes a sentence but the sentence does not answer the question asked.	Uses a correct operation to solve the problem and explains clearly the thinking done in solving the problem. Writes a sentence that answers the question asked.
Present the following situation to the student: **"You save 3 times as much money this year as you saved last year. If you saved $128 last year, how much money did you save this year? Show your work and write your answer in a complete sentence."** Present the following solution to the student and ask him or her to explain why the solution to the problem makes sense or not: **"3 × 128 = (3 × 100) + (3 × 20) + (3 × 5) = 600 + 12 + 30 = 642** **Answer: I saved $642 this year."**	Does not understand the solution so is unable to critique it. Disagrees with the solution and thinks a different operation should be used; e.g., 128 divided by 3. Understands the use of place value in the application of the distributive property but fails to notice the error.	Explains clearly how place value is used in the application of the distributive property. States that there is an error and explains why it is an error, using strong conceptual understanding of place value. Corrects the error efficiently and justifies why the adjusted answer is the right answer.
Tell the student: **"Create a problem that can be represented by the number sentence 72 ÷ 3 = □ .** **Explain how you know your problem matches the number sentence."**	Does not create a problem that can be solved using division as indicated by the number sentence. Creates a division problem but does not explain why the created problem matches the number sentence.	Creates a division problem represented by the number sentence provided. Explains clearly why the created problem matches the number sentence.

B. Choosing Instructional Strategies

Consider the following guidelines for teaching multiplication and division:

- Teach in a problem-solving context. Research shows that by solving problems using multiplication and division, students create personal strategies for computing and develop understanding about the relationship between the operations and their properties (NCTM 2000, p. 153).
- Choose problems that relate to the students' own lives (Van de Walle 2001).
- Provide a variety of problems representing the different multiplication and division situations with varying degrees of difficulty to differentiate instruction.
- Work with the whole group initially and have the students paraphrase the problem to enhance understanding (Willis et al. 2006) and to recognize whether the numbers in the problem refer the whole, the number of groups or the quantity in each group. Discuss whether the unknown refers to the whole, the number of groups or the quantity in each group, thereby facilitating thinking about which operation to use in solving the problem.
- Have the students estimate the answer to the problem before calculating so that they are better able to determine the reasonableness of their answers.
- Make base ten materials available for the students to use as needed.
- Provide time for the students to create their personal strategies to solve the problem and share these strategies with members of their groups or with the entire class.
- Guide the discussion by asking questions to encourage thinking about number relationships, the connections among the operations and their personal strategies.
- Have the students compare their answers to the estimates they made before solving the problems.
- Challenge the students to solve the problem another way, do a similar problem without models or clarify the explanation of their personal strategies.
- Have the students critique their personal strategies as well as those of their classmates to decide which strategy works best for them and why.
- Have the students create problems for a variety of number sentences illustrating multiplication and division, including examples of equal sharing, equal grouping, comparison problems and combination problems.

C. Choosing Learning Activities

The following learning activities are examples of activities that could be used to develop student understanding of the concepts identified in Step 1.

Sample Activities:

Teaching Personal Strategies for Multiplying and Dividing Whole Numbers

1. **Multiplying Two 2-digit Numbers Using Personal Strategies, Concrete Materials Connected to Diagrams and Symbolic Representations, Arrays and the Distributive Property** (p. 15)
2. **Multiplying Two 2-digit Numbers Using Personal Strategies without Concrete Materials** (p. 19)
3. **Dividing 3-digit Numbers by 1-digit Numbers with and without Remainders Using Personal Strategies, Concrete Materials Connected to Diagrams and Symbolic Representations, and Connections to Multiplication** (p. 21)
4. **Dividing 3-digit Numbers by 1-digit Numbers Using Personal Strategies without Concrete Materials** (p. 25)

Teaching Solving Problems Involving Multiplication and Division

1. **Four Corner Strategy** (p. 28)
2. **Whole, the Number of Groups or the Quantity in Each Group** (p. 29)
3. **Thumbs Up, Thumbs Down, Thumbs Sideways** (p. 30)
4. **Choosing Number Sentences** (p. 31)
5. **Classifying Problems: Open and Closed Sorts** (p. 32)
6. **Similarities and Differences** (p. 33)

Teaching Personal Strategies for Multiplying and Dividing Whole Numbers

Sample Activity 1: Multiplying Two 2-digit Numbers Using Personal Strategies, Concrete Materials Connected to Diagrams and Symbolic Representations, Arrays and the Distributive Property

Draw on prior knowledge by reviewing arrays and some personal strategies used by students in solving problems by multiplying 2- and 3-digit numbers by 1-digit numbers. Have students share their ideas.

Explain that students will explore various ways to multiply two 2-digit numbers.

Present the following problem to the students:
John and Judy set up 12 rows of chairs with 14 chairs in each row in the gymnasium for an assembly. How many chairs did they set up altogether?

Guide discussion as to whether the numbers in the problem and the unknown refer to the whole, the number of groups or the quantity in each group. Discuss which operation would be used to solve the problem and what would be a good *estimate* for the answer. Have the students justify all their answers.

Look For …

Do students:

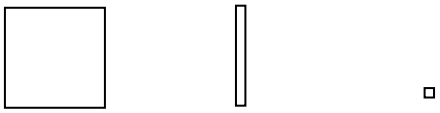
- ☐ know and draw on number facts and other number relationships?
- ☐ estimate the answer to determine the reasonableness of the calculated solution?
- ☐ solve the problem with base-ten materials but need guidance in using symbols to show what they did?
- ☐ explain why the steps that they use work?

