Mathematics

Planning Guide

Grade 5 *Working with Decimal Numbers*

Number

Specific Outcomes 8, 9, 10 and 11

This Planning Guide can be accessed online at:
http://www.learnalberta.ca/content/mepg5/html/pg5_workingwithdecimalnumbers/index.html

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Planning Guide: *Grade 5 Working with Decimal Numbers*

Strand: Number
Specific Outcomes: 8, 9, 10 and 11

This *Planning Guide* addresses the following outcomes from the Program of Studies:

Strand: Number	
Specific Outcomes:	8. Describe and represent decimals (tenths, hundredths, thousandths), concretely, pictorially and symbolically. 9. Relate decimals to fractions and fractions to decimals (to thousandths). 10. Compare and order decimals (to thousandths) by using: • benchmarks • place value • equivalent decimals. 11. Demonstrate an understanding of addition and subtraction of decimals (limited to thousandths).

Curriculum Focus

The changes to the curriculum targeted by this sample include:

- The general outcome focuses on number sense; whereas the previous math curriculum specified demonstrating number sense for whole numbers and exploring proper fractions and decimals.
- The specific outcome includes representing decimals to thousandths; whereas the previous curriculum included reading and writing decimals to thousandths in Grade 6.
- The specific outcome includes relating fractions to decimals (to thousandths); whereas the previous curriculum included only decimals to hundredths in Grade 5.
- The specific outcome includes comparing and ordering decimals (to thousandths) with specific strategies outlined; whereas the previous math curriculum focused on comparing and ordering fractions and decimals to hundredths with no strategies suggested.
- The specific outcome includes adding and subtracting decimals (limited to thousandths); whereas the previous math curriculum focused on adding and subtracting decimals to hundredths with emphasis on concrete, pictorial and symbolic connections.

What Is a Planning Guide

Planning Guides are a tool for teachers to use in designing instruction and assessment that focuses on developing and deepening students' understanding of mathematical concepts. This tool is based on the process outlined in *Understanding by Design* by Grant Wiggins and Jay McTighe.

Planning Steps

The following steps will help you through the Planning Guide:

Step 1: Identify Outcomes to Address

Guiding Questions

- What do I want my students to learn?
- What can my students currently understand and do?
- What do I want my students to understand and be able to do based on the Big Ideas and specific outcomes in the program of studies?

Big Ideas

Conceptual understanding of decimals requires that the students connect decimals to whole numbers and to fractions. Decimals are shown as an extension of the whole number system by introducing a new place value, the tenths place, to the right of the ones place, separated by the decimal point. The tenths place follows the pattern of the base ten number system by iterating one tenth ten times to make one whole or a unit (Wheatley and Abshire 2002, p. 152). Similarly, the hundredths place to the right of the tenths place iterates one hundredth ten times to make one-tenth. Following this pattern, the thousandths place to the right of the hundredths place iterates one thousandth ten times to make one hundredth. Van de Walle and Lovin (2006) suggest that the concepts of whole number place value be reviewed prior to considering decimal numerals with students and they state:

> "The base-ten place-value system extends infinitely in two directions: to tiny values as well as to large values. Between any two place values, the ten-to-one ratio remains the same. The decimal point is a convention that has been developed to indicate the units position. The position to the left of the decimal point is the unit that is being counted as singles or ones" (p. 107).

If a decimal numeral represents a quantity or a measure less than 1 unit, then a zero must be placed in the ones place to identify that there are no complete units in this numeral; e.g., one thousandth is written as 0.001 and is read as "one thousandth."

The connection between decimals and fractions is developed conceptually when students read decimals as fractions and represent them using the same visuals. For example, 0.8 is read as eight tenths and can be represented using fraction strips or decimal strips (Wheatley and Abshire 2002). Similarly, 0.008 is read as eight thousandths and can be represented using a thousandth square. Van de Walle and Lovin (2006) state, "Decimal numbers are simply another way of writing fractions" (p. 107).

As students use the same concrete representations for fractions and decimals and connect them to the same pictorial representations, they understand the connections between these two ways to represent part of a whole. The difference between fractions and decimals is in the symbolic representation where a fraction has a numerator and denominator while a decimal has a decimal point and place value (an extension of the whole numbers).

Various strategies are used in comparing and ordering decimals, including benchmarks, place value and equivalent decimals. Decimals could be sorted initially as being greater than, less than or equal to a benchmark such as 0.5. To facilitate the sorting process, the decimals should be all written with the same number of digits after the decimal; i.e., using equivalent decimals where necessary. For example, 0.52 is equivalent to 0.520. Conceptual understanding of equivalent decimals is based on connecting equivalent fractions to equivalent decimals. Therefore, 0.52 = 0.520 because $\frac{52}{100} = \frac{520}{1000}$.

An efficient and accurate way to order decimals is to use place value, as is done in ordering whole numbers. By building on students' prior knowledge about ordering whole numbers using place value and place value charts, the ordering of decimals is seen as continuing a pattern rather than something entirely new.

Estimation is crucial in the addition and subtraction of decimals. In fact, "students should become adept at estimating decimal computations well before they learn to compute with pencil and paper" (Van de Walle and Lovin 2006, p. 124). Through estimation, the students use number sense to determine if the answer is reasonable. By building on students' understanding of addition and subtraction of whole numbers, the pattern of place value is extended to decimals and the importance of adding the same place values continues; i.e., adding tenths to tenth, hundredths to hundredths and so on. Van de Walle and Lovin (2006) state,

> Addition and subtraction with decimals are based on the fundamental concept of adding and subtracting the numbers in like position values—a simple extension from whole numbers (p. 107).

