Planning Guide: *Grade 7 Addition and Subtraction of Positive Fractions and Mixed Numbers*

Strand: Number
Outcome: 5

Curriculum Highlights

This sample targets the following changes in the curriculum:

- The General Outcome focuses on developing number sense, whereas the previous mathematics curriculum focused on applying arithmetic operations on decimals and integers, and illustrating their use in solving problems.
- The Specific Outcome focuses on understanding adding and subtracting positive fractions and mixed numbers concretely, pictorially and symbolically, whereas the previous mathematics curriculum did not include operations with fractions until Grade 8.

Step 1: Identify Outcomes to Address

Guiding Questions

- What do I want my students to learn?
- What can my students currently understand and do?
- What do I want my students to understand and be able to do, based on the Big Ideas and specific outcomes in the program of studies?

Big Ideas

Operations with fractions build on operations with whole numbers. Van de Walle and Lovin (2006, p. 66) reaffirm this in the following excerpt:

- The meanings of each operation on fractions are the same as the meanings for the operations on whole numbers. Operations with fractions should begin by applying these same meanings to fractional parts. For addition and subtraction, it is critical to understand that the numerator tells the number of parts and the denominator the type of part.
- Estimation of fraction computations is tied almost entirely to concepts of the operations and of fractions. A computation algorithm is not required for making estimates. Estimation should be an integral part of computation development to keep students' attention on the meanings of the operations and the expected size of the results.

Reproduced from Van de Walle, John A., LouAnn H. Lovin, *Teaching Student-Centered Mathematics, Grades 5–8* (p. 66). Published by Allyn and Bacon, Boston, MA. Copyright © 2006 by Pearson Education. Reprinted by permission of the publisher.

Just as integers are an extension of the whole number system to include the opposite of every number, so also fractions are an extension of the whole number system to represent numbers between the whole numbers. Fractions include the whole numbers so the same properties apply but, in addition, fractions are closed under division.

Students need a solid conceptual foundation in fractions as a necessary prerequisite for fraction computation. They must first understand the meaning of fractions, using different models—region, set and length or measurement. To help students add and subtract fractions correctly and with understanding, teachers must help them develop understanding of the numerator and denominator, equivalence and the relation between mixed numbers and improper fractions. Teachers must also encourage students to use benchmarks and estimations (NCTM 2000, p. 218).

Students develop understanding of operations with fractions by making sense of the ideas internally. To guide this process, it is necessary to encourage flexibility in thinking and provide learning opportunities in connecting:

- operations with whole numbers to operations with fractions
- subtraction of fractions to addition of fractions
- concrete, pictorial and symbolic representations
- operations with fractions to real world problems (Alberta Education 2004, p. 3).

Sequence of Outcomes from the Program of Studies

See (Web address) for the complete program of studies.

Grade 6	**Grade 7**	**Grade 8**
Specific Outcomes 3. Demonstrate an understanding of factors and multiples by: • determining multiples and factors of numbers less than 100 • identifying prime and composite numbers • solving problems using multiples and factors. 4. Relate improper fractions to mixed numbers and mixed	Specific Outcomes 5. Demonstrate an understanding of adding and subtracting positive fractions and mixed numbers, with like and unlike denominators, concretely, pictorially and symbolically (limited to positive sums and differences).	Specific Outcomes 6. Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially and symbolically.

numbers to improper fractions.	

Step 2: Determine Evidence of Student Learning

Guiding Questions

- What evidence will I look for to know that learning has occurred?
- What should students demonstrate to show their understanding of the mathematical concepts, skills and Big Ideas?

As you begin planning lessons and learning activities, keep in mind ongoing ways to monitor and assess student learning. One starting point for this planning is to consider the achievement indicators listed in *The Alberta K–9 Mathematics Program of Studies with Achievement Indicators* (Alberta Education 2007). You may also generate your own indicators and use these to guide your observation of students.

The following achievement indicators may be used to determine whether students have met this specific outcome.

- Use benchmarks to estimate the sum or difference of two positive fractions or mixed numbers.
- Model addition and subtraction of two given positive fractions or mixed numbers, using concrete representations, and record symbolically.
- Determine the sum of two given positive fractions or mixed numbers with like denominators.
- Determine the difference of two given positive fractions or mixed numbers with like denominators.
- Determine a common denominator for a given set of positive fractions or mixed numbers.
- Determine the sum of two given positive fractions or mixed numbers with unlike denominators.
- Determine the difference of two given positive fractions or mixed numbers with unlike denominators.
- Write the sum or difference of two positive fractions or mixed numbers in simplest form, dividing the numerator and the denominator by the greatest common factor.
- Simplify the solution to a given problem involving the sum or difference of two positive fractions or mixed numbers.
- Solve a given problem involving the addition or subtraction of positive fractions or mixed numbers and determine if the solution is reasonable.

Some sample behaviours to look for in relation to these indicators are suggested for many of the instructional activities in Step 3, Section C, Choosing Learning Activities.

Step 3: Plan for Instruction

Guiding Questions

- What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should I use?
- How will I meet the diverse learning needs of my students?

A. Assessing Prior Knowledge and Skills

Before introducing new material, consider ways to assess and build on students' knowledge and skills related to factors, multiples, improper fractions and mixed numbers. For example:

- Philly runs around a racetrack every 6 minutes while Lightning runs around the same racetrack every 8 minutes. If both horses start the race at the same time and continue at the same pace, in how many minutes will they be side by side again on the racetrack? Explain your thinking.
- Jamie said that the same number can be both a multiple and a factor of a given number. Is Jamie correct? Explain.
- Explain how 24 and 18 are different from 29 and 17.
- There are 24 candies of one kind in the first bag and 32 candies of another kind in the second bag. What is the greatest number of people who could share equally the candies from both bags? Explain.
- There are $\frac{17}{8}$ pizzas left after a party. Express this number as a mixed number. Draw a diagram to show the pizzas.
- Susie has $4\frac{3}{4}$ Kit Kat chocolate bars. Write this mixed number as an improper fraction to show how many quarter bars (Kit Kat pieces) she has in all. Draw a diagram to show the chocolate bars.

If a student appears to have difficulty with these tasks, consider further individual assessment, such as a structured interview, to determine the student's level of skill and understanding. See Sample Structured Interview: Assessing Prior Knowledge and Skills.

B. Choosing Instructional Strategies

Consider the following strategies when planning lessons.

