Mathematics

Planning Guide

Grade 5 *Quadrilaterals*

Shape and Space
(3-D Objects and 2-D Shapes)
Specific Outcomes 6 and 7

This Planning Guide can be accessed online at:
http://www.learnalberta.ca/content/mepg5/html/pg5_quadrilaterals/index.html

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Planning Guide: *Grade 5 Quadrilaterals*

Strand: Shape and Space (3-D Objects and 2-D Shapes)
Specific Outcomes: 6, 7

This *Planning Guide* addresses the following outcomes from the Program of Studies:

Strand: Shape and Space (3-D Objects and 2-D Shapes)

Specific Outcomes:

6. Describe and provide examples of edges and faces of 3-D objects, and sides of 2-D shapes that are:
   - parallel
   - intersecting
   - perpendicular
   - vertical
   - horizontal.
7. Identify and sort quadrilaterals, including:
   - rectangles
   - squares
   - trapezoids
   - parallelograms
   - rhombuses

   according to their attributes.

Curriculum Focus

The changes to the curriculum targeted by this sample include:

- The general outcome focuses on describing the characteristics of 3-D objects and 2-D shapes, and analyzing the relationships among them; whereas the previous math curriculum focused on describing, classifying, constructing and relating 3-D objects and 2-D shapes, using mathematical vocabulary.
- The specific outcomes focus on most of the same concepts related to quadrilaterals and the relationship between line segments as the previous math curriculum; however, the previous curriculum included these concepts in Grade 4 not in Grade 5.
- The specific outcomes focus on parallel, intersecting, perpendicular, vertical and horizontal edges (line segments) and faces; whereas the previous curriculum focused primarily on the line segments.

What Is a Planning Guide?

Planning Guides are a tool for teachers to use in designing instruction and assessment that focuses on developing and deepening students' understanding of mathematical concepts. This tool is based on the process outlined in *Understanding by Design* by Grant Wiggins and Jay McTighe.

Planning Steps

The following steps will help you through the Planning Guide:

Step 1: Identify Outcomes to Address

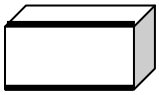
Guiding Questions

- What do I want my students to learn?
- What can my students currently understand and do?
- What do I want my students to understand and be able to do based on the Big Ideas and specific outcomes in the program of studies?

Big Ideas

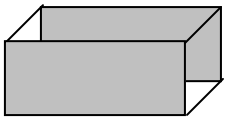
The terms *parallel*, *intersecting*, *perpendicular*, *vertical* and *horizontal* can be used to describe the relationship between two edges or faces of 3-D objects, and between two sides of 2-D shapes. For example, the first 3-D object has a pair of parallel edges darkened and the second 3-D object has a pair of parallel faces shaded.

3-D object #1
with parallel edges darkened



The face of the 3-D object is a rectangle with the opposite sides parallel.

3-D object #2
with parallel faces shaded



Van de Walle and Lovin (2006) explain the role of geometric properties in describing the similarities and differences among various shapes:

> What makes shapes alike and different can be determined by an array of geometric properties. For example, shapes have sides that are parallel, perpendicular, or neither; they have line symmetry, rotational symmetry, or neither; they are similar, congruent, or neither (p. 204).

Principles and Standards for School Mathematics states that instructional programs relating to the geometry standard should enable all students to "analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships" (NCTM 2000, p. 164). It goes on to explain that students should use "drawings, concrete materials, and geometry software to develop and test their ideas . . . about why geometric relationships are true" (NCTM 2000, p. 166).

The van Hiele levels of understanding in geometry describe the development in students' learning:

Level 0: Visualization
The objects of thought at level 0 are shapes and what they "look like."… It is the appearance of the shape that defines it for the student. . . . The products of thought at level 0 are classes or groupings of shapes that seem to be "alike."

Level 1: Analysis
The objects of thought at level 1 are classes of shapes rather than individual shapes. . . . At this level, students begin to appreciate that a collection of shapes goes together because of properties; e.g., all cubes have six congruent faces, and each of the faces is a square. . . . The products of thought at level 1 are the properties of shapes.

Level 2: Informal Deduction
The objects of thought at level 2 are the properties of shapes. . . . For example, four congruent sides and at least one right angle can be sufficient to define a square. Rectangles are parallelograms with a right angle. . . . The products of thought at level 2 are relationships among properties of geometric objects.

Level 3: Deduction
The objects of thought at level 3 are relationships among properties of geometric objects. . . . The products of thought at level 3 are deductive axiomatic systems for geometry.

Level 4: Rigor
The objects of thought at level 4 are deductive axiomatic systems for geometry. . . . The products of thought at level 4 are comparisons and contrasts among different axiomatic systems of geometry.

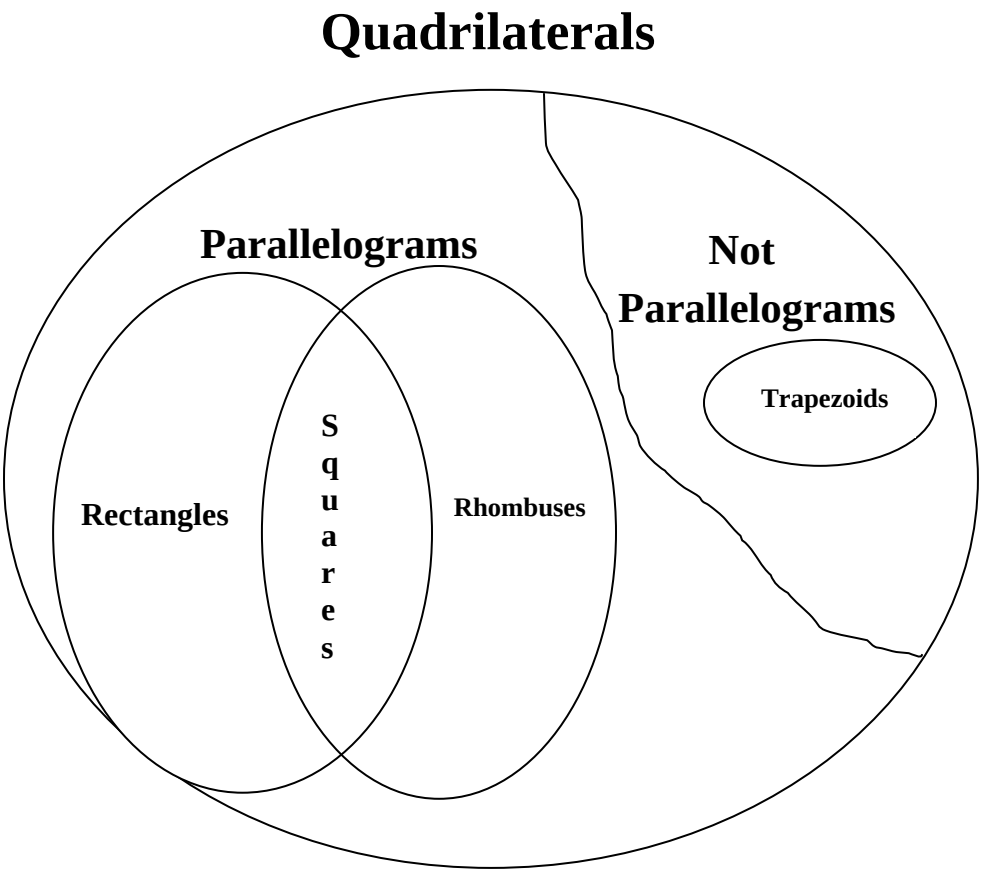
The focus in Grade 5 is on levels 1 and 2.