Encourage the students to use a personal strategy to solve the problem and share their ideas with one another. Make base ten materials available for students to use as needed. Through class discussion, have the students decide which personal strategy is efficient, correct mathematically and works in similar multiplication problems. Have the students compare the calculated answer to the estimate.

Capitalize on the sharing done with base ten materials. If no one uses base ten materials, then suggest that the base ten materials would provide another option for explaining the process of multiplication. Have other students share what they did with base ten materials or guide the students as they use the base ten materials in an array to solve the problem. For example:

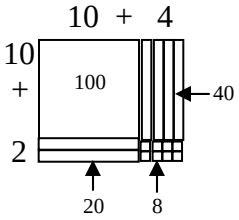
Provide base ten grid paper for the students to draw diagrams that represent the work done with the base ten materials.

Base ten materials solution:



Flat is 100. Rod is 10. Unit is 1.

The base ten materials below represent an array of 12 rows with 14 chairs in each row. The 12 rows are written in expanded form as 10 + 2. The 14 chairs in each row are written in expanded form as 10 + 4.

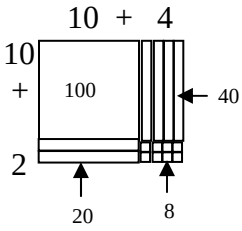


The four partial products can be seen in the array.

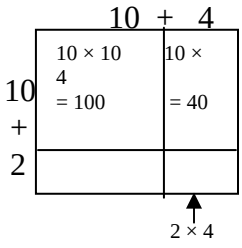
To begin, have the students draw a diagram of the array to represent the concrete materials by using base ten grid paper that shows the 100 squares, 40 squares and so on. A blackline master for base ten grid paper can be downloaded from http://www.ablongman.com/vandewalleseries. Under Volume 3, the blackline master is labelled "BLM 1."

Later, the diagram can be drawn without grid paper. Have the students draw a rectangle, divide it into four sections to represent the four partial products, label the dimensions of the rectangle as the sum of the tens and the ones for each factor, and label each section with the appropriate product. Encourage the students to connect the symbolic representation to the concrete and pictorial representations as shown in the following diagrams.

Concrete Materials



Diagram



Symbolic Representation

left to right		right to left
14		14
× 12	or	× 12
8		100
20		40
40		20
100		8
168		168

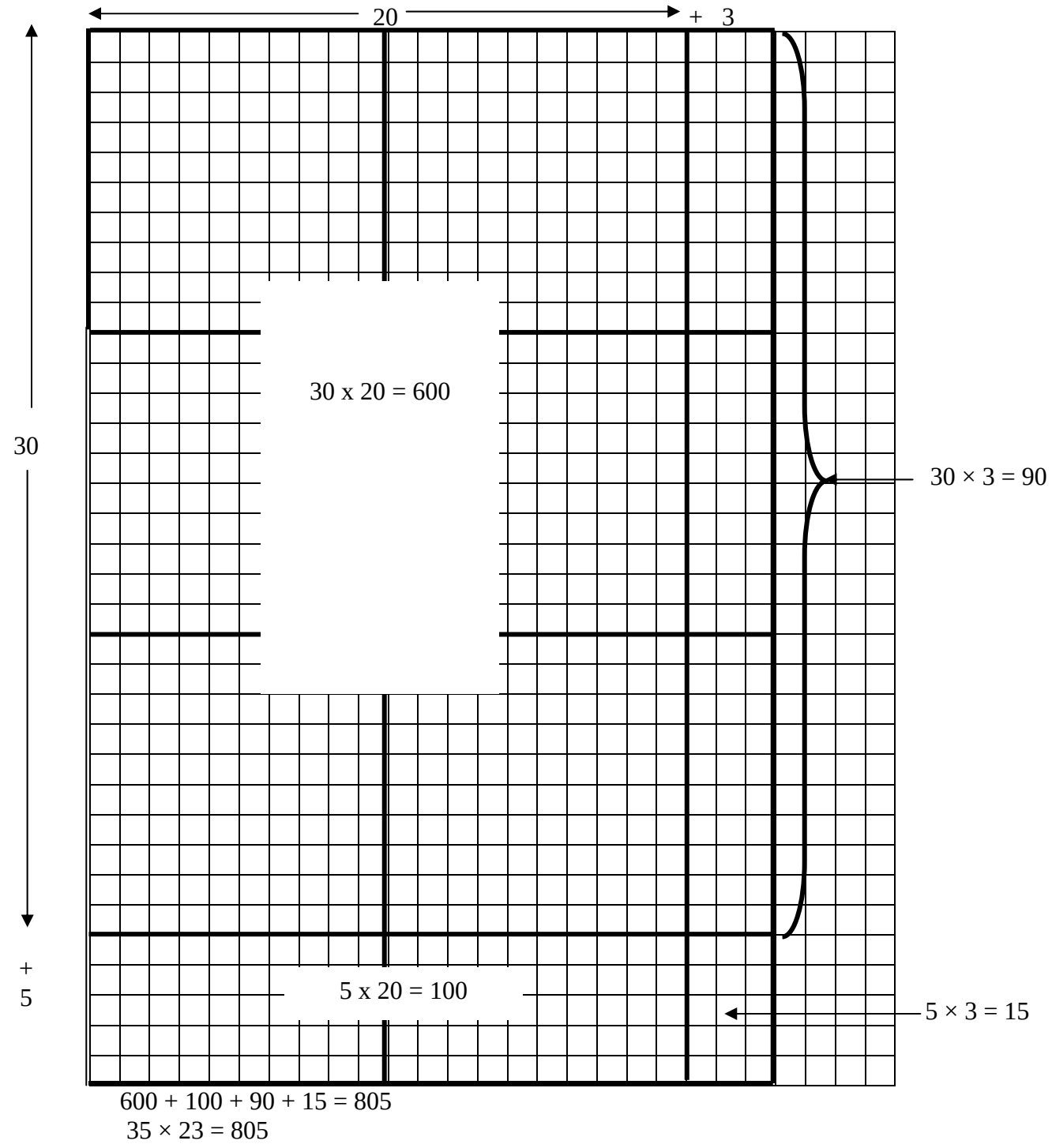
Guide the students in connecting the work done with concrete materials to the distributive property showing the partial products:
$12 \times 14 = (10 + 2) \times (10 + 4) = 10 \times 10 + 10 \times 4 + 2 \times 10 + 2 \times 4 = 100 + 40 + 20 + 8 = 168$

Answer: They set up 168 chairs altogether.