- Access prior knowledge on fractions outlined in the achievement indicators for grades 5 and 6.
- Focus on the meaning of a fraction—the numerator counts and the denominator shows what is counted.
- Use a problem-solving context that relates to students and applies the addition and subtraction of fractions.

- Connect problems applying the addition and subtraction of fractions to similar problems with whole numbers. Review the types of problems for addition and subtraction, such as part-part-whole, comparison and join or separate.
- Connect the subtraction of fractions to the addition of fractions.
- Estimate sums and differences of fractions before calculating by using benchmarks such as $\frac{1}{2}$, 1, $1\frac{1}{2}$ and 2.
- Encourage the use of informal methods in developing strategies for finding sums and differences of fractions.
- Have students explore sums and differences of fractions by using a variety of manipulatives, such as fraction strips and fraction blocks.
- Emphasize that students connect the concrete, pictorial and symbolic representations for sums and differences of fractions.
- Encourage students to look for similarities between different types of problems and between solution strategies to add and subtract fractions (Mack 2004).
- Have students justify the strategies they use in finding sums and differences of fractions and critique strategies used by others.

Adapted from Alberta Education, *Fractions: Learning Strategies to Enhance Understanding* (unpublished workshop handout) (Edmonton, AB: Alberta Education, 2004), pp. 8–9.

C. Choosing Learning Activities

The following learning activities are examples of activities that could be used to develop student understanding of the concepts identified in Step 1.

Sample Activities for Teaching Adding and Subtracting Positive Fractions and Mixed Numbers

1. Estimating Sums and Differences

   Develop students' number sense by having them think about the meaning of fractions as they respond to open-ended questions, such as, "What can you tell me about two fractions that have a sum between 0 and 1?" (Reys 1992, p. 6).

   Build on students' understanding of fractions and using benchmarks on a number line to place specific fractions. In the following activity, have students think about each fraction relative to $\frac{1}{2}$ in making their estimates of sums.

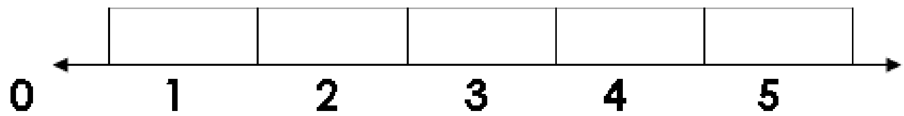
Look For …

Do students:

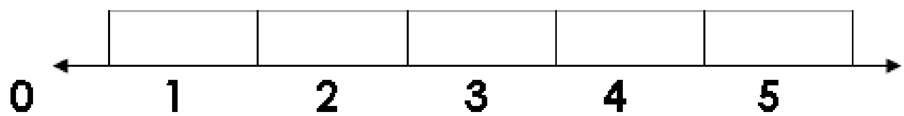
- ☐ understand the meaning of a fraction?
- ☐ use the relative size of fractions in terms of benchmarks to estimate sums and differences?
- ☐ use the associative property in estimating sums of mixed numbers?
- ☐ clearly explain the strategies used in estimating?

Place the following number lines on the board or overhead projector. Provide students with cut-out arrows that can be moved along the number line and placed in an appropriate position to mark the sum of the two fractions in each case. Put each addition into a contextual problem and have students describe their thinking in solving the problem.

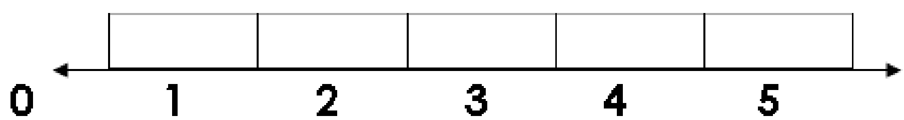
$\frac{1}{2} + \frac{3}{4}$



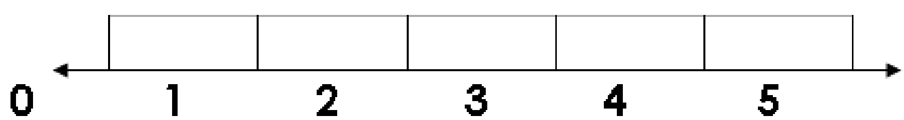
$\frac{7}{8} + \frac{2}{5}$



$1\frac{5}{9} + \frac{2}{3}$



$2\frac{3}{5} + 1\frac{1}{2}$



Students might explain their reasoning for the last problem as follows:

"I place $2\frac{3}{5}$ just past $2\frac{1}{2}$ on the number line because 3 out of 5 is a little more than half of 5. Then I move 1 unit to the right, placing the arrow a little past $3\frac{1}{2}$. Finally, I move $\frac{1}{2}$ unit further to the right, placing the arrow just past 4. I estimate the sum to be a little more than 4 units."

Adapted from Barbara J. Reys, *Developing Number Sense in the Middle Grades: Curriculum and Evaluation Standards for School Mathematics Addenda Series, Grades 5–8* (Reston, VA: The National Council of Teachers of Mathematics, 1992), p. 33. Adapted with permission of the National Council of Teachers of Mathematics.

A similar procedure could be used to estimate differences of fractions.

2. Adding and Subtracting Fractions with Like Denominators

Access prior knowledge of adding whole numbers and the meaning of fractions, in particular, the meaning of the numerator and the denominator; i.e., the numerator counts and the denominator shows what is counted. Remind students that fractions may represent regions (e.g., pattern blocks), sets (e.g., buttons) or lengths (e.g., fraction strips). Present the following problem to students:

Celina eats $\frac{3}{5}$ of a licorice and Jeff eats $\frac{4}{5}$ of another licorice the same size.

a) Estimate whether they eat more or less than 1 licorice in total.
b) How much licorice do they eat in all?

a) To estimate, students may wish to use benchmarks on a number line and reason that each person eats more than half a licorice so they would eat more than 1 licorice in total.
b) Provide students with fractions strips to use as needed in solving the problem.

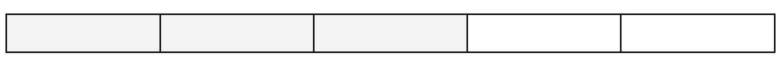
Look For …

Do students:

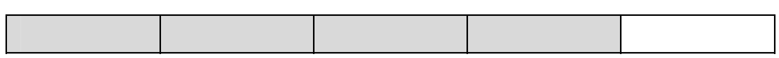
- ☐ estimate sums or differences, using appropriate strategies?
- ☐ explain their thinking as they estimate and calculate?
- ☐ recognize that the whole region or whole set is the same size for all the fractions in any given problem?
- ☐ explain that the numerator counts and the denominator shows what is counted?
- ☐ relate addition to subtraction of fractions in a similar way that was done with whole numbers?

