

# Planning Guide: *Grade 7 Addition and Subtraction of Integers*

**Strand:** Number

**Outcome:** 6

## Curriculum Highlights

This sample targets the following changes in the curriculum:

- The General Outcome focuses on developing number sense, whereas the previous mathematics curriculum focused on applying arithmetic operations on decimals and integers, and illustrating their use in solving problems.
- The Specific Outcome focuses on understanding addition and subtraction of integers concretely, pictorially and symbolically, whereas the previous mathematics curriculum included multiplication and division of integers as well in Grade 7.

## Step 1: Identify Outcomes to Address

### Guiding Questions

- What do I want my students to learn?
- What can my students currently understand and do?
- What do I want my students to understand and be able to do, based on the Big Ideas and specific outcomes in the program of studies?

### Big Ideas

Operations with integers build on operations with whole numbers. Just as decimals are an extension of the whole number place value system to represent numbers between the whole numbers, so also the set of integers is an extension of the whole number system to include the opposite of every whole number. The properties for whole numbers apply to integers and, in addition, the integers are closed under subtraction. Van de Walle and Lovin (2006, p. 131) reaffirm this in the following excerpt:

Integers add to number the idea of opposite, so that every number has both size and a positive or negative relationship to other numbers. A negative number is the opposite of the positive number of the same size.

Since integers include the whole numbers and their opposites (negative numbers), it can be said that integers are numbers that deal with direction as well as magnitude (Cathcart 1997, p. 349). In everyday life there are many uses of integers, so problem solving plays a major role in developing understanding of adding and subtracting integers.

Students need a solid conceptual foundation in whole numbers as a necessary prerequisite for integer computation. Strategies in problem solving with whole numbers can be transferred to solving problems with integers. Subtraction is related to addition in both the whole numbers and the integers. As students make these connections, they develop number sense. Number sense is defined as follows:

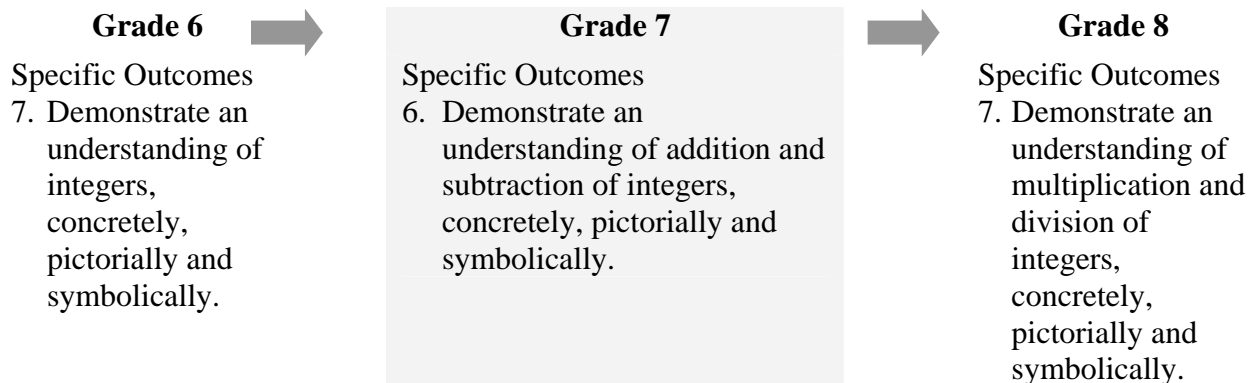
"Number sense refers to an intuitive feeling for numbers and their various uses and interpretations; an appreciation for various levels of accuracy when figuring; the ability to detect arithmetical errors; and a common-sense approach to using numbers" (Reys 1992, p. 3).

Students develop understanding of operations with integers by making sense of the ideas internally. To guide this process, it is necessary to encourage flexibility in thinking and provide learning opportunities in connecting:

- operations with whole numbers to operations with integers
- subtraction of integers to addition of integers
- concrete, pictorial and symbolic representations
- operations with integers to real world problems.

### Sequence of Outcomes from the Program of Studies

See (Web address) for the complete program of studies.



## Step 2: Determine Evidence of Student Learning

### Guiding Questions

- What evidence will I look for to know that learning has occurred?
- What should students demonstrate to show their understanding of the mathematical concepts, skills and Big Ideas?

As you begin planning lessons and learning activities, keep in mind ongoing ways to monitor and assess student learning. One starting point for this planning is to consider the achievement indicators listed in *The Alberta K–9 Mathematics Program of Studies with Achievement Indicators* (Alberta Education 2007). You may also generate your own indicators and use these to guide your observation of students.

The following achievement indicators may be used to determine whether students have met this specific outcome.

- Explain, using concrete materials such as integer tiles and diagrams, that the sum of opposite integers is zero.

- Illustrate, using a number line, the results of adding or subtracting negative and positive integers; e.g., a move in one direction followed by an equivalent move in the opposite direction results in no net change in position.
- Add two given integers, using concrete materials or pictorial representations, and record the process symbolically.
- Subtract two given integers, using concrete materials or pictorial representations, and record the process symbolically.
- Illustrate the relations between adding integers and subtracting integers.
- Solve a given problem involving the addition and subtraction of integers.

Some sample behaviours to look for in relation to these indicators are suggested for many of the instructional activities in [Step 3, Section C, Choosing Learning Activities](#).

## Step 3: Plan for Instruction

### Guiding Questions

- What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should I use?
- How will I meet the diverse learning needs of my students?

### A. Assessing Prior Knowledge and Skills

Before introducing new material, consider ways to assess and build on students' knowledge and skills related to counting. For example:

Provide students with 30-cm rulers.

- Draw a number line showing the integers.
- Place the following numbers on a number line: 3, -4, 0, 1, -5. Explain how these numbers are ordered.
- Write  $<$ ,  $=$  or  $>$  in the boxes to complete the sentences below correctly. Verify by placing the numbers on a number line.
 

a)  $-15 \square -20$                       b)  $-46 \square -31$                       c)  $-2 \square 5 - 5$
- Name the negative integers that are greater than  $-20$  and divisible by 3. Explain your answer.
- The temperature is  $-15^{\circ}$  Celsius on Monday and  $-8^{\circ}$  Celsius on Tuesday. Which day shows the higher temperature? How do you know?
- Explain why one negative integer is less than another negative integer by using a real world context.