Definitions:
Polygon: a closed figure with three or more sides.
Quadrilateral: a four-sided polygon.
Rectangle: a quadrilateral with four right angles.
Square: a quadrilateral with four equal sides and four right angles.
Trapezoid: a quadrilateral with only one pair of parallel sides.
Parallelogram: a quadrilateral with two pairs of parallel sides.
Rhombus: a quadrilateral with two pairs of parallel sides and four equal sides.

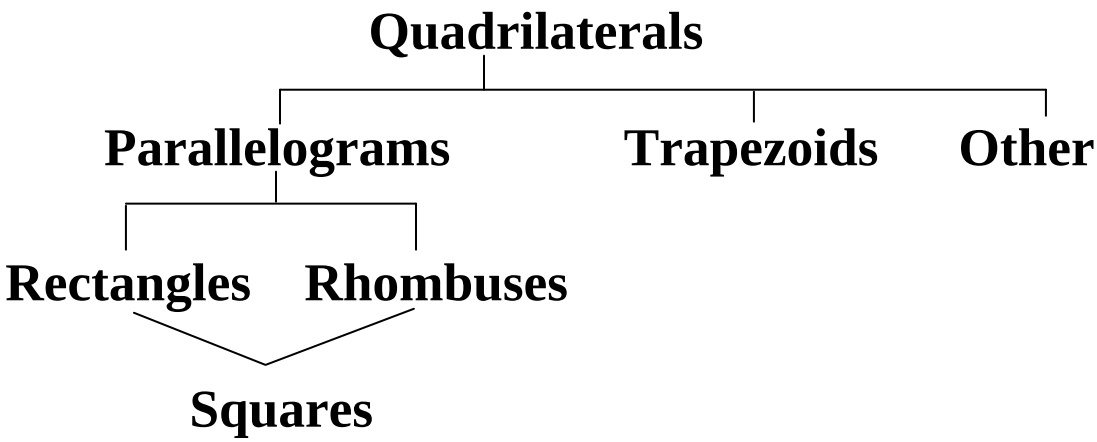
Another source for definitions is the *Mathematic Glossary* at http://www.ronblond.com/MathGlossary/.

The relationship among the various quadrilaterals is shown in the Venn diagram and the tree diagram that follow.

Venn Diagram



Tree Diagram



Sequence of Outcomes from Program of Studies

See http://education.alberta.ca/teachers/core/math/programs.aspx for the complete program of studies.

Grade 4	Grade 5	Grade 6
Specific Outcomes	**Specific Outcomes**	**Specific Outcomes**
4. Describe and construct right rectangular and right triangular prisms.	6. Describe and provide examples of edges and faces of 3-D objects, and sides of 2-D shapes that are: • parallel • intersecting • perpendicular • vertical • horizontal. 7. Identify and sort quadrilaterals, including: • rectangles • squares • trapezoids • parallelograms • rhombuses according to their attributes.	4. Construct and compare triangles, including: • scalene • isosceles • equilateral • right • obtuse • acute in different orientations. 5. Describe and compare the sides and angles of regular and irregular polygons.

Step 2: Determine Evidence of Student Learning

Guiding Questions

- What evidence will I look for to know that learning has occurred?
- What should students demonstrate to show their understanding of the mathematical concepts, skills and Big Ideas?

Using Achievement Indicators

As you begin planning lessons and learning activities, keep in mind ongoing ways to monitor and assess student learning. One starting point for this planning is to consider the achievement indicators listed in the *Mathematics Kindergarten to Grade 9 Program of Studies with Achievement Indicators*. You may also generate your own indicators and use them to guide your observation of the students.

The following indicators may be used to determine whether or not students have met this specific outcome. Can students:

- identify parallel, intersecting, perpendicular, vertical and horizontal edges and faces on 3-D objects?
- identify parallel, intersecting, perpendicular, vertical and horizontal sides on 2-D shapes?
- provide examples from the environment that show parallel, intersecting, perpendicular, vertical and horizontal line segments?
- find examples of edges, faces and sides that are parallel, intersecting, perpendicular, vertical and horizontal in print and electronic media, such as newspapers, magazines and the Internet?
- draw 2-D shapes that have sides that are parallel, intersecting, perpendicular, vertical or horizontal?
- draw 3-D objects that have edges and faces that are parallel, intersecting, perpendicular, vertical or horizontal?
- describe the faces and edges of a given 3-D object, using terms such as parallel, intersecting, perpendicular, vertical or horizontal?
- describe the sides of a given 2-D shape, using terms such as parallel, intersecting, perpendicular, vertical or horizontal?
- identify and describe the characteristics of a pre-sorted set of quadrilaterals?
- sort a given set of quadrilaterals and explain the sorting rule?
- sort a given set of quadrilaterals according to the lengths of the sides?
- sort a given set of quadrilaterals according to whether or not opposite sides are parallel?
- use intersecting diagonals (perpendicular and not perpendicular, bisected or not bisected) to identify and sort quadrilaterals, including rectangles, squares, trapezoids, parallelograms and rhombuses?