Have students explain their thinking in finding the sum. Ask them how they used the numerators and the denominators of the fraction in their solutions. Have students draw a diagram and write a number sentence to represent the problem; e.g.,

Celina eats $\frac{3}{5}$ of a pizza. To illustrate this fraction, 3 out of 5 equal parts of the fraction strip below are shaded.



Jeff eats $\frac{4}{5}$ of a pizza. To illustrate this fraction, 4 out of 5 equal parts of the fraction strip below are shaded.

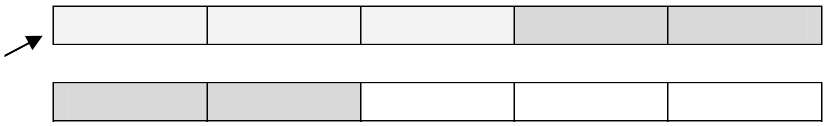


$\frac{3}{5} + \frac{4}{5} = \frac{?}{5}$ How many fifths did they eat in all? $\frac{3}{5} + \frac{4}{5} = \frac{7}{5}$

They eat seven fifths or $\frac{7}{5}$ pizzas in all.

We can show this addition with the fraction strips.

$\frac{7}{5}$ shaded



They eat $\frac{7}{5}$ or $1\frac{2}{5}$ licorices in all.

Review the relation between addition and subtraction of whole numbers and have students transfer this understanding to fractions. Transform the addition problem into a related subtraction problem, such as:

Jeff has $1\frac{2}{5}$ licorices and he eats $\frac{4}{5}$ of a licorice. How much licorice is left?

Provide other similar related addition and subtraction problems with like denominators, using a variety of manipulatives including fraction strips, pattern blocks and buttons. Reinforce that the whole, whether it is region or a set, must be the same size for all the fractions in any given problem.

Through discussion, have students verbalize a generalization about adding and subtracting fractions with like denominators; e.g., when the denominators of two fractions are the same, the parts of the whole being counted are the same size so you just add the numerators to obtain the sum.

3. Adding and Subtracting Fractions with Unlike Denominators (Changing One Denominator)

   Present students with the following problem:

   David eats $\frac{3}{4}$ of a bag of candies. Marnie eats $\frac{3}{8}$ of a same size bag of candies.

   a) Estimate whether they eat more or less than l complete bag of candies in total.
   b) How many bags of candy do they eat in all?

   a) To estimate, students may use benchmarks saying that $\frac{3}{4}$ is $\frac{1}{4}$ away from one whole and $\frac{3}{8}$ is more than $\frac{1}{4}$ so they will eat more than one full bag of candies in total.

Look For …

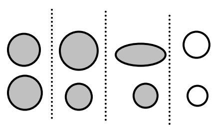
Do students:

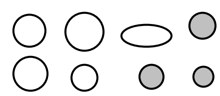
- ☐ explain that the numerators can be added or subtracted only if they represent the same size parts of the whole, i.e., the denominators are the same?
- ☐ apply their knowledge of equivalent fractions in finding common denominators?
- ☐ use invented strategies to add or subtract fractions?
- ☐ use the same size sets to represent all the fractions in the problem?

b) Have students suggest how they might solve this problem by connecting it to the problems with like denominators. They might explain how the numbers in this problem can be changed so that the parts (denominators) are the same. Have them suggest how they might draw a diagram to represent the problem. How many candies could they use in one whole bag? Why?

Students may represent the problem with a set of 8 candies in the whole bag, where 8 represents the denominator (the number of equal parts in a whole) because 4 is a factor of 8.

Review equal parts of a set, emphasizing that each part is equal if it has the same number of items in it.





Review equivalent fractions:

$\frac{3}{4} = \frac{6}{8}$

$\frac{3}{8}$

$\frac{6}{8} + \frac{3}{8} = \frac{9}{8} = 1\frac{1}{8}$

They eat $\frac{9}{8}$ or $1\frac{1}{8}$ bags of candies in total.

Transform the addition problem into a related subtraction problem, such as the following:

David has $1\frac{1}{8}$ bags of candies. If he eats $\frac{3}{4}$ of a bag of candies, what fraction of a bag of candies is left?

Provide other similar related addition and subtraction problems with unlike denominators (changing one denominator), using a variety of manipulatives, including fraction strips, pattern blocks and buttons.

Through discussion, have students verbalize that the parts of the whole being counted must be the same size so you must first have common denominators before adding the numerators.

Look For …

Do students:

- ☐ apply their knowledge about solving comparison problems with whole numbers?
- ☐ use the simpler problem strategy and change the fractions to whole numbers as a method to decide which operation to use in the problem?
- ☐ use a variety of manipulatives to explain their thinking and connect the concrete to the pictorial and symbolic representations?

4. Adding and Subtracting Fractions with Unlike Denominators (Changing Both Denominators)

Present students with the following problem:

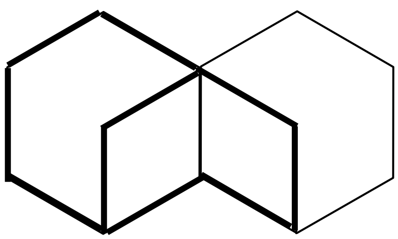
Lori eats $\frac{2}{3}$ of a cake and Nicholas eats $\frac{1}{4}$ of the same size cake.

a) Estimate the total amount of cake eaten by both people.
b) What is the total amount of cake that they ate?

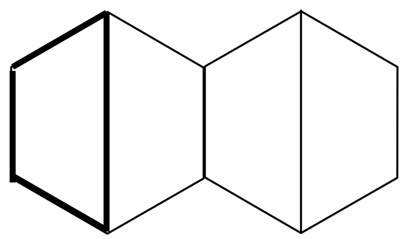
a) Students might estimate by thinking that $\frac{2}{3}$ is more than $\frac{1}{2}$ and it is $\frac{1}{3}$ away from one whole. Since $\frac{1}{4}$ is less than $\frac{1}{3}$, the total amount will be slightly less than one whole.

b) Have students suggest how they might solve the problem, keeping in mind that the parts of the whole must be the same size before they can be added. Applying what they learned from the previous activity, students may use equivalent fractions to find a solution to the problem. Provide fraction strips and fractions blocks for students to use as needed.