Sequence of Outcomes from Program of Studies

See http://education.alberta.ca/teachers/core/math/programs.aspx for the complete program of studies.

Grade 4	Grade 5	Grade 6
Specific Outcomes	**Specific Outcomes**	**Specific Outcomes**
9. Represent and describe decimals (tenths and hundredths), concretely, pictorially and symbolically. 10. Relate decimals to fractions and fractions to decimals (to hundredths). 11. Demonstrate an understanding of addition and subtraction of decimals (limited to hundredths) by: • using personal strategies to determine sums and differences • estimating sums and differences • using mental mathematics strategies to solve problems.	8. Describe and represent decimals (tenths, hundredths, thousandths), concretely, pictorially and symbolically. 9. Relate decimals to fractions and fractions to decimals (to thousandths). 10. Compare and order decimals (to thousandths) by using: • benchmarks • place value • equivalent decimals. 11. Demonstrate an understanding of addition and subtraction of decimals (limited to thousandths).	2. Solve problems involving whole numbers and decimal numbers. 8. Demonstrate an understanding of multiplication and division of decimals (1-digit whole number multipliers and 1-digit natural number divisors).

Step 2: Determine Evidence of Student Learning

Guiding Questions

- What evidence will I look for to know that learning has occurred?
- What should students demonstrate to show their understanding of the mathematical concepts, skills and Big Ideas?

Using Achievement Indicators

As you begin planning lessons and learning activities, keep in mind ongoing ways to monitor and assess student learning. One starting point for this planning is to consider the achievement indicators listed in the *Mathematics Kindergarten to Grade 9 Program of Studies with Achievement Indicators.* You may also generate your own indicators and use these to guide your observation of students.

The following indicators may be used to determine whether or not students have met this specific outcome. Can students:

- write a decimal for a given concrete or pictorial representation of part of a set , part of a region or part of a unit of measure?
- represent a given decimal using concrete materials or a pictorial representation?
- represent an equivalent tenth, hundredth or thousandth for a given decimal, using a grid?
- express a given tenth as an equivalent hundredth and thousandth?
- express a given hundredth as an equivalent thousandth?
- describe the value of each digit in a given decimal?
- write a given decimal in fraction form?
- write a given fraction with a denominator of 10, 100 or 1000 as a decimal?
- express a given pictorial or concrete representation as a fraction or decimal; e.g., 250 shaded squares on a thousandth grid can be expressed as 0.250 or $\frac{250}{1000}$?
- order a given set of decimals by placing them on a number line that contains the benchmarks 0.0, 0.5 and 1.0?
- order a given set of decimals including only tenths using place value?
- order a given set of decimals including only hundredths using place value?
- order a given set of decimals including only thousandths using place value?
- explain what is the same and what is different about 0.2, 0.20 and 0.2000?
- order a given set of decimals, including tenths, hundredths and thousandths, using equivalent decimals; e.g., 0.92, 0.7, 0.9, 0.876, 0.925 in order is 0.700, 0.876, 0.900, 0.920, 0.925?
- place the decimal point in a sum or difference using front-end estimation; e.g., for 6.3 + 0.25 + 306.158, think 6 + 306, so the sum is greater than 312?
- correct errors of decimal point placements in sums and differences without using paper and pencil?
- explain why keeping track of place value positions is important when adding and subtracting decimals?

- predict sums and differences of decimals using estimation strategies?
- solve a given problem that involves addition and subtraction of decimals, limited to thousandths?

Sample behaviours to look for related to these indicators are suggested for some of the activities listed in **Step 3, Section C: Choosing Learning Activities** (p. 14).

Step 3: Plan for Instruction

Guiding Questions

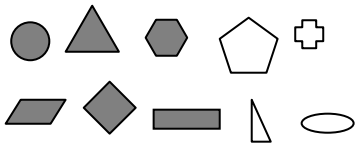
- What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should I use?
- How will I meet the diverse learning needs of my students?

A. Assessing Prior Knowledge and Skills

Before introducing new material, consider ways to assess and build on students' knowledge and skills related to counting. For example:

1. Write a fraction and a decimal to show the shaded part of each of the following diagrams:

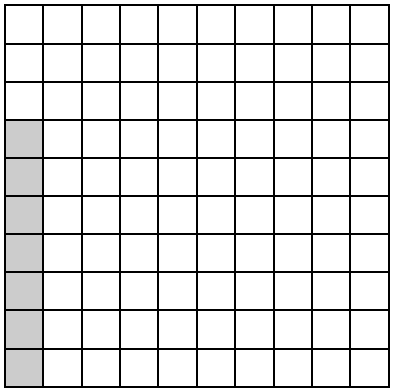
   a.



   Fraction: _____

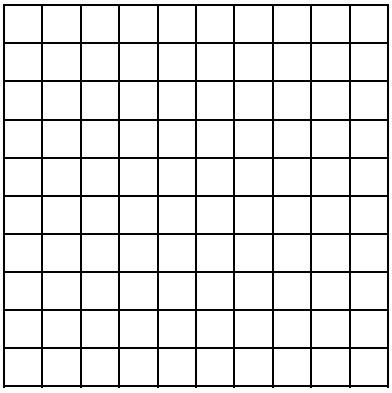
   Decimal: _____

   b.



   Fraction: _____ Decimal: _____

2. a. Shade the following diagram to show 0.32.



   b. Write the words you would say if you read this decimal.

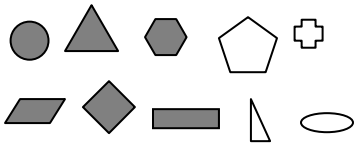
3. Write $\frac{3}{100}$ as a decimal.

