Mathematics

Planning Guide

Grade 8 *Congruence of Polygons*

Shape and Space (Transformations)

Specific Outcome 6

This Planning Guide can be accessed online at:
http://www.learnalberta.ca/content/mepg8/html/pg8_congruenceofpolygons/index.html

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Planning Guide: *Grade 8 Congruence of Polygons*

Strand: Shape and Space (Transformations)
Specific Outcome: 6

This *Planning Guide* addresses the following outcomes from the Program of Studies:

Strand: Shape and Space (Transformations)	
Specific Outcome:	6. Demonstrate an understanding of the congruence of polygons.

Curriculum Focus

This sample targets the following changes in the curriculum:

- The general outcome focuses on describing and analyzing position and motion of objects and shapes; whereas the previous mathematics curriculum focused on applying coordinate geometry and pattern recognition to predict the effects of translations, rotations, reflections and dilatations on 1-D lines and 2-D shapes in the Grade 9 curriculum.
- The specific outcome focuses on the understanding of the congruence of polygons; whereas the previous curriculum focused on the congruence of a triangle in the Grade 9 curriculum.

What Is a Planning Guide?

Planning Guides are a tool for teachers to use in designing instruction and assessment that focuses on developing and deepening students' understanding of mathematical concepts. This tool is based on the process outlined in *Understanding by Design* by Grant Wiggins and Jay McTighe.

Planning Steps

The following steps will help you through the Planning Guide:

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Step 1: Identify Outcomes to Address

Guiding Questions

- What do I want my students to learn?
- What can my students currently understand and do?
- What do I want my students to understand and be able to do based on the Big Ideas and specific outcomes in the program of studies?

Big Ideas

- Shapes are alike and different by determining an array of geometric properties. These properties could include: shapes have sides that are parallel, perpendicular or neither; they have line symmetry, rotational symmetry or neither; they are similar, congruent or neither.
- Shapes can be moved in a plane. These movements can be described in terms of translations (slides), reflections (flips) and rotations (turns).
- Coordinate systems can be used to describe the precise location of a shape in a plane. The coordinate view of shape is also useful in understanding the property of transformation (changes in position) of shapes.
- Transformation includes a study of translations, reflections, rotations (slides, flips and turns) and the study of symmetries.
- Congruent objects have the same size and same shape. The corresponding angles and corresponding sides of two congruent objects are equal. Congruency is denoted by the symbol $\cong$; i.e., "quadrilateral ABCD is congruent to quadrilateral DEFG" is shown by $ABCD \cong DEFG$.

Adapted from John A. Van de Walle, LouAnn H. Lovin, *Teaching Student-Centered Mathematics: Grades 5–8*, 1e (pp. 179, 180). Published by Allyn and Bacon, Boston, MA. Copyright © 2006 by Pearson Education. Reprinted by permission of the publisher.

Sequence of Outcomes from the Program of Studies

See http://education.alberta.ca/teachers/core/math/programs.aspx for the complete program of studies.

Grade 7	Grade 8	Grade 9
Specific Outcomes	**Specific Outcomes**	**Specific Outcomes**
There are no directly related specific outcomes in Grade 7.	6. Demonstrate an understanding of the congruence of polygons.	3. Demonstrate an understanding of similarity of polygons.

Step 2: Determine Evidence of Student Learning

Guiding Questions

- What evidence will I look for to know that learning has occurred?
- What should students demonstrate to show their understanding of the mathematical concepts, skills and Big Ideas?

Using Achievement Indicators

As you begin planning lessons and learning activities, keep in mind ongoing ways to monitor and assess student learning. One starting point for this planning is to consider the achievement indicators listed in the *Mathematics Kindergarten to Grade 9 Program of Studies with Achievement Indicators*. You may also generate your own indicators and use them to guide your observation of the students.

The following indicators may be used to determine whether or not students have met this specific outcome. Can students:

- determine the coordinates of the vertices of an image following a given combination of transformations of the original figure?
- draw the original figure and determine the coordinates of its vertices given the coordinates of the image's vertices and a description of the transformation (translation, rotation, reflection)?

Sample behaviours to look for related to these indicators are suggested for some of the activities listed in **Step 3, Section C: Choosing Learning Activities** (p. 10).

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Step 3: Plan for Instruction

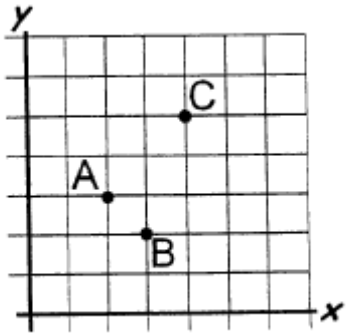
Guiding Questions

- What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should I use?
- How will I meet the diverse learning needs of my students?