- explain the relationship among the set of quadrilaterals, using properties to describe the essential characteristics of each quadrilateral; e.g., all squares are rectangles because all squares have four sides and four right angles like rectangles but they also have four equal sides, making them special rectangles?

Sample behaviours to look for related to these indicators are suggested for some of the activities listed in **Step 3, Section C: Choosing Learning Activities** (p. 12).

Step 3: Plan for Instruction

Guiding Questions

- What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should I use?
- How will I meet the diverse learning needs of my students?

A. Assessing Prior Knowledge and Skills

Before introducing new material, consider ways to assess and build on students' knowledge and skills related to counting. For example:

Provide the students with pattern blocks, modelling clay, a set of 3-D solids, including right rectangular prisms and right triangular prisms and nets for the 3-D solids including right rectangular prisms and right triangular prisms.

1. Using pattern blocks, modelling clay or nets, construct a model of a:
   a. right rectangular prism
   b. right triangular prism

2. Describe at least four similarities and at least two differences between right rectangular prisms and right triangular prisms:

   Similarities Differences

3. Trina said that a cube is a right rectangular prism. Is Trina correct? Explain why or why not.

If a student appears to have difficulty with these tasks, consider further individual assessment, such as a structured interview, to determine the student's level of skill and understanding. See **Sample Structured Interview: Assessing Prior Knowledge and Skills** (p. 11).

Sample Structured Interview: Assessing Prior Knowledge and Skills

Directions	Date:	
	Not Quite There	**Ready to Apply**
Provide the students with pattern blocks, modelling clay, a set of 3-D solids including right rectangular prisms and right triangular prisms, and nets for the 3-D solids, including right rectangular prisms and right triangular prisms. Say, **"Using pattern blocks, modelling clay or nets, construct a model of a:** **a. right rectangular prism** **b. right triangular prism."**	Makes errors in constructing at least one of the prisms and/or the prisms are inaccurate.	Constructs both prisms correctly and accurately.
Say, **"Describe at least four similarities and at least two differences between right rectangular prisms and right triangular prisms."** Place the following headings on a piece of paper before the student: Similarities/Differences.	Describes some but not the required number of similarities and differences between right rectangular prisms and right triangular prisms. Descriptions are vague and incomplete; e.g., the differences include descriptions of only one type of prism but not both—the triangular prism has two bases in the shape of triangles.	Describes all the required number of similarities and differences between right rectangular prisms and right triangular prisms. Descriptions are clear and use correct mathematical language; e.g., one difference might be that right triangular prisms have two parallel and congruent triangular bases, whereas right rectangular prisms have two parallel and congruent rectangular bases (which includes square bases).
Say, **"Trina said that a cube is a right rectangular prism. Is Trina correct? Explain why or why not."**	Says that Trina is incorrect or says that she is correct but provides no explanation or a vague explanation.	Says that Trina is correct and provides a clear explanation using correct mathematical language; e.g., Trina is correct because all squares are rectangles. A cube has six congruent square faces, the bases are parallel and meet the lateral sides at right angles, so a cube is a right rectangular prism.

B. Choosing Instructional Strategies

Consider the following general strategies for teaching the characteristics of 3-D objects and 2-D shapes:

- Connect new geometric concepts to previous concepts learned.
- Connect the geometric concepts to the real world.
- Integrate the specific outcomes; e.g., focus on parallel, intersecting, perpendicular, vertical and horizontal sides of quadrilaterals.
- Provide hands-on experiences by having students construct various 3-D objects and 2-D shapes and then focus on their properties.
- Use concept attainment with examples and non-examples of a given geometric concept to promote discussion and critical thinking so that students construct their own meanings and write their own definitions.
- Have the students discuss the similarities and differences between two geometric concepts.
- Provide various activities to consolidate learning of the various geometric concepts; e.g., Frayer Model, Venn diagram, tree diagram and semantic feature chart.

C. Choosing Learning Activities

The following learning activities are examples of activities that could be used to develop student understanding of the concepts identified in Step 1.

Sample Activities:

1. **Concept Attainment for Geometric Concepts** (p. 13)
2. **Connections Among Geometric Concepts** (p. 16)
3. **Venn Diagrams and Tree Diagrams** (p. 17)
4. **Open and Closed Sorts** (p. 18)
5. **Property Lists for Quadrilaterals** (p. 19)
6. **Minimal Defining Lists** (p. 20)
7. **What Is My Shape?** (p. 21)
8. **Frayer Model** (p. 22)
9. **True or False** (p. 24)
10. **Semantic Feature Chart** (p. 25)
11. **Using Technology to Draw 2-D Shapes and 3-D Objects** (p. 26)