If a student appears to have difficulty with these tasks, consider further individual assessment, such as a structured interview, to determine the student's level of skill and understanding. See [Sample Structured Interview: Assessing Prior Knowledge and Skills](#).

## B. Choosing Instructional Strategies

Consider the following strategies when planning lessons.

- Access prior knowledge on integers outlined in the achievement indicators for Grade 6.
- Use a problem-solving context that relates to students and applies the addition and subtraction of integers.
- Connect problems applying addition and subtraction of integers to similar problems with whole numbers. Review the types of problems for addition and subtraction, such as part-part-whole, comparison and join or separate.
- Connect the subtraction of integers to the addition of integers.
- Have students explore sums and differences of integers by using a variety of manipulatives, such as integer tiles and arrows on a number line.
- Emphasize that students connect the concrete, pictorial and symbolic representations for sums and differences of integers.
- Have students justify the strategies they use in finding sums and differences of integers and critique strategies used by others.

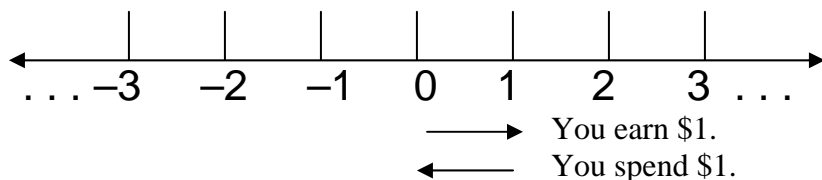
## C. Choosing Learning Activities

The following learning activities are examples of activities that could be used to develop student understanding of the concepts identified in Step 1.

### Sample Activities for Teaching Addition and Subtraction of Integers

#### 1. Opposite Integers/Concept of Zero

Provide students with everyday examples of gaining and losing to end up where you started, at zero; e.g.,  
You earn \$1 and then spend \$1. How much is your profit or loss?  
Represent the gain and loss on a number line as shown.



Your profit or loss is zero.

Have students share other similar examples. Introduce the term *opposite integers* and relate it to the example provided above.

Earning \$1 can be represented by +1. Spending \$1 can be represented by -1. +1 and -1 are called opposite integers. Symbolically,  $(+1) + (-1) = 0$ .

Have students draw a number line and place pairs of opposite integers on the number line. Through discussion, have them verbalize that opposite integers are equidistant from zero on the number line and when you add two opposite integers the sum is always zero.

#### Look For ...

Do students:

- relate to everyday contexts, using opposite integers?
- demonstrate flexibility in representing opposite integers, using a number line and integer tiles?

Introduce the term *additive inverse*, explaining that integers include the whole numbers and the additive inverses of the whole numbers.

Provide students with integer tiles and explain that these tiles can be used to represent integers. The red tiles represent positive integers and the white tiles represent negative integers.



Have students use the tiles to represent zero, draw diagrams and write number sentences. Encourage them to share their answers.

To consolidate understanding of zero, present students with the following question:  
Which of the following would represent zero, using integer tiles?



Through discussion, have students generalize that zeros are made by using tiles that are congruent and opposite in colour.

Have students use their integer tiles to answer questions; e.g.,

$(+2)$  and  $(-2) = 0$

$(+3)$  and  $(-3) = \underline{\quad}$

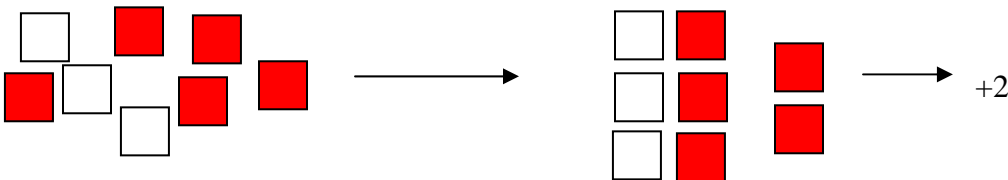
$(-5)$  and  $(\quad) = 0$

$(\quad)$  and  $(+4) = 0$

Adapted from Alberta Education, *Integers: Learning Strategies to Enhance Understanding* (unpublished workshop handout) (Edmonton, AB: Alberta Education, 2005), pp. 19–21.

## 2. Representing Integers

Provide students with integer tiles. Place the following tiles on the overhead projector and have students explain which integer the tiles represent. If necessary, remind students to make pairs of one positive and one negative to make zeros.



Have students represent +2 in other ways, using the integer tiles, and share their ideas. Ask how many different ways this can be done and take the opportunity to discuss the concept of infinity.

Use a similar process to explore various representations, using integer tiles for other integers such as  $-2$ ,  $+3$  and  $-5$ . Suggest that students look for patterns. They may wish to record their findings in a chart; e.g.,

<b>Look For ...</b>	
Do students:	
<input type="checkbox"/>	apply the concept of zeros in representing other integers?
<input type="checkbox"/>	interpret patterns to determine if a given number of integer tiles can be used to represent a given integer?

Integer	+2	-2	+3	-5
Number of integer tiles	2, 4, 6, 8, ...	2, 4, 6, 8, ...	3, 5, 7, 9...	5, 7, 9, 11...

Have students use the patterns they find to decide whether they could represent  $+3$  with 121 tiles and explain why or why not (Morrison and Morrison 1995, p. 59).

Guide the discussion to explore the sum of even numbers and the sum of odd numbers. Review that an even number of tiles is needed to make the zeros that form part of the number. Through discussion, have students verbalize the following generalization:

If the integer is an even number, then an even number of tiles greater than or equal to the absolute value of the integer must be used to represent it because an even number plus an even number is an even number.

If the integer is an odd number, then an odd number of tiles greater than or equal to the absolute value of the integer must be used to represent it because an odd number plus an even number is an odd number.

### 3. Adding Integers

Present students with real-world problems that involve the addition of integers; e.g.,

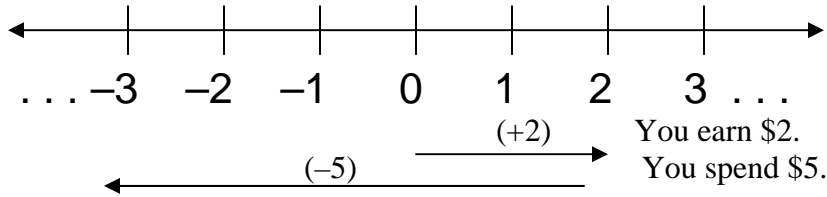
You earn \$2 and then spend \$5. How much is your profit or loss?