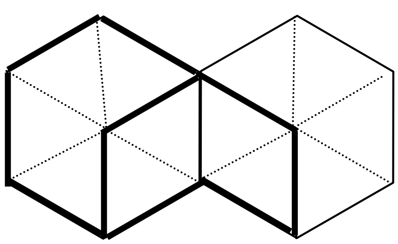
For example, students might represent the problem by using fraction blocks as follows:



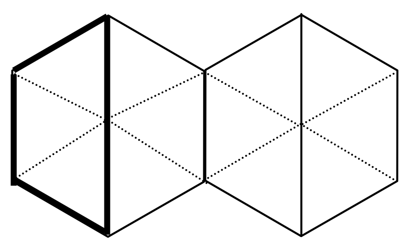
$\frac{2}{3}$ of a cake



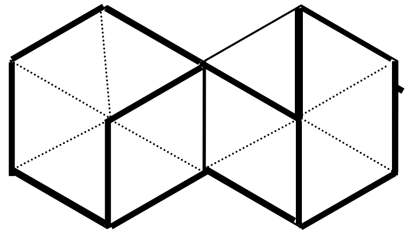
$\frac{1}{4}$ of a cake



$\frac{2}{3} = \frac{8}{12}$



$\frac{1}{4} = \frac{3}{12}$



$\frac{8}{12} + \frac{3}{12} = \frac{11}{12}$

They ate $\frac{11}{12}$ of the cake.

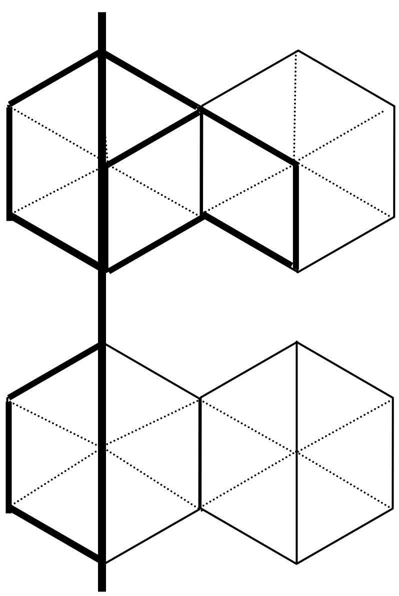
Relate to subtracting fractions by changing the problem to the following:

Lori eats $\frac{2}{3}$ of a cake and Nicholas eats $\frac{1}{4}$ of the same size cake.

a) Estimate how much more cake is eaten by Lori than by Nicholas.
b) How much more cake did Lori eat than Nicholas?

a) Students might estimate by thinking that $\frac{2}{3}$ is equivalent to $\frac{4}{6}$ and is therefore $\frac{1}{6}$ away from $\frac{1}{2}$ or $\frac{3}{6}$. Since $\frac{1}{4}$ is more than $\frac{1}{6}$, the difference between $\frac{2}{3}$ and $\frac{1}{4}$ will be a little less than $\frac{1}{2}$.

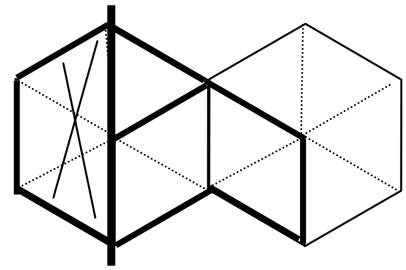
b) Have students suggest how to find the difference by applying what they know about adding fractions with unlike denominators (changing both denominators). Provide fraction strips and fraction blocks as needed. For example, the student might represent the problem by using fraction blocks as follows:



$\frac{2}{3} = \frac{8}{12}$

$\frac{1}{4} = \frac{3}{12}$

$$\frac{8}{12} - \frac{3}{12} = \frac{5}{12}$$



Lori eats $\frac{5}{12}$ of a cake more than Nicholas.

Provide other similar problems with unlike denominators (changing both denominators) and have students suggest strategies in solving the problems. Use different problem contexts and have students use a variety of manipulatives connected to appropriate diagrams and number sentences.

Have students generalize that the numerators of fractions can be added or subtracted only if these numerators are counting the same size parts of the whole; i.e., the denominators are the same.

5. Adding and Subtracting Mixed Numbers

Build on students' understanding of adding and subtracting whole numbers as well as proper fractions. To add mixed numbers, use the associative property and show that the whole numbers can be added together first and then the proper fractions can be added. To complete the addition, the two sums are added together. For example:

Johnny bikes for $2\frac{3}{5}$ hours and hikes for $1\frac{1}{2}$ hours.

a) About how much time does he spend biking and hiking?
b) How much time does he spend biking and hiking?

a) Students might estimate the total time by adding the whole numbers (2 + 1) and then use benchmarks to estimate the sum of the fractional parts. $\frac{3}{5}$ is a little more than $\frac{1}{2}$.

Therefore, $\frac{3}{5} + \frac{1}{2}$ is a little more than 1. Adding the whole and fractional parts together, a good estimate would be about 4 hours.

Look For …

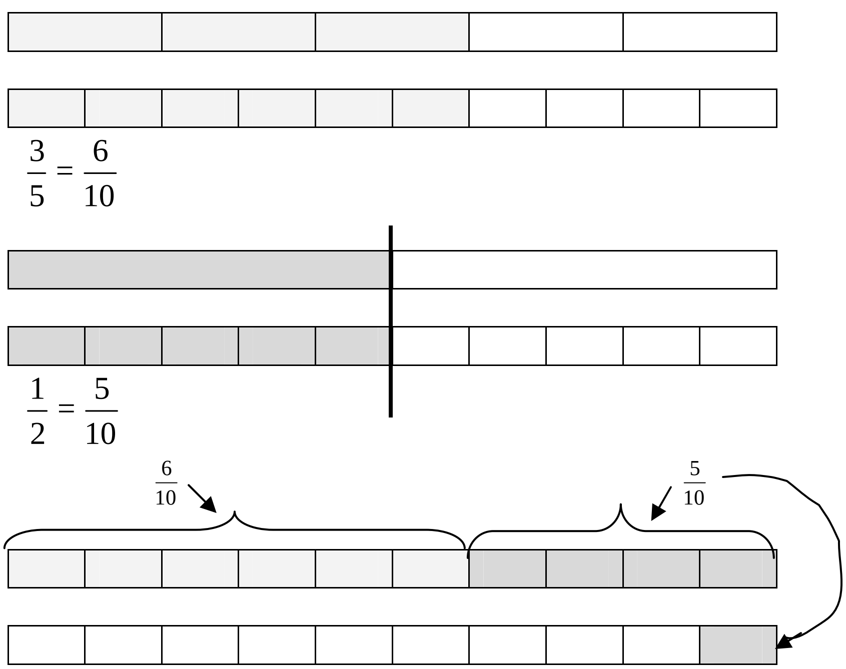
Do students:

- ☐ apply their knowledge of the relation between mixed numbers and improper fractions in solving the problems?
- ☐ use the associative property and commutative property or addition to add the whole numbers and then the fractions?
- ☐ transfer learning about adding mixed numbers to subtracting mixed numbers?

b) Have students suggest how to find the sum by using their background knowledge of operations with whole numbers as well as fractions. You might suggest that the mixed numbers be rewritten as $2 + \frac{3}{5} + 1 + \frac{1}{2}$. Then 2 and 1 could be added to obtain 3. This sum is then added to the sum of the two proper fractions to obtain the total sum. Have students use fraction strips to represent the sum of $\frac{3}{5}$ and $\frac{1}{2}$, reviewing that a common denominator would be 10. Both fractions can be written equivalent to fractions in tenths.

Students draw diagrams and write appropriate symbols to show the addition process.

Using fractions bars to show the addition of $\frac{3}{5}$ and $\frac{1}{2}$:



Symbolic representation:

$$2\frac{3}{5} + 1\frac{1}{2} = 2 + \frac{3}{5} + 1 + \frac{1}{2}$$
$$= 2 + \frac{6}{10} + 1 + \frac{5}{10}$$
$$= 3 + \frac{11}{10}$$
$$= 3 + 1\frac{1}{10}$$
$$= 4\frac{1}{10}$$

Johnny spends $4\frac{1}{10}$ hours biking and hiking.

Alternative method:
Convert the mixed numbers to improper fractions and then add them.

Have students solve subtraction problems, using similar numbers such as the following:

Johnny bikes for $2\frac{2}{5}$ hours and hikes for $1\frac{1}{2}$ hours.

a) About how much more time does he spend biking than hiking?
b) How much more time does he spend biking than hiking?

a) Students could estimate the answer by simply subtracting the whole numbers because the fractional parts are about the same.
b) Have students suggest ways to subtract the mixed numbers. They may wish to convert the mixed numbers to improper fractions, find common denominators and subtract to calculate the answer. Have fraction strips available for use as needed. Encourage students to draw diagrams and write number sentences to explain their thinking.

Adapted from Alberta Education, *Fractions: Learning Strategies to Enhance Understanding* (unpublished workshop handout) (Edmonton, AB: Alberta Education, 2004), pp. 21–25.

6. Concept Definition Maps for Adding and Subtracting Fractions

Have students complete concept definition maps to consolidate their understanding of adding and subtracting fractions. A concept definition map for adding fractions could be done together as a class. Then students could work in groups or independently to create another concept definition map for subtracting fractions.

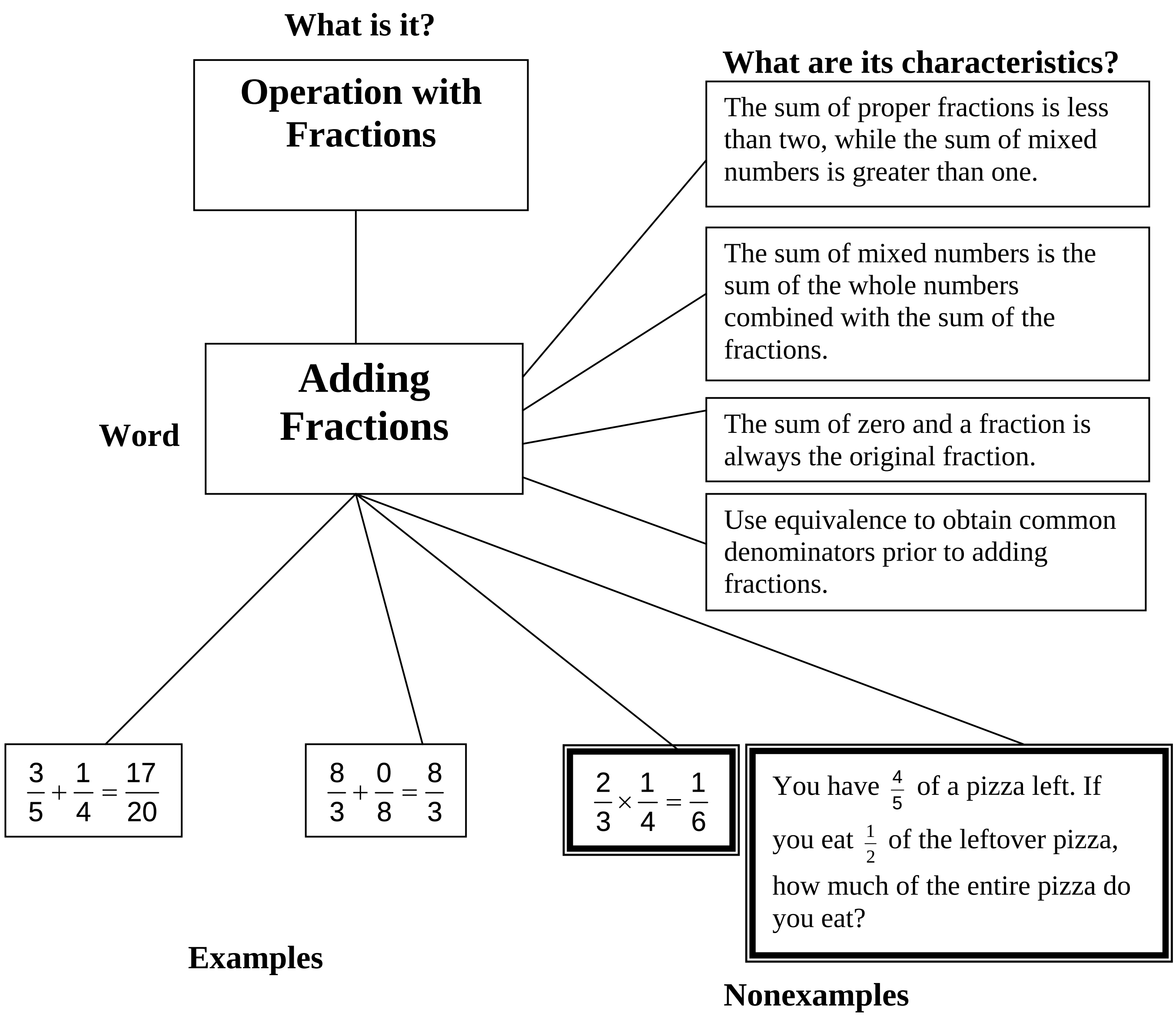
Sample concept definition maps are shown below.

