Planning Guide: *Grade 7 Circles*
Strand: Shape and Space (Measurement)
Outcome: 1

Curriculum Highlights

This sample targets the following changes in the curriculum:
- The General Outcome focuses on using direct or indirect measurement to solve problems, whereas the previous mathematics curriculum focused on solving problems involving the properties of circles and their connections with angles.
- The Specific Outcome is very similar to the previous mathematics curriculum, but also includes the construction of circles, the sum of the central angles of a circle and specific reference to pi.

Step 1: Identify Outcomes to Address

Guiding Questions

- What do I want my students to learn?
- What can my students currently understand and do?
- What do I want my students to understand and be able to do, based on the Big Ideas and specific outcomes in the program of studies?

Big Ideas

A circle is "a plane figure that has all its points the same distance from a fixed point called the center of the circle" (Cathcart 1997, p. 185). The radius is the distance from the centre of the circle to the edge of that circle while the diameter is a line segment passing through the centre of the circle with both endpoints on the circle. The circumference of a circle is the distance around or the perimeter of a circle.

In using any type of measurement, such as length or angle, it is important to discuss the similarities in developing understanding of the different measures: first identify the attribute to be measured, then choose an appropriate unit and finally compare that unit to the object being measured (NCTM 2000, p. 171).

When measuring the circumference, radius and diameter of circles, the attribute of length is being measured and appropriate units to measure length include millimetres, centimetres and metres. When finding the sum of the central angles of a circle, the attribute of angle measure is being used and the appropriate unit to measure angles is the degree.

The circumference of any circle divided by the diameter of that circle is a constant called pi and is written as π. "Pi (rather than some other Greek letter like alpha or omega) was chosen as the letter to represent the number 3.141592 ... because the letter [π] in Greek, pronounced like our letter 'p,' stands for 'perimeter'" (The Math Forum@Drexel).
Pi is an irrational number, a nonrepeating, nonterminating decimal.
Pi = 3.14159265358979323846264338327950288419716939937510582097494 4 ...

The value of pi is often approximated as 3.14 or $\frac{22}{7}$.

Since pi is irrational, it cannot be expressed as a whole number divided by another whole number. Only rational numbers can be expressed as $\frac{a}{b}$, where *a* and *b* are integers and *b* is not equal to zero. Since pi is the ratio of the circumference to the diameter of a given circle, then the measure of either the circumference or the diameter or both cannot be whole numbers.

All measurements are approximate. When measuring the circumference and diameter of a circle, we use an approximate measure for each one, recognizing that at least one of the measures is an approximation for an irrational number.

Irrational numbers have a place on the number line and are therefore real numbers; i.e., real measures. For example, the measure of the hypotenuse of a right isosceles triangle with the congruent sides each one metre in length is $\sqrt{2}$ metres, a real measure, a real number that is irrational.

Sequence of Outcomes from the Program of Studies

See (Web address) for the complete program of studies.

Grade 6	Grade 7	Grade 8
Specific Outcomes 1. Demonstrate an understanding of angles by: • identifying examples of angles in the environment • classifying angles according to their measure • estimating the measure of angles, using 45°, 90° and 180° as reference angles • determining angle measures in degrees • drawing and labelling angles when the measure is specified. 2. Demonstrate that the sum of interior angles is: • 180^{o} in a triangle • 360^{o} in a quadrilateral. 3. Develop and apply a formula for determining the: • perimeter of polygons • area of rectangles • volume of right rectangular prisms.	Specific Outcomes 1. Demonstrate an understanding of circles by: • describing the relationships among radius, diameter and circumference • relating circumference to pi • determining the sum of the central angles • constructing circles with a given radius or diameter • solving problems involving the radii, diameters and circumferences of circles.	Specific Outcomes 3. Determine the surface area of: • right rectangular prisms • right triangular prisms • right cylinders to solve problems 4. Develop and apply formulas for determining the volume of right rectangular prisms, right triangular prisms and right cylinders.

Step 2: Determine Evidence of Student Learning

Guiding Questions

- What evidence will I look for to know that learning has occurred?
- What should students demonstrate to show their understanding of the mathematical concepts, skills and Big Ideas?