Sample Activity 1: Concept Attainment for Geometric Concepts

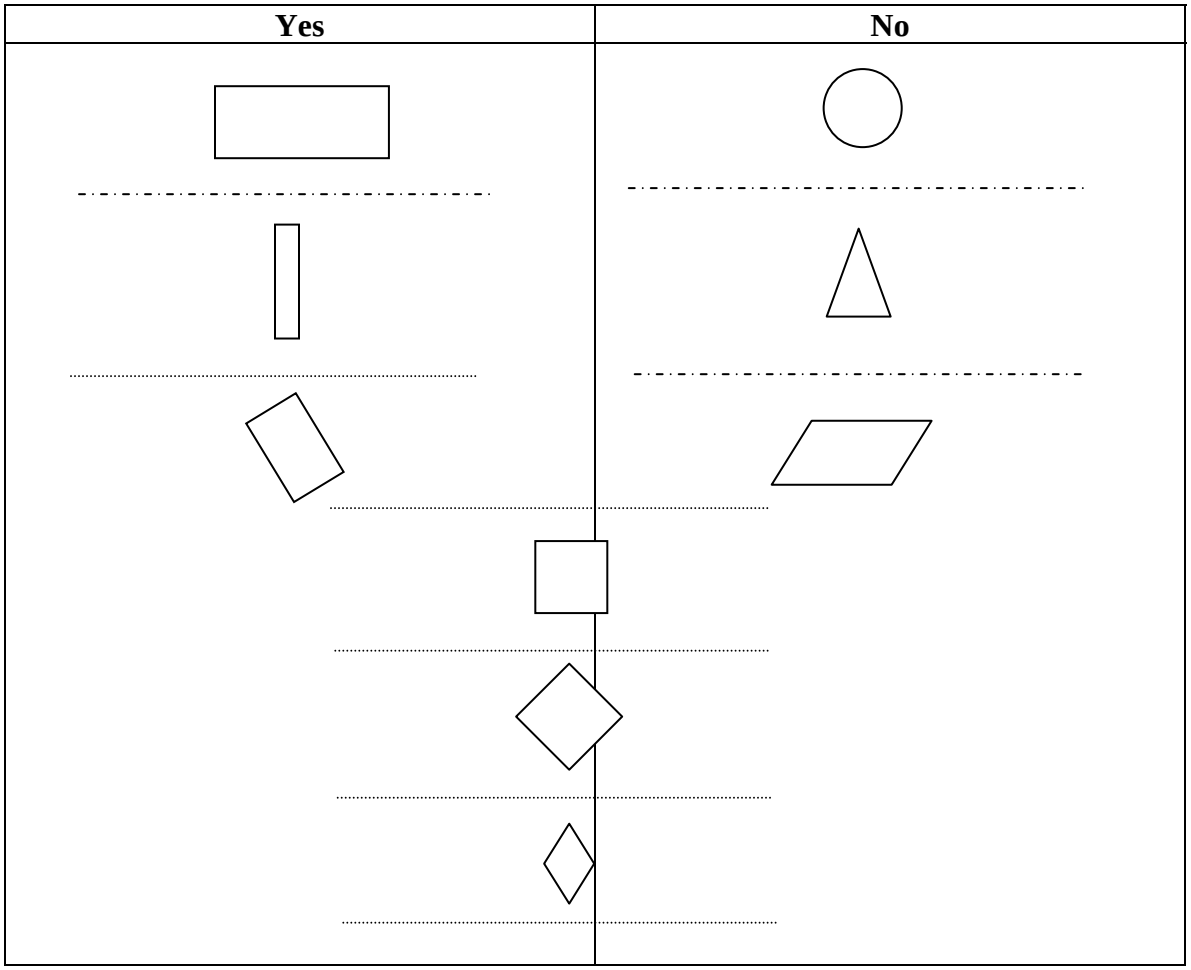
Use concept attainment to stimulate students' thinking about what visual representations show the various concepts by focusing on examples and non-examples.

Use a Yes/No chart on the overhead projector or drawn one on the board.

Tell the students that you are thinking of a geometric concept and will place examples of this concept in the "Yes" column and non-examples in the "No" column.

Have the students explain the differences between the figures in the two columns by describing the properties, not naming the shapes. Use the students' responses to decide on the next example and non-example to use.

The following example of concept attainment focuses on rectangles.



Place a rectangle in the "Yes" column and a circle in the "No" column. Students might say that the figure in the "Yes" column has straight sides and the one in the "No" column has no sides. Since you also want to focus on the number of sides, show the next example of a different rectangle in the "Yes" column and a triangle in the "No" column. Have the students explain how the two examples in the "Yes" column are the same and how they are different from the two examples in the "No" column; e.g., the two figures in the "Yes" column each have four sides, whereas the two figures in the "No" column do not have four sides.

To focus on right angles, place the third rectangle in the "Yes" column and a parallelogram with no right angles in the "No" column. Again, have the students describe the properties of all the examples in the "Yes" column that do not apply to the non-examples in the "No" column.

Look For …

Do students:

- ☐ explain how the examples in the "Yes" column are the same by focusing on the characteristics they have in common but are different from the examples in the "No" column?
- ☐ decide into which column to place a given set and justify their choice?
- ☐ transfer the learning about properties to write definitions in their own words?

Place the next example, a square, on the line between the "Yes" and the "No" columns and have the students decide where they think the square should go and justify their responses. Since you are focusing on the properties of a rectangle, place the square in the "Yes" column because it has four sides and four right angles and so is a rectangle.

Continue with other figures placed on the line between the "Yes" and "No" columns. After discussion, place each example in the correct column to continue developing the concept of a rectangle. Proceed with examples and non-examples until the class discussion summarizes that all the examples in the "Yes" column have the following properties that do not apply to the non-examples in the "No" column:

- 4 sides
- 4 right angles.

At the very end, invite the students to suggest a name that applies to all the examples in the "Yes" column but does not apply to any of the non-examples in the "No" column; i.e., rectangles.

Then have the students write their own definitions of a rectangle by describing the necessary and sufficient conditions as well as drawing diagrams.

Sample Activity 2: Connections Among Geometric Concepts

Once the students understand various geometric concepts through the use of examples and non-examples in concept attainment (see Activity 1), have them discover and explain the relationships among them.

Examples:

- When a vertical and a horizontal edge, face or side intersect, they result in perpendicular edges, faces or sides.
- All perpendicular edges, faces or sides intersect; however, not all intersecting edges, faces or sides are perpendicular.
- Two vertical edges, faces or sides are parallel. Two horizontal edges, faces or sides are parallel.
- All squares are rectangles because all squares have exactly four sides and four right angles, which are the characteristics that define a rectangle.
- All squares are rhombuses because all squares are quadrilaterals with equal sides, which are the characteristics that define a rhombus.

Look For …

Do students:

- ☐ apply what they learned in the concept attainment activity to writing mathematically correct statements about geometric concepts?
- ☐ critique statements written by others and explain their reasoning?
- ☐ transfer their learning about properties to write definitions in their own words?

Sample Activity 3: Venn Diagrams and Tree Diagrams

Instruct the students to show the connections among geometric concepts by drawing a tree diagram or a Venn diagram. Explain that these diagrams show the hierarchy of the concepts. See Step 1 for an example of a Venn diagram and a tree diagram showing the relationship among the set of quadrilaterals.

Look For …

Do students:

- ☐ demonstrate their understanding of the relationship among quadrilaterals by drawing Venn or tree diagrams?
- ☐ interpret and critique Venn or tree diagrams drawn by others?

