

## Planning Guide

### Grade 5 *One-step Equations*

#### Patterns and Relations (Variables and Equations) Specific Outcomes 2 and 3

**This Planning Guide can be accessed online at:**

[http://www.learnalberta.ca/content/mepg5/html/pg5\\_onestepequations/index.html](http://www.learnalberta.ca/content/mepg5/html/pg5_onestepequations/index.html)

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## Planning Guide: *Grade 5 One-step Equations*

**Strand:** Patterns and Relations (Variables and Equations)

**Specific Outcomes:** 2 and 3

This *Planning Guide* addresses the following outcomes from the Program of Studies:

**Strand:** Patterns and Relations (Variables and Equations)

**Specific Outcomes:**

2. Express a given problem as an equation in which a letter variable is used to represent an unknown number (limited to whole numbers).
3. Solve problems involving single-variable, one-step equations with whole number coefficients and whole number solutions.

### Curriculum Focus

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The changes to the curriculum targeted by this sample include:

- The general outcome focuses on representing algebraic expressions in multiple ways; whereas the previous math curriculum focused on constructing, extending and summarizing patterns, including those found in nature, using rules, charts, mental mathematics and calculators.
- The specific outcome focuses on using letter variables in one-step equations to represent and solve problems; whereas the previous curriculum did not include solving equations using letter variables.

### What Is a Planning Guide

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**Planning Guides** are a tool for teachers to use in designing instruction and assessment that focuses on developing and deepening students' understanding of mathematical concepts. This tool is based on the process outlined in *Understanding by Design* by Grant Wiggins and Jay McTighe.

## Planning Steps

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The following steps will help you through the Planning Guide:

- **Step 1: Identify Outcomes to Address** (p. 5)
- **Step 2: Determine Evidence of Student Learning** (p. 7)
- **Step 3: Plan for Instruction** (p. 8)
- **Step 4: Assess Student Learning** (p. 30)
- **Step 5: Follow-up on Assessment** (p. 38)

## Step 1: Identify Outcomes to Address

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### Guiding Questions

- What do I want my students to learn?
- What can my students currently understand and do?
- What do I want my students to understand and be able to do based on the Big Ideas and specific outcomes in the program of studies?

### Big Ideas

Mathematics is often referred to as the science of patterns. Patterns permeate every mathematical concept and are found in everyday contexts. The various representations of patterns, including symbols and variables, provide valuable tools in making generalizations of mathematical relationships. Some characteristics of patterns include:

- There are different types of patterns that can be modelled in a variety of ways.
- Patterns using concrete and pictorial representations of a problem can be translated into patterns using numbers and symbols or letters in equations.
- Patterns are used to generalize relationships; e.g., numbers may be added or multiplied in any order without changing the sum or product.
- Equality is used to express relationships. The symbols used on either side of the equals sign represent a quantity. The equals sign is "a symbol of equivalence and balance" (NCTM 2000, p. 39).
- Patterns using symbols and variables provide a means of describing change mathematically; e.g., five more than, three less than.
- Variables and equations are used to mathematically model everyday problems that describe quantitative relationships.

Letter variables and symbols stand for any one of a set of numbers or objects and have multiple meanings. They are used as:

- representations of specific unknown values; e.g.,  $3 + \square = 15$ ,  $3 + n = 15$
- representations of quantities that vary or change; e.g.,  $\square + \triangle = 15$ ,  $x + y = 15$
- placeholders in a generalized expression or formula; e.g.,  $c = 100$  m, where  $c$  represents the number of centimetres and  $m$  represents the number of metres.

Adapted from John A. Van de Walle, LouAnn H. Lovin, *Teaching Student-Centered Mathematics: Grades 3–5*, 1e (pp. 290, 307). Published by Allyn and Bacon, Boston, MA. Copyright © 2006 by Pearson Education. Adapted by permission of the publisher.

Mathematician's rules or conventions dictate the use of variables for quantities that change or are dynamic.

- If the same variable or symbol is used repeatedly in the same equation, then there is only one possible solution for that variable or symbol; e.g.,  $n + n = 20$ ; the unique solution is  $n = 10$ .

- If different variables or symbols are used in the same equation, then there are many different possible solutions, one of which may be the same number replacing each variable or symbol; e.g.,  $x + y = 20$ , other solutions include  $0 + 20$ ,  $10 + 10$ ,  $\frac{1}{2} + 19\frac{9}{36}$ , and  $25 - 5$ .

The majority of the work done by young students in using variables and symbols is focused on specific unknowns. They "use symbolic representations and drawings to help them solve problems and communicate their ideas" (Van de Walle and Lovin 2006, p. 303).

Algebraic reasoning is directly related to patterns because this reasoning focuses on making generalizations based on mathematical experiences and recording these generalizations using symbols or variables (Van de Walle and Lovin 2006).

### Sequence of Outcomes from Program of Studies

See <http://education.alberta.ca/teachers/core/math/programs.aspx> for the complete program of studies.

Grade 4	➔	Grade 5	➔	Grade 6
<p><b>Specific Outcomes</b></p> <p>5. Express a given problem as an equation in which a symbol is used to represent an unknown number.</p> <p>6. Solve one-step equations involving a symbol to represent an unknown number.</p>		<p><b>Specific Outcomes</b></p> <p>2. Express a given problem as an equation in which a letter variable is used to represent an unknown number (limited to whole numbers).</p> <p>3. Solve problems involving single-variable, one-step equations with whole number coefficients and whole number solutions.</p>		<p><b>Specific Outcomes</b></p> <p>3. Represent generalizations arising from number relationships, using equations with letter variables.</p> <p>4. Express a given problem as an equation in which a letter variable is used to represent an unknown number.</p> <p>5. Demonstrate and explain the meaning of preservation of equality, concretely and pictorially.</p>

## Step 2: Determine Evidence of Student Learning

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### Guiding Questions

- What evidence will I look for to know that learning has occurred?
- What should students demonstrate to show their understanding of the mathematical concepts, skills and Big Ideas?

### Using Achievement Indicators

As you begin planning lessons and learning activities, keep in mind ongoing ways to monitor and assess student learning. One starting point for this planning is to consider the achievement indicators listed in the program of studies. You may also generate your own indicators and use them to guide your observation of the students.

The following indicators may be used to determine whether or not students have met this specific outcome. Can students:

- explain the purpose of the letter variable in a given addition, subtraction, multiplication or division equation with one unknown; e.g.,  $36 \div n = 6$ ?
- express a given pictorial or concrete representation of an equation in symbolic form?
- identify the unknown in a problem and represent the problem with an equation?
- create a problem for a given equation with one unknown?
- express a given problem as an equation where the unknown is represented by a letter variable?
- solve a given single-variable equation with the unknown in any of the terms; e.g.,  $n + 2 = 5$ ,  $4 + a = 7$ ,  $6 = r - 2$ ,  $10 = 2c$ ?
- identify the unknown in a problem, represent the problem with an equation and solve the problem concretely, pictorial or symbolically?
- create a problem for a given equation?

Sample behaviours to look for related to these indicators are suggested for some of the activities listed in **Step 3, Section C: Choosing Learning Activities** (p. 13).



## Step 3: Plan for Instruction

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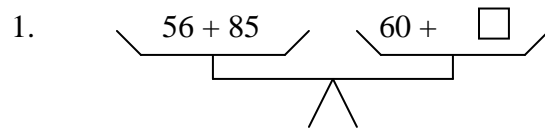
### Guiding Questions

- What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should I use?
- How will I meet the diverse learning needs of my students?