As you begin planning lessons and learning activities, keep in mind ongoing ways to monitor and assess student learning. One starting point for this planning is to consider the achievement indicators listed in *The Alberta K–9 Mathematics Program of Studies with Achievement Indicators* (Alberta Education 2007). You may also generate your own indicators and use these to guide your observation of students.

The following achievement indicators may be used to determine whether students have met this specific outcome.

- Illustrate and explain that the diameter is twice the radius in a given circle.
- Illustrate and explain that the circumference is approximately three times the diameter in a given circle.
- Explain that, for all circles, pi is the ratio of the circumference to the diameter (C/d), and its value is approximately 3.14.
- Explain, using an illustration, that the sum of the central angles of a circle is 360^{o}.
- Draw a circle with a given radius or diameter with and without a compass.
- Solve a given contextual problem involving circles.

Some sample behaviours to look for in relation to these indicators are suggested for many of the instructional activities in Step 3, Section C, Choosing Learning Activities.

Step 3: Plan for Instruction

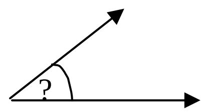
Guiding Questions

- What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should I use?
- How will I meet the diverse learning needs of my students?

A. Assessing Prior Knowledge and Skills

Before introducing new material, consider ways to assess and build on students' knowledge and skills related to understanding circles. For example:

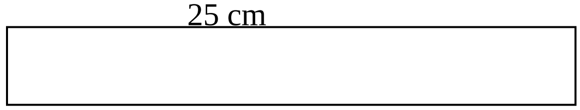
- Estimate the measure of the following angle. Explain your thinking.



- Explain, using a model, that the sum of the interior angles of a quadrilateral is 360^o.
- Write a rule for finding the perimeter of:
  a) any polygon
  b) a rectangle that is not a square
  c) a square.
- How many metres of fencing are required to enclose, completely, a rectangular yard that is 25 m by 35 m? Explain your thinking.
- If the perimeter of a square garden is 24 m, what is the length of each side of the garden? Explain your thinking.
- Find the perimeter of the rectangle shown below. Use the formula for finding the perimeter of a rectangle.

25 cm

40 mm



If a student appears to have difficulty with these tasks, consider further individual assessment, such as a structured interview, to determine the student's level of skill and understanding. See Sample Structured Interview: Assessing Prior Knowledge Skills.

B. Choosing Instructional Strategies

Consider the following strategies when planning lessons.

- Access prior knowledge on perimeter and angles outlined in the achievement indicators for Grade 6.
- Use exploratory activities that stimulate students' thinking and provide a foundation for generalizing Big Ideas about circles.
- Use literature to stimulate students' thinking about circles and their properties.
- Connect the circumference of circles to perimeters of polygons.
- Emphasize that students connect the concrete, pictorial and symbolic representations as they explore the properties of circles.
- Have students justify the strategies they use in solving problems related to circles and critique strategies used by others.

C. Choosing Learning Activities

The following learning activities are examples of activities that could be used to develop student understanding of the concepts identified in Step 1.

Sample Activities for Teaching Understanding of Circles

Look For …
Do students: ☐ construct their meaning of a circle by standing the same distance from a fixed point (centre)? ☐ relate radius to diameter visually? ☐ relate diameter to circumference visually?

1. Concrete Circle

   a) Creating the Circle

   Have students position themselves to make a human circle with one student as the centre of the circle. Pose the question:

   > How can you use the person at the centre of the circle to check that the circle made is really circular?

   Guide discussion about one of the characteristics of a circle; i.e., any point on the circle is the same distance from a point at its centre.

   Provide the student at the centre with a string about 3 metres long. Have him or her give the other end of the string to a student on the circle. The student must adjust his or her position so the string is taut. Then the student passes the string to the person next to him or her on the circle, who adjusts his or her position so that the string is taut. Have students continue the process until the string has been passed to all students on the circle. As the string is passed from student to student, the student in the centre of the circle must stay in position but may need to turn his or her body with the movement of the string.

   b) Relating the Diameter and Radius of a Circle

   Introduce the term *radius* as the distance from the centre of the circle to a point on the circle. Ask students what represents the radius in their concrete circle. After they understand that the length of the string represents the radius, have them predict how many lengths of string are needed to extend from one student to another student on the opposite side of the circle, passing through the centre. Have students show that double the radius equals the distance across the circle through the centre.