Sample Activity 4: Open and Closed Sorts

Provide the students with a variety of quadrilaterals as well as 2-D shapes that are not quadrilaterals. Blackline masters 20 to 26 can be downloaded from the Web site at http://www.ablongman.com/vandewalleseries.

Open Sort
Instruct the students to sort the 2-D shapes into a certain number of groups and describe the sorting rule. Then have the sort them a different way and describe the sorting rule.

Closed Sort
Provide the students with the categories into which they are to sort the 2-D shapes; e.g., parallelograms, trapezoids, other 2-D shapes. Have the students sort the 2-D shapes into the given categories and explain their reasoning.

Look For …

Do students:

- ☐ sort 2-D shapes into two or more groups and describe the sorting rule?
- ☐ sort 2-D shapes into two or more groups in a different way and describe the sorting rule?
- ☐ critique sorting rules created by other students?
- ☐ sort 2-D shapes into given categories and explain their thinking?

Sample Activity 5: Property Lists for Quadrilaterals

Provide the students with three or four examples of a given quadrilateral; e.g., a rhombus. Have the students work in groups to find as many properties of the given quadrilateral that they can. Explain that each property listed must apply to all the diagrams of the quadrilateral provided to them. Blackline masters 30 to 33 for this activity are available from the Web site at http://www.ablongman.com/vandewalleseries.

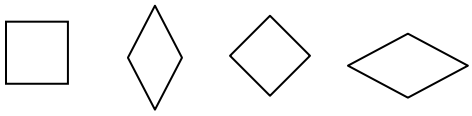
Look For …

Do students:

- ☐ list properties for a given quadrilateral by examining the similarities of a given group of diagrams?
- ☐ explain how each property listed applies to all the diagrams in a given group?

A sample is provided below.

Rhombuses



Properties of sides:

Properties of angles:

Properties of symmetry:

Properties of diagonals:

Sample Activity 6: Minimal Defining Lists

Look For …

Do students:

- ☐ apply the property lists for quadrilaterals from Activity 5 to create minimal defining lists for a given quadrilateral?
- ☐ choose defining or necessary properties for a given quadrilateral and explain their thinking?
- ☐ choose minimal or the least number of properties that are sufficient in describing a given quadrilateral and explain their thinking?

This activity is a follow-up to the activity "Property Lists for Quadrilaterals."

After the property lists for quadrilaterals have been completed, discussed and corrected as a class, post them in the classroom or duplicate them for distribution to the students.

Discuss the meaning of *defining* and *minimal*.
Defining: necessary properties that describe a particular quadrilateral; e.g., a necessary property of a square is that all sides are congruent.

Minimal: the shortest list possible of the necessary properties that is sufficient to completely describe a particular quadrilateral; e.g., sufficient properties of a square are that it is a rectangle in which all sides are congruent.

Present the following problem to the students:

Problem:
Find "minimal defining lists" for each quadrilateral; i.e., a subset of the properties for a quadrilateral that is "defining" and "minimal." Find at least two "minimal defining lists" for each quadrilateral: rectangle, square, trapezoid, parallelogram and rhombus.

Sample Activity 7: What Is My Shape?

Draw various quadrilaterals, one on each sticky label. Place one label on the back of each student. Present the following problem to the students.

Problem:
Identify the shape on the label on your back by asking questions with the following guidelines:

- Ask only two questions of any one student. Then move on to another student.
- The questions must have a "yes" or "no" answer.
- There are four possible answers to a question:
  - Yes
  - No
  - I don't understand. Please ask it another way.
  - I don't know how to answer that.

Look For …

Do students:

- ☐ ask questions about the properties of their mystery shape?
- ☐ use answer from one question to guide the next question ?
- ☐ correctly answer the questions asked of them?
- ☐ draw a diagram of their mystery shape to apply the knowledge they obtained through questioning their classmates?

Instruct the students to move the label from the back to the front when they have correctly identified the quadrilateral. Model some questioning strategies as needed and provide guidance as necessary.

Extensions:

- Ask the students to find a partner that has a quadrilateral that matches theirs in some way. Share the matches with the class and have the students explain their thinking.
- Instruct the students to form groups and explain their rules for grouping. For example, one group may say, "Our quadrilaterals all have four right angles."

Sample Activity 8: Frayer Model

Provide the students with a template for the Frayer Model and have them fill in the sections individually or as a group to consolidate their understanding of a geometric concept such as rhombus.

A sample of a Frayer Model follows.

Look For …

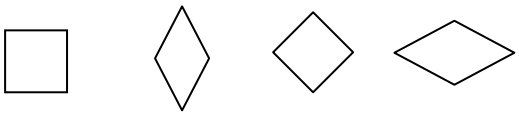
Do students:

- ☐ define rhombus in their own words?
- ☐ describe the essential characteristics of a rhombus?
- ☐ provide examples of a rhombus by drawing diagrams?
- ☐ provide non-examples of a rhombus by drawing diagrams?

Frayer Model for a Rhombus

Definition	Characteristics
A rhombus is a quadrilateral with four congruent sides.	• a polygon: a closed 2-D figure with sides and no curves • a four-sided figure • has four congruent sides • has two lines of symmetry • diagonals may or may not be equal • diagonals bisect each other • diagonals intersect at right angles

Rhombus

Examples	Non-examples
	

Format adapted from D. A. Frayer, W. C. Frederick and H. J. Klausmeier, *A Schema for Testing the Level of Concept Mastery* (Working Paper/Technical Report No. 16) (Madison, WI: Research and Development Center for Cognitive Learning, University of Wisconsin, 1969). Adapted with permission from the Wisconsin Center for Education Research, University of Wisconsin-Madison.

Sample Activity 9: True or False

Provide the students with statements that pertain to the relationship among lines and also the relationship among quadrilaterals.
Have the students explain their reasoning and change the false statements to make them true.

Sample statements:

- All quadrilaterals are trapezoids.
- All rhombuses are parallelograms.
- All squares are rectangles.
- All squares are rhombuses.
- All parallelograms are rhombuses.
- All intersecting lines are perpendicular.
- All perpendicular lines are intersecting.
- All perpendicular lines have vertical and horizontal lines.
- All intersecting vertical and horizontal lines are perpendicular.