Look For …

Do students:

- ☐ identify the main characteristics of adding and subtracting fractions?
- ☐ create examples and nonexamples of addition and subtraction problems?

Concept Definition Map



Format adapted from Robert M. Schwartz, "Learning to Learn Vocabulary in Content Area Textbooks," *Journal of Reading* 32, 2 (1988), p. 110, Example 1. Adapted with permission from International Reading Association.

Concept Definition Map

What is it?

Operation with Fractions

What are its characteristics?

- The difference of proper fractions is less than one.
- Use equivalence to obtain common denominators prior to subtracting fractions.
- When you subtract zero from a fraction, the answer is always the original fraction.
- To find the difference of mixed numbers, you can first change them to improper fractions.

Word

Subtracting Fractions

Examples

$\frac{3}{5} - \frac{1}{4} = \frac{7}{20}$

$\frac{8}{3} - \frac{0}{8} = \frac{8}{3}$

Nonexamples

$\frac{2}{3} \times \frac{1}{4} = \frac{1}{6}$

You have $\frac{4}{5}$ of a pizza left. If you eat $\frac{1}{2}$ of the leftover pizza, how much of the entire pizza do you eat?

Format adapted from Robert M. Schwartz, "Learning to Learn Vocabulary in Content Area Textbooks," *Journal of Reading* 32, 2 (1988), p. 110, Example 1. Adapted with permission from International Reading Association.

Step 4: Assess Student Learning

Guiding Questions

- Look back at what you determined as acceptable evidence in Step 2.
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

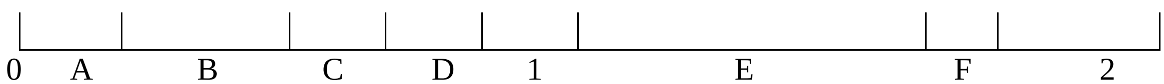
In addition to ongoing assessment throughout the lessons, consider the following sample activities to evaluate students' learning at key milestones. Suggestions are given for assessing all students as a class or in groups, individual students in need of further evaluation and individual or groups of students in a variety of contexts.

A. Whole Class/Group Assessment

Note: Performance-based assessment tasks are under development.

Provide fraction strips, fraction blocks and counters for students to use as needed. Instruct students to show all their work. Emphasize that all fractions are to be written in the simplest form.

1. Use estimation skills and the number line provided to answer the parts of this question. Explain your thinking for each answer.



   a) If the fractions represented by A and B are added, what point on the number line best represents the sum?
   b) If the fractions represented by C and D are added, what point on the number line best represents the sum?
   c) What point on the number line best represents the difference: F – C?
   d) What point on the number line best represents the difference: E – D?

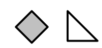
2. **Without** calculating, write <, = or > to complete each sentence correctly. Explain your thinking for each sentence.

   a) $\frac{5}{8} + \frac{3}{5} \square 1$ b) $\frac{2}{5} + \frac{1}{3} \square 1$ c) $\frac{1}{4} + \frac{1}{6} \square \frac{1}{2}$

   d) $\frac{1}{3} + \frac{1}{4} \square \frac{1}{2}$ e) $1\frac{2}{3} + 1\frac{1}{6} \square 2$

3. Trevor reads for $\frac{3}{4}$ of an hour on Monday and $1\frac{3}{4}$ hours on Tuesday. What is the total length of time that he reads on these two days?
4. Hungry Harry eats $\frac{5}{6}$ of a pizza. Ravenous Rita eats $\frac{2}{3}$ of a same size pizza. Harry eats what fraction of a pizza more than Rita? Include a diagram in your solution.
5. Marcy had $2\frac{1}{4}$ boxes of crayons. She lost some of her crayons and now has $1\frac{2}{3}$ boxes of crayons left. What fraction of a box of crayons did Marcy lose?
6. Peter, the pie man, has some pies, all the same size. He sells $\frac{3}{8}$ of a pie and has $2\frac{5}{6}$ left.
   a) Estimate how many pies Peter had at the beginning. Explain your thinking.
   b) How many pies did he have at the beginning?
7. Ben thinks that $\frac{2}{3} + \frac{1}{2} = \frac{3}{5}$. He reasons as follows:

$\frac{2}{3}$ shaded



$\frac{1}{2}$ shaded

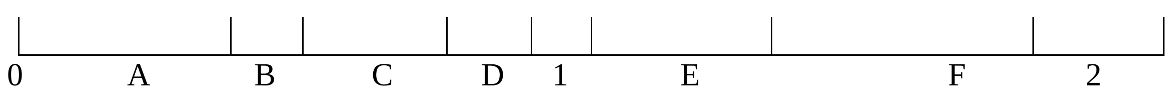
3 out of 5 shapes or $\frac{3}{5}$ of the shapes are shaded. Therefore, when you add one-half and two-thirds you get three-fifths of all the shapes shaded.
Critique Ben's reasoning.

B. One-on-One Assessment

Assessment activities can be used with individual students, especially students who may be having difficulty with the outcome.

Provide fraction strips, fraction blocks and counters for the student to use as needed. Instruct the student that he or she is to explain his or her thinking for each of the questions/problems. Emphasize that all fractions are to be written in the simplest form.

1. Tell the student to use estimation skills and the number line provided to answer the following questions.



   a) If the fractions represented by A and B are added, what point on the number line best represents the sum?
   b) If the fractions represented by B and C are added, what point on the number line best represents the sum?
   c) What point on the number line best represents the difference: F – E?
   d) What point on the number line best represents the difference: D – A?