   Introduce the term *diameter* to represent this distance. Have students verbalize the relation between the radius and the diameter and write it symbolically; i.e., $d = 2r$.

   c) Relating Diameter, Radius and Circumference

   Introduce the term *circumference* as the distance around the circle and relate it to the perimeter of polygons. Through discussion, have students verbalize that the circumferences of circles vary, depending on the length of the radii and the diameters. Have students shorten or lengthen the string used to make a smaller or larger circle.

Connect the length of the diameter to the circumference of a circle by posing the following question:

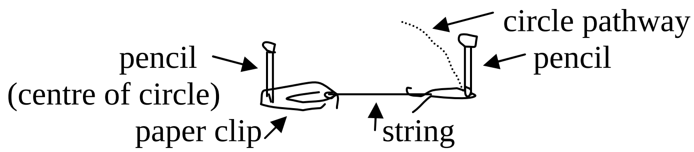
> About how many lengths the size of the diameter would fit around the perimeter or circumference of the circle?

After students make their predictions, have the student in the centre take the string and repeatedly use it as a measure of the circumference. (Note: if done accurately, a little more than 6 radii are needed to measure the circumference of the circle.) Pose the following question:

> If about 6 lengths of string, each representing the radius of the circle, are needed to measure the circumference of the circle, how many lengths of string the size of the diameter of the circle would be needed ? Explain your thinking.

2. Drawing Circles with and without a Compass

Provide students with 30-cm rulers, paper clips, string and pencils. Have them work in groups to devise a way to construct a circle. One student might hold the pencil in a paper clip for the centre of the circle while another student holds the other pencil that traces the pathway of the circle. Have them label the radius, diameter and circumference.



Look For …
Do students:
- ☐ transfer their understanding of a concrete circle using a string to a pictorial circle?
- ☐ measure the length of string accurately for the given radius?
- ☐ relate diameter to radius when given the diameter of a circle that they are to construct?

Then have students construct circles of various sizes with given radii and diameters; e.g., radius is 5 cm, diameter is 12 cm. Instruct them to label the lengths of the radius and diameter for each circle. Encourage them to predict the circumference of each circle and then use string to measure it and verify the prediction.

Provide students with compasses. Have them explore constructing circles, using a compass and ruler. Remind them to label the length of the radius and the diameter of each circle. If necessary, model the construction of a circle with a compass by using an overhead projector or a compass on the board.

3. Exploring Pi

Introduce this activity by reading part of the book *Sir Cumference and the Dragon of Pi: A Math Adventure* (1999) by Cindy Neuschwander. Provide students with labelled lids of various sizes and string to use as they are actively involved in simulating the story.

The main character in the book is Radius, son of Sir Cumference. During the story, Radius is watching his sister bake pies. She uses pie dough to make lattice strips as the top crust. Each strip fits all the way across the pie through the centre. Have students cut string to simulate a strip that is the diameter of the lid. Explain that Radius is laying the strips around the edge of the pie. Have students predict how many strips Radius will need to go around the edge of the pie (or to complete the circumference of the pie).

After students make their predictions, have them verify their predictions by cutting a piece of string the length of the diameter of their lid and repeatedly using it to measure the circumference of that lid. Have students share their results. They should conclude that it takes a little more than 3 lengths of string the size of the diameter of a given circle to complete the circumference of that circle.

Look For …

Do students:

- ☐ translate the action in the story to the action of measuring various circular lids?
- ☐ predict the relation between the diameter and the circumference by using their background knowledge from previous activities?
- ☐ accurately record their measures for the diameter and circumference of each circle?
- ☐ discover the constant—the relationship between the circumference and the diameter of any circle?

Have students measure the length of the diameter and the circumference of each lid and record the results in a class chart posted on the overhead or the board. To measure the circumference, have students place a string around the circumference of the lid and then measure the length of string. Ask students to predict the entries for the circumference ÷ diameter in the chart.