4. Explain the meaning of each digit in $8.88.

5. Mary wants to buy a CD for $14.98 and a drink for $0.85.
   a. Will $15 cover the cost of both items? Explain your thinking.
   b. Find the exact total cost of the two items. Explain your thinking.

   If a student appears to have difficulty with these tasks, consider further individual assessment, such as a structured interview, to determine the student's level of skill and understanding. See **Sample Structured Interview: Assessing Prior Knowledge and Skills** (p. 11).

Sample Structured Interview: Assessing Prior Knowledge and Skills

Directions	Date:	
	Not Quite There	**Ready to Apply**
a. Place before the student the following diagram:  Say, **"Write a fraction and a decimal to show the shaded part of the whole set."** b. Place before the student a hundred grid with 0.07 shaded; i.e., 7 squares are shaded in one column on the grid. Say, **"Write a fraction and a decimal to show the shaded part of the whole region."**	Does not write the fraction or the decimal correctly. Writes either the fraction or the decimal correctly, but not both.	Writes both the fraction and the decimal correctly for both parts of the question; i.e., a. 6/10 and 0.6 and b. 7/100 and 0.07.
Place before the student a hundreds grid. a. Say, **"Shade this grid to show 0.32."** b. Say, **"Write the words you would say if you read this decimal."**	Does not shade the hundreds grid correctly to show 0.32. Does not know how to write the decimal in words or writes "point three two," "decimal thirty-two," or "decimal three two" rather than "thirty-two hundredths."	Shades the hundreds grid correctly to show 0.32. Writes "thirty-two hundredths" as the correct way to read this decimal.
Say, **"Write $\frac{3}{100}$ as a decimal."**	Does not write the decimal equivalent of $\frac{3}{100}$ correctly; e.g., may write 0.3 or .03.	Writes the decimal equivalent of $\frac{3}{100}$ correctly; i.e., 0.03.

Say, **"Explain the meaning of each digit in $8.88."**	Does not explain the meanings of any of the digits correctly. Explains the meanings of one or two but not all three of the digits correctly.	Explains the meaning of all three digits correctly; i.e., the first 8 means 8 whole dollars, the second 8 means 8 dimes or $\frac{8}{100}$ = 0.8 of a dollar and the third 8 means 8 pennies or $\frac{8}{100}$ = 0.08 of a dollar.
Present before the student the following problem: **"Mary wants to buy a CD for $14.98 and a drink for $0.85.** **a. Will $15 cover the cost of both items? Explain your thinking.** **b. Find the exact total cost of the two items. Explain your thinking."**	a. Thinks that $15 will cover the cost because he or she uses the front-end strategy without using compensation. Or, thinks that $15 will not cover the cost but is unable to explain his or her thinking. b. Does not add the two decimals correctly. Or, adds the two decimals correctly but is unable to explain his or her thinking.	a. Explains that $15 will not cover the cost using a mathematically correct estimation strategy. b. Calculates the total cost of the two items correctly and explains the process clearly.

B. Choosing Instructional Strategies

Consider the following general strategies for teaching fractions and decimals:

- Access students' prior knowledge of fractions and decimals and build on this understanding.
- Develop understanding of decimals by relating them to whole numbers and to fractions.
- Use everyday contexts such as units of measurement to facilitate understanding of decimals.
- To develop understanding, use a variety of concrete representations (e.g., base ten materials, metre sticks, transparent decimal grids that can be superimposed on one another) and connect them to pictorial and symbolic representations.
- To demonstrate understanding, have the students represent the symbolic fractions and decimals concretely and pictorially.
- Emphasize that all fractions can be written as decimals that represent the same part of the whole region or whole set. Build on students' understanding of equivalent fractions to show that ½ is equivalent to $\frac{5}{10}$ or $\frac{50}{100}$ or $\frac{500}{1000}$, which can be written as 0.5 or 0.50 or 0.500. Similarly, emphasize that all decimals can be written as fractions.
- Provide many examples of the three models for fractions and decimals: part of a region, part of a length or measurement and part of a set.
- Reinforce the relationship between the symbolic and pictorial modes (symbolic fraction name, pictorial parts, pictorial whole) by posing problems in which two of these are provided and the student determines the third by using their models (Van de Walle and Lovin 2006).
- Emphasize the meaning of a decimal as the various ways to compare decimals are explored. Encourage flexibility in thinking as students compare decimals.
- Connect ordering decimals to ordering whole numbers and fractions:
  - Equivalent fractions are used to find common denominators in ordering fractions. Similarly, equivalent decimals are used to change all decimals to the same number of place values after the decimal in ordering decimals.
  - Benchmarks are used in ordering decimals just as benchmarks are used in ordering whole numbers and fractions.
  - Place value is used in ordering whole numbers and is also used in ordering decimals.
- Access students' prior knowledge of adding and subtracting decimals to hundredths and connect it to adding and subtracting decimals to thousandths.
- Emphasize estimating sums and differences of decimals to thousandths prior to calculating the exact answers.

C. Choosing Learning Activities

The following learning activities are examples of activities that could be used to develop student understanding of the concepts identified in Step 1.