If the student has difficulty, have the student use different strategies, such as mentally adding or subtracting the two lengths provided or placing appropriate fractions on the number line to aid in using benchmarks such as $\frac{1}{2}$, 1 or $1\frac{1}{2}$.

2. Instruct the student to write <, = or > to complete each sentence correctly, without calculating the answer. Ask the student to explain his or her thinking for each sentence.

a) $\frac{3}{8} + \frac{2}{5} \square 1$ b) $\frac{3}{4} + \frac{1}{3} \square 1$ c) $\frac{3}{8} + \frac{1}{6} \square \frac{1}{2}$

If the student has difficulty estimating the answer, suggest that he or she use benchmarks for each fraction, such as both $\frac{3}{8}$ and $\frac{2}{5}$ are less than $\frac{1}{2}$ so the sum of these two fractions must be less than one.

3. Present the following problem to the student.

Terry jogs for $\frac{5}{6}$ of a hour on Monday and $1\frac{5}{6}$ hours on Friday. What is the total length of time that he jogs on these two days?

If the student has difficulty solving any of the problems, suggest that he or she replace the fractions with whole numbers to aid in deciding which operation to use. To check the reasonableness of the answer, have the student estimate the answer first by using benchmarks. For example, $\frac{5}{6}$ is close to one and $1\frac{5}{6}$ is close to two so the answer should be close to three.

Then provide the student with the appropriate fraction strips or fraction blocks to represent the problem concretely. Encourage the student to draw diagrams to represent the problem, even though diagrams are not required.

4. Present the following problem to the student.

Hungry Harry eats $\frac{7}{8}$ of a pizza. Ravenous Rita eats $\frac{2}{3}$ of a same size pizza. Harry eats what fraction of a pizza more than Rita? Include a diagram in your solution.

5. Present the following problem to the student.

Marcy had $2\frac{1}{5}$ boxes of crayons. She lost some of her crayons and now has $1\frac{3}{10}$ boxes of crayons left. What fraction of a box of crayons did Marcy lose?

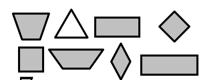
6. Present the following problem to the student.

   Peter, the pie man, has some pies, all the same size. He sells $\frac{3}{5}$ of a pie and has $2\frac{3}{4}$ left.

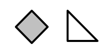
   a) Estimate how many pies Peter had at the beginning. Explain your thinking. (Prompt the student to use benchmarks if he or she has trouble estimating the sum of the two fractions.)
   b) How many pies did he have at the beginning?

7. Present the following situation to the student and have him or her critique the reasoning.

   Ben thinks that $\frac{7}{8}+\frac{1}{2}=\frac{8}{10}$. He reasons as follows:

   $\frac{7}{8}$ shaded

   $\frac{1}{2}$ shaded

   8 out of 10 shapes or $\frac{8}{10}$ of the shapes are shaded. Therefore, when you add $\frac{7}{8}+\frac{1}{2}$ you get $\frac{8}{10}$ of all the shapes shaded.

   If difficulty arises in explaining the fallacy, prompt the student to think about the size of the whole in each case. Remind him or her that the fractions in a given problem must all relate to same whole set or whole region.

C. Applied Learning

Provide opportunities for students to use their addition and subtraction of fractions and mixed numbers in a practical situation and notice whether or not the understanding transfers. For example, have the student explain if he or she reads more or less than an hour in total if he or she reads $\frac{3}{5}$ of an hour one day and $\frac{2}{3}$ of an hour another day. Then have the student calculate the number of hours that he or she read in these two days. Ask the student to use the fractions in the problem to create a subtraction problem and solve it.

Does the student:

- display number sense by using benchmarks in making estimates of sums or differences of fractions?
- explain his or her thinking in making estimates and calculating answers?
- apply his or her knowledge of whole numbers in working with fractions?
- understand the relative size of the numbers so that he or she creates a subtraction problem with a positive solution?
- solve a similar problem, using mixed numbers, and explain the solution?
- use diagrams if needed to aid in explaining the solution?

Step 5: Follow-up on Assessment

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction?

A. Addressing Gaps in Learning

- Use manipulatives of various kinds to establish a solid foundation for the meaning of fractions and equivalent fractions.
- Review the relation between mixed numbers and improper fractions, using concrete materials, diagrams and symbols.
- Relate adding and subtracting fractions to adding and subtracting whole numbers. The types of problems involving the addition or subtraction of whole numbers are the same for fraction problems; e.g., part-part-whole, comparison and join or separate problems. Substituting whole numbers for fractions helps students decide which operation to use in solving the problems.
- Encourage students to estimate before calculating to ensure that their answer makes sense and also to develop number sense.
- Use problem contexts that students can relate to and keep the fractions simple initially; i.e., use fractions with like denominators, then use unlike denominators with only one denominator needing to be changed and finally use unlike denominators with both denominators needing to be changed.
- Relate addition to subtraction to help students see the connection.
- Connect the concrete, pictorial and symbolic modes.
- Provide the option for students to use manipulatives as long as needed, even on tests.
- Encourage students to explain their thinking orally if they have difficulty explaining it in writing.

B. Reinforcing and Extending Learning

Students who have achieved or exceeded the outcomes will benefit from ongoing opportunities to apply and extend their learning. These activities should support students in developing a deeper understanding of the concept and should not progress to the outcomes in subsequent grades.

Consider strategies, such as the following.

- Provide tips for parents on adding and subtracting fractions and mixed numbers at home or in the community.
  - Find the total amount of pizza eaten by adding the fractional amounts eaten by each family member.

- The child has 3 hours before dinner and needs $\frac{3}{4}$ of an hour to help with dinner, so how much time does the child have to visit friends before dinner?
- If the child reads $2\frac{1}{5}$ hours on Monday and $1\frac{3}{4}$ hours on Tuesday, ask questions related to these fractions, such as:
  - Did the child read more or less than four hours on the two days? How do they know without calculating the answer?
  - What was the total time that the child read on the two days?
  - How much longer did the child read on one day than the other?
  - How much more time would the child have to read to have read five hours?

- Reinforce adding and subtracting fractions by using two-way tasks. The three numbers in any row or column must form a correct addition sentence. The self-checking nature of these tasks is an asset in providing reinforcement. The tasks can be made more or less challenging by changing the numbers. Students should be encouraged to create two-way tasks for other students to complete.