### A. Assessing Prior Knowledge and Skills

Before introducing new material, consider ways to assess and build on students' knowledge and skills related to counting. For example:

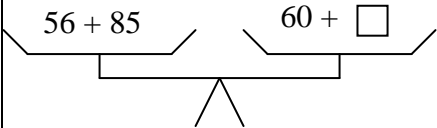
Have balance scales and centicubes available for the students to use as needed.



- Write an equation to describe the quantities on the balance scale.
  - Solve the equation. Explain your thinking.
2. Joey has 8 times as many stickers as Minnie. If Joey has 56 stickers, how many stickers does Minnie have?
- Write two different equivalent equations to represent the problem.
  - Solve the equation. Show your work.
  - Answer the question asked in the problem.
3.  $\triangle + 63 = 82$
- Draw a diagram to represent the equation.
  - Solve the equation. Show your work.
  - Write another equation that is equivalent to  $\triangle + 63 = 82$ .
4. Joey reads for 146 minutes and Jane reads for 225 minutes. Jane reads how many more minutes than Joey?
- Write two different equivalent equations to represent the problem.
  - Solve the equations. Show your work.
  - Answer the question asked in the problem.
5.  $3 \times \square = 24$
- Draw a diagram to represent the equation.
  - Solve the equation. Show your work.

If a student appears to have difficulty with these tasks, consider further individual assessment, such as a structured interview, to determine the student's level of skill and understanding. See **Sample Structured Interview: Assessing Prior Knowledge and Skills** (p. 10).

## Sample Structured Interview: Assessing Prior Knowledge and Skills

Directions	Date:	
	Not Quite There	Ready to Apply
<p>Place the following diagram before the student:</p>  <p>Say, <b>"Write an equation to describe the quantities on the balance scale."</b> Then say, <b>"Solve the equation. Explain your thinking."</b></p>	<p>Writes an incorrect equation to describe the quantities on the balance scale.</p> <p>Solves the equation with errors.</p> <p>Provides no explanation or a vague explanation of his or her thinking.</p>	<p>Writes the correct equation and solves it, explaining the process in detail.</p>
<p>Present the following problem before the student: Joey has 8 times as many stickers as Minnie. If Joey has 56 stickers, how many stickers does Minnie have?</p> <p>Say: <b>"Write 2 different equivalent equations to represent the problem. Solve the equation. Show your work. Answer the question asked in the problem."</b></p>	<p>Writes an incorrect equation to represent the problem or writes one correct equation but no equivalent equation.</p> <p>Makes errors in solving the equation and does not show work or shows confusing work.</p> <p>Has difficulty answering the question asked correctly or answer the question incorrectly.</p>	<p>Writes two different equivalent equations to represent the problem; e.g., <math>8 \times n = 56</math> and <math>56 \div 8 = n</math>.</p> <p>Solves the equation and explains the process.</p> <p>Correctly answers the question asked in the problem.</p>
<p>Place the following equation before the student:</p> $\triangle + 63 = 82$ <p>Say, <b>"Draw a diagram to represent the equation. Solve the equation. Show your work. Write another equation that is equivalent to <math>\triangle + 63 = 82</math>."</b></p>	<p>Does not draw a diagram to represent the equation or draws an incorrect diagram.</p> <p>Has difficulty solving the equation, making errors to show misunderstanding.</p> <p>Does not write another equation that is equivalent to the original equation.</p>	<p>Draws a diagram to correctly represent the equation.</p> <p>Solves the equation using an appropriate process.</p> <p>Write another equation that is equivalent to the original equation.</p>

<p>Present the following problem before the student:          Joey read for 146 minutes and Jane read for 225 minutes. Jane read for how many more minutes than Joey?</p> <p>Say,  <b>"Write two different equivalent equations to represent the problem. Solve the equations. Show your work. Answer the question asked in the problem."</b></p>	<p>Writes an incorrect equation to represent the problem or writes one correct equation but no equivalent equation.</p> <p>Makes errors in solving the equation and does not show work or shows confusing work.</p> <p>Has difficulty answering the question asked correctly or answer the question incorrectly.</p>	<p>Writes two different equivalent equations to represent the problem; e.g., <math>225 - 146 = n</math> and <math>146 + n = 225</math>.</p> <p>Solves the equation and explains the process.</p> <p>Answers the question asked in the problem.</p>
<p>Place the following equation before the student:</p> <p><math>3 \times \diamond = 24</math></p> <p>Say,  <b>"Draw a diagram to represent the equation. Solve the equation. Show your work."</b></p>	<p>Does not draw a diagram to represent the equation or draws an incorrect diagram.</p> <p>Has difficulty solving the equation, making errors to show misunderstanding.</p>	<p>Draws a diagram to correctly represent the equation; e.g., arrays.</p> <p>Solves the equation using an appropriate process.</p>

## B. Choosing Instructional Strategies

Consider the following general strategies for teaching the use of equations involving a letter variable to represent an unknown number.

- Build on students' knowledge from the previous grade in using equations with symbols to write addition, subtraction, multiplication and division equations.
- Discuss the similarities between using a symbol such as  $\Delta$  to represent an unknown in an equation and using a letter variable such as  $n$ .
- Connect the concrete, pictorial and symbolic representations consistently as students develop and demonstrate their understanding of equations.
- Use everyday contexts for problems that students can relate to so that they can translate the meaning of the problem into an appropriate equation using a letter variable to represent the unknown number.
- Provide a variety of problems for students to express as equations. Include problems that illustrate the various types of addition and subtraction (e.g., change, part-part-whole and comparison) as well as multiplication and division (e.g., of equal sharing, equal grouping, comparison problems and combination)
- Review the relationship between addition and subtraction number sentences as well as the relationship between multiplication and division number sentences.
- Have the students create problems for a variety of number sentences illustrating addition and subtraction, including examples of change, part-part-whole and comparison problems.
- Have the students create problems for a variety of number sentences illustrating multiplication and division, including examples of equal sharing, equal grouping, comparison problems and combination problems.
- Encourage the students to write equations in various ways to represent the meaning of a given problem.
- Include examples of equations in which the unknown is on the left or the right side of the equation; e.g.,  $n + 7 = 12$ ,  $7 + b = 15$ ,  $17 = r - 8$ ,  $48 = 6c$ .
- Emphasize that the equals sign is a symbol of equivalence or balance of the two quantities on either side of the equation.
- Use balance scales to represent the equation concretely and focus on the meaning of the equal sign.

## **C. Choosing Learning Activities**

The following learning activities are examples of activities that could be used to develop student understanding of the concepts identified in Step 1.

### **Sample Activities:**

#### **Teaching Students to Express a Given Problem as an Equation Using a Symbol to Represent an Unknown Quantity**

1. **Story Translations for Addition and Subtraction Problems** (p. 15)
2. **Story Translations for Multiplication and Division Problems** (p. 18)
3. **Creating Problems from Equations** (p. 22)

#### **Teaching Students to Solve One Step Equations Involving a Symbol to Represent an Unknown Quantity**

1. **Balance Scales and Number Sentences** (p. 24)
2. **Balance Scales and the Guess and Test Strategy for Solving Equations** (p. 25)
3. **Pan Balance Scales Using Known and Unknown Weights and Related Equations** (p. 27)
4. **Virtual Pan Balance—Shapes** (p. 28)
5. **Virtual Pan Balance—Numbers** (p. 29)
6. **Open Number Sentences** (p. 29)

## **Teaching Students to Express a Given Problem as an Equation Using a Symbol to Represent an Unknown Quantity**

## Sample Activity 1: Story Translations for Addition and Subtraction Problems

Build on students' understanding of writing equations using a symbol to represent the unknown.