Look For …

Do students:

- ☐ apply their knowledge of related geometric concepts to decide whether the statements are true or false and explain their thinking?
- ☐ correct the false statements to make them true and explain their thinking?
- ☐ create other statements for students to answer true or false?

Sample Activity 10: Semantic Feature Chart

Explain the purpose of the semantic feature chart to the students:

The semantic feature chart is a graphic organizer that helps you understand a concept by comparing its features to those of other concepts that fall into that same category. The chart helps you understand the similarities and differences between the various concepts. The completed chart provides a visual reminder of how certain concepts are alike or different. Through using this strategy, you are led to discover both the shared and unique characteristics of the concepts and associated vocabulary words.

Provide the students with a template of a semantic feature chart and tell them to write the names of the different quadrilaterals in the first column. Then have them suggest some properties of quadrilaterals that can be placed along the top of the chart. Have the students complete the chart by using an "X" to indicate when a property applies to the term. Encourage the students to explain the rationale for their choices.

Note: Explain that the property listed must apply to <u>all</u> types of any given quadrilateral before they mark the X to indicate that it applies to the term.

Example:

Properties / Terms	Has 4 sides	2 pairs of parallel sides	Opposite sides are congruent	All sides are congruent	Has 4 right angles
Quadrilateral	X				
Parallelogram	X	X	X		
Rhombus	X	X	X	X	
Rectangle	X	X	X		X
Square	X	X	X	X	X
Trapezoid	X				

Sample Activity 11: Using Technology to Draw 2-D Shapes and 3-D Objects

Have the students use a variety of technology tools to draw 2-D shapes and highlight or colour sides that are parallel, intersecting, perpendicular, vertical or horizontal.

Similarly, have the students draw 3-D objects and highlight or colour edges or faces that are parallel, intersecting, perpendicular, vertical or horizontal.

Examples of technology:

- Word processing programs have autoshapes, including 2-D shapes and 3-D objects along with a draw and paint program.
- Geometer's Sketchpad® can be used to draw 2-D shapes and 3-D objects using commands such as parallel, intersecting or perpendicular.

Look For …

Do students:

- ☐ use technology appropriately to draw 2-D shapes and 3-D objects?
- ☐ use technology to highlight or colour the sides of 2-D shapes that are parallel, intersecting, perpendicular, vertical or horizontal?
- ☐ use technology to highlight or colour the faces or edges of 3-D objects that are parallel, intersecting, perpendicular, vertical or horizontal?

Other ideas for teaching the relationship among quadrilaterals can be found on pages 137–140 of the *Diagnostic Mathematics Program, Division II, Geometry*.

Step 4: Assess Student Learning

Guiding Questions

- Look back at what you determined as acceptable evidence in Step 2.
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

In addition to ongoing assessment throughout the lessons, consider the following sample activities to evaluate students' learning at key milestones. Suggestions are given for assessing all students as a class or in groups, individual students in need of further evaluation, and individual or groups of students in a variety of contexts.

A. Whole Class/Group Assessment

Quadrilaterals and Relationships Between Sides; 3-D Objects and Perpendicular Faces
In this assessment task, students will demonstrate their understanding of perpendicular faces by shading a pair of perpendicular faces on a diagram of a 3-D object. This is only a sample of concepts in specific outcome 6 related to 3-D objects. Then the students will draw quadrilaterals and identify parallel, intersecting, perpendicular, vertical and horizontal sides. They will sort a given set of quadrilaterals into groups: rectangles, squares, trapezoids, parallelograms and rhombuses. Finally, they will describe the essential properties of each type of quadrilateral.

Materials required: a box (rectangular prism) similar to the diagram in Question 1.

Each student will:

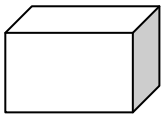
- identify perpendicular faces on a diagram of a 3-D object and explain why the faces are perpendicular.
- draw quadrilaterals and identify parallel, intersecting, perpendicular, vertical and horizontal sides.
- sort quadrilaterals into groups: rectangles, squares, trapezoids, parallelograms and rhombuses.
- describe the essential properties of rectangles, squares, trapezoids, parallelograms and rhombuses.

As students sort the quadrilaterals into the groups provided, they should demonstrate an understanding of the hierarchy within the set of quadrilaterals. For example, they must show that all squares are rectangles, all squares are rhombuses, all rectangles are parallelograms, all rhombuses are parallelograms, and that trapezoids are quadrilaterals but not parallelograms.

Early finishers can provide a different way to describe the essential properties for each quadrilateral in Question 4. They can also draw a Venn diagram or a tree diagram to show the relationships among the quadrilaterals.

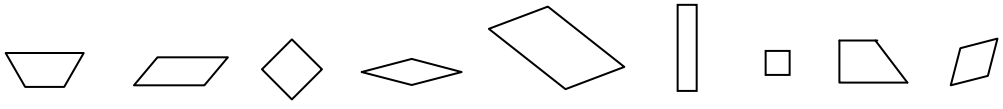
3-D Objects and Perpendicular Faces; Quadrilaterals and Relationships Between Sides—Student Assessment Task

1. On the diagram of the right rectangular prism shown below, shade two perpendicular faces. Explain your thinking.



2. Draw at least two different quadrilaterals that have parallel, intersecting, perpendicular, vertical and horizontal sides. Label the quadrilaterals ABCD, EFGH and so on. Write appropriate letters of the alphabet to identify:
   a. parallel sides
   b. intersecting sides
   c. perpendicular sides
   d. vertical sides
   e. horizontal sides

3. Identify by number which of the following quadrilaterals are:
   a. parallelograms
   b. squares
   c. rectangles
   d. rhombuses
   e. trapezoids



1 2 3 4 5 6 7 8 9

4. Explain the essential properties of each quadrilateral listed in Question 3.

Student _____

SCORING GUIDE
3-D Objects and Perpendicular Faces; Quadrilaterals and Relationships Between Sides