Have the students apply their personal strategies to solve a similar problem with different numbers such as setting up 18 rows of chairs with 26 chairs in each row.

Sample personal strategies for finding the product of 35 × 23:

- Using a diagram on base ten grid paper to represent concrete materials:



- Connecting the diagram to the distributive property:
  $35 \times 23 = (30 + 5) \times (20 + 3) = 30 \times 20 + 30 \times 3 + 5 \times 20 + 5 \times 3 = 805$
- Connecting the diagram and the distributive property to another symbolic representation using partial products:

$$\begin{array}{r} 23 \\ \times\ 35 \\ \hline 15 \\ 100 \\ 90 \\ \underline{600} \\ 805 \end{array}$$

Sample Activity 2: Multiplying Two 2-digit Numbers Using Personal Strategies without Concrete Materials

Draw on prior knowledge by reviewing some personal strategies used by students in solving problems by multiplying 2- and 3-digit numbers by 1-digit numbers. Have the students share their ideas.

Explain that students will explore various ways to multiply two 2-digit numbers.

Present the following problem to the students:
A book has 62 pages. How many pages are there in 98 of these books?

Guide discussion as to whether the numbers in the problem and the unknown refer to the whole, the number of groups or the quantity in each group. Discuss which operation would be used to solve the problem and what would be a good *estimate* for the answer. Have the students justify all their answers.

Look For …

Do students:

- ☐ know and draw on number facts and other number relationships?
- ☐ estimate the answer to determine the reasonableness of the calculated solution?
- ☐ use personal strategies that do not work in all situations (not generalizable)?
- ☐ explain why the steps that they use work?

Provide enough time for the students to solve the problem by writing a number sentence and their personal strategies to show calculations for finding the product.

To address different learning styles and abilities, have base ten materials available for students who still need the concrete or visual representation before using the symbolic personal strategies.

Have the students explain how their strategies work by relating to place value and have them decide on which strategy is most efficient for them to use in calculating other problems.
Have the students apply their personal strategies to solve a similar problem with different numbers such as finding the total number of pages in 36 books if each book has 25 pages. Then extend to include different problem contexts with different numbers.

Sample personal strategies:

First sample (distributive property of multiplication over addition):

$98 \times 62 = (90 + 8) \times (60 + 2) = 90 \times 60 + 90 \times 2 + 8 \times 60 + 8 \times 2 = 5400 + 180 + 480 + 16 = 6076$
The 98 books have 6076 pages in all.

Second sample (distributive property of multiplication over subtraction):

$98 \times 62 = 62 \times 98$ using the commutative property of multiplication
$62 \times 98 = 62 (100 – 2)$ because $98 = 100 – 2$
$62 (100 – 2) = 62 \times 100 – 62 \times 2 = 6200 – 124 = 6076$
The 98 books have 6076 pages in all.

Sample Activity 3: Dividing 3-digit Numbers by 1-digit Numbers with and without Remainders Using Personal Strategies, Concrete Materials Connected to Diagrams and Symbolic Representations, and Connections to Multiplication

Draw on prior knowledge by reviewing some personal strategies used by students in solving problems by dividing 1- and 2-digit numbers by 1-digit numbers. Have the students share their ideas by finding the quotients to equal sharing and equal grouping problems.

Explain that students will explore various ways to divide 3-digit numbers by 1-digit numbers.

Present the following problem to the students:
Danny collected $138 in 3 days. If he collected the same amount of money each day, how much was his daily collection?

Guide discussion as to whether the numbers in the problem and the unknown refer to the whole, the number of groups or the quantity in each group. Discuss which operation would be used to solve the problem and what would be a good **estimate** for the answer. Have the students justify all their answers.

Look For …

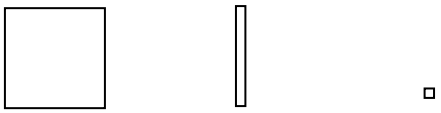
Do students:

- ☐ know and draw on number facts and other number relationships?
- ☐ estimate the answer to determine the reasonableness of the calculated solution?
- ☐ solve the problem with base-ten materials but need guidance in using symbols to show what they did?
- ☐ explain why the steps that they use work?
- ☐ interpret the remainder by using the context of the problem?
- ☐ relate division to multiplication?
- ☐ show, by using concrete materials, that repeated subtraction can be used to solve equal grouping problems but not equal sharing problems?

Encourage the students to use a personal strategy to solve the problem and share their ideas with one another. Make base ten materials available for the students to use as needed. Through class discussion, have the students decide which personal strategy is efficient, correct mathematically and works in similar multiplication problems. Have the students compare the calculated answer to the estimate.