Provide the students with part-part-whole and comparison word problems and have them explore the idea of a letter variable representing a specific, unknown quantity as they translate the problems into written equations. Review that the meaning of the equals sign is "equivalence or balance of the two quantities on either side of the equation."

Examples of problems:

<b>Part-part-whole</b>	<b>Whole Unknown</b>		<b>Part Unknown</b>	
	Connie has 15 red marbles and 28 blue marbles. How many marbles does she have?		Connie has 43 marbles. 15 are red and the rest are blue. How many blue marbles does Connie have?	
<b>Comparison</b>	<b>Difference Unknown</b>	<b>Unknown Big Quantity</b>		<b>Unknown Small Quantity</b>
	Connie has 15 red marbles and 28 blue marbles. How many more blue marbles than red marbles does Connie have?	Connie has 15 red marbles and some blue marbles. She has 13 more blue marbles than red ones. How many blue marbles does Connie have?		Connie has 28 blue marbles. She has 13 more blue marbles than red ones. How many red marbles does Connie have?

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### a. Part-part-whole problem

Cut strips of cash register tape into the following lengths:

- 46 cm and 54 cm
- 48 cm and 52 cm
- 52 cm and 48 cm
- 56 cm and 44 cm

Provide each student with one strip of the pre-cut cash register tape. Have them measure their strips and record these measurements on sticky notes they stick on the fronts of their shirts. Their challenge is to find a partner who has a strip that, when added to theirs, makes a strip that is exactly 1 m long.

### Look For ...

Do students:

- apply previous knowledge in understanding the four operations?
- apply previous knowledge of using symbols to represent unknowns in equations?
- draw appropriate diagrams to represent the types of problems for the four operations?
- relate addition to subtraction and multiplication to division in writing equations and solving problems?
- show flexibility in using a variety of letter variables to represent an unknown number in an equation?

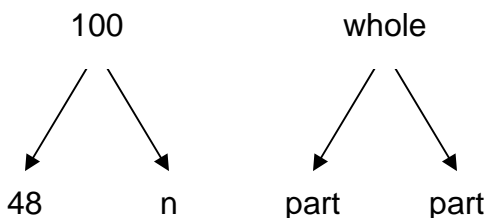


Encourage the students to use mental mathematics strategies to predict how long their partner's strip will have to be. Once they find their partners, have them verify their solutions by placing the strips together and comparing the combined length to a metre stick.

Ask the students to draw a diagram to represent the situation, perhaps using a solid line to represent their string and a dotted line to represent their partner's string.



Model this part–part–whole situation:



Through discussion, have the students verbalize various ways that equations can be written to represent the situation. Encourage the students to use a variety of letter variables. Ensure that students include equations in which the letter variable to represent the unknown quantity is on the left side and other equations in which the unknown quantity is on the right side.

$$48 + n = 100 \quad y + 48 = 100 \quad 100 = m + 48 \quad 100 = 48 + p$$

$$100 - 48 = s \quad f = 100 - 48 \quad 100 - c = 48 \quad 48 = 100 - z$$

Provide the students other part–part–whole problems to solve, with either the whole or one of the parts as the unknown. Have the students solve these problems using manipulatives, then draw appropriate diagrams and write equations with a letter variable for the unknown.

b. Comparison Situations

Provide the students with interlocking cubes (white and two other colours). Pose the following problem:

Connie has 15 red marbles and 24 blue marbles. How many more blue marbles than red marbles does Connie have?

Have the students model this situation by building two columns with the cubes, one representing the red marbles and the other representing the blue marbles. To find the difference between the two columns, white cubes are added to the red column to represent the difference between the two quantities. Have the students draw a diagram to represent the situation.

Encourage the students to write as many different equations as they can using letter variables to represent this situation. Ask the students to share their equations and ensure that a wide variety of equations are included.

$$15 + m = 24$$

$$q + 15 = 24$$

$$24 = 15 + d$$

$$24 = e + 15$$

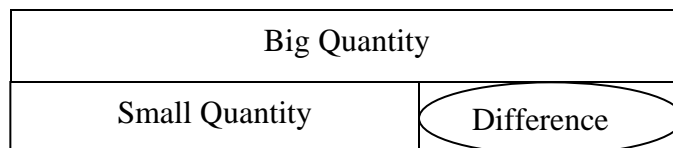
$$24 - g = 15$$

$$24 - 15 = h$$

$$15 = 24 - i$$

$$j = 24 - 15$$

Summarize the comparison problems by showing the students a general model that they can use to think about other comparison problems.



Note: Model suggested by Karen Fuson in "Meaning of Numerical Operations through Word Problem Solving: Access to All through Student Situational Drawings within an Algebraic Approach," a presentation at the Annual Conference for the National Council of Supervisors of Mathematics, St. Louis, April 26, 2006.

Have the students solve other comparison problems, with either the big quantity, small quantity or difference as the unknown. Encourage the students to draw diagrams and write equations with letter variables to represent the situations.

Note: In the primary grades, students have worked with symbols as specific unknowns in join and separate problems. Samples of these problems can also be provided as review.

Examples of problems:

	<b>Result Unknown</b>	<b>Change Unknown</b>	<b>Start Unknown</b>
<b>Join</b>	Connie had 15 marbles. Juan gave her 28 more marbles. How many marbles does Connie have altogether?	Connie has 15 marbles. How many more marbles does she need to have 43 marbles altogether?	Connie had some marbles. Juan gave her 15 more marbles. Now she has 43 marbles. How many marbles did Connie have to start with?
<b>Separate</b>	Connie had 43 marbles. She gave 15 to Juan. How many marbles does Connie have left?	Connie had 43 marbles. She gave some to Juan. Now she has 15 marbles left. How many marbles did Connie give to Juan?	Connie had some marbles. She gave 15 to Juan. Now she has 28 marbles left. How many marbles did Connie have to start with?

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Emphasize that the power of patterns and relations in solving problems is that the same equation can be used to represent a variety of different addition and subtraction problems.

## Sample Activity 2: Story Translations for Multiplication and Division Problems

Provide the students with equal grouping, equal sharing, comparison, area or array and combination word problems and have them explore the idea of a variable representing a specific, unknown quantity as they translate the problems into written equations. Review that the meaning of the equals sign is "equivalence or balance of the two quantities on either side of the equation."

Examples of problems:

<b>Problem Type</b>	<b>Multiplication</b> (given: number of groups and number of objects in each group)	<b>Measurement Division</b> (given: total number of objects and the number of objects in each group)	<b>Partitive Division</b> (given: total number of objects and the number of groups)
<b>Grouping/ Partitioning</b>  <b>Equal Groups</b>	Gene has 4 tomato plants. There are 6 tomatoes on each plant. How many tomatoes are there altogether?	Gene has some tomato plants. There are 6 tomatoes on each plant. Altogether there are 24 tomatoes. How many tomato plants does Gene have?	Gene has 4 tomato plants. There is the same number of tomatoes on each plant. Altogether there are 24 tomatoes. How many tomatoes are on each tomato plant?
<b>Multiplicative Comparison</b>	<p>The giraffe in the zoo is 3 times as tall as the kangaroo. The kangaroo is 6 feet tall. How tall is the giraffe?</p> <p>The giraffe is 18 feet tall. The kangaroo is 6 feet tall. The giraffe is how many times taller than the kangaroo?</p> <p>The giraffe is 18 feet tall. She is 3 times as tall as the kangaroo. How tall is the kangaroo?</p>		
<b>Area and Array</b>	<p>A farmer plants a rectangular vegetable garden that measures 2 m along one side and 5 m along an adjacent side. How many <math>m^2</math> of garden did the farmer plant?</p> <p>A baker has a pan of fudge that measures 8 inches on one side and 9 inches on another side. If the fudge is cut into square pieces 1 inch on a side, how many pieces of fudge does the pan hold?</p> <p>A farmer plans to plant a rectangular vegetable garden. She has enough room to make the garden 5 m on one side. How long does she have to make the adjacent side in order to have <math>10 m^2</math> of garden?</p>		
<b>Combinations</b>	The Friendly Old Ice Cream Shop has 2 types of cones (waffle and plain). They have 5 flavours of ice cream (chocolate, vanilla, strawberry, rainbow and tiger). How many one-scoop combinations of an ice cream flavour and cone type can you get at the Friendly Old Ice Cream Shop?		