Level / Criteria	4 Excellent	3 Proficient	2 Adequate	1 Limited *	Insufficient / Blank *
Identifies Perpendicular Faces on 3-D Objects **Question #1**	The student shades two perpendicular faces on the diagram and explains clearly and with correct mathematical language why these faces are perpendicular.	The student shades two perpendicular faces on the diagram and explains clearly why these faces are perpendicular.	The student shades two perpendicular faces on the diagram and provides a limited explanation as to why these faces are perpendicular.	The student shades one or two faces on the diagram that may or may not be perpendicular and makes no effort to explain why these faces are perpendicular.	No score is awarded because there is insufficient evidence of student performance based on the requirements of the assessment task.
Draws Quadrilaterals and Identifies Parallel, Perpendicular, Intersecting, Vertical and Horizontal Sides **Question #2**	The student draws more than two quadrilaterals and correctly identifies parallel, perpendicular, intersecting, vertical and horizontal sides.	The student draws two quadrilaterals and correctly identifies parallel, perpendicular, intersecting, vertical and horizontal sides.	The student draws one quadrilateral and correctly identifies parallel, perpendicular, intersecting, vertical and horizontal sides.	The student draws one quadrilateral and identifies parallel, perpendicular, intersecting, vertical or horizontal sides, but not an example of each type.	No score is awarded because there is insufficient evidence of student performance based on the requirements of the assessment task.
Sorts Quadrilaterals into Given Groups **Question #3**	The student correctly sorts all the quadrilaterals into the groups provided.	The student correctly sorts all the quadrilaterals into the groups provided.	The student correctly sorts most of the quadrilaterals into the groups provided.	The student correctly sorts a few quadrilaterals into the groups provided.	No score is awarded because there is insufficient evidence of student performance based on the requirements of the assessment task.
Describes the Essential Properties of Various Types of Quadrilaterals **Question #4**	The student clearly describes the essential properties of each type of quadrilateral using precise mathematical language.	The student describes the essential properties of each type of quadrilateral.	The student describes the properties of each type of quadrilateral but may include properties that are redundant or nonessential.	The student provides limited descriptions of some properties of some quadrilaterals but may be confused about how the properties relate to the hierarchy of the quadrilaterals.	No score is awarded because there is insufficient evidence of student performance based on the requirements of the assessment task.

***** When work is judged to be limited or insufficient, the teacher makes decisions about appropriate intervention to help the student improve.

B. One-on-one Assessment

Parallel, Intersecting, Perpendicular, Vertical and Horizontal
Provide the student with a variety of 3-D objects and 2-D shapes. Have him or her identify edges and faces of the 3-D objects, and sides of 2-D shapes that are parallel, intersecting, perpendicular, vertical or horizontal.

If the student has difficulty, review the meanings of *edges*, *faces* and *sides* by using examples on the objects and shapes provided. Relate the meanings of *parallel*, *intersecting*, *perpendicular*, *vertical* and *horizontal* to real world examples; e.g., railroad tracks are parallel, the streets are intersecting.

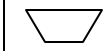
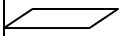
Have the student explain the similarities and differences between a pair of terms such as intersecting and perpendicular using objects, diagrams and words. Encourage the student to explain how vertical and horizontal relate to perpendicular edges, faces and sides. Students should conclude that sides may be perpendicular without one being vertical and the other horizontal, but that intersecting vertical and horizontal sides are always perpendicular.

Quadrilaterals
Have the student construct the different quadrilaterals (rectangles, squares, trapezoids, parallelogram, rhombuses) by using straws and string or paper strips and brass fasteners. Encourage the student to explain why a given quadrilateral such as a square is classified as a square by describing the properties of a square. Have the student manipulate the square to make it into a rhombus that is not a square. Similarly, have the student manipulate a rectangle to make a parallelogram that is not a rectangle. More information is available on pages 137–139 of the *Diagnostic Mathematics Program, Division II, Geometry*.

Another option is to provide the student with the following chart and have him or her complete the chart and explain his or her reasoning.

Instructions:
Look at the column under parallelogram. Check the figures in the column that are parallelograms. Do the same for each of the 2-D shapes in the chart.

Circle the check mark that indicates the **best** name for each 2-D shape.

	Parallelogram	Square	Trapezoid	Rhombus	Rectangle	Quadrilateral
						
						
						
						
	✓			✓		✓

If the student has difficulty including more than one check mark for certain 2-D shapes, remind the students of the properties of the shape being discussed. For example, the last shape in the chart is a rhombus because it has four sides, two pairs of parallel sides and four congruent sides. However, since it has four sides, it is also a quadrilateral. Since it has two pairs of parallel sides, it is also a parallelogram. Explain that *rhombus* is the best name for this 2-D shape because it is defined by all the properties: four sides, two pairs of parallel sides and four congruent sides.

See pages 73–75 of the *Diagnostic Mathematics Program, Division II, Geometry* for more assessment ideas on quadrilaterals.

C. Applied Learning

Provide opportunities for students to use their understanding of parallel, intersecting, perpendicular, vertical and horizontal as well as the different quadrilaterals in practical situations and notice whether or not this understanding transfers. For example, have the students find a 3-D object in the real world that has a face shaped like a square. Does the student:

- find a suitable object?
- explain why the face on the object is a square?
- explain the connection between a square corner and perpendicular sides of a square when asked?
- find two parallel edges on the object when asked?
- provide other names for a square when asked?
- find two parallel faces on the object or explain why two faces are not parallel?

Step 5: Follow-up on Assessment

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction?

A. Addressing Gaps in Learning

If the student has difficulty describing and providing examples of edges and faces of 3-D objects, and sides of 2-D shapes that are parallel, intersecting, perpendicular, vertical or horizontal, have the student construct 3-D objects and 2-D shapes using a variety of manipulatives such as straws and string, polydrons, nets and play dough.

By using straws and string construction, the focus is on the edges of 3-D objects and the sides of 2-D shapes and this provides a visual for students to see the relationships between pairs of edges or pairs of sides.

By using polydrons, nets and play dough for construction, the focus is on the faces of 3-D objects and this provides a visual for students to see the relationships between pairs of faces. Blackline masters for nets are available on pages 207–223 of the *Diagnostic Mathematics Program, Division II, Geometry*.

Encourage the students to use real world objects to describe and provide examples of edges and faces of 3-D objects that are parallel, intersecting perpendicular, vertical or horizontal.