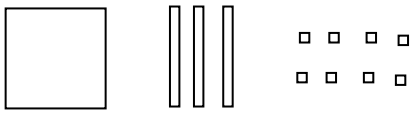
Capitalize on the sharing done with base ten materials. If no one uses base ten materials, then suggest that the base ten materials would provide another option for explaining the process of multiplication. Have other students share what they did with base ten materials or guide the students as they use the base ten materials in an array to solve the problem. For example:

Base ten materials solution:

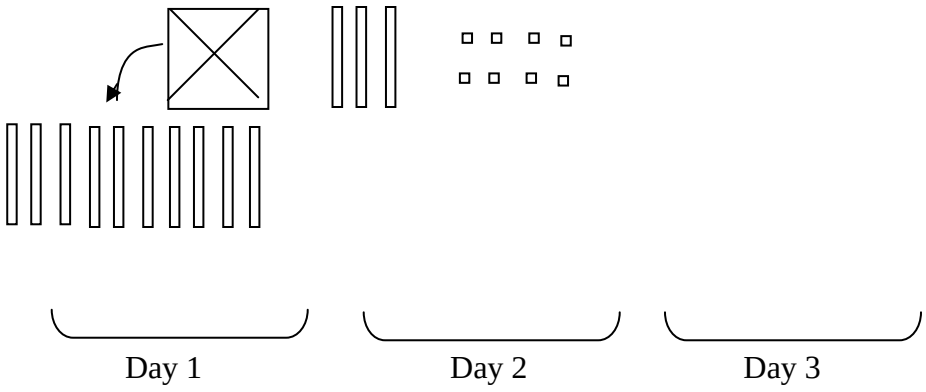


Flat is 100. Rod is 10. Unit is 1.

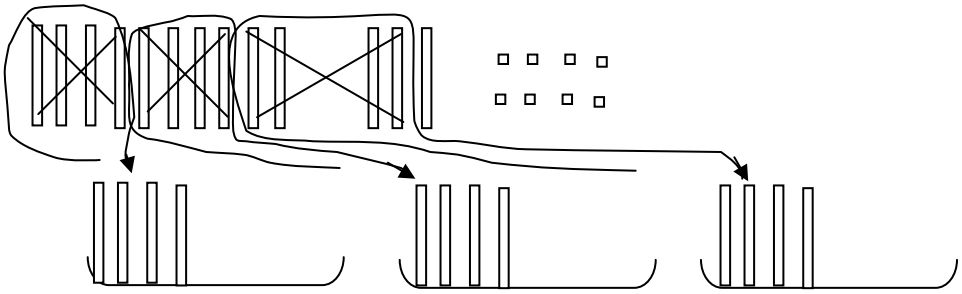
The base ten materials below represent 138.



These base ten materials must be shared equally among the 3 days. Let each day be represented by a tray. Since the flat or 100 cannot be shared among the 3 trays, it is traded for 10 rods or 10 tens.



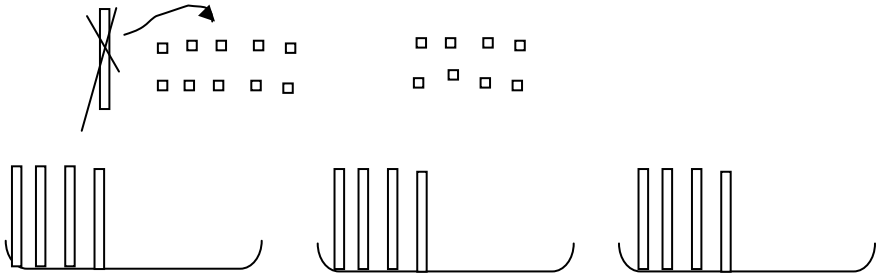
The 13 rods are shared equally among the 3 trays with 1 rod left over.



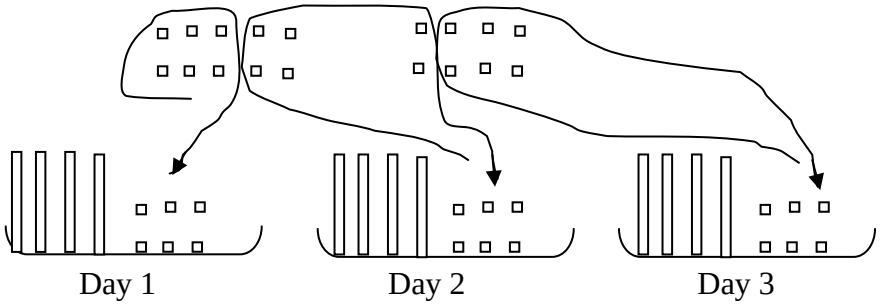
The rod that is left over is traded for 10 ones.

Symbolic representation

$$3\overline{)138}\quad 40$$
$$\underline{12\,0}$$
$$18$$



Finally, the 18 ones are equally shared among the 3 days.



Symbolic representation

$$3\overline{)138}\quad 40$$
$$\underline{12\,0}\quad\quad 6$$
$$18$$
$$\underline{18}$$
$$46$$

Answer to the problem: Danny collected $46 dollars each day.

Provide time for the students to solve the problem by writing a number sentence and their personal strategies to show calculations of the quotient and then compare them to their estimates. Emphasize the connections between the concrete representation and the symbolic personal strategies.

Encourage the students to connect division to multiplication by using multiplication to check that the quotient is correct.

Have the students discuss whether the problem could be represented by the number sentence $3 \times \square = 138$. Why or why not?

Have the students create a multiplication problem using the same context. For example: Andre collects $46 a day for 3 days. How much money does he collect in all?

Have the students explain how their strategies work by relating to place value and have them decide on which strategy is most efficient for them to use in calculating other quotients.

Have the students apply their personal strategies to solve a similar problem with different numbers, such as:
Kodie collected $175 in 2 days. If she collected the same amount of money each day, how much was her daily collection? Encourage discussion about what to do with the dollar left over as a remainder.