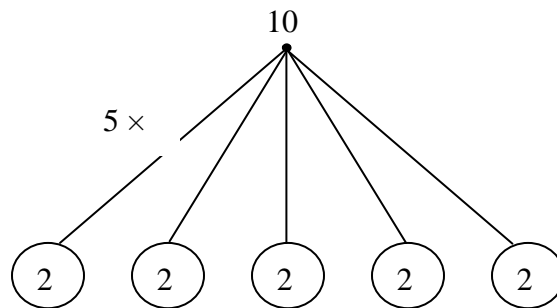
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## Models

Note: Models suggested by Karen Fuson in "Meaning of Numerical Operations through Word Problem Solving: Access to All through Student Situational Drawings within an Algebraic Approach," presentation at the Annual Conference of the National Council of Supervisors of Mathematics, St. Louis, April 26, 2006.

### Equal Grouping and Equal Sharing

1. Amy has 5 cousins. She is making 2 puppets for each cousin. How many puppets will Amy need to make?
2. Amy made 10 puppets to divide equally among her 5 cousins. How many puppets will each cousin get?
3. Amy made 10 puppets for her cousins. Each cousin will get 2 puppets. How many cousins does Amy have?



Have the students write equations to represent the situations, using as many different ways as possible. The equations for problem 2 could include the following:

$$10 \div 5 = a$$

$$t = 10 \div 5$$

$$10 \div u = 5$$

$$5 = 10 \div m$$

$$5 \times s = 10$$

$$10 = 5 \times w$$

$$r \times 5 = 10$$

$$10 = p \times 5$$

### Comparisons

1. Bill has 2 apples. Kim has **5 times** as many apples as Bill. How many apples does Kim have?
2. Kim has 10 apples. Bill has  $\frac{1}{5}$  as many apples as Kim. How many apples does Bill have?
3. Bill has 2 apples. Kim has 10 apples. Kim has how many times as many apples as Bill?

B 

2
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K 

2	2	2	2	2
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Have the students write equations to represent the situations, using as many different ways as possible. The equations for problem 3 could include the following:

$10 \div 2 = c$

$k = 10 \div 2$

$10 \div r = 2$

$2 = 10 \div b$

$2 \times e = 10$

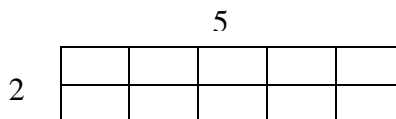
$10 = 2 \times y$

$q \times 2 = 10$

$10 = n \times 2$

### Array and Area

1. a. A garden has 2 rows and 5 columns of bean plants. How many plants are there in all?  
b. The garden is 2 m wide and 5 m long. What is its area?
2. a. A garden has 10 bean plants in 2 equal rows. How many columns does it have?  
b. The garden is  $10 \text{ m}^2$  in area. It is 2 m wide. How long is it?
3. a. A garden has 10 bean plants in 5 equal columns. How many rows does it have?  
b. The garden is  $10 \text{ m}^2$  in area. It is 5 m long. How wide is it?



Have the students write equations to represent the situations using as many different ways as possible. The equations for problem 2 could include the following:

$10 \div 2 = a$

$y = 10 \div 2$

$10 \div e = 2$

$2 = 10 \div c$

$2 \times m = 10$

$10 = 2 \times n$

$q \times 2 = 10$

$10 = p \times 2$

### Combinations

1. Paco is making sandwiches on white bread and rye bread. The fillings are cheese, tuna, ham, peanut butter and egg salad. How many combinations can he make?
2. Paco made 10 different sandwiches. He used 5 kinds of fillings. How many kinds of bread did he use?
3. Paco made 10 different sandwiches. He used 2 kinds of bread. How many kinds of fillings did he use?

	C	T	H	P	E
W	WC	WT	WH	WP	WE
R	RC	RT	RH	RP	RE

Have the students write equations to represent the situations using as many different ways as possible. The equations for problem 3 could include the following:

$$10 \div 2 = a$$

$$z = 10 \div 52$$

$$10 \div w = 2$$

$$2 = 10 \div b$$

$$2 \times d = 10$$

$$10 = 2 \times g$$

$$r \times 2 = 10$$

$$10 = u \times 2$$

Emphasize that the power of patterns and relations in solving problems is that the same equation can be used to represent a variety of different multiplication and division problems.

## Sample Activity 3: Creating Problems from Equations

Build on students' understanding of creating problems from equations using a symbol to represent the unknown. Then have the students create problems from the equation using a letter variable to represent the unknown. Discuss the similarities and differences between using a symbol and a letter variable to represent an unknown in an equation.

Provide the students with an equation using a letter variable to represent the unknown; e.g.,  $48 \div n = 6$ . Discuss and write equations that are equivalent to the equation provided. Instruct the students to create a problem that could be represented by the given equation or one of the equivalent equations. Then tell the students to draw and label a diagram to represent the situation described in the problem. Encourage the students to create more than one problem that could be represented by the given equation; e.g., equal grouping and equal sharing. Have the students share their problems and critique them for clarity and mathematical content.

### Look For ...

Do students:

- apply the four operations to everyday contexts?
- apply previous knowledge of using symbols to represent unknowns in equations?
- provide examples of equivalent equations?
- communicate clearly their examples of problems related to the four operations?

## **Teaching Students to Solve One Step Equations Involving a Symbol to Represent an Unknown Quantity**



## Sample Activity 1: Balance Scales and Number Sentences

Build on students' understanding of using balance scales and writing equations using a symbol to represent the unknown. Progress to using a letter variable to represent the unknown.