Build on students' understanding of adding and subtracting whole numbers on the number line and extend it to include the negative integers. Discuss that the numbers in the problem could be represented as  $+2$  and  $-5$ .

<b>Look For ...</b>	
Do students:	
<input type="checkbox"/>	interpret real-world problems as combining quantities to represent addition?
<input type="checkbox"/>	explain how adding integers is similar to adding whole numbers?
<input type="checkbox"/>	explain how adding integers is different from adding whole numbers?
<input type="checkbox"/>	demonstrate flexibility in using number lines and integer tiles in representing addition problems?

a) Number Line

Have students suggest how the problem could be represented on the number line.



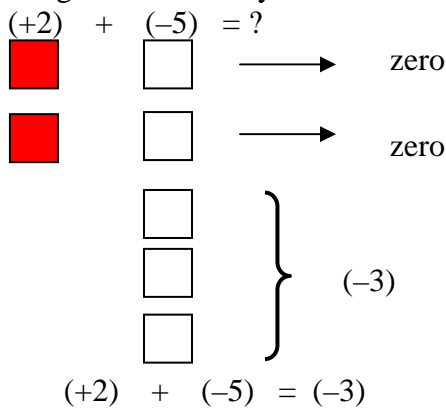
$(+2) + (-5) = (-3)$   
 Your loss is \$-3.

Provide more addition problem examples and have students represent the problems on a number line. Through discussion, have students verbalize that positive numbers go to the right on the number line and negative numbers go to the left.

b) Integer Tiles

Provide students with integer tiles and have them explore how to represent the problem with the tiles. Remind students that the red tiles are positive and the white tiles are negative. Have students share their strategies.

Earning \$2 is shown by +2.  
 Spending \$5 is shown by -5.



Have students add other integers using the integer tiles, drawing diagrams and writing the number sentences.



$(+2) + (+3) = \underline{\quad}$        $(-1) + (+4) = \underline{\quad}$   
 $(-1) + (-3) = \underline{\quad}$        $(+2) + (-4) = \underline{\quad}$

Encourage students to summarize their explorations by making generalizations about adding integers; e.g., the sum of two negative integers is a negative integer.

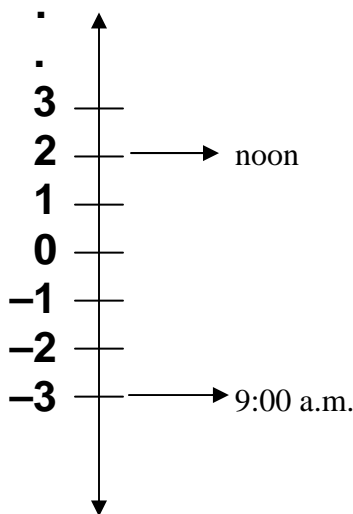
#### 4. Subtracting Integers

Present students with real-world problems that involve the addition of integers; e.g.,

At 9:00 a.m., the temperature was  $-3^{\circ}\text{C}$  and three hours later it was  $+2^{\circ}\text{C}$ . What was the increase in temperature during that time?

##### a) Vertical Number Line

Provide a vertical number line for students to illustrate the problem.



Have students solve the problem by using the number line. The increase in temperature is  $5^{\circ}\text{C}$ .

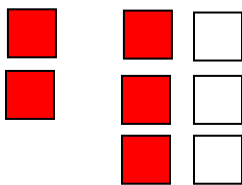
Discuss how a number sentence could be written to show the subtraction. If difficulty arises, work with whole numbers first; i.e., the increase from  $1^{\circ}\text{C}$  to  $3^{\circ}\text{C}$  would be written as  $3 - 1 = 2$ . Similarly, this problem could be written as  $3 - (-2) = 5$ . Have students generalize that you must subtract the initial temperature from the final temperature to find an increase in temperature.

##### b) Integer Tiles

Provide students with integer tiles and have them explore how to represent the problem in the previous activity with the tiles; i.e.,  $(+2) - (-3) = ?$  Remind students that the red tiles are positive and the white tiles are negative.

Review the subtraction of whole numbers, using the tiles, such as  $5 - 3 = 2$ . Then pose the questions: How is it possible to take away 3 white tiles when you have only 2 red tiles? How might we include some white tiles without changing the value of the equation? What is another way to represent  $+2$ , using some white tiles? Refer students to the activity on Representing Integers in which zeros were added. Have students share their strategies, such as the following:

$$(+2) - (-3) = ?$$



3 zeros added

Since we have  $(+2)$  or 2 red tiles, we must add some zeros so that we can subtract  $(-3)$  or 3 white tiles.

##### Look For ...

Do students:

- interpret real-world problems as comparison or take away to represent subtraction?
- explain the similarities and differences between subtraction of whole numbers and integers?
- demonstrate flexibility in using number lines and integer tiles in representing subtraction problems?
- generalize that subtraction is adding the opposite of the number that is subtracted?



If we remove  $(-3)$  or 3 white tiles, we have 5 red tiles or  $(+5)$  remaining.  
 $(+2) - (-3) = (+5)$  or  $(+2) + (+3) = (+5)$

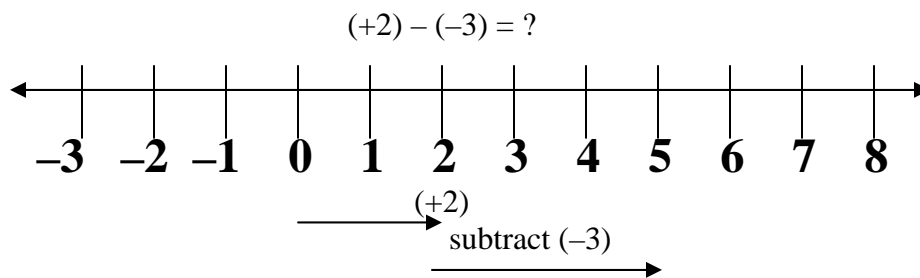


Have students solve a variety of subtraction problems and generalize that when you subtract, you add the opposite of the number that you are subtracting.

c) Horizontal Number Line

Provide a horizontal number line to represent the problem. Review subtracting whole numbers on the number line, such as  $5 - 3 = 2$ . Move 5 units right from 0 and then 3 units left from 5 to arrive at 2. Also, review adding integers such as  $(+2) + (-3) = (-1)$ . Move 2 units to the right from 0 and then 3 units to the left to arrive at  $-1$ .