Circle or Lid	Diameter in cm	Circumference in cm	Circumference ÷ Diameter (to 2 decimal places)
A			
B			
C			

Encourage students to use calculators to complete the last column of the chart. Then continue on with the story to determine if Radius' explorations lead to the same conclusions. If necessary, have students record more data, using different lids to verify that the quotient of the circumference divided by the diameter is always a little more than 3.

Introduce the term *pi* and the Greek symbol for it—π. Explain that an approximate value for pi is 3.14. You may wish to introduce irrational numbers at this point, explaining that the value for pi is nonrepeating and nonterminating and therefore not rational (of the form a/b, where a and b are integers and b is not equal to zero).

Encourage students to use the π button on the calculator to explore the decimal notation for π.

4. Research Activity

Continue the discussion of π by reviewing that π is the ratio of the circumference of a circle to the diameter of that circle and can be written symbolically as $\pi = \frac{C}{d}$. Review that π cannot be written as a repeating or terminating decimal because it is an irrational number.

Pose the following question:

> Since $\pi = \frac{C}{d}$, a fraction, then $\frac{C}{d}$ can be represented by a repeating or terminating decimal. Why is π not equal to a repeating or terminating decimal and yet it is equal to a fraction?

Look For …

Do students:

- ☐ actively research pi and explain how pi can be a nonrepeating, nonterminating decimal and also be a ratio or a fraction (C/d)?
- ☐ communicate clearly the results of the research related to the question posed about pi?

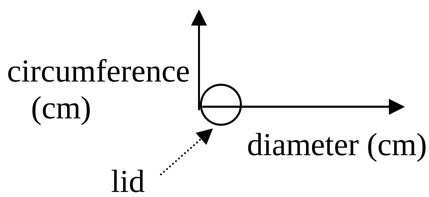
Have students do some research on this question by interviewing staff members, their family or community members. They may also use the Internet to research this question; e.g., The Math Forum Web site.

Through discussion, have students verbalize that for π to be equal to a nonrepeating, nonterminating decimal, then either the diameter or the circumference or both must be a measure that is a nonrepeating, nonterminating decimal. Remind students that all measures are approximate so they are not aware when a certain measure may be a nonrepeating, nonterminating decimal.

5. Graphing the Circumference and the Diameter

Have students construct a scatter plot or a line graph, labelling the horizontal axis "diameter," and the vertical axis "circumference."

Students may use their lids from Activity 3 to graph the diameter and the circumference of each lid. They can place the lid along the horizontal axis and mark the diameter, then roll the lid along the vertical axis and mark the circumference.



Alternately, students can use the data collected in their charts in Activity 3 to plot the points (diameter, circumference) on the graph.

Have students write an equation for the line graph that relates the circumference of a circle to its diameter; i.e., $C = \pi d$ or $\pi = \frac{C}{d}$.

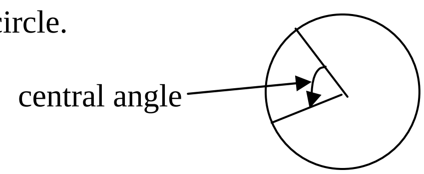
Look For …
Do students:
- ☐ graph the points representing the diameter and circumference for circles so that a linear relation is shown visually?
- ☐ use the formula for the circumference of a circle to add more points to the graph?
- ☐ interpret the graph by finding the diameter or circumference, given one of these measures?

Explore various problems in which the circumference, diameter or radius is provided and the other two measures must be calculated.

6. Sum of the Central Angles

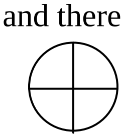
Provide students with scrap paper, compasses or traceable lids, pencils and paper. Have them construct circles by tracing lids or using a compass. Pose the following problem:

> Explain how you would find the sum of the central angles of any circle without using a protractor. A central angle is an angle with its vertex at the centre of the circle and its arms intersecting the circle.