If the students are having difficulty identifying and sorting quadrilaterals, have them construct the various quadrilaterals using straw and string or paper strips and brass fasteners. By changing the lengths of the straws or paper strips and manipulating the 2-D shape; e.g., flattening a square somewhat to create a rhombus that is not a square, students see visually the various quadrilaterals and can describe the properties and relationship among the shapes (see pages 137–140 of the *Diagnostic Mathematics Program, Division II, Geometry*).

Using concept attainment (see Activity 1 in Step 3) provides an opportunity for students to construct understanding of the geometric concepts and also the relationships among the concepts. Since this activity focuses the student's attention on the properties of a given quadrilateral, the student is better able to identify and sort quadrilaterals and justify their answers using sound mathematical reasoning.

B. Reinforcing and Extending Learning

Students who have achieved or exceeded the outcomes will benefit from ongoing opportunities to apply and extend their learning. These activities should support students in developing a deeper understanding of the concept and should not progress to the outcomes in subsequent grades.

Consider strategies such as.

- Provide tips for parents on helping their children to reinforce understanding of *parallel*, *intersecting*, *perpendicular*, *vertical* and *horizontal* as well as the various types of quadrilaterals. For example:
  - Capitalize on your outings with your children and have them find examples in the real world of edges and faces of 3-D objects and sides of 2-D shapes that are parallel, intersecting, perpendicular, vertical horizontal.
  - Chose real world objects and have the children describe the faces of these objects with particular attention on quadrilaterals (rectangles, squares, trapezoids, parallelograms and rhombuses).
  - In magazines, commercial drawings or drawings created by the children, have the children identify edges and faces of 3-D objects, and sides of 2-D shapes that are parallel, intersecting, perpendicular, vertical or horizontal. Have them identify different quadrilaterals: rectangles, squares, trapezoids, parallelograms and rhombuses.

- **Explorations Using a Geoboard**
  Provide each pair of students with a geoboard and a set of cards on which are written different lists of properties that apply to the various quadrilaterals. Have one student read the list of properties and evaluate the work done by the other student as he or she creates a quadrilateral that has the properties listed. Encourage the students to create as many different quadrilaterals as possible for a given set of properties. Have the students exchange roles periodically.

  If necessary, model this activity by reading a set of properties from a card while the students create appropriate quadrilaterals on their transparent geoboards. Have the students share their ideas of which quadrilaterals are described by the given properties by placing on the overhead their geoboards with the quadrilaterals outlined. Encourage the students to evaluate one another's work as it is presented to the class.

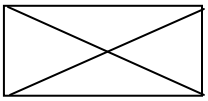
- **Using Diagonals to Sort Quadrilaterals (Connecting Intersecting Line Segments to Quadrilaterals)**
  Provide the students with three strips of construction paper about 2 cm wide. Cut two strips each 30 cm long and the third strip 20 cm long. Punch nine equally-spaced holes along each strip. Provide butterfly clips or brass fasteners to join two strips at a time.

Explain that these intersecting strips represent the diagonals of a quadrilateral. Through discussion and examples, have the students define a diagonal; e.g., "a line segment joining any two nonadjacent vertices of a figure with the line segment completely in the interior of the figure" (Cathcart 1997, p. 185).

Problem:
Connect the diagonals in different ways to produce different quadrilaterals. For each set of diagonals, consider:

- the length of each
- where they intersect
- the angles between them.

Draw a diagram of each quadrilateral with its two intersecting diagonals and explain the diagram in words.
Example:



A rectangle has diagonals that are congruent and cut each other in half; i.e., bisect each other.

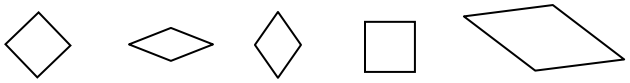
Have the students share their explorations with one another and describe the diagonals for each quadrilateral; e.g.,

- The diagonals of a rectangle are congruent and bisect each other.
- The diagonals of squares are congruent, bisect each other and meet at right angles.
- The diagonals of a trapezoid may or may not be congruent and they do not bisect each other.
- The diagonals of a parallelogram may or may not be congruent and bisect each other.
- The diagonals of a rhombus may or may not be congruent, bisect each other and meet at right angles (Van de Walle and Lovin 2006).

- **Mystery Definition**
  Present the students with three groups of quadrilaterals on the overhead, chalkboard or in a handout.

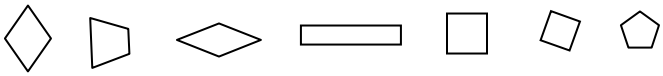
  First Group:
  Choose examples of the concept that allow for all the possible variations of the mystery definition or properties.
  Second Group:
  Choose non-examples to be as close to the examples as is necessary to provide the needed information for an accurate definition.
  Third Group:
  Choose a mixture of examples and non-examples of the concept. Include non-examples with which students are most likely to be confused.

Example:
First Group
All of these have something in common.



Second Group:
None of these have it.

Third Group:
Which of these have it? Explain your thinking.



Have students create other mystery definitions and share them with the class.

Adapted from John A. Van de Walle, LouAnn H. Lovin, *Teaching Student-Centered Mathematics: Grades 3–5*, 1e (p. 225). Published by Allyn and Bacon, Boston, MA. Copyright © 2006 by Pearson Education. Reprinted by permission of the publisher.

- **Make a Pair**
  Provide the students with two decks of cards. The first deck of cards has a quadrilateral drawn on each card. The second deck of card has some properties listed on each card.

  Instructions:
  Play this game with a partner.
  Place each of the two decks of cards face down.
  Decide on who starts the game by rolling a die. The one who rolls the highest number starts the game.
  The first player turns over a card from each pile of cards. If the cards match, the player takes the two cards and has another turn. If the cards do not match, the cards are left as is and the second player turns over one card from each pile. The cards match if the properties listed on one card describe the quadrilateral drawn on the other card.

  The winner is the player who has the most pairs of cards at the end of the game.

  Adaptations:

  – Write the name of a quadrilateral (e.g., rectangle, square, trapezoid, parallelogram or rhombus) on each card instead of drawing a diagram of a quadrilateral.
  – Include parallel, intersecting, perpendicular, vertical or horizontal sides (e.g., at least one pair of parallel sides) on the cards that describe the properties as well as other properties such as four right angles or four congruent sides.
  – Play "Concentration" with the two decks of cards.

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