Pose the question: If we move 3 units left when we add  $-3$ , what do you think we should do when we subtract  $-3$ ? Remind students that subtraction is the inverse or the opposite of addition.



We start at zero and move  $(+2)$  to the right. In subtracting  $(-3)$  we move 3 units to the right, the opposite of what we would do if we added  $(-3)$ .

Therefore,  $(+2) - (-3) = (+5)$  or  $(+2) + (+3) = (+5)$

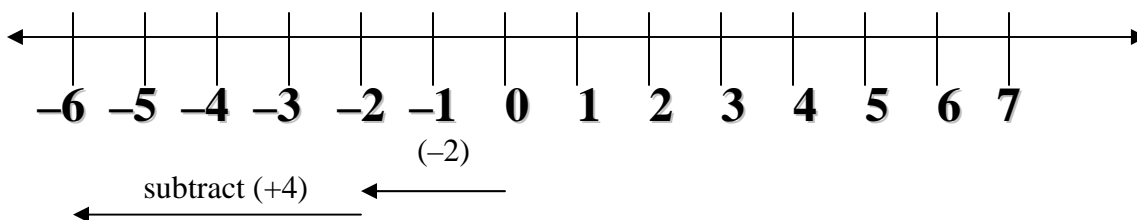
Guide the discussion so that students verbalize the following:

When you subtract a number on a number line, you move in the opposite direction than you would if you added that number. Addition and subtraction are opposite operations. When you subtract, add the opposite of the number that you are subtracting.

Provide students with other examples of subtraction problems, using integers such as the following. Have them use the number line or integer tiles, draw diagrams and write number sentences.

You are \$2 in debt and spend \$4 more. What is your debt now?

$$(-2) - (+4) = ?$$



We start at zero and move  $(-2)$  to the left. In subtracting  $(+4)$  we move 4 units to the left, the opposite of what we would do if we added  $(+4)$ .

Therefore,  $(-2) - (+4) = (-6)$  or  $(-2) + (-4) = (-6)$

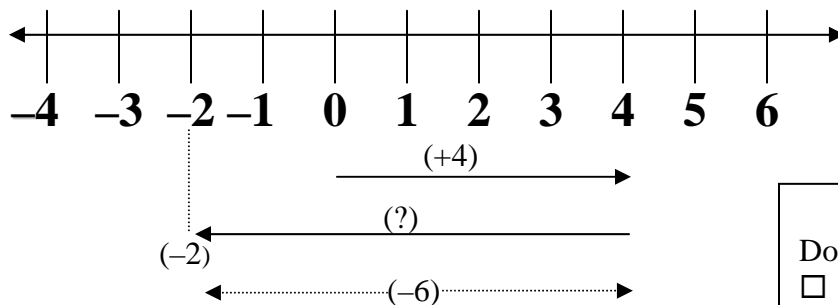
My debt now is  $\$-6$ .

Adapted from Alberta Education, *Integers: Learning Strategies to Enhance Understanding* (unpublished workshop handout) (Edmonton, AB: Alberta Education, 2005), pp. 22–25.

d) Writing Related Number Sentences for Addition and Subtraction

Review related number sentences for addition and subtraction of whole numbers; e.g.,  $5 - 3 = ?$  can be written as  $3 + ? = 5$ . Suggest that students try this strategy to solve the previous problem.

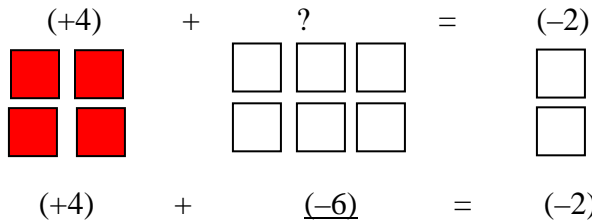
$(-2) - (+4) = ?$  can be written as  $(+4) + ? = (-2)$



$(+4) + (-6) = (-2)$  My debt is now  $\$-6$ .

Adapted from W. George Cathcart, Yvonne M. Pothier and James H. Vance, *Learning Mathematics in Elementary and Middle Schools* (2<sup>nd</sup> ed.) (Scarborough, ON: Prentice-Hall Canada, 1997), p. 350. Adapted with permission from Pearson Education Canada.

Have students use integer tiles to solve the same problem:



Provide practice in subtracting integers. Connect the concrete, pictorial and symbolic modes.

- $(+2) - (+3) = \underline{\quad}$
- $(+2) - (-2) = \underline{\quad}$
- $(-1) - (+3) = \underline{\quad}$  I am  $\$1$  in debt and I spend  $\$3$  more. What is my debt/profit now?
- $(-1) - (-3) = \underline{\quad}$
- $(-3) - (-2) = \underline{\quad}$  I am  $\$3$  in debt and someone cancels a  $\$2$  debt that I owe him. What is my debt/profit now?

**Look For ...**

Do students:

- apply related number sentences for adding and subtracting whole numbers and integers?
- represent the related number sentences by using a number line or integer tiles?

**Look For ...**

Do students:

- apply the appropriate process for adding and subtracting integers?
- demonstrate flexibility in solving addition and subtraction questions, using a variety of strategies?
- explain clearly the strategies used in adding and subtracting integers?

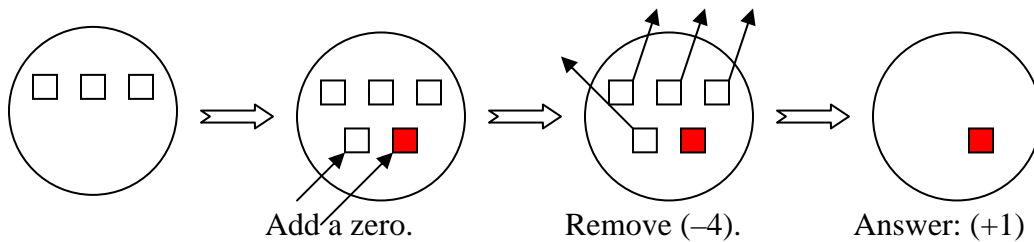
## 5. Adding and Subtracting Integers

Provide mixed practice for students to add and subtract integers by connecting the concrete, pictorial and symbolic representations. Have students use integer tiles to model their work and draw diagrams as in the examples below or have them choose an appropriate strategy and explain the process.