Sample Activities:

Teaching the Meaning Decimals and Related Fractions

1. **Concrete Decimals Using Base Ten Materials and Place Value Mats** (p. 16)
2. **Decimal Grids and Equivalent Decimals** (p. 18)
3. **Decimals and Metric Measures of Length** (p. 20)

Teaching the Comparing and Ordering Decimals (to Thousandths)

1. **Benchmarks** (p. 22)
2. **Place Value and Decimal Grids** (p. 23)
3. **Equivalent Decimals** (p. 25)

Teaching the Addition and Subtraction of Decimals (to Thousandths)

1. **Estimating Sums and Differences** (p. 27)
2. **Personal Strategies** (p. 28)
3. **Place Value Charts** (p. 29)
4. **Placing the Decimal Point** (p. 30)

Teaching the Meaning Decimals and Related Fractions

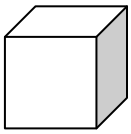
Sample Activity 1: Concrete Decimals Using Base Ten Materials and Place Value Mats

Provide the students with base ten materials and build on their prior knowledge of the using these materials to represent decimals to hundredths. Encourage flexibility in thinking by changing the unit. If the flat is one whole, guide the discussion to conclude that the long is one tenth and the small cube is one hundredth.

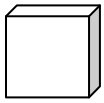
Then use the block as one whole and have the students determine which base ten materials represent one tenth and one hundredth. Through discussion, have the students verbalize that if the block is one whole, then the flat is one tenth and the long is one hundredth. Review the relationships between the place values and have the students predict what place value is directly to the right of the hundredths place and which of the base ten materials would represent that amount. Following a pattern, students should conclude that the thousandths place is to the right of the hundredths place and can be represented by a small cube because a small cube is one tenth of the long.

Have the students draw and label the base ten materials as follows.

Base ten materials:



Block: 1 whole



Flat: $\frac{1}{10}$ or 0.1

Long: $\frac{1}{100}$ or 0.01

Small cube: $\frac{1}{1000}$ or 0.001

Have the students practise representing various decimals with the base ten materials and writing the appropriate decimal and fraction symbols. The overhead base ten materials are very useful as a means of showing various base ten representations to the whole class for them to discuss and critique.

Have the students use a place value chart showing hundreds, tens, ones, tenths, hundredths and thousandths to reinforce the connections between the concrete (base ten materials) and the symbolic representation for decimals.

Place Value Chart

Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths

To develop understanding, have the students practise in first using the concrete materials and then writing the symbolic representations on the place value mat. To demonstrate understanding, have the students show the meaning of decimals written on the place value chart by using the appropriate base ten materials.

A more detailed description of using base ten materials for teaching fractions and decimals is provided on pages 239–245 of the *Diagnostic Mathematics Program, Division II, Numeration*.

Look For …

Do students:

- ☐ communicate clearly the pattern of the base ten system to thousandths?
- ☐ represent decimals to thousandths by using concrete materials and then write the appropriate symbols?
- ☐ explain the importance of place value in reading and writing decimals?

Sample Activity 2: Decimal Grids and Equivalent Decimals

Meaning of Decimals
Build on students' prior knowledge of using the hundred grid to represent hundredths. Review decimals to hundredths by shading in part of a hundredths grid and having the students write the appropriate decimal to represent the fraction of the whole grid that is shaded. Then reverse the procedure and have the students shade the hundredths grid to represent the symbolic decimals.

Have the students suggest how the hundredth grid could be subdivided to show thousandths. If necessary, review the place value pattern that one whole is divided into ten equal parts to make one tenth, one tenth is divided into ten equal parts to make one hundredth, therefore, one hundredth must be divided into ten equal parts to make one thousandth. See pages 246–253 of the *Diagnostic Mathematics Program, Division II, Numeration* for visuals and detailed description.

Present the students with decimal grids showing thousandths. Shade part of the thousandths grid on the overhead projector and have the students write the appropriate decimal and fraction to represent the fraction of the whole grid that is shaded. Then reverse the procedure and have the students shade the thousandths grid to represent the symbolic decimals and fractions; e.g., 0.012 or $\frac{12}{1000}$.

Equivalent Decimals

Review equivalent fractions and build on this understanding to explain the meaning of equivalent decimals; e.g., $\frac{2}{10} = \frac{20}{100} = \frac{200}{1000}$.

Provide each group of students with a set of transparent decimal grids: tenths, hundredths and thousandths. Use another set of transparent grids on the overhead projector to guide discussion. Have the students write $\frac{2}{10}$ or 0.2 and colour this part of the tenths grid. Then have them superimpose the tenths grid on the hundredths grid to show $\frac{20}{100}$ or 0.20. Have the students write the appropriate fraction and decimal. Finally, have them superimpose the hundredths grid onto the thousandths grid to show $\frac{200}{1000}$ or 0.200 and write the fraction and decimal.

Look For …

Do students:

- ☐ write the appropriate decimal to thousandths when to represent the shaded part of a thousandths grid?
- ☐ shade a thousandth grid to represent a given decimal to thousandths?
- ☐ connect equivalent fractions to equivalent decimals?
- ☐ use decimal grids to show the meaning of equivalent decimals?

Through discussion, have the students conclude that equivalent decimals are decimals that represent the same quantity with different sized parts of the whole. Have the students write 0.2 = 0.20 = 0.200. Have the students practise representing other equivalent decimals. See pages 96–100, 103–106 and 248 of the *Diagnostic Mathematics Program, Division II, Numeration* for a more detailed description along with visuals as well as assessment tasks.

Sample Activity 3: Decimals and Metric Measures of Length

Provide the each group of students with a metre stick. Review the metric measures and relate the ten-to-one ratio to the base ten number system. Have the students summarize: one tenth of a metre is a decimetre, one tenth of a decimetre is a centimetre, and one tenth of a centimetre is a millimetre.

Have the student apply their knowledge of the metric system in explaining the meaning of each digit in the measure 1.325 m. Then have a group of volunteers draw this length on the board and label each part: 1 m, 0.3 m or 3 dm, 0.02 m or 2 cm and 0.005 m or 5 mm.