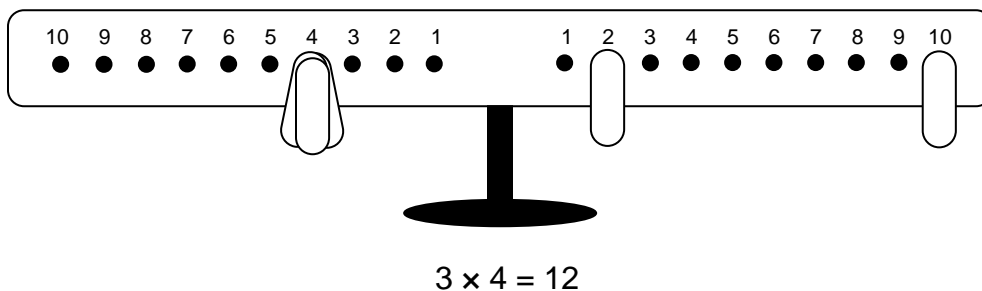
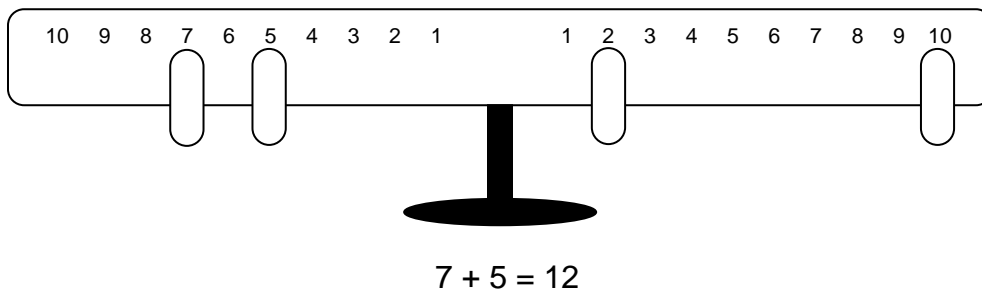
Provide the students with a pan balance and blocks (centicubes) or another type of balance scale to show equality. The students will find the value of specific unknown numbers as they explore the meaning of the equal sign in the context of both additive and multiplicative relationships.

Have the students use the balance scale to find different ways to express the numbers 12, 18, 24 and 32 as either 2 addends or a multiplicative expression. Tell the students to write an appropriate equation for each representation on the balance scale.

### Look For ...

Do students:

- apply previous knowledge of using symbols to represent unknowns in equations?
- represent equations on a balance scale?
- write equations that are represented on a balance scale by using letter variables?



## Sample Activity 2: Balance Scales and the Guess and Test Strategy for Solving Equations

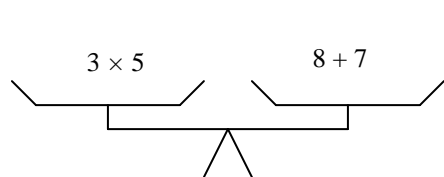
Provide the students with balance scales pictured in the previous activity or with pan balances and centicubes to use in solving one step equations involving a letter variable to represent an unknown number; e.g.,  $3 \times n = 18$ .

Model the guess and test strategy as one strategy that may be used to solve the equation, for example:

Think aloud that  $3 \times n = 18$  means three equal groups of something must balance 18. Say that you will try 3 groups of 4 on one side of the balance scale and 18 on the other side of the scale. Since the scale tips to show that 3 groups of 4 are not heavy enough, say that you must adjust the guessed answer to make it higher; e.g., 3 groups of 6. Show that 3 groups of 6 balance 18 on the scale, this situation represents the equation and can be written as  $3 \times \underline{6} = 18$ ,  $18 = 3 \times \underline{6}$ ,  $18 \div 3 = \underline{6}$  and  $8 \div \underline{6} = 3$ . Therefore, the solution to the equation is  $n = 6$ .

Explain that for larger numbers, students should keep track of their guesses in a table.

To provide further reinforcement, draw a simple 2-pan balance scale with a numeric expression in each pan. Have the students decide whether the pan will tilt or balance and write a corresponding equation to illustrate the situation. Students may test their hypothesis using a real balance scale.



$$3 \times 5 = 8 + 7$$

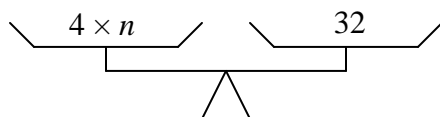
Equation



$$3 \times 5 > 2 \times 6$$

Inequality

Then include a letter variable on one side of the balance scale as shown below:

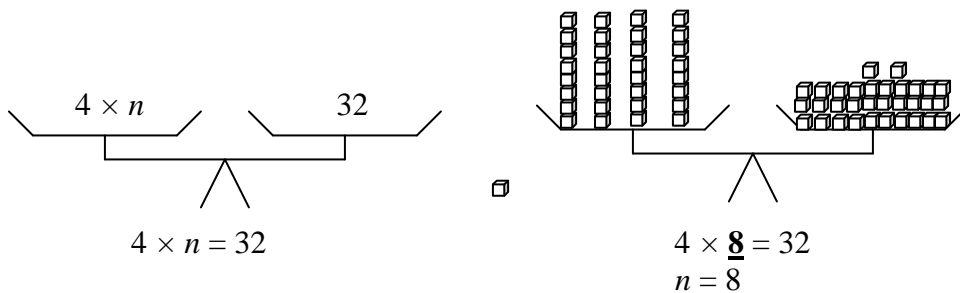


Remind the students that since the scale is balanced, an equation can be written to represent the situation illustrated. Have the students replicate the situation using blocks (centicubes) and a balance scale. Then have them write the equation and the solution.

### Look For ...

Do students:

- apply their knowledge of the four operations in solving equations using guess and test?
- apply previous knowledge of using symbols to represent unknowns in equations?
- show that equality means the quantities on either side of the scale are balanced?
- use logical reasoning in applying the guess and check strategy; i.e., use the results of the previous guess to make the next guess?

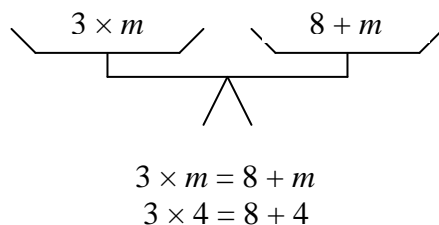


Four groups of 8 is the same quantity as 32.

Encourage the students to write the equation in equivalent ways with the unknown on the left side of the equation in some examples and on the right side of the equation in other examples.

Include other examples using letter variables with varying levels of difficulty. Encourage the students to write the equations and then use the blocks and the balance scales to solve them. If necessary, model the use of guess and test as one strategy in finding the value for the unknown that will balance both sides of the equation. Keep track of the guesses made.

Example:



Have the students create their own equation puzzles using one unknown represent by a letter variable, and challenge their peers to solve the problem using the balance scale to verify their equations.

### Sample Activity 3: Pan Balance Scales Using Known and Unknown Weights and Related Equations

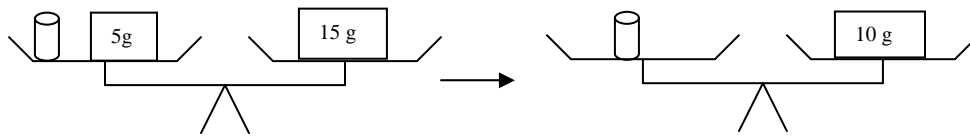
Provide the students with pan balance scales, weights and containers filled with pinto beans. The containers with pinto beans should all weigh the same and the weight depends on the equations that students are to solve by using the balance scales. Reminder: 1 centicube weighs 1 g.

For example, if the students are to solve the equation  $n + 5 = 15$ , then  $n$  represents the weight of the container and pinto beans, which is 10 g.

Present the students with a problem such as the following:  
If you add 5 g to the weight of an object (in this case the container filled with pinto beans) you get a total weight of 15 g. What is the weight of the object? Use the balance scales to solve the problem. Draw a diagram and write equations to show your work.

<p><b>Look For ...</b></p> <p>Do students:</p> <ul style="list-style-type: none"><li><input type="checkbox"/> apply prior knowledge in using balance scales?</li><li><input type="checkbox"/> apply previous knowledge of using symbols to represent unknowns in equations?</li><li><input type="checkbox"/> use related number sentences solve equations and maintain equality?</li><li><input type="checkbox"/> translate easily among the concrete, pictorial and symbolic modes?</li></ul>
--

Sample student work:



$$\begin{aligned} w + 5 &= 15 \\ w &= 15 - 5 \quad \text{using a related equation with subtraction} \\ w &= 10 \end{aligned}$$

The weight of the object is 10 g.