Have the students apply their personal strategies to solve a variety of division problems that include equal sharing and equal grouping with and without remainders. Note that the traditional division algorithm is built on the process involved with fair-share problems (Van de Walle 2001). To solve equal grouping problems, repeated subtraction can be used but is not efficient if the quotient is a fairly large number (Russell 2000).

Look For …

Do students:

- ☐ know and draw on number facts and other number relationships?
- ☐ estimate the answer to determine the reasonableness of the calculated solution?
- ☐ use personal strategies that will not work in all situations (not generalizable)?
- ☐ explain why the steps that they use work?
- ☐ interpret the remainder by using the context of the problem?
- ☐ use appropriate number sentences and personal strategies for equal grouping and equal sharing problems to suit the context of the problems?

Sample Activity 4: Dividing 3-digit Numbers by 1-digit Numbers Using Personal Strategies without Concrete Materials

Draw on prior knowledge by reviewing some students' personal strategies used in solving problems by dividing 1- and 2-digit numbers by 1-digit numbers. Have the students share their ideas by finding the quotients to equal sharing and equal grouping problems.

Explain that students will explore various ways to divide 3-digit numbers by 1-digit numbers.

Present the following problem to the students:
You have 258 beads to make 4 necklaces. If each necklace has the same number of beads, how many beads are on each necklace? Will any beads be left over? If so, how many?

Guide discussion as to whether the numbers in the problem and the unknown refer to the whole, the number of groups or the quantity in each group. Discuss which operation would be used to solve the problem and what would be a good *estimate* for the answer. Have the students justify all their answers.

Provide enough time for the students to solve the problem by writing a number sentence and their personal strategies to show calculations for finding the quotient.

To address different learning styles and abilities, make base-ten materials available for students who still need the concrete or visual representation before using the symbolic personal strategies. Have the students explain how their strategies work by relating to place value and have them decide on which strategy is most efficient for them to use in calculating other quotients.
Have the students apply their personal strategies to solve a similar problem with different numbers, such as having 508 beads to make 5 necklaces.

Also include equal grouping problems by changing the problem as follows: You have 450 beads. Each necklace uses 8 beads. How many necklaces can you make? Will any beads be left over? If so, how many?

Then extend the activity to include different problem contexts with different numbers, including answers with and without remainders.

Sample personal strategies:

<table>
<tr>
<td>First sample with equal sharing:

You have 258 beads to make 4 necklaces. If each necklace has the same number of beads, how many beads are on each necklace? Will any beads be left over? If so, how many?

258 ÷ 4 = ☐ or 258 ÷ ☐ = 4

4 × 60 = 240
258 – 240 = 18
18 ÷ 4 = 4 with 2 remaining
60 + 4 = 64

Each necklace has 64 beads with 2 beads left over.</td>
<td>Second sample with equal sharing:

You have 258 beads to make 4 necklaces. If each necklace has the same number of beads, how many beads are on each necklace? Will any beads be left over? If so, how many?

258 ÷ 4 = ☐ or 258 ÷ ☐ = 4

$\begin{array}{r|l} 4\overline{)258} & \\ -\underline{240} & 60 \\ 18 & \\ -\underline{16} & 4 \\ 2 & 64 \end{array}$ OR $4\overline{)258}$ $60 + 4 = 64$
$-\underline{240}$
18
$-\underline{16}$
2
Answer: 24 remainder 2
Each necklace has 24 beads with 2 beads left over.</td>
</tr>
</table>

Teaching Solving Problems Involving Multiplication and Division

Sample Activity 1: Four Corner Strategy

Draw on prior knowledge by reviewing multiplication and division problems. Emphasize the connections among the story problems, the models/diagrams, the number sentences and the personal strategies used in calculations.
Have the students divide a page into four sections to make graphic organizers and label them as follows:

Four Corner Strategy

Story Problem	Models/Diagrams
Number Sentence	Personal Strategy

Present the students with a problem or a number sentence involving multiplication or division, such as:

- A ribbon that is 285 cm long is divided into 5 equal parts. What is the length of each part?
- 5 × ☐ = 285

Have the students complete the graphic organizer by writing the story problem or the number sentence in one corner and filling in the other corners appropriately.

Adaptations:

- Students work in groups and fill in the graphic organizer on large chart paper that can be displayed and discussed with other students and the whole class.
- Use different labels for the four corners of the graphic organizer, such as "word problem," "number sentence," "estimation" and "personal strategy."
- Have the students interpret the remainder if it exists for each of the division problems.

Look For …

Do students:

- ☐ exhibit flexibility in writing number sentences in more than one way to show the relationship between multiplication and division?
- ☐ clearly explain their personal strategies and adjust them through discussion to make them more efficient?
- ☐ create problems to illustrate number sentences as well as write number sentences for problems?

Sample Activity 2: Whole, the Number of Groups or the Quantity in Each Group

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Look For …

Do students:

- ☐ correctly label the numbers in each problem as the whole, the number of groups or the quantity in each group?
- ☐ identify whether the whole, the number of groups or the quantity in each group is not known in the problem?
- ☐ solve the problems and justify why the solution is correct?

Draw on prior knowledge by reviewing multiplication and division problems. Guide discussion to whether the numbers in the problems and the unknown refer to the whole, the number of groups or the quantity in each group. Have the students solve the problems and explain their thinking.

Problem examples:

- How many different single-scoop ice cream cones can be made with 4 different kinds of cones and 125 different flavours of ice cream?
- Star practises the piano each day for 75 minutes. How long does she practise the piano in 2 weeks?
- You travelled 584 km in 4 days. If you travelled the same distance each day, how far did you travel each day?
- You have 130 flowers to put into bouquets of 8 flowers each. How many bouquets can you make with these flowers?