Have the students estimate and then measure a variety of different lengths and record their measures to the nearest millimetre. Encourage the students to write their answers in metres, centimetres and millimetres to emphasize the importance of establishing what the whole is when working with decimals and measurements. For example, 0.485 m = 48.5 cm = 485 mm.

Look For …

Do students:

- ☐ communicate the similarities between the base ten number system and the metric system?
- ☐ use a metric stick correctly to measure lengths to the nearest millimetre?
- ☐ write the measure of a length in metres and explain the meaning of each digit in the decimal?
- ☐ draw a line segment given the length as a part of metre using a decimal to thousandths?

Teaching the Comparing and Ordering Decimals (to Thousandths)

Sample Activity 1: Benchmarks

Build on students' prior knowledge of using benchmarks to order fractions. Instead of identifying which of the benchmarks, 0, ½ or 1, is closer to a given fraction, have the students identify which of the benchmarks, 0, 0.5 or 1, is closer to a given decimal. Encourage the students to use base ten materials or decimal grids to represent the decimals if they have difficulty deciding which benchmark the decimal is closest to. They may also wish to use the fraction equivalents for each decimal when using the benchmarks to order the decimals; e.g., 0.135 = $\frac{135}{1000}$, 0.5 = $\frac{500}{1000}$.

Look For …

Do students:

- ☐ connect decimals to their equivalent fractions with denominators of 10, 100 and 1000?
- ☐ connect the use of benchmarks to order fractions with ordering decimals?
- ☐ communicate clearly how benchmarks are used to order decimals to thousandths?

Problem:
Order the following decimals by placing them on the number line below that contains the benchmarks 0, 0.5 and 1.

0.685, 0.9, 0.42, 0.135

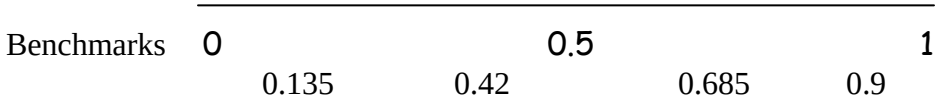
Sample Solution:

0.135 or $\frac{135}{1000}$ is closer to 0 than to 0.5 or $\frac{500}{1000}$, so it is on the number line close to 0.

0.42 or $\frac{42}{100}$ is closer to 0.5 or $\frac{50}{100}$ than 0, so it is on the number line a little less than 0.5.

0.685 or $\frac{685}{1000}$ is closer to 0.5 or $\frac{500}{1000}$ than 1, so it is on the number line a little more than 0.5.

0.9 or $\frac{9}{10}$ is closer to 1 than to 0.5 or $\frac{5}{10}$, so it is on the number line close to 1.



Sample Activity 2: Place Value and Decimal Grids

Build on students' prior knowledge of using place value and decimal grids to order decimals to hundredths. Provide each student with a place value chart showing thousandths.

Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths

Review that in ordering whole numbers and decimals, the place values to the left indicate larger amounts than the place values to the right. If necessary, provide base ten materials and have the students represent decimals with the base ten materials and then write the decimals in the place value chart.

Begin by comparing two decimals using the place value chart and then have the students order more than two decimals.

To verify that their answers are correct, encourage the students to shade decimal grids showing thousandths for each decimal that is to be ordered. The students can see visually that the least decimal has the least amount of shading in the thousandths grid and the greatest decimal has the greatest amount of shading in the grid. See page 278 of the *Diagnostic Mathematics Program, Division II, Numeration* for a detailed description of ordering decimals using decimal grids.

Look For …

Do students:

- ☐ represent decimals with base ten materials and then write the symbols for the decimal in a place value chart?
- ☐ explain the important role of place value in ordering decimals?
- ☐ order decimals symbolically and then verify by shading decimal grids?

Problem:
Order the following decimals from least to greatest by using the place value chart below.
0.36, 0.058, 0.375, 0.4

Sample Solution:

Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
		0	3	6	
		0	0	5	8
		0	3	7	5
		0	4		

The place values decrease from left to right, so look at the largest place value first.

None of the decimals have digits other than zero in the whole number part; i.e., in the place values to the left of the decimal. Therefore, compare the tenths in each decimal.
0.058 has 0 tenths and the rest of decimals have some tenths, so 0.058 is the least decimal.
0.36 and 0.375 each have 3 tenths, so look at the hundredths place in each decimal.
0.3<u>6</u> has fewer hundredths than 0.3<u>7</u>5, so 0. 36 is less than 0.375.
0.4 is the greatest decimal because it has the greatest number of tenths.

The order of the decimals from least to greatest is: 0.058, 0.36, 0.375 and 0.4.

These decimals can be represented on decimal grids to verify that the order is correct.

Sample Activity 3: Equivalent Decimals

Look For …

Do students:

- ☐ connect equivalent fractions to equivalent decimals?
- ☐ communicate clearly how to write equivalent decimals?
- ☐ explain how to use equivalent decimals when ordering decimals to thousandths?
- ☐ shade decimal grids to verify the ordering of decimals done by using equivalent fractions

Build on students' prior knowledge of using equivalent fractions to order fractions with unlike denominators. Review the concept of equivalent decimals (see Activity #2). Explain that by using equivalent decimals, all the decimals can be written as thousandths, so the size of the parts in each decimal is the same. The focus then is on how many of these same sized parts are included in each decimal. Review the connections between decimals and fractions so the students can easily convert decimals to fractions with denominators of 10, 100 or 1000.