□ White (−)      ■ Red (+)

Example:

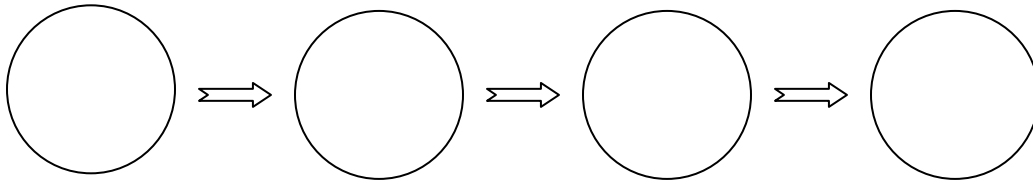
$$(-3) - (-4) = +1$$



Reproduced from Barbara and Harry Morrison, *Soar with Integers: A Complete Learning and Teaching Resource* (Barrie, ON: Exclusive Educational Products, 1995), p. 143. Reproduced with permission from Exclusive Educational Products.

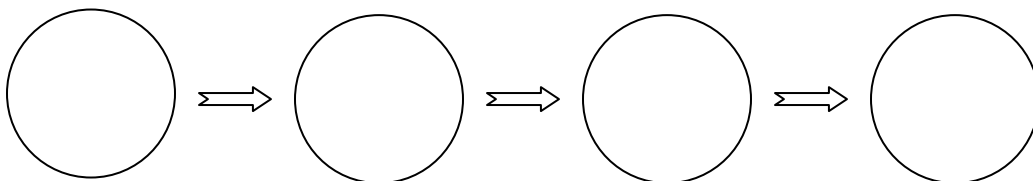
Question 1

$$(-2) - (-4) = \underline{\quad}$$



Question 2

$$(+3) + (-5) = \underline{\quad}$$



## 6. Concept Definition Map for Adding or Subtracting Integers

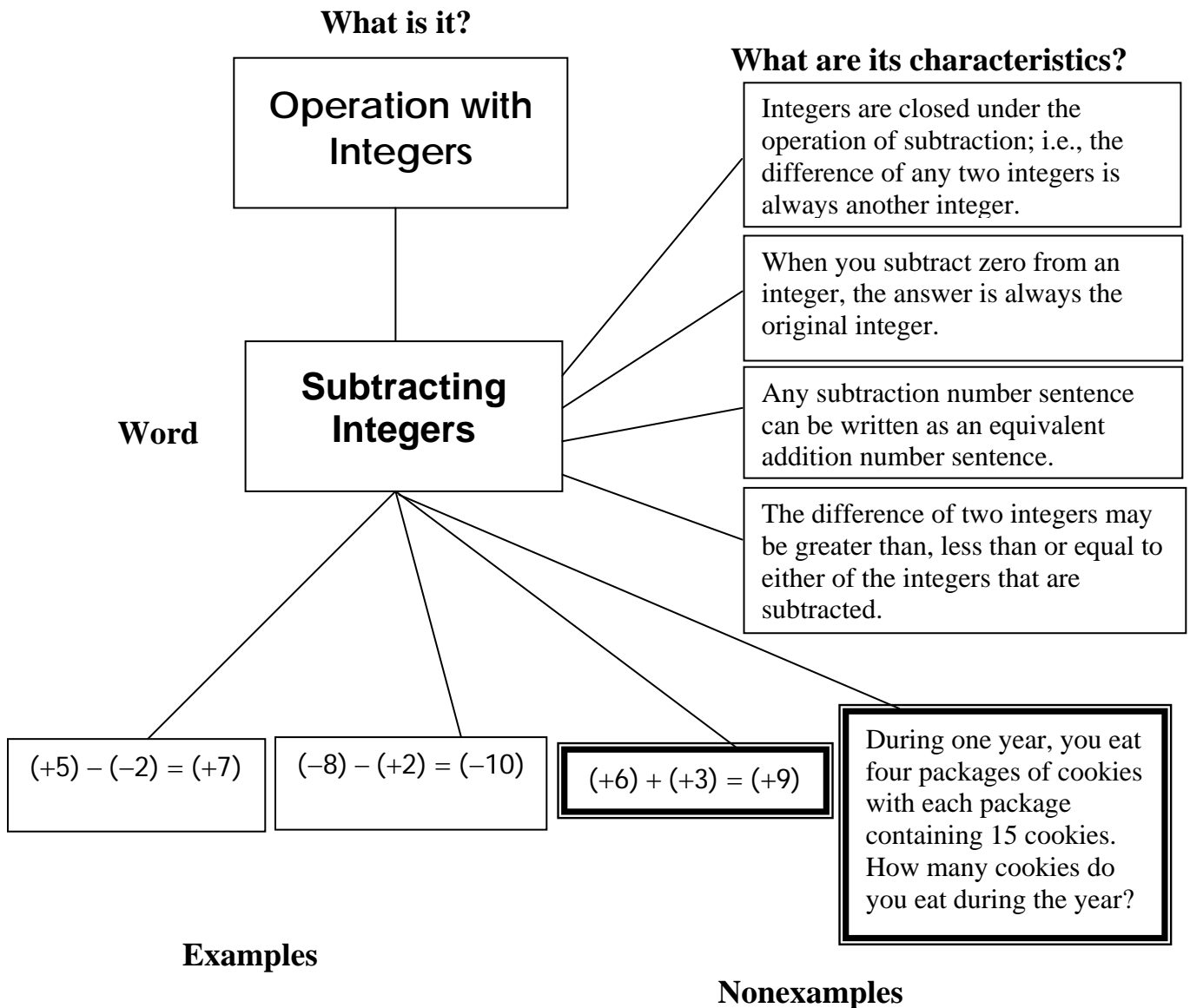
Have students construct a concept definition map for adding or subtracting integers to consolidate understanding. The map may be constructed together as a whole class, in small groups or individually, depending on the needs of students. A sample concept definition map for subtracting integers is provided below.

### **Look For ...**

Do students:

- identify the main characteristics of adding and subtracting integers?
- create examples and nonexamples of addition and subtraction problems?

# Concept Definition Map



Format adapted from Robert M. Schwartz, "Learning to Learn Vocabulary in Content Area Textbooks," *Journal of Reading* 32, 2 (1988), p. 110, Example 1. Adapted with permission from International Reading Association.