Sample Activity 3: Thumbs Up, Thumbs Down, Thumbs Sideways

Present the students with a variety of multiplication and division problems. Reading the problems orally and also having them displayed on the white board or the overhead projector may help to address the different learning styles of the students.

For each problem, ask the students to put their thumbs up if multiplication can be used to solve the problem, thumbs down if division can be used and thumbs sideways if both multiplication and division can be used. Have the students justify their choices, either in small groups or with the entire class.

Finally, have the students write number sentences to support their choices. Emphasize the relationship between multiplication and division as the students suggest different number sentences.

Look For …

Do students:

- ☐ correctly classify problems as involving multiplication, division or both operations?
- ☐ exhibit flexibility in writing number sentences in more than one way to show the relationship between multiplication and division?
- ☐ justify their choice of operation(s) to solve a problem?

Adaptations:

- Provide the students with written copies of the problems and have them work in pairs or individually to classify the problems as multiplication, division or both multiplication and division. Students then write the appropriate number sentences for each problem.
- Have different groups of students take turns creating and classifying multiplication and division problems.
- Have students write equivalent number sentences for a given problem. Through discussion, have the students generalize that the semantic number sentence (the one that shows the meaning of the problem) is often rearranged to expedite calculation.
  For example, 185 ÷ 5 = ☐ can be written as 5 × ☐ = 185
  The first number sentence shows dividing, but students might prefer to use multiplication to solve the problem and therefore rearrange the number sentence in the rewritten form.
- Have the students classify the problems according to whether the unknown is the whole, the number of groups or the quantity in each group.

Sample Activity 4: Choosing Number Sentences

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Present the students with a problem and have them choose which of the number sentences provided could be used to solve the problem. Ask why the number sentences chosen can be used to solve the problem.

Example:
Diego saved $184 this month by doing odd jobs for the neighbours. Last month, he saved $8. How many times as much money did he save this month as last month?

184 × 8 = ☐	☐ = 8 × 184	8 × ☐ = 184
184 × ☐ = 8	184 ÷ 8 = ☐	8 ÷ 184 = ☐
☐ ÷ 8 = 184	184 ÷ ☐ = 8	8 ÷ ☐ = 184

Look For …

Do students:

- ☐ correctly choose equivalent number sentences that relate to the problem?
- ☐ justify their choices by relating to the meaning of the problem and the relationship between multiplication and division?

Sample Activity 5: Classifying Problems: Open and Closed Sorts

Look For …

Do students:

- ☐ create categories for the problems based on the meaning of operations?
- ☐ exhibit flexibility in creating more than one way to sort the problems?
- ☐ justify why a given problem was placed in a specific category?

Open Sort

Present the students with a variety of multiplication and division problems that are written on separate pieces of paper. Have the students work in groups to organize the problems, label the categories and explain why the problems fit where they have been placed. Explain that some problems may fit in more than one category. Challenge the students to:

- find another way to classify the problems
- create other problems and place them into the categories.

Some categories used by students may include the following:

- multiplication, division, both multiplication and division
- equal grouping, equal sharing (division problems)
- equal-group problems, comparison problems, combinations problems
- only estimation is needed, both estimation and a calculated answer are needed
- the unknown is the whole, the number of groups or the quantity in each group

The division problems can be sorted into the following categories:

- ignore the remainder; e.g., making teams of 6 from 155 people
- round up the quotient; e.g., the number of 8 passenger vans to transport 110 people
- express remainders as fraction; e.g., 15 pizzas shared equally between two people
- express remainders as decimals; e.g., $155 shared equally between two people

Closed Sort

Present the students with a variety of multiplication and division problems that are written on separate pieces of paper and also provide them with the categories into which they are to organize the problems. See the examples of categories given above.
The students sort the problems into the categories provided and justify their choices.

Sample Activity 6: Similarities and Differences

Provide the students with two problems using the same numbers but different meanings for multiplication or division, such as one showing equal grouping and one showing the equal sharing. Ask the students to explain how the problems are the same and how they are different. They may wish to put their explanations in a graphic organizer such as the T-chart shown below:

Similarities	Differences

An example of a problem showing equal grouping:
You have 175 pictures to put into albums. If each album page holds 4 pictures, how many pages do you need? Explain how you know.

An example of a problem showing equal sharing:
You have 175 marbles to share equally among 4 friends. How many marbles will each friend receive? Explain how you know.

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Other strategies for teaching multiplication and division problems, using arrays and showing the connections between the operations are available on pages 217–225 of the *Diagnostic Mathematics Program, Division II, Operations*.

Look For …

Do students:

- ☐ explain that in both problems the whole is known and one of the parts is unknown?
- ☐ explain that both problems can be represented by the same number sentence?
- ☐ explain that the concrete or visual representation in each problem is different?
- ☐ explain that the 4 represents the number in each group in the first problem but it represents the number of groups in the second problem?
- ☐ explain that the remainder in each problem is handled differently?
- ☐ explain that the units are different in the two problems?
- ☐ explain that the problems describe different real-life situations?

Step 4: Assess Student Learning

Guiding Questions

- Look back at what you determined as acceptable evidence in Step 2.
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

In addition to ongoing assessment throughout the lessons, consider the following sample activities to evaluate students' learning at key milestones. Suggestions are given for assessing all students as a class or in groups, individual students in need of further evaluation, and individual or groups of students in a variety of contexts.