Problem:
Order the following decimals from least to greatest by using the place value chart below.
0.36, 0.058, 0.375, 0.4

Sample Solution:
The decimals that are to be ordered include decimals to thousandths, so all the decimals will be written as equivalent decimals showing thousandths.

0.36 or $\frac{36}{100}$ is equivalent to $\frac{360}{1000}$ or 0.360.

0.058 or $\frac{58}{1000}$ is already written in thousandths.

0.375 or $\frac{375}{1000}$ is already written in thousandths.

0.4 or $\frac{4}{10}$ is equivalent to $\frac{400}{1000}$ or 0.400.

Looking at the number of thousandths in each decimal, the order of the decimals from least to greatest is 0. 058, 0.360, 0.375 and 0.400.

These decimals can be represented on decimal grids to verify that the order is correct.

Teaching the Addition and Subtraction of Decimals (to Thousandths)

Sample Activity 1: Estimating Sums and Differences

Review estimating sums and differences of whole numbers and decimals to hundredths and connect this process to decimals to thousandths. Clarify that students should estimate their answers before calculating the exact answer to determine the reasonableness of the answer. Explain that estimating focuses on the meaning of the decimals and the operations and not on how many digits are after the decimal in each number. Provide examples in which front-end estimation can be used.

Front-end Estimation

Front-end estimation focuses on the whole number value of each decimal and ignores the decimal part of the number. This process results in an estimated answer that is always less than the exact answer. For example, to estimate the sum of 3.686 and 28.97, think 3 + 28 = 31. The estimated answer is slightly more than 31. Compensation can be used to refine the estimate by adjusting the answer to compensate for the values to the right of the decimal. The final compensated estimate might be about 32 or 33.

Look For …

Do students:

- ☐ estimate first and then calculate the exact sums and differences of decimals to thousandths?
- ☐ communicate understanding of the meaning of decimals and operations?
- ☐ use the front-end strategy to estimate sums and differences and explain the process?

Look For …

Do students:

- ☐ estimate prior to calculating sums and differences of decimals to thousandths?
- ☐ use personal strategies that make sense to them and explain the process?
- ☐ test a personal strategy on other sums and differences involving decimals to thousandths?
- ☐ critique the personal strategies of other students?
- ☐ choose personal strategies that are efficient and mathematically sound?
- ☐ compare the exact answer with the estimated answer to determine reasonableness of the answer?

Sample Activity 2: Personal Strategies

Provide the students with a sum involving numbers that have a different number of decimal places; e.g., 74.34 + 0.289 + 5.1. Have the students estimate the sum first and explain their thinking. Then have them calculate the sum by devising a method that will work for the sum of any decimals. Finally, have the students share their strategies for computation and test them on other computations involving decimals to thousandths.

Sample Activity 3: Place Value Charts

Provide the students with place value charts to thousandths. Review the process of adding whole numbers and adding decimals to hundredths and emphasize that the numbers in like-position place values must be added. This process is clarified by using place value charts. Provide the students with appropriate problems and have them estimate and then calculate the answers.

Problem:
Jimmy and Sam each time his own quarter-mile run with a stopwatch. Jimmy ran the distance in 85.6 seconds. Sam recorded his time with more precision as 92.345 seconds. How many seconds faster did Sam run than Jimmy?

This problem adapted from John A. Van de Walle, LouAnn H. Lovin, *Teaching Student-Centered Mathematics: Grades 5–8,* 1e (p. 125). Published by Allyn and Bacon, Boston, MA. Copyright © 2006 by Pearson Education. Reprinted by permission of the publisher.

Look For …

Do students:

- ☐ estimate the sum or difference before calculating the exact answer?
- ☐ explain the importance of place value and the need to add or subtract numbers in "like position values"?
- ☐ use a place value chart when needed to clarify the place value of each digit?
- ☐ use base-ten materials as needed when carrying out their personal strategies, such as regrouping?
- ☐ compare the exact answer with the estimated answer to determine reasonableness of the answer?

Sample Solution:
Use front-end estimation to approximate the answer as a difference of 92 – 85 = 7 seconds.

To find the exact answer, place the decimals on a place value chart to aid in aligning the place values. Then encourage the students to use a variety of strategies to calculate the answer. One of these strategies (regrouping) is shown below.

Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
	$\cancel{9}$ 8 – 8	$\cancel{2}$ 11 5	$\cancel{3}$ 13 6	4	5
Answer:		6	7	4	5

Answer to the problem: Sam ran 6.745 seconds faster than Jimmy.

Another strategy would be to count on from 85.6 by adding 6.7 to get 92.3 seconds and then adding on the remaining 0.045 seconds, making a difference of 6.745 seconds.

Provide other appropriate problems and have the students estimate the answer and then calculate the exact answer using a variety of strategies. Encourage the students to explain how place value plays a major role in each of the strategies they choose.

See pages 122–133, 254–256 of the *Diagnostic Mathematics Program, Division II, Operations and Properties* for other ideas on adding and subtracting decimals as well as assessment tasks.

Sample Activity 4: Placing the Decimal Point

Provide the students with a selection of addition and subtraction problems using decimals to thousandths and include the calculated answer without the decimal point. Have the students decide where the decimal point should be placed in each answer and explain how they know. Encourage the use of a variety of estimation strategies.

Students can then share their answers with the whole class and correct any errors in the decimal point placements in the sums and differences.

Problem:
Joanne bought 3.537 m of ribbon and used 0.48 m of it to wrap presents. How many metres of ribbon does Joanne have left?

Answer:
Joanne has 3057 m of ribbon left.
Place the decimal point in the number to answer the problem correctly. Explain your thinking.