## Step 4: Assess Student Learning

### Guiding Questions

- Look back at what you determined as acceptable evidence in Step 2.
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

In addition to ongoing assessment throughout the lessons, consider the following sample activities to evaluate students' learning at key milestones. Suggestions are given for assessing all students as a class or in groups, individual students in need of further evaluation and individual or groups of students in a variety of contexts.

### A. Whole Class/Group Assessment

Note: Performance-based assessment tasks are under development.

Provide students with integer tiles and 30-cm rulers.

1. You earn \$4 and then spend \$4. How much is your profit or loss? Draw diagrams with integer tiles to represent this problem.
2. At 10:00 a.m., the temperature was  $-5^{\circ}$  C and 10 hours later it was  $+4^{\circ}$  C. What was the increase in temperature during this time? Show all your work, including a diagram and a number sentence.
3. Marcy has \$16 and spends \$3 per day. Johnny has \$20 and spends \$4 per day. What is the total amount of money or debt that they have at the end of one week? Show all your work.
4. Phu has a debt of \$45. He earns \$20 on each of three days and spends \$29 on a pair of pants. How much money or debt does he have now? Show all your work.
5. Find the answers to the following addition and subtraction of integers. Explain your thinking by drawing diagrams as indicated for each question.
  - a)  $(+4) + (-2) = ?$  Draw integer tiles to represent this question.
  - b)  $(-4) + (-2) = ?$  Draw a number line and arrows to represent this question.
  - c)  $(-2) - (+4) = ?$  Draw integer tiles to represent this question
  - d)  $(+2) - (+4) = ?$  Draw a number line and arrows to represent this question.
  - e)  $(+4) - (-2) = ?$  Draw integer tiles to represent this question.
  - f)  $(-4) - (-2) = ?$  Draw a number line and arrows to represent this question.
6. Write number sentences equivalent to the number sentence below by using addition.
  - a)  $(+2) - (-4) = ?$  is equivalent to  $(+2) + ( \underline{\quad\quad} ) = ?$
  - b)  $(+2) - (-4) = ?$  is equivalent to  $(-4) + \underline{\quad\quad} = ?$
  - c) What is the solution?

## B. One-on-One Assessment

Assessment activities can be used with individual students, especially students who may be having difficulty with the outcome.

Provide students with integer tiles and 30-cm rulers.

1. Present the following problem to the student:

You earn \$5 and then spend \$5. How much is your profit or loss? Draw diagrams with integer tiles to represent this problem.

If the student has difficulty, have him or her read the problem orally and decide whether the problem involves the addition or subtraction of integers and explain why. Remind the student that the red integer tiles represent positive numbers and the white integer tiles represent negative numbers. Ask the student what number is represented by the combination of one red tile and one white tile.

2. Present the following problem to the student:

At 8:00 a.m., the temperature was  $-8^{\circ}\text{C}$  and 10 hours later it was  $+2^{\circ}\text{C}$ . What was the increase in temperature during this time? Show all your work, including a diagram and a number sentence.

If the student has difficulty, have him or her identify the needed information and place it on a vertical number line to represent a thermometer. Remind the student that addition and subtraction are opposite operations and either operation can be used to represent this problem symbolically.

3. Present the following problem to the student:

Debbie has \$20 and spends \$4 per day. Danny has \$12 and spends \$2 per day. What is the total amount of money or debt that they have at the end of one week? Show all your work.

If the student has difficulty, have him or her read the problem orally to determine the needed information. Ask the student how much money Debbie has at the end of one week or 7 days. Remind the student that she spends \$4 per day, not per week. Then ask how much money Danny has at the end of one week or 7 days. Finally have the student combine the two amounts. Have the student write appropriate number sentences. If necessary, use smaller numbers in the problem and have the student represent the problem on a number line or using tiles. Then he or she can transfer the process to the larger numbers in the original problem.

4. Present the following problem to the student:

Tran has a debt of \$55. He earns \$30 on each of two days and spends \$39 on a pair of pants. How much money or debt does he have now? Show all your work.

If the student has difficulty, have him or her read the problem orally and represent each number as an integer, either positive or negative. Point out that the total earnings is determined by multiplying \$30 by the number of days worked. Remind the student that when adding positive and negative numbers, you are going in opposite directions on the number line and hence subtraction is involved. If necessary, use smaller numbers and have the student represent the problem on a number line or using tiles. Then he or she can transfer the process to the original problem with larger numbers.

5. Present each of the following questions to the student, one at a time. Have the student explain his or her thinking by drawing a diagram, either a number line or integer tiles, as specified for each question.
- a)  $(+5) + (-3) = ?$  Draw integer tiles to represent this question.
  - b)  $(-5) + (-3) = ?$  Draw a number line and arrows to represent this question.
  - c)  $(-3) - (+5) = ?$  Draw integer tiles to represent this question.
  - d)  $(+3) - (+5) = ?$  Draw a number line and arrows to represent this question.
  - e)  $(+5) - (-3) = ?$  Draw integer tiles to represent this question.
  - f)  $(-5) - (-3) = ?$  Draw a number line and arrows to represent this question.

If the student has difficulty with any of the questions, review how positive and negative integers are represented on a number line or by using integer tiles. You may need to do a similar problem, using different numbers, and represent it by drawing a number line and by using integer tiles. Have the student use the concrete integer tiles for the questions requiring the diagrams of integer tiles.

6. Instruct the student to write a number sentence equivalent to the first number sentence by using addition as shown. Do the same for part (b).
- a)  $(+3) - (-5) = ?$  is equivalent to  $(+3) + ( \quad ) = ?$
  - b)  $(+3) - (-5) = ?$  is equivalent to  $(-5) + \quad = ?$
  - c) What is the solution?

If the student has difficulty with part (a), remind the student that the subtraction sign can be replaced with the addition sign by adding the opposite of the number subtracted. Review opposite integers; e.g., +5 is opposite to -5.

If the student has difficulty with part (b), provide an example of a related number sentence for addition and subtraction, using whole numbers; e.g.,  $5 - 3 = 2$ , therefore,  $2 + 3 = 5$ .

If the student has difficulty with part (c), encourage the student to represent the problem using a number line or integer tiles.