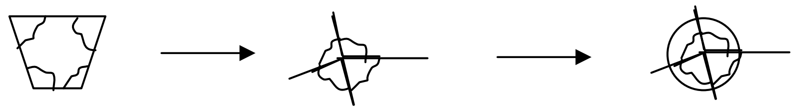


Look For …
Do students:
- ☐ apply their knowledge of angles and degrees in finding the sum of the central angles of a circle?
- ☐ explain, using a variety of strategies, why the sum of the central angles is always equal to 360^o?

Encourage students to share their strategies. Students should build on their understanding of 180^o being a semi-circle; therefore, the complete circle will be 360^o. Other students may build on their understanding that a right angle is 90^o and there are four 90^o central angles in a circle; therefore, $4 \times 90^o = 360^o$.



Still other students may apply their knowledge that the sum of the angles of any quadrilateral is 360^{o} by tearing off the vertices of any quadrilateral and arranging them so the vertices all meet in the centre of a circle.



Quadrilateral   Rearrange vertices   Vertices as central angles in a circle

7. Concept Definition Map

Provide students with a template of a concept definition map to complete for circles. The map can be done together as a class, a small group or individually, depending on the needs of students. As students complete the map, they consolidate their understanding of circles and review some of the big ideas. A sample concept definition map for circles is provided below.

Look For …

Do students:

- ☐ write a definition for a circle in their own words?
- ☐ explain the essential characteristics of circles, based on explorations of a circle?
- ☐ provide examples and nonexamples to clarify the characteristics of circles?

Concept Definition Map

Category
What is it?

2-dimensional Figure

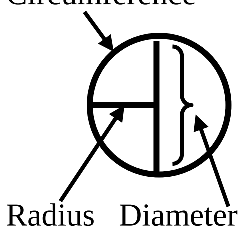
Properties
What are its characteristics?

- The circumference is the distance around a circle.
- The diameter of a circle is always twice the radius of that circle.
- The ratio of the circumference to the diameter of any circle is constant and is named pi (π).
- Multiplying the diameter of a circle by pi (π) results in obtaining the circumference of that circle.
- The sum of the central angles of any circle is 360^{o}.

Word

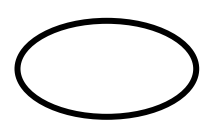
Circle

Examples



Find the length of fencing needed to enclose a circular lot with a radius of 10 m.

Nonexamples



Find the length of fencing needed to enclose a rectangular lot with an area of 12 m^2.

Format adapted from Robert M. Schwartz, "Learning to Learn Vocabulary in Content Area Textbooks," *Journal of Reading* 32, 2 (1988), p. 110, Example 1. Adapted with permission from International Reading Association.

8. Pop Cans and Tennis Balls

Provide a pop can for each group of students. Present students with the following problem:

> Predict whether the distance around a pop can is more than, less than or the same as the height of the pop can. Explain your thinking.

Have students share their predictions and explanations. Guide the discussion to use the length of one finger as the distance across the pop can (the diameter). Since the circumference of the pop can is about three times the diameter, it is necessary to decide if the height of the pop can is more than, less than or equal to three finger lengths with each finger length equivalent to the diameter. Of course, the pop can is less than three finger lengths with each finger length equivalent to the diameter of the can.

Look For …

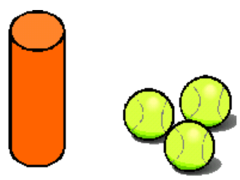
Do students:

- ☐ apply their understanding of the relation between the diameter and the circumference of a circle to make accurate predictions?
- ☐ justify their predictions?
- ☐ use previous predictions and verifications to refine their predictions and justify them?

Provide students with string to measure the circumference of the pop can and verify their predictions.

Provide students with other cylindrical containers to predict and then determine if the circumference of the container is more than, less than or equal to the height of the container.

Adapted from Marilyn Burns, "A Can of Coke Leads to a Piece of Pi," *Journal of Staff Development* 25, 4 (Fall 2004), pp. 16, 18. Used with permission of the National Staff Development Council, www.nsdc.org, 2007. All rights reserved.



Include the cylindrical container for three tennis balls as one example. Through discussion, students should explain that the tennis ball container would be about as tall as the distance around the container. The three balls (representing the height of the container) are three times the diameter of the container (Alberta Education 2005, p. 53).