Provide other problems that students solve by using the balance scales and weights, writing the related equations and answering in a sentence with a numerical value and the proper units.

## **Sample Activity 4: Virtual Pan Balance—Shapes**

Provide students with computers that are linked to the Internet and have them access the following Web site: <http://illuminations.nctm.org/ActivityDetail.aspx?ID=33>, and follow the instructions.

## **Sample Activity 5: Virtual Pan Balance—Numbers**

Provide students with computers that are linked to the Internet and have them access the following Web site: <http://illuminations.nctm.org/ActivityDetail.aspx?ID=26>, and follow the instructions.

## Sample Activity 6: Open Number Sentences

Use open number sentences as short warm-ups to start a mathematics class. They help the students understand letter variables as specific unknowns as students develop efficient mental mathematics strategies for computation.

Write an open number sentence on the board and ask the students how to make the statement true. Have the students justify their responses; e.g., using balance models, comparing distances on a number line.

$$\begin{aligned}23 + 15 &= b \\38 &= 23 + q \\38 &= r \\33 + 25 &= t + 23 \\23 + 15 &= g + 14\end{aligned}$$

By alternating the letter variable used in these equations, students understand that changing the letter variable used does not change the equation.

$$\begin{aligned}23 + e &= 40 \\23 + h &= 40 \\23 &= 40 - k\end{aligned}$$



### Look For ...

Do students:

- apply the properties of operations in solving open number sentences?
- apply previous knowledge of using symbols to represent unknowns in equations?
- use compensation in solving open number sentences?
- show flexibility in using a variety of letter variables to represent an unknown number in open number sentences?

## Step 4: Assess Student Learning

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### Guiding Questions

- Look back at what you determined as acceptable evidence in Step 2.
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

In addition to ongoing assessment throughout the lessons, consider the following sample activities to evaluate students' learning at key milestones. Suggestions are given for assessing all students as a class or in groups, individual students in need of further evaluation, and individual or groups of students in a variety of contexts.

### A. Whole Class/Group Assessment

#### Apple Picking Problems and Equations

In this assessment task, students will demonstrate their understanding of representing a problem with an equation using a letter variable for the unknown. They will solve a problem about picking apples by first identifying the unknown and writing an equation. They will then explain the solution of the equation by connecting the pictorial and symbolic representations. Finally, students will create a problem given an equation with a letter variable representing the unknown and justify their choice of problem.

Materials required if needed by the students: balance scale, centicubes.

Task-specific Criteria:

- to assist later in developing the scoring guide
- should be written in a way that answers "how well" the student achieved the outcome(s)

Each student will:

- represent a problem with an equation where the unknown is represented by a letter variable and justify his or her choice of equation
- draw a diagram to represent and solve the problem
- connect the pictorial and symbolic solutions to solve the problem
- explain in words the process of solving a problem by using an equation
- create a problem that can be represented by a given equation with the unknown represented by a letter variable
- explain why a created problem represents a given equation.

Since an equation has an equal sign, then a pictorial representation to illustrate the equals sign includes a balance scale. Students should draw objects on the balance scale to keep the scale balanced as they solve the equation. With each diagram drawn, students write the math symbols for the solution of the equation.

Suggestions for early finishers:

- Students can write equations that are equivalent to the equation that they wrote in Question 1 and explain why they are equivalent.
- Students can write equations that are equivalent to the equation provided in Question 3 and explain why they are equivalent.
- Students can create other problems along with appropriate equations using letter variables.



## Apple Picking Problems and Equations—Student Assessment Task

Jenny and Johnny are picking apples from the tree in their back yard. Jenny picks 3 times as many apples as Johnny. If Jenny picks 24 apples, how many apples does Johnny pick?



1. Express the problem as an equation using a letter variable to represent the unknown number. Explain how you know the equation represents the problem.
2. Solve the problem. Explain your thinking by using diagrams, symbols and words.
3. Create another problem about picking apples that can be represented by the following equation:  $25 = n + 7$ . Explain how you know that your problem can be represent by this equation.

Student \_\_\_\_\_

<p><b>SCORING GUIDE</b></p> <p><b>Apple Picking Problems and Equations</b></p>
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Level  Criteria	<b>4</b> <b>Excellent</b>	<b>3</b> <b>Proficient</b>	<b>2</b> <b>Adequate</b>	<b>1</b> <b>Limited *</b>	<b>Insufficient /</b> <b>Blank *</b>
<p><b>Representing a problem with an equation using a letter variable</b></p> <p><b>Question #1</b></p>	<p>Represents the problem with an equation using a letter variable and justifies the choice of equation with sound mathematical thinking.</p>	<p>Represents the problem with an equation using a letter variable and justifies the choice of equation clearly.</p>	<p>Represents the problem with an equation using a letter variable and provides limited justification for the choice of equation.</p>	<p>Represents the problem with an incorrect equation and provides no justification for his or her choice of equation.</p>	<p>No score is awarded because there is insufficient evidence of student performance based on the requirements of the assessment task.</p>
<p><b>Solving an equation – connecting pictorial and symbolic representations</b></p> <p><b>Question #2</b></p>	<p>Solves the problem by drawing accurate diagrams, connects these diagrams to appropriate symbols and explains the process clearly and in detail.</p>	<p>Solves the problem by drawing accurate diagrams, connects these diagrams to appropriate symbols and explains the process clearly .</p>	<p>Solves the problem by drawing diagrams with some inaccuracy, connects these diagrams to appropriate symbols and provides limited explanation of the process.</p>	<p>Solves the problem incorrectly by drawing inaccurate, incorrect or no diagrams, shows minimal or no connection between the diagrams and symbols and provides a vague explanation or no explanation of the process.</p>	<p>No score is awarded because there is insufficient evidence of student performance based on the requirements of the assessment task.</p>
<p><b>Creating a problem for a given equation</b></p> <p><b>Question #3</b></p>	<p>Creates an insightful and well-worded problem for the given equation and explains clearly and in detail how the problem relates to the equation.</p>	<p>Creates a correctly worded problem for the given equation and explains clearly and how the problem relates to the equation.</p>	<p>Creates a problem with some wording difficulties for the given equation and provides a limited explanation of how the problem relates to the equation.</p>	<p>Creates a problem with vague and inaccurate wording for the given equation and provides a vague explanation or no explanation of the process.</p>	<p>No score is awarded because there is insufficient evidence of student performance based on the requirements of the assessment task.</p>

\* When work is judged to be limited or insufficient, the teacher makes decisions about appropriate intervention to help the student improve.

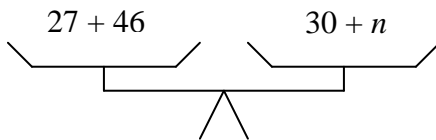
<p><i>Student Learning Goals</i></p> <hr style="border: 0; border-top: 1px solid black; margin: 5px 0;"/> <hr style="border: 0; border-top: 1px solid black; margin: 5px 0;"/> <hr style="border: 0; border-top: 1px solid black; margin: 5px 0;"/>
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## B. One-on-one Assessment

Have base ten materials, a balance scale and centicubes available for the student to use as needed.

Draw on the student's understanding of representing the meaning of a problem by writing an equation with a symbol for the unknown problem. In the following problems, the letter can be replaced with a symbol initially and then changed to a letter variable to illustrate that solving an equation with a symbol to represent the unknown or a letter variable are the same except for the symbol or letter used.