### C. Applied Learning

Provide opportunities for students to use their understanding of adding and subtracting integers in a practical situation and notice whether or not the understanding transfers. For example, have students find the increase in temperature from two time periods in which at least one of the temperatures is negative. Does the student:



- use appropriate strategies to calculate the increase in temperature?
- find the increase and write a number sentence if requested?
- relate addition to subtraction of integers by representing the problem, using either operation?
- apply the process to other similar problems with larger or smaller integers?

## Step 5: Follow-up on Assessment

### Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction?

### A. Addressing Gaps in Learning

- Use manipulatives of various kinds to develop and demonstrate understanding of adding and subtracting integers.
- Emphasize the connection between adding integers and subtracting integers.
- Relate adding and subtracting integers to adding and subtracting whole numbers. The types of problems involving the addition or subtraction of whole numbers are the same for fraction problems; e.g., part-part-whole, comparison and join or separate problems. Substituting whole numbers for integers helps students to decide which operation to use in solving the problems.
- Use problem contexts that students can relate to and keep the integers simple initially; i.e., use integers close to zero so that they can easily be represented by using integer tiles or a number line.
- Connect the concrete, pictorial and symbolic modes.
- Provide the option for students to use manipulatives as long as needed, even on tests.
- Encourage students to explain their thinking orally if they have difficulty explaining it in writing.

### B. Reinforcing and Extending Learning

Students who have achieved or exceeded the outcomes will benefit from ongoing opportunities to apply and extend their learning. These activities should support students in developing a deeper understanding of the concept and should not progress to the outcomes in subsequent grades.

Consider strategies, such as the following.

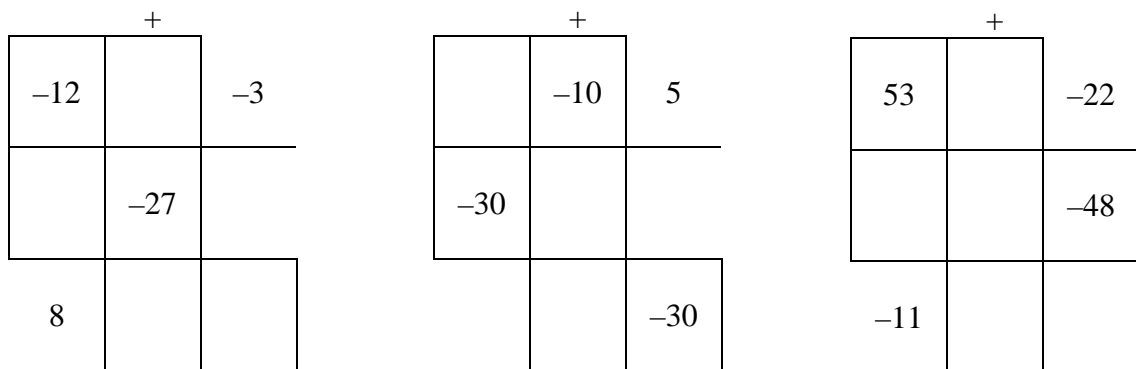
- Provide tips for parents on adding and subtracting integers at home or in the community.
  - Balance the checkbook, using deposits and withdrawals.
  - Play card games in which positive and negative scores must be totalled; e.g., bridge or 500.
  - Find the difference in temperatures on some winter days to apply negative integers.

- Reinforce adding and subtracting integers by using a deck of cards or overhead playing cards. Have students find the sum of a series of cards drawn in which the black cards represent positive numbers and the red cards represent negative numbers.
- Solve problems, such as the following:
  - You have \$36 and spend \$4 per day for two weeks. How much money or debt do you have at the end of two weeks?
  - You have \$25 and spend \$5 per day. Your friend has \$12 and spends \$3 per day. Find the total amount of money or debt at the end of
    - a) four days.
    - b) a week.
  - Lauren has a debt of \$36. She earns \$25 on each of two days and spends \$19 on a shirt. How much money or debt does she have now?
  - Ted has \$15. He spends \$8 on a book and would like to buy a CD for \$16. How much money will he have to borrow to buy the CD?
  - Find the increase in temperature from  $-16^{\circ}\text{C}$  to  $+18^{\circ}\text{C}$ .

Last three problems reproduced from Alberta Education, *Integers: Learning Strategies to Enhance Understanding* (unpublished workshop handout) (Edmonton, AB: Alberta Education, 2005), p. 48.

- Solve integer riddles, such as the following:
  - a multiple of 3
  - a negative integer
  - greater than the difference between  $-36$  and  $-24$
  - less than the sum of  $-4$  and  $-2$ .
- Reinforce adding and subtracting integers by using two-way tasks. The three numbers in any row or column must form a correct addition sentence. The self-checking nature of these tasks is an asset in providing reinforcement. The tasks can be made more or less challenging by changing the numbers. Students should be encouraged to create two-way tasks for other students to complete.

Examples of two-way tasks:



Adapted from Grayson H. Wheatley and George Abshire, *Developing Mathematical Fluency: Activities for Grades 5–8* (Tallahassee, FL: Mathematics Learning, 2002), p. 207. Adapted with permission from Mathematics Learning, [www.mathematicslearning.org](http://www.mathematicslearning.org).

- Reinforce adding and subtracting integers by using magic squares. Have students complete magic squares by finding the missing integers so that the sum of the integers in each row, column and diagonal is the same; e.g.,

5		2
	3	
4		

	-7	
		-11
-9	-1	-2

		-30
-44	-30	
-30		

Adapted from Barbara and Harry Morrison, *Soar with Integers: A Complete Learning and Teaching Resource* (Barrie, ON: Exclusive Educational Products, 1995), p. 77. Adapted with permission from Exclusive Educational Products.

- Reinforce adding and subtracting integers by playing the game, I Have . . . Who Has? This is a whole class activity. Provide each student with at least one card as shown below. The person who receives the card with "Start" printed on it begins the game. The game ends when the student answers the last question that has "End" printed on the card. There are 36 cards provided in the resource indicated.

<p>I have <b>START</b> who has . . .</p> <p><math>-6 + 3</math></p>
---

<p>I have <b>- 3</b> who has . . .</p> <p><math>7 + 3</math></p>
--

<p>I have <b>10</b> who has . . .</p> <p><math>10 - 6</math></p>
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Adapted from Barbara and Harry Morrison, *Soar with Integers: A Complete Learning and Teaching Resource* (Barrie, ON: Exclusive Educational Products, 1995), pp. 188, 189. Adapted with permission from Exclusive Educational Products.