A. Whole Class/Group Assessment

A Field Trip Using Multiplication and Division of Whole Numbers
In this assessment task, students will demonstrate their understanding of multiplying and dividing whole numbers. Students will use a personal strategy to find the total cost of the students' tickets. They will then explain the multiplication process by connecting the pictorial and symbolic representations.

Materials required: paper and pencil, base ten grid paper, if necessary.

Each student will:

- solve a multiplication problem using personal strategies and clearly explain the process
- solve a division problem using personal strategies and clearly explain the process
- connect the pictorial and symbolic representations of multiplication, and also of division, using sound mathematical understanding
- interpret correctly the remainder in a division problem and clearly explain why this interpretation was made.

Suggestions for early finishers:

1. Find all the single digit divisors that will make the following number sentence a true statement.
   355 ÷ ? = a quotient that is 60 or less.

2. Jody said that you can switch the digits in the ones places when multiplying two numbers and the product will remain unchanged.
   For example, she says that 16 × 48 is the same as 18 × 46.
   Is Jody correct? Explain.

A Field Trip Using Multiplication and Division of Whole Numbers – Student Assessment Task

The grades 5 and 6 students are going on a field trip to the Telus World of Science. These 139 students will be travelling in vans that can each take a maximum of 6 students. The options provided to the students include:

	Both the Imax and the Margaret Ziedler Star Theatres	Either the Imax or the Margaret Ziedler Star Theatre and Exploration of Free Displays
Cost per student, including tax	$18	$12

1. Use a personal strategy to calculate how many vans are needed to transport the 139 students to the Telus World of Science. Explain in detail the process you used to solve the problem.

2. Draw a diagram to describe the solution procedure for solving question #1 with concrete materials. Explain how your diagram relates to a paper-and-pencil solution for this problem.

3. Calculate the cost for the student tickets if 85 students go to both the Imax and the Star Theatres while the remaining 54 students go to only one of the theatres and explore the free displays for the remaining time. Explain in detail the process you used to solve this problem.

4. Draw a diagram to describe the solution procedure for determining the cost of tickets for the 54 students going to only one theatre. Explain how your diagram relates to a paper-and-pencil solution for this problem.

Student _____

SCORING GUIDE
A Field Trip Using Multiplication and Division of Whole Numbers

Level / Criteria	4 Excellent	3 Proficient	2 Adequate	1 Limited *	Insufficient / Blank *
Division of whole numbers **Question #1**	Uses a personal strategy to obtain a correct solution (including interpretation of the remainder) and explains the process with sound mathematical support. ☐	Uses a personal strategy to obtain a correct solution (including interpretation of the remainder) and explains the process clearly. ☐	Uses a personal strategy with vague interpretation of the remainder to obtain a solution that has minor errors and explains the process with minimal support. ☐	Uses a personal strategy without interpreting the remainder to obtain a solution that has major errors and is unable to explain the process. ☐	No score is awarded because there is insufficient evidence of student performance based on the requirements of the assessment task. ☐
Division of whole numbers – connecting pictorial and symbolic representations **Question #2**	Shows a thorough understanding of solving the division problem by connecting the pictorial and symbolic representations. ☐	Shows a clear understanding of solving the division problem by connecting the pictorial and symbolic representations. ☐	Shows a limited understanding of solving the division problem by connecting the pictorial and symbolic representations. ☐	Shows little or no understanding of solving the division problem by connecting the pictorial and symbolic representations. ☐	No score is awarded because there is insufficient evidence of student performance based on the requirements of the assessment task. ☐
Multiplication of whole numbers **Question #3**	Uses a personal strategy to obtain a correct solution and explains the process with sound mathematical support. ☐	Uses a personal strategy to obtain a correct solution and explains the process clearly. ☐	Uses a personal strategy to obtain a solution that has minor errors and explains the process with minimal support. ☐	Uses a personal strategy to obtain a solution that has major errors and is unable to explain the process. ☐	No score is awarded because there is insufficient evidence of student performance based on the requirements of the assessment task. ☐
Multiplication of whole numbers – connecting pictorial and symbolic representations **Question #4**	Shows a thorough understanding of solving the multiplication problem by connecting the pictorial and symbolic representations. ☐	Shows a clear understanding of solving the multiplication problem by connecting the pictorial and symbolic representations. ☐	Shows a limited understanding of solving the multiplication problem by connecting the pictorial and symbolic representations. ☐	Shows little or no understanding of solving the multiplication problem by connecting the pictorial and symbolic representations. ☐	No score is awarded because there is insufficient evidence of student performance based on the requirements of the assessment task. ☐

***** When work is judged to be limited or insufficient, the teacher makes decisions about appropriate intervention to help the student improve.

B. One-on-one Assessment

1. Ask the student to explain the connection between multiplication and division by using counters or base ten materials. If necessary, coach the student to make an array and show how the array shows both multiplication and division. Start with smaller numbers and then gradually move to using larger numbers required at the grade level.

2. Present the following problem to the student and have him or her read it orally. Have base ten materials available to use as needed.

   A bottle contains 182 mL of medicine. Jerry takes 8 mL of medicine every 4 hour. How many hours before all the medicine is gone?

   Use the following prompts to guide thinking, if necessary:

   - State the problem in your own words.
   - What do each of the numbers in the problem represent—the whole, the number of groups or the quantity in each group?
   - What is the unknown in the problem—the whole, the number of groups or the quantity in each group?
   - What number sentence could you write to show the meaning of the problem?
   - Does the problem use multiplication or division or both? Explain.
   - *About* how hours before the medicine is all gone? Explain your thinking. Hint: Remind the student that the medicine is taken every 4 hours.
   - Use a strategy that makes sense to you to find the answer to the problem. Explain your thinking as you write the numbers.
   - Will any medicine be left over? Explain.
   - Explain how you know your answer makes sense and is reasonable.
   - Would you solve the problem another way? Explain your thinking.