1. Present the following labelled diagram of a balance scale to the student and ask him or her to answer the questions.



- a. Write an equation to describe the quantities on the balance scale.
- b. Solve the equation. Explain your thinking.

The student may have difficulty interpreting the diagram, so suggest that he or she use a balance scale and centicubes to show what is in the diagram. Encourage the student to rearrange the centicubes to show  $30 + 43$  on the left side of the balance scale (take 3 away from 46 and add them to 27, keeping the total number of centicubes the same).

2. Present the following problem to the student and have him or her answer the questions. Terry has 8 times as many marbles as Sammy. If Terry has 56 marbles, how many marbles does Sammy have?
  - a. Write two different equivalent equations using letter variables to represent the problem.
  - b. Solve the equations. Show your work.
  - c. Answer the question asked in the problem.

If difficulty arises, replace the numbers in the problem with smaller numbers. This usually helps the students determine which operation to use and how to write the number sentence. Remind the student to represent the unknown quantity with a letter variable.

Provide the student with base ten blocks or counters to use if he or she is struggling with number facts. Help the student snap together 56 blocks and have the student explore how these blocks can be separated into 8 equal groups. Encourage the student to draw an appropriate diagram.

If difficulty still exists and the student writes  $8 \times 56 = y$ , ask him or her who has more marbles, Terry or Sammy. Remind the student that multiplying a number by 8 makes the answer larger. Suggest that the student write 8 groups of some unknown number (representing Sammy's marbles) is 56. Remind the student that putting equal groups together can be written as a multiplication sentence.

If the student has difficulty writing an equivalent equation, remind him or her that division and multiplication are related. Another option to consider is that the unknown can be written on either side of the equals sign to make an equivalent equation.

3. Present the following equation to the student and have him or her answer the questions.

$$b - 23 = 48$$

- Draw a diagram to represent the equation.
- Solve the equation.
- Write another equation that is equivalent to  $x - 23 = 48$ .

If the student has difficulty drawing a diagram, have him or her make up a real world problem and then illustrate the problem with a diagram; e.g., 23 cm is cut off a ribbon and 48 cm remain. What was the length of the ribbon at the beginning?

If the student has difficulty solving the equation, remind him or her that addition is related to subtraction so the student could write an equivalent addition statement. This would also help with part c.

4. Present the following problem to the student and have him or her answer the questions. Emma jogs for 66 minutes on Monday and for 93 minutes on Tuesday. She jogs for how many more minutes on Tuesday than on Monday?

- Write two different equivalent equations using letter variables to represent the problem.
- Solve the equations.
- Answer the question asked in the problem.

If the student has difficulty deciding which operation to use in writing equivalent equations, change the numbers to smaller numbers and have the student write an equation with smaller numbers. The larger numbers can then be substituted for the smaller numbers to complete the correct equations.

If the student writes a subtraction equation but is unable to write an equivalent equation, remind him or her that addition is related to subtraction.

If the student has difficulty solving the equation, provide base ten materials and have him or her represent 93 with the materials. Beneath these materials, have the student represent 66. Then have the student count to determine how many blocks must be added to 66 to obtain 93. Encourage the student to draw an appropriate diagram.

5. Present the following equation to the student and have him or her answer the questions.

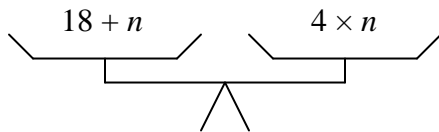
$$4 \times y = 32$$

- Draw two different diagrams to represent the equation.
- Solve the equation.

If the student has difficulty drawing the diagrams, suggest that he or she create a real world problem that involves multiplication. If the student cannot create a problem, create contexts for him or her. Some examples include:

- There are 4 equal groups of birds that make a total of 32 birds. How many birds are in each group? The student draws 4 groups and places an equal number of birds in each group until he or she obtains a total of 32 birds.
- There are 4 rows of tiles with same number of tiles in each row making a total of 32 tiles. How many tiles are in each row? The student draws an array to show the tiles.
- You have 4 times as long a ribbon as your friend. If your ribbon is 32 cm long, how long is your friend's ribbon? The student uses a ruler and draws a line segment 32 cm long and breaks it up into 4 equal lengths.

6. Present the following labelled diagram of a balance scale to the student and have him or her answer the questions.



- Write an equation to represent the situation shown on the balance scale.
- Solve the equation. Explain your thinking.

If the student has difficulty writing the equation, remind him or her that the balance scale is balanced so it represents the same quantity on both sides so an equals sign can be used to connect the quantity in each pan.

If the student has difficulty solving the equation, suggest that he or she use guess and test starting with a simple number like two. Have the student try out the various guesses using the balance scale and the centicubes. Remind the student that whatever number he or she chooses, that number must be used in place of the unknown on both sides of the equation. Encourage the student to keep track of his or her guesses by recording them in a table.

7. Present the student with the following equation:

$$8 = 96 \div r$$

Ask the student to create a problem that can be represented by this equation.

Ask the student to solve the problem that he or she created and to show all his or her work.

If the student has difficulty creating a problem, ask him or her to give examples of division used in everyday life. Also, start with smaller numbers and the student might be successful in creating a problem. Then have the student substitute the larger numbers for the smaller numbers to complete the question. If the student has difficulty with division, remind him or her that division is related to multiplication so he or she could think of 8 equal groups of something totalling 96.

### **C. Applied Learning**

Provide opportunities for students to use equations with letter variables in practical situations and notice whether or not the understanding of equations transfers. For example, ask how many booklets can be made to create stories for primary students if there are 126 pages and each booklet has 9 pages. Does the student:

- represent the problem by using an appropriate equation using a letter variable?
- recognize that the answer to the problem must be less than 126?
- need to use smaller numbers initially to determine what operation to use when solving the problem?
- represent the problem by counting the pages and removing groups of 9 until all the pages have been used?
- use a variety of division and multiplication equations with letter variables to represent the problem when asked to do so?
- respond that more booklets can be made if asked to determine the effect of using 6 pages per booklet instead of 9?

## Step 5: Follow-up on Assessment

### Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction?

### A. Addressing Gaps in Learning

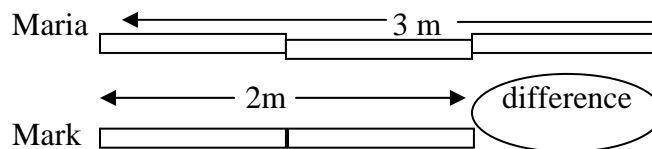
If the student appears to be having difficulty representing the meaning of a problem by writing an equation with a letter variable for the unknown problem, provide scaffolding, for example:

- Build on students' understanding of representing the meaning of a problem by writing an equation with a symbol. Then replace the symbol with a letter variable.
- Use problems with everyday contexts that are meaningful to the students so that they are motivated and are able to understand and explain the meaning of the problem.
- Use friendly numbers in the problems initially so that students can easily use manipulatives and/or draw diagrams to represent the problem and then connect them to the symbolic representation.
- Encourage the students to solve a simpler problem by substituting smaller whole numbers in the problem so that it is easier for them to represent it using manipulatives and/or diagrams and determine which operation to use when writing the equation and solving the problem.

For example, given the following comparison problem:

Maria runs 350 m in the same time that Mark runs 250 m. How much farther does Maria run than Mark in the given time?

Change the problem to read, "Maria runs 3 m in the same time that Mark runs 2 m. How much farther does Maria run than Mark in the given time?"



$3 - 2 = n$  or  $2 + n = 3$ . Therefore,  $n = 1$ .