- Play the following games:
  - Make Twenty

Players: 2

Materials: a number line from  $-20$  to  $+20$

A die labelled with integers such as  $+3, +2, +1, -1, -2, -3$

Description: Each player places a different coloured marker at 0 on the number line. Players take turns rolling the die. The player decides whether to add or subtract the number on the die to determine the new position of his or her marker on the number line.

The player moves his or her marker to the right when adding a positive number or subtracting a negative number. The player moves his or her marker to the left when subtracting a positive number or adding a negative number.

Goal: The first person to reach or pass  $-20$  or  $+20$  is the winner.

Variations:

- ✓ The players add the number on the die to the marked number on the number line rather than deciding whether to add or subtract the number on the die.
- ✓ The number line could include more or less numbers, e.g., from  $-30$  to  $+30$ .
- ✓ The players could multiply the number on the die by the number on the number line identified by the marker instead of adding or subtracting.
- ✓ Use a spinner divided into six congruent sectors instead of a die.

- Operation Integers

Players: 2 to 4

Materials: a deck of cards (no face cards)

Description: Deal all the cards face down to the players. Black suits are positive and red suits are negative. Each player turns over two cards and decides whether to add or subtract the two numbers on the cards. The player who has the greatest sum or difference wins all the cards that are face up.

Goal: The play continues until one person (the winner) has all the cards.

Variations:

- ✓ Use fewer cards or cards with only certain numbers.
- ✓ Use fewer operations.
- ✓ Turn over three or four cards instead of two cards for each play.
- ✓ The winner of the game could be the one who has no cards left.
- ✓ The player who has the least sum or difference wins all the cards that are face up.
- ✓ Each player rolls two (or more) dice with integers on each face rather than using playing cards. The player with the greatest (or least) number resulting from the operations could score one point. The winner is the player with the most points.

Adapted from Alberta Education, *Integers: Learning Strategies to Enhance Understanding* (unpublished workshop handout) (Edmonton, AB: Alberta Education, 2005), pp. 51–52.

- Have students draw a Frayer Model for adding or subtracting integers, using the following template, and encourage them to share their models.

Definition	Characteristics
Visual Representation	Real-world Problem

- Sample Structured Interview: Assessing Prior Knowledge and Skills

<b>Directions</b> Provide the student with a 30-cm ruler.	Date:	
	<b>Not Quite There</b>	<b>Ready to Apply</b>
Instruct the student, "Draw a number line showing the integers."	<ul style="list-style-type: none"> <li>• Draws a number line and places positive and negative integers and zero on the number line but does not space them out properly and/or does not indicate that the set of integers is infinite (arrows on both ends of the number line).</li> </ul>	<ul style="list-style-type: none"> <li>• Draws a number line and places positive and negative integers and zero on the number line spaced out correctly and includes arrows on the number line to depict a set of numbers that gets infinitely larger and infinitely smaller.</li> </ul>
Instruct the student, "Place the following numbers on a number line: 3, -4, 0, 1, -5. Explain how these numbers are ordered."	<ul style="list-style-type: none"> <li>• Draws a number line and places the numbers on the number line but interchanges the position for -4 and -5 and/or lacks correct spacing between the numbers.</li> <li>• Has difficulty explaining how to order the negative numbers.</li> </ul>	<ul style="list-style-type: none"> <li>• Draws the number line and places the numbers correctly on it.</li> <li>• Explains clearly how the numbers are ordered, indicating that the smaller numbers are placed farther to the left on the number line; i.e., -5 is placed to the left of -4 because -5 is less than -4.</li> </ul>

<p>Place the open number sentences before the student.</p> <p>a) <math>-15 \square -20</math>  b) <math>-46 \square -31</math></p> <p>c) <math>-2 \square 5 - 5</math></p> <p>Say, "Write <math>&lt;</math>, <math>=</math> or <math>&gt;</math> in the boxes to complete the number sentences correctly. Verify your answers by placing the numbers on a number line."</p>	<ul style="list-style-type: none"> <li>• Makes errors in using the inequality sign because the signs are mixed up or there is a misunderstanding of the order of integers.</li> <li>• Draws a number line but makes the same errors on the number line as in the number sentences; e.g., <math>-15</math> is placed to the left of <math>-20</math> on the number line instead of to the right.</li> </ul>	<ul style="list-style-type: none"> <li>• Completes the number sentences correctly and draws number lines to verify the answers.</li> </ul>
<p>Present the following problem to the student orally and in written form:  "Name the negative integers that are greater than <math>-20</math> and divisible by 3. Explain your answer."</p>	<ul style="list-style-type: none"> <li>• Lists multiples of 3 that are less than <math>-20</math> or lists multiples of 3 greater than <math>-20</math>, including positive numbers.</li> <li>• Has difficulty explaining the answer.</li> </ul>	<ul style="list-style-type: none"> <li>• Lists the negative integers greater than <math>-20</math> that are multiples of 3 and explains why the numbers are greater than <math>-20</math> and why they are multiples of 3.</li> </ul>
<p>Present the following problem to the student, orally and in written form:  "The temperature is <math>-15^{\circ}</math> C on Monday and <math>-8^{\circ}</math> C on Tuesday. Which day shows the higher temperature? How do you know?"</p>	<ul style="list-style-type: none"> <li>• Chooses the incorrect day.</li> <li>• Chooses the correct day but is unable to explain why <math>-8</math> shows a higher temperature than <math>-15</math>.</li> </ul>	<ul style="list-style-type: none"> <li>• Chooses the correct day and explains that <math>-8</math> is greater than <math>-15</math> and therefore shows the higher temperature; the weather would be warmer at <math>-8^{\circ}</math> C rather than <math>-15^{\circ}</math> C.</li> </ul>
<p>Present the following problem to the student, orally and in written form:  "Explain why one negative integer is less than another negative integer by using a real world context."</p>	<ul style="list-style-type: none"> <li>• Does not provide a real-world context for using negative integers.</li> <li>• Provides a real-world context for integers but has difficulty explaining the order of integers.</li> </ul>	<ul style="list-style-type: none"> <li>• Provides a real-world context for integers and uses an example to show the order of two chosen negative numbers.</li> </ul>

## **BIBLIOGRAPHY—Planning Guide: *Grade 7 Addition and Subtraction of Integers***

**Strand:** Number

**Outcome:** 6

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### **Step 4 References**

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## Step 5 References

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