Step 4: Assess Student Learning

Guiding Questions

- Look back at what you determined as acceptable evidence in Step 2.
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

In addition to ongoing assessment throughout the lessons, consider the following sample activities to evaluate students' learning at key milestones. Suggestions are given for assessing all students as a class or in groups, individual students in need of further evaluation and individual or groups of students in a variety of contexts.

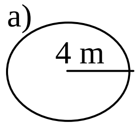
A. Whole Class/Group Assessment

Note: Performance-based assessment tasks are under development.

Provide students with a compass and straight edge. Present them with the following problems.

1. For each of the following circles, find the missing measures. Show your work.

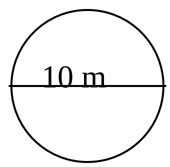
a)



circumference: ______

diameter: _______

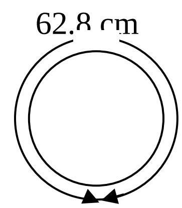
b)



circumference: _______

radius: _______

c)



diameter: ______

radius: ______

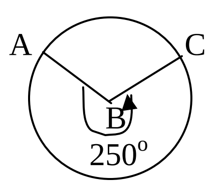
2. Construct a circle with a diameter of 7 cm. Label the diameter, radius and circumference.

3. Three of the four central angles of a circle are congruent and each measures 80^{o}. What is the measure of the fourth central angle? Explain your thinking.

4. A circular mirror has a diameter of 15 cm. What is the circumference to the nearest tenth of a centimetre?

5. A dog is tethered to a stake in a yard and can walk or run in a circle. The largest circumference of his runway is 56.52 m. What is the length of the dog's tether rope? Explain your thinking.

6. Without using a protractor, determine the measure of the obtuse angle ABC in the following diagram. Show your work.



7. The diameter of one circular lid is four times the diameter of another circular lid.
   a) The radius of one lid is how many times greater than the radius of the other lid?

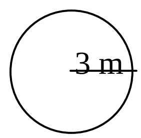
b) The circumference of one lid is how many times greater than the circumference of the other lid?

B. One-on-One Assessment

Assessment activities can be used with individual students, especially students who may be having difficulty with the outcome.

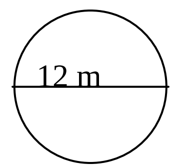
1. Present the following parts of this question to the student, one at a time. Have the student show his or her work or explain his or her thinking for each part.

a)



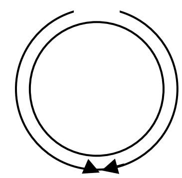
circumference: _____
diameter: ______

b)



circumference: ______
radius: ______

c) 31.4 cm



diameter: _____
radius: _____

If the student has difficulty completing any part of this question, remind the student of the relationship between the radius and the diameter, the radius and the circumference, and the diameter and the circumference. For part c, also review related number sentences for multiplication and division. This will provide scaffolding for the student to understand that multiplying the diameter by pi to get the circumference is the opposite of dividing the circumference by pi to get the diameter.

2. Instruct the student to construct a circle with a diameter of 5 cm and label the diameter, radius and circumference.

If the student has difficulty constructing the circle, have him or her explain how he or she would construct any circle using a compass. Remind the student that the point of the compass is the centre of the circle. Ask if the diameter or the radius is needed in constructing a circle with a compass. Review the relationship between the diameter and the radius.

3. Present the following problem to the student.
Four of the five central angles of a circle are congruent and each measures 60^{o}. What is the measure of the fifth central angle? Explain your thinking.

If the student has difficulty with some of the terminology in the question, review central angles by using a diagram and also review congruency. Have the student sketch a diagram to represent the question. Remind the student that the measures of the central angles of any circle total 360^{o}.

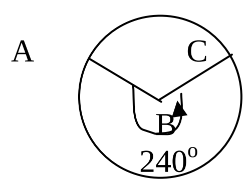
4. Present the following problem to the student.
   A compact disk has a diameter of 12 cm. What is the circumference to the nearest tenth of a centimetre?