Maria runs 1 m farther than Mark in the given time.

Using the original numbers in problem,  $350 - 250 = y$  or  
 $250 + y = 350$ . Therefore,  $y = 100$ .

Maria runs 100 m farther than Mark in the given time.

- Use a variety of manipulatives, including balance scales, to represent the meaning of equality so that students are able to apply this understanding in writing equations with an unknown to represent different kinds of problems.
- Encourage the students to explain orally the meaning of the problem and justify why they choose to represent the meaning of the problem with a specific equation. Provide guidance as necessary to ensure that the equation does represent the meaning of the problem.

- Reinforce that a variety of letter variables may be used to represent an unknown in a problem; e.g.,  $45 + n = 350$  or  $45 + y = 350$ . Clarify that the symbol used to represent the unknown can change without changing the relationship between the numbers in the problem.
- Clarify the mathematician's rule:
  - An equation with two identical symbols has only one possible solution, that is, the same symbol used within one equation must represent the same number; e.g.,  $n \times n = 36$ ,  $n = 6$ .
  - An equation with two different symbols means that these symbols represent quantities that can vary rather than a specific unknown,  $n \times m = 36$ ,  $n$  equals 6 and  $m$  also equals 6; also, these symbols represent other values whose product is 36 such as 4 and 9.
- Review the relationship between addition and subtraction problems and between multiplication and division problems by representing them with manipulatives and diagrams before writing the related equations.
- Encourage the students to write equivalent equations using a letter variable for the unknown; e.g.,  $8 + n = 15$ ,  $15 = 8 + n$ ,  $15 - n = 8$ ,  $8 = 15 - n$ .
- Provide the students with an equation using a letter variable and have them create a variety of problems that could be represented by that equation.

If the student appears to be having difficulty solving a one-step equation with a letter variable for the unknown provide scaffolding as follows:

- Build on students' understanding of number sentences and equations using a symbol for the unknown from previous grades.
- Encourage the students to use manipulatives to represent the equation and then draw a corresponding diagram.
- Have the students explain their thinking and provide guidance to overcome any misconceptions or misunderstandings.
- Translate the equation into a problem with an everyday context that the student understands and reinforce why the problem represents the equation.
- Have the students use manipulatives and diagrams to show the relationship between addition and subtraction, emphasizing that they are opposite operations, and then have the students write the related number sentences, using letter variables. Similarly, review the relationship between multiplication and division.
- Encourage the students to solve a simpler equation by substituting smaller whole numbers in the equation so that it is easier for them to represent it using manipulatives and/or diagrams and determine which operation to use when solving the equation.
- Review the role of letter variables in equations, including the mathematician's rule, as described in the problem-solving section located just prior to these statements.
- Reinforce understanding of equations by integrating them in every strand and emphasize the power of equations in mathematics.



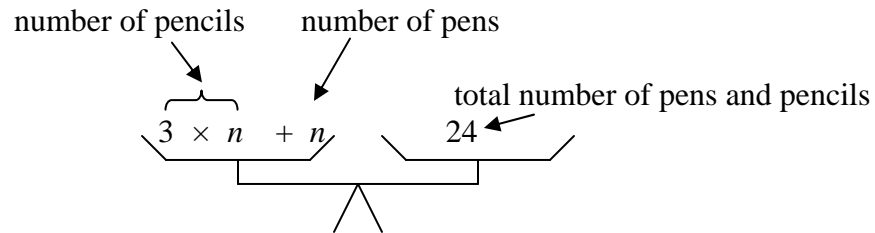
## B. Reinforcing and Extending Learning

Students who have achieved or exceeded the outcomes will benefit from ongoing opportunities to apply and extend their learning. These activities should support students in developing a deeper understanding of the concept and should not progress to the outcomes in subsequent grades.

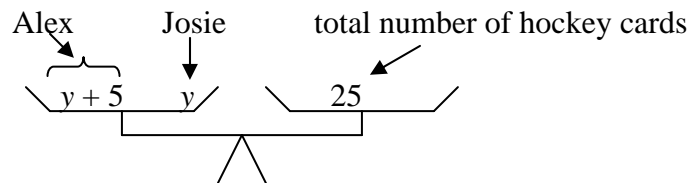
Consider strategies such as:

- Provide tips for parents on practising writing and solving equations at home or in the community. For example, encourage the student to write equations using letter variables for each of the following as they solve the problems:
  - Ask what the weight of each bag of dog food is if the total weight of 5 bags is given.
  - Ask how many pieces of cutlery are needed in all for a dinner party of 8 people if each person has five pieces of cutlery.
  - Ask how much longer one student reads than another student, given the number of minutes each student reads.
  - Include problems with money; e.g., if your student receives \$24, which is 3 times what his or her sibling receives, ask how much money the sibling would receive.
- Have the students create problems showing the various types of addition and subtraction problems (change, part-part-whole, comparison) and write appropriate equations for each one, using letter variables to represent the unknowns. These problems can be displayed on the bulletin board.
- Have the students create problems showing the various types of multiplication and division problems (equal group problems including equal grouping and equal sharing, comparison problems, combination problems) and write appropriate equations for each one, using letter variables to represent the unknowns. These problems can be displayed on the bulletin board.
- Have the students create problems with different contexts but using the same numbers such as 72 and 8. Follow this up by having the class decide which of the problems could be solved using a given number sentence such as  $72 \div 8 = m$ .
- Provide the students with a variety of story problems and have them draw diagrams and write equations to represent the situations, using letter variables for the unknowns. Encourage the students to write many equivalent equations to represent each situation. Examples of story problems:
  - You have 3 boxes of pencils with the same number of pencils in each box. There are 36 pencils in all.
  - You can make 24 different outfits by matching one shirt with one pair of pants. If you have 8 different shirts, how many different pairs of pants do you have?
  - A red ribbon is 36 cm long and a blue ribbon is 63 cm long. The blue ribbon is how much longer than the red ribbon?
  - You have 23 coins. Eight of the coins are quarters and the rest are dimes. How many dimes do you have?
  - You picked 48 apples. You picked 6 times as many apples as Tiny Tim. How many apples did Tiny Tim pick?
  - You have 78 cookies to share equally among 6 friends. How many cookies does each of your friends receive?

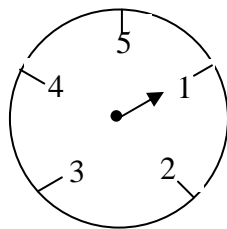
- Provide opportunities for the students to use the guess and test strategy together with appropriate diagrams and charts as they solve problems in which a letter variable is used to represent an unknown number. Examples:
  - You have three times as many pencils as pens. If you have 24 pens and pencils altogether, how many of each do you have?



- Two students have a collection of hockey cards. Alex has 5 more cards than Josie. If they have 25 hockey cards altogether, how many cards do they each have (Proulx 2006)?



- Use clock arithmetic to reinforce writing and solving equations in which a letter variable is used to represent an unknown number. Clock arithmetic uses a clock with a given number of hours on it, such as the standard 12-hour clock or a 5-hour clock. The clock has only one hand.



Provide equations for the students to solve using the 5-hour clock. Examples:

- $4 + 3 = n$       Answer: 2
- $m - 4 = 3$       Answer: 2
- $3 \times p = 2$       Answer: 4
- $2 \div 3 = r$       Answer: 4 To divide, look for a related multiplication sentence; e.g.,  $3 \times r = 2$ .

Have the students write equations for other students to solve. Change the clock to include a different number of hours such as 6 hours, 8 hours or 12 hours (Cathcart 1997).

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