  Examples of two-way tasks:

	+	
	$\frac{2}{5}$	$6\frac{1}{5}$
$\frac{27}{100}$		
		$6\frac{1}{2}$

	+	
	$\frac{1}{5}$	$\frac{29}{50}$
	$\frac{2}{25}$	
$\frac{2}{5}$		

	+	
$4\frac{4}{5}$		7
$5\frac{1}{10}$	$2\frac{9}{10}$	

Adapted from Grayson H. Wheatley and George Abshire, *Developing Mathematical Fluency: Activities for Grades 5–8* (Tallahassee, FL: Mathematics Learning, 2002), p. 207. Adapted with permission from Mathematics Learning, www.mathematicslearning.org.

- Provide students with pattern blocks and fraction blocks. Have them find all the different ways they can build the yellow hexagon pattern block from different combinations of pattern blocks. Instruct them to use fractions to record the different ways that they find. For example, the green triangle is $\frac{1}{6}$ of the hexagon and the trapezoid is 1/2 of the hexagon so one way to write the fractions would be $\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{2}$. Similarly, students could find all the different ways to build the pink double hexagon fraction block and write the fractions to record the ways.

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- Challenge students with fraction riddles, such as the following:

  A rectangle is $\frac{1}{2}$ red, 1/5 green, 1/10 blue and the rest yellow. How much of the rectangle is yellow? Draw the rectangle on squared paper and record the fraction that tells which part is yellow.

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- Challenge students to solve multi-step problems involving the addition and subtraction of fractions and mixed numbers, such as the following. Encourage students to create problems to share with others.

  Pete has 2 1/2 pepperoni pizzas left over from a party as well as 1 5/8 ham and pineapple pizzas. He gives 2 3/4 of these pizzas away to friends to take home after the party. If the pizzas are all the same size, how much pizza does Pete have left for himself?

- Have students play the following games:
  - Make One Whole
    Players: small group or whole class
    Materials: three sets of fraction cards: $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{2}{3}, \frac{2}{4}, \frac{3}{4}, \frac{3}{2}, \frac{4}{3}$
    Description: Shuffle the cards and give one card to each person in the class. Each student finds a partner who has a fraction that can be added to his or her fraction to make a whole number or subtracted from his or her fraction to make a whole number.
    Goal: find as many partners as possible to produce a whole number.

    Variations:
    ✓ Players can work in teams of three or four with one set of fraction cards. The team that finds the most ways to add or subtract the cards is the winner.
    ✓ Each team is given a set of cards and is asked to use any of the four operations with fractions to arrive at the greatest (or least) answer.
    ✓ Change the fractions on the fraction cards to include more (or less) variety.

    Adapted from Alberta Education, *Fractions: Learning Strategies to Enhance Understanding* (unpublished workshop handout) (Edmonton, AB: Alberta Education, 2004), p. 64.

  - Operation Fractions
    Players: 2 to 4
    Materials: a set of fraction cards; e.g., $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{2}{3}, \frac{2}{4}, \frac{3}{4}, \frac{3}{2}, \frac{4}{3}$
    Description: Deal all the fraction cards face down to the players. Each player turns over two cards and decides whether to add or subtract the two fractions on the cards. The player who has the greatest sum or difference wins all the cards that are face up.

    Goal: The play continues until one person (the winner) has all the cards.

Variations:

- ✓ Use fewer cards or more cards with different fractions
- ✓ Use cards with only certain numbers.
- ✓ Use more operations.
- ✓ Turn over three or four cards instead of two cards for each play.
- ✓ The winner of the game could be the person with no cards left.
- ✓ The player who has the least sum or difference wins all the cards that are face up.
- ✓ Each player rolls two (or more) dice with fractions on each face rather than using playing cards. The player with the greatest (or least number) resulting from the operations could score one point. The winner is the player with the most points.

Adapted from Alberta Education, *Fractions: Learning Strategies to Enhance Understanding* (unpublished workshop handout) (Edmonton, AB: Alberta Education, 2004), p. 65.

- Sample Structured Interview: Assessing Prior Knowledge and Skills

Directions Present the following problems to the student.	Date: **Not Quite There**	**Ready to Apply**
Philly runs around a racetrack every six minutes while Lightning runs around the same racetrack every eight minutes. If both horses start the race at the same time and continue at the same pace, in how many minutes will they be side by side again on the racetrack? Explain your thinking.	• Does not use multiples to solve the problem. • Solves the problem but is unable to explain the process.	• Uses multiples to solve the problem correctly and illustrates the multiples by listing the multiples of each number, using patterns in a chart, arrows on a number line or some other appropriate method.
Jamie said that the same number can be both a multiple and a factor of a given number. Is Jamie correct? Explain.	• States that Jamie is not correct. • Says Jamie is correct but is unable to give an example to explain why.	• Says that Jamie is correct and substantiates this viewpoint with an example, such as six is a factor of six and also a multiple of six.

There are 24 candies of one kind in the first bag and 32 candies of another kind in the second bag. What is the greatest number of people who could share equally the candies from both bags? Explain.	• Has difficulty understanding the problem and resorts to adding or subtracting the numbers or some other operation that does not lead to a correct solution. • Solves the problem but is unable to explain the process.	• Solves the problem and explains the process clearly by referring to factors of 24 and 32 of which 8 would be the greatest number that would divide evenly into both 24 and 32.
There are $\frac{17}{8}$ pizzas left after a party. Express this number as a mixed number. Draw a diagram to show the pizzas.	• Does not express the improper fraction as a mixed number. • Expresses the improper fraction as a mixed number but is unable to explain the process using a diagram.	• Expresses the improper fraction as a mixed number and draws an appropriate diagram.
Susie has $4\frac{3}{4}$ Kit Kat chocolate bars. Write this mixed number as an improper fraction to show how many quarter bars (Kit Kat pieces) she has in all. Draw a diagram to show the chocolate bars.	• Does not write the improper fraction. • Writes the improper fraction but does not draw an appropriate diagram to illustrate the problem.	• Writes the improper fraction and draws an appropriate diagram showing one equal-sized chocolate bars, each divided into four equal pieces, and one more chocolate bar of the same size with three out of four equal pieces left.

BIBLIOGRAPHY—Planning Guide: ***Grade 7 Addition and Subtraction of Positive Fractions and Mixed Numbers***

Strand: Number
Outcome: 5

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