   If the student has difficulty relating the diameter of a circle to the circumference, review the relationship by measuring the diameter of a lid or the base of a cup and then finding out how many of these lengths go around the circumference of the object. Remind the student that for any circle, this relationship remains constant; i.e., circumference = about 3 times the diameter. Review that the constant is known as pi, which is about 3.14. Review the formula for the circumference of a circle.

5. Present the following problem to the student.
   A dog is tethered to a stake in a yard and can walk or run in a circle. The largest circumference of his runway is 50.24 m. What is the length of the dog's tether rope? Explain your thinking.

   If the student has difficulty understanding the problem, have him or her read it orally. Have the student draw a diagram to represent the problem. If the student needs prompting regarding the relation between the circumference and the diameter, review this relationship by using objects—see comments for question 4. If the student has difficulty manipulating the multiplication sentence $C = 3.14\ d$ into a division sentence, $C/3.14 = d$, review related number sentences for multiplication and division, using whole numbers. Have the student check his or her work to make sure that the radius found is half the diameter, because the dog's tether rope is the radius of the circle with circumference 50.24 m.

6. Present the following problem to the student.
   Without using a protractor, determine the measure of the obtuse angle ABC in the following diagram. Show your work.



   If the student has difficulty interpreting the diagram, remind the student that there are two central angles shown in the diagram; one is 240^{o} and the other is unknown. Ask the student what the total number of degrees is for all the central angles of any circle. This total can then be used to find the missing central angle.

7. Present the following problem to the student.
   The diameter of one circular lid is twice the diameter of another circular lid.
   a) The radius of one lid is how many times greater than the radius of the other lid?
   b) The circumference of one lid is how many times greater than the circumference of the other lid?

If the student has difficulty, use numbers in the problem; e.g., diameter of one lid is 10 cm, diameter of the other lid is 20 cm, therefore the radius of one lid is 5 cm and the radius of the other lid is 10 cm. Have the student explain that 10 cm is how many times greater than 5 cm. Use similar reasoning for the circumference.

C. Applied Learning

Provide opportunities for students to use their understanding of circles in a practical situation and notice whether or not this understanding transfers. For example, have students find the amount of fencing needed for a circular plot or the length of lace needed for the edging of a circular tablecloth. Does the student:

- estimate about how much fencing or lace is needed by using an estimate for the length of the diameter?
- measure the diameter, using appropriate units of measure?
- multiply the diameter of the circle by π or 3.14 to obtain the circumference of that circle?
- apply the understanding of circumference when the problem is adapted; e.g., doubling the diameter would double the circumference, doubling the length of the radius of a circle would quadruple both the diameter and the circumference of that circle?

Step 5: Follow-up on Assessment

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction?

A. Addressing Gaps in Learning

- Continue work with hands-on experiences in relating radius, diameter and circumference. Have students measure the radius, diameter and circumference of various circular objects, such as lids.
- Provide practice for students to use a compass in creating art designs with circles.
- Review the relationship between multiplication and division by using number sentences with whole numbers. Transfer this learning to $C = 3.14\ d$ and $C/3.14 = d$.
- Create problems, using circumference, radius and diameter with everyday contexts that students can relate to.
- Encourage students to read the problem orally, identify the given information, draw a diagram to represent the problem and write appropriate number sentences that relate the radius, diameter, circumference and/or pi.
- Encourage students to use the calculator if the calculations are difficult in solving problems.

B. Reinforcing and Extending Learning

Students who have achieved or exceeded the outcomes will benefit from ongoing opportunities to apply and extend their learning. These activities should support students in developing a deeper understanding of the concept and should not progress to the outcomes in subsequent grades.

Consider strategies, such as the following.

- Provide tips for parents on working with circles at home or in the community.
  - Find the length of lace needed for a circular tablecloth.
  - Find the length of fencing needed for a circular garden.
  - Find the distance across a circular lake if you know the number of steps it takes to walk around it.

- Use the Geometer's Sketchpad to draw circles and explore the relationship among radius, diameter and circumference. Use the Sketchpad to explore the quotient of the circumference and the diameter of various circles (Bennett 1999).