Sample Solution:
Using front-end estimation, the difference between the two numbers is $3 – 0 = 3$. Therefore, the decimal point must be placed directly behind the 3 in 3057.
Correct answer to the problem: Joanne has 3.057 m of ribbon left.

Step 4: Assess Student Learning

Guiding Questions

- Look back at what you determined as acceptable evidence in Step 2.
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Sample Assessment Tasks

In addition to ongoing assessment throughout the lessons, consider the following sample activities to evaluate students' learning at key milestones. Suggestions are given for assessing all students as a class or in groups, individual students in need of further evaluation, and individual or groups of students in a variety of contexts.

A. Whole Class/Group Assessment

Comparing Lengths of Shelves
In the context of a problem about building two different sized shelves, the student will demonstrate understanding about the meaning of decimals (to thousandths) by using concrete materials and/or drawing diagrams to represent the two decimals in a problem. Then he or she will decide which shelf is longer by using one or more appropriate strategies. Finally, the student will find the difference between the lengths of the two shelves and explain the process used.

Materials required: Base ten materials, metre sticks, decimal grids (to thousandths), place value charts (to thousandths).

Each student will:

- explain the meaning of a decimal by representing it using concrete materials or a diagram
- use and explain an appropriate strategy for ordering decimals
- solve a given problem that involves addition and subtraction of decimals, limited to thousandths, and explain the process used.

Early finishers can create other problems about decimals (to thousandths) for the class to solve; e.g., track times, comparing weights.

Comparing Lengths of Shelves—Student Assessment Task

Brooklyn is helping her dad build shelves for the den. One shelf is 0.6 m long. The other shelf is 0.475 m long.

1. Represent each decimal by drawing a diagram.

2. Which shelf is longer? Explain your thinking by using one or more strategies.

3. Exactly how much longer is one shelf than the other? Explain your thinking in detail.

Student _____

SCORING GUIDE
Comparing Lengths of Shelves

Level / Criteria	4 Excellent	3 Proficient	2 Adequate	1 Limited *	Insufficient / Blank *
Demonstrates the meaning of decimals by drawing diagrams **Question #1**	Draws a diagram that clearly shows the meaning of each digit in each decimal.	Draws a diagram that shows the meaning of each digit in each decimal.	Draws a diagram that shows the meaning of each digit in each decimal with some inaccuracy.	Draws a diagram to show the meaning of tenths but has difficulty showing hundredths and/or thousandths.	No score is awarded because there is insufficient evidence of student performance based on the requirements of the assessment task.
Compares two decimals (to thousandths) using an appropriate strategy **Question #2**	Compares the two decimals using at least two appropriate strategies with sound mathematical reasoning.	Compares the two decimals using at least one appropriate strategy with a clear explanation.	Compares the two decimals using one strategy with some explanation.	Compares the two decimals using one strategy with no explanation. Or, incorrectly compares the two decimals with or without a limited explanation.	No score is awarded because there is insufficient evidence of student performance based on the requirements of the assessment task.
Solves a problem by adding or subtracting two decimals using an appropriate strategy **Question #3**	Solves the problem by using an appropriate strategy with detailed and sound mathematical reasoning.	Solves the problem using an appropriate strategy with a clear explanation including some detail.	Solves the problem using an appropriate strategy but makes a slight error and includes some explanation.	Solves the problem using an appropriate strategy but makes errors and may or may not include a limited explanation.	No score is awarded because there is insufficient evidence of student performance based on the requirements of the assessment task.

B. One-on-one Assessment

Have base ten materials, a metre stick, place value charts and thousandths decimal grids for students to use as needed.

Present the student with the following problem:
Joanne bought 3.5 m of ribbon and used 0.482 m of it to wrap presents. How many metres of ribbon does Joanne have left?

Understand the Problem
Have the student read the problem orally and restate the problem in his or her own words. Provide guidance to ensure that the key ideas are explained and the question is understood.

Ask the student what operation can be used to solve this problem. If the student has difficulty answering this question, simplify the problem by changing the numbers in the problem to simple whole numbers. Then go back to using the decimals after the student recognizes what operation can be used to solve the problem.

Estimate First
Ask the student to estimate the answer first as a way to check for reasonableness of the calculated answer. If necessary, provide guidance by covering the decimal part of the each number and have the student focus on the whole number part.

Calculate the Exact Answer Using a Variety of Strategies
Ask the student to calculate the exact answer using a method that makes sense to him or her. Have the student answer the question asked in the complete sentence once the calculation is completed and compare the exact answer with the estimated answer.

Using a Metre Stick
Have the student measure a length of 3.5 m on the floor and review the meaning of each digit; i.e., 3 whole metres and five-tenths of a metre or 5 decimetres. Encourage the student to mark the metre stick to show the subtraction process; i.e., mark off 0.482 m, which is 4 dm, 8 cm and 2 mm.

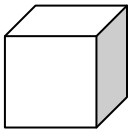
Using a Place Value Chart and Equivalent Decimals
Provide the student with a place value chart to thousandths and have him or her write the decimals from the problem in the chart and explain how this is done. Encourage the student to use equivalent decimals; i.e., 3.5 = 3.500. Then they may use a strategy of choice to solve the problem symbolically, pictorially (decimal grids) or concretely base-ten materials.

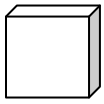
Using the Counting On Strategy
If the student uses a counting on strategy to solve the problem, provide guidance as necessary by skip counting from 482 thousandths to 492 thousandths, which is 10 thousandths or 0.010. Then add 8 thousandths or 0.008 to make 500 thousandths.