3. Use a similar procedure as outlined in question 2 with the following problem:
   You read for 45 minutes every day. How many minutes do you read in 3 weeks?

4. Create a problem that can be shown by the number sentence $245 \div 6 = \square$.
   Solve the problem you created by using a strategy that makes sense to you.

C. Applied Learning

Provide opportunities for students to use multiplication and division in a practical situation and notice whether or not the strategies transfer. For example, ask the students to estimate and then calculate the number of baseball teams of 9 players that can be made from 175 people. Does the student:

- use an estimate that is reasonably close to the calculated answer?
- interpret the remainder correctly by ignoring it because the remaining four people will not make up a complete team?
- use a personal strategy that makes sense in calculating the answer?

Step 5: Follow-up on Assessment

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction?

A. Addressing Gaps in Learning

Students who have difficulty solving multiplication and division problems using a personal strategy with and without concrete materials will enjoy more success if one-on-one time is provided in which there is open communication to diagnose where the learning difficulties lie. Assessment by observing a student solving problems will provide valuable data to guide further instruction. Success in problem solving depends on a positive climate in which the students are confident in taking risks. By building on the existing understandings of each student and accommodating the individual learning styles, success will follow.

If the difficulty lies in understanding the problem, use the following strategies:

- Provide problems that relate to the students' interests; use the student's name in the problem.
- Use smaller numbers in the problem initially.
- Have the student paraphrase the problem.
- Guide the student to determine if the numbers refer to the whole, the number of groups or the quantity in each group.
- Ask the student if the unknown in the problem refers to the whole, the number of groups or the quantity in each group.
- Provide base ten materials for the students to represent the problem as needed.
- Have the student decide which operation should be used and why.
- Ask guiding questions to show the connections between multiplication and division and the possible option of using either operation in solving the problem.
- Provide a graphic organizer such as the K–N–W–S chart (see blackline master on page 29).

If the difficulty lies in using personal strategies to solve multiplication and division problems, use the following strategies:

- Use smaller numbers in the problems initially.
- Review place value and number facts.
- Provide base ten materials as needed.
- Think aloud a personal strategy that you would use to solve the problem and explain why this strategy is more efficient than another one that you describe.
- Emphasize flexibility in choosing a personal strategy; a strategy that is efficient for one student may not be efficient for another student.
- Build on the student's understanding of place value and number facts to guide him or her in finding a strategy that works.

- Provide ample time for the student to think and ask questions to clarify his or her thinking.
- Have the students work in groups so that they learn strategies from one another.
- Guide the students to critique various personal strategies to find one that can be used on a variety of problems efficiently.
- Have the students explain their personal strategies to the class so others can hear how they work in kid-friendly language.
- Post various personal strategies in the classroom for students to share and critique.
- Encourage the student to check the reasonableness of the answer by comparing the answer to the estimated answer provided earlier.

B. Reinforcing and Extending Learning

Students who have achieved or exceeded the outcomes will benefit from ongoing opportunities to apply and extend their learning. These activities should support students in developing a deeper understanding of the concept and should not progress to the outcomes in subsequent grades.

Consider strategies such as:

- Provide tips for parents on practising multiplication and division at home or in the community. For example,
  - Take the children shopping and have them estimate the total cost if you buy multiple copies of the same item.
  - Involve the children in real-life problem-solving activities such as calculating the total time spent reading if 45 minutes are spent reading each day. Convert the problem into a division problem by providing the total number of minutes spent reading for a given number of days and asking for the number of minutes spent reading each day if the time is the same for each day.
  - Talk to your children about data in the newspaper and magazines and encourage them to estimate products and quotients and explain how they are doing it.
- Have the students create problems showing the various types of multiplication and division problems (equal group problems including equal grouping and equal sharing, comparison problems, combination problems) and write appropriate number sentences for each one. These problems can be displayed in a chart on the bulletin board.
- Have the students create problems with different contexts but using the same numbers, such as 378 and 8. They could follow this up by having the class decide which of the problems could be solved using a given number sentence such as $378 \div 8 = \square$.
- Have the students solve multi-step problems that involve the multiplication or division of more than two numbers. Examples:
  - Joey places 15 rows of 24 chairs in the gym. Tracy places twice as many chairs as Joey. How many chairs are placed in the gym?
  - Eight hundred ninety-six apples are evenly distributed among 8 boxes. If one of the boxes is shared equally among 7 people, how many apples does each person receive?
  - Marcy and Tina buy a dozen pieces of chocolate fudge that each weigh 45 grams. If they share the fudge equally, how grams of fudge does each person receive?

  - How many different outfits can Ragan make if she has 15 different shirts, 4 different pairs of pants and 9 different pairs of socks?
  - You read 95 minutes a week. At that rate, how many minutes will you read in 126 days?
- Have the students convert single-step multiplication and division problems into multi-step problems and explain to other students how to solve them.
- Have the students critique other students' personal strategies and explain why they work or not. Which would be the most efficient and why?
- Have the students write an explanation for a personal strategy so that everyone in the class can understand it.

K What facts do I **KNOW** from the information in the problem?	**N** Which information do I **NOT** need?	**W** **WHAT** does the problem ask me to find?	**S** What **STRATEGY** will I use to solve the problem?

Adapted with permission from Mary Lee Barton and Clare Heidema, *Teaching Reading in Mathematics: A Supplement to Teaching Reading in the Content Areas Teacher's Manual* (2nd ed.) (Aurora, CO: McREL (Mid-continent Research for Education and Learning), 2002), p. 113.

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