- Provide students with rolls of masking tape. Have them take a piece of tape and measure the diameter of the roll, placing this length of tape on a piece of paper. Have them take a piece of tape off the roll that represents the circumference of the role and place the tape parallel to the other piece of tape with the bottoms of the tapes aligned. Encourage students to use their visual skills to predict how many times longer one piece of tape is than the other piece. Then have them measure each piece to check their predictions. Instruct students to do this same activity for other rolls of masking tape and compare the results.

  Adapted from Alberta Education, *Measurement: Activities to Develop Understanding* (unpublished workshop handout) (Edmonton, AB: Alberta Education, 2005), p. 57.

- Have students research the origin of radius, diameter, circumference and pi.

- Provide students with a variety of problems in which two of the following three measures are provided and the student must find the third measure: radius, diameter, circumference.

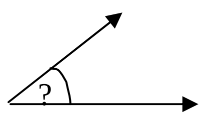
- Have students create problems about circles.

- Challenge students to compare the graph for the circumference and the diameter of various circles ($C = 3.14\ d$) with the graph for the perimeter and the length of a side for various squares ($P = 4s$). Encourage them to discuss the similarities and differences between the two graphs.

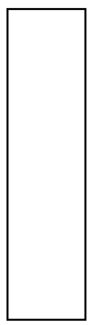
- Have students draw a Frayer model for circles, using the following template, and encourage them to share their models.

Definition	Characteristics
Visual Representation	Real World Problem

- Sample Structured Interview: Assessing Prior Knowledge and Skills

Directions	Date:	
	Not Quite There	**Ready to Apply**
Place the following angle before the student and say, "Estimate the measure of this angle. Explain your thinking." 	• Estimates more than 50 degrees or less than 40 degrees. • Does not use referents to explain his or her thinking.	• Estimates about 45 degrees and uses the square corner as a referent, thinking that the angle is about half a right angle.
Say, "Explain, using a model, that the sum of the interior angles of a quadrilateral is 360^o."	• Draws a quadrilateral and attempts to measure the four angles but is unable to get a sum of 360^o. • Draws a quadrilateral and cuts off the four corners but is unsuccessful in arranging the four angles to make a complete circle.	• Draws a quadrilateral, cuts off the four corners and arranges them carefully to make a complete circle. • Explains that since half a circle is 180^o, then the entire circle must be 360^o.

Say, "Write a rule for finding the perimeter of: a) any polygon b) a rectangle that is not a square c) a square."	• Does not write a rule and is unable to generalize appropriate formulas. • Does not explain what perimeter means in attempting to find a rule.	• Writes a rule for each perimeter, recognizing that perimeter is the distance around a 2-D shape.
Present the following problem to the student orally and in writing: "How many metres of fencing are required to enclose, completely, a rectangular yard that is 25 m by 35 m? Explain your thinking."	• Adds the two numbers but fails to double the answer to obtain the perimeter of the entire yard. • Finds the perimeter but is unable to explain the process.	• Finds the perimeter of the yard correctly and explains the process by using a diagram and/or appropriate number sentences.
Present the following problem to the student orally and in writing: "If the perimeter of a square garden is 24 m, what is the length of each side of the garden? Explain your thinking."	• Is unable to find the length of each side. • Guesses the answer. • Finds the length of each side but is unable to explain the process.	• Finds the length of each side and explains the process clearly; e.g., a square has four congruent sides, therefore, the length of each side is one quarter of the perimeter.

Present the following problem to the student, orally and in writing: "Find the perimeter of the rectangle shown below. Use the formula for finding the perimeter of a rectangle."  **25 cm**	• Does not convert the units so that the units of measure are the same for all sides of the rectangle. • Converts the units to the same units for all sides of the rectangle but only adds two sides or calculates incorrectly. • Finds the correct answer but is unable to apply the formula correctly.	• Converts the units so that they are the same for all sides of the rectangle. • Uses the formula for finding the perimeter and calculates the correct answer; e.g., $P = 2\ (L + W)$ $P = 2\ (25 + 4) = 58$ cm.

BIBLIOGRAPHY—Planning Guide: *Grade 7 Circles*

Strand: Shape and Space (Measurement)
Outcome: 1

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