Using the Thousandths Decimal Grids
If necessary, have the student represent 3.500 on thousandths decimal grids by shading the grids. Review the parts of the thousandths grid if necessary to show tenths, hundredths and thousandths. Use one of the grids to show that 0.482 is 0.018 less than 0.500. On the grid showing 0.500 shaded, have the student show the subtraction of 0.482 by crossing out 4 tenths, 8 hundredths and 2 thousandths of the shaded part. This visual will show that the final answer is 3.018 m left.

Using Base Ten Materials
If the student uses the base ten materials to solve the problem, review the meaning the decimal, 3.5 as represent by base ten materials.

Base ten materials:


Block: 1 whole


Flat: $\frac{1}{10}$ or 0.1


Long: $\frac{1}{100}$ or 0.01


Small cube: $\frac{1}{1000}$ or 0.001

To subtract 0.482, provide guidance as needed in regrouping one flat into ten longs and regrouping one long into ten small cubes. Have the student write the symbols that correspond to the regrouping of the base ten materials.

See pages 122–125 of the *Diagnostic Mathematics Program, Division II, Operations and Properties* for a structured interview on adding and subtracting decimals to thousandths.

Comparing Decimals
To address the concept of comparing decimals, have the student explain, using concrete materials and/or diagrams, which is greater, 3.5 or 0.482.

C. Applied Learning

Does the student:

- indicate that 0.45 m is longer than 0.395 m and explain why, using a variety of strategies such as:

    a. use a metre stick to measure lengths of string to represent each measure
    b. apply his or her understanding of place value in comparing the decimals
    c. apply his or her understanding of equivalent decimals (0.45 = 0.450) in comparing the decimals
    d. apply his or her understanding of benchmarks by explaining that both decimals are less than 0.5 m and that 0.45 m is closer to half a metre than 0.395 m and is therefore the larger decimal?

- apply his or her understanding of decimals to order a series of decimals in a real world context?

Step 5: Follow-up on Assessment

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction?

A. Addressing Gaps in Learning

- Draw on the prior knowledge of students, spending time reviewing simple fractions as part of a region and part of a set. Review the meaning of a fraction and how it relates to a part and to a whole.
- Emphasize the similarities and differences between a fraction of a region and a fraction of a set.
- Provide everyday contexts for fractions and decimals that students can relate to.
- Use concrete materials such as counters, decimal grids and metre sticks. Connect the concrete to diagrams and symbols.
- Allow the students to use concrete materials as long as necessary to establish an understanding of the concepts.
- Connect the concrete, pictorial and symbolic representations.
- Build on students' prior knowledge of using benchmarks on a number line to order fractions and connect it to ordering decimals.
- Have the students sort a set of decimals into groups and explain the sorting process. One way to group the decimals could be: greater than 0.5, less than 0.5 or equal to 0.5.
- Ask guiding questions to direct the student's thinking. See the examples provided on the one-on-one assessment.
- Provide time for students to explore and construct their own meaning rather than being told.
- Encourage flexibility in thinking as students describe various ways to order decimals.
- Draw on the prior knowledge of students about adding and subtracting decimals to hundredths. Review the process using base ten materials, fraction bars, grids, counters and other appropriate concrete materials.
- Emphasize that students estimate the sum or difference of decimals before calculating the answer. Review that front-end estimating is useful and focuses only on the digits to the left of the decimal.
- Have the students share their thinking with others so that students having some difficulty hear how another person thinks about fractions and decimals in kid-friendly language.

B. Reinforcing and Extending Learning

Students who have achieved or exceeded the outcomes will benefit from ongoing opportunities to apply and extend their learning. These activities should support students in developing a deeper understanding of the concept and should not progress to the outcomes in subsequent grades. For example, in Grade 3 you might want to explore perimeter of more irregular shapes, but you would not extend this to connecting perimeter to area, which is a Grade 4 outcome.

Consider strategies such as:

- Provide parents with suggestions for using fractions and decimals with their children, such as:
  - Ask your child if 0.6 m of string is greater than, less than or the same length as 0.538 m of string. Have them explain why or why not.
  - Have your child measure lengths to the nearest thousandth of a metre; i.e., to the nearest millimetre.
  - Have your child use data from newspapers or magazines that is presented in thousandths to create and solve problems involving addition and subtraction of decimals.
- Have students make a set of cards that connect the diagram to the fraction to the decimal. For example, have the students draw a diagram on one card to show three thousandths, write the fraction $\frac{3}{1000}$ on second card and write the decimal 0.003 on a third card. When the set of cards is completed, students could use them to play "Memory" (pick up triplets rather than pairs), Go Fish, Rummy (focusing on three of a kind) or some other appropriate game that requires matching.
- Use everyday contexts for decimals to thousandths such as batting averages, track times, weights and lengths. Encourage the students to create and solve problems using a variety of contexts and a variety of strategies. Then they can share their ideas with others in the class.
- Ask the students to compare two decimals such as 0.507 and 0.498 by determining which decimal is closest to 0.5 and explain why.
- Challenge the students to find another decimal between any two given decimals such as 0.618 and 0.619. Have the students explain why the set of decimals is dense; i.e., between any two decimals there is always another decimal.
- Challenge the student to solve multi-step problems such as:
  - Three children raced to the flagpole. Chris got there in 24.057 seconds. Mary got there in 1.8 seconds less than Chris. Danny got there in 0.42 seconds more than Mary. How much longer did it take Chris to reach the flagpole than Danny? Explain your thinking.
  - Three children raced to the flagpole. Jaden got there in 0.089 seconds longer than Maren. Tanya got there in 0.5 seconds less than Jaden. If Maren reached the flagpole in 5.12 seconds, how long did it take Tanya to reach the flagpole? Explain your thinking.

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