A. Assessing Prior Knowledge and Skills

Before introducing new material, consider ways to assess and build on students' knowledge and skills related to the Cartesian plane. For example:

Activity 1:



Use the coordinate system above to answer the following questions:

a. Give the coordinates for point B.
b. How far is B from the vertical or *y*-axis?
c. How far is B from the horizontal or *x*-axis?
d. If the x-coordinate of a point is defined as the distance from the vertical or *y*-axis, define *y*-coordinate.
e. What is the value of the *y*-coordinate for point C?

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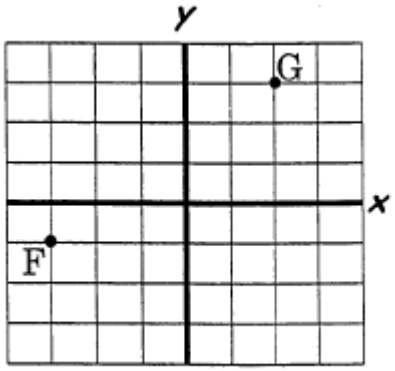
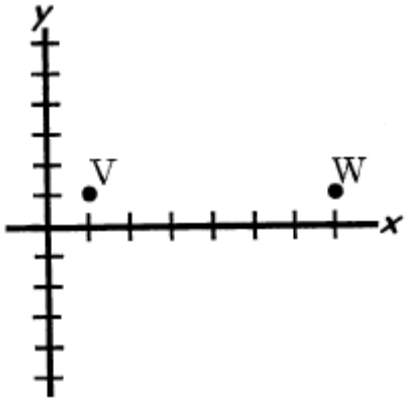
Activity 2:



Consider points F and G as shown. Reflect G in the y-axis to get a new point, G'. What are the coordinates of G'?

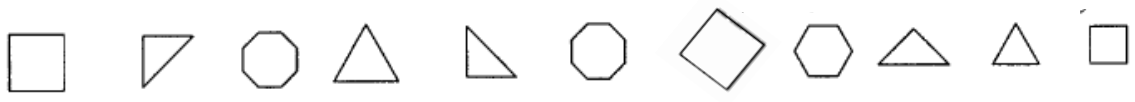
Activity 3:



a. An equilateral triangle has all sides equal in length. Draw a possible equilateral triangle with vertices at V (1,1) and W (7,1).
b. Draw a reflection of triangle UVW over side VW and label it U'VW.
c. From your sketch, what can you say for sure about the coordinates of U and U'?

Activity 4:

Sorting Congruent Shapes
Provide diagrams of 2-D shapes, some of which are congruent, such as:



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Ask the students to:

- put a check mark on shapes that are congruent to 
- put an X on shapes congruent to 
- shade in shapes that are congruent to 

Have the students explain the strategies they used to determine if the shapes were congruent.

If a student appears to have difficulty with these tasks, consider further individual assessment, such as a structured interview, to determine the student's level of skill and understanding. See **Sample Structured Interview: Assessing Prior Knowledge and Skills** (p. 9).

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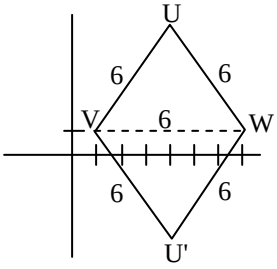
Sample Structured Interview: Assessing Prior Knowledge and Skills

Directions	Date:	
	Not Quite There	**Ready to Apply**
1. Use the coordinate system shown in Step 3, Part A, Activity 1 to help answer the following questions: a. Give the coordinates for point B. b. How far is B from the vertical or *y*-axis? c. How far is B from the horizontal or *x*-axis? d. If the *x*-coordinate of a point is defined as the distance from the vertical or *y*-axis, define *y*-coordinate. e. What is the value of the *y*-coordinate for point C?	1. Student responds with a correct answer for a. but is unable to respond to the rest of the question.	1. Student responds with the following answers: a. (3,2) b. 3 units c. 2 units d Possible answers: – The distance a point is from the *x*-axis or the horizontal axis. – How far you travel from the *x*-axis in the vertical direction to get to the point. e. 5
2. Consider points F and G as shown in Step 3, Part A, Activity 2. Reflect G in the *y*-axis to get a new point, G'. What are the coordinates of G'?	2. Student is unable to reflect point G.	2. Student responds with an answer G' = (–2,3).
3. a. An equilateral triangle has all sides equal in length. Draw a possible equilateral triangle with vertices at V (1,1) and W (7,1). b. Draw a reflection of triangle UVW over side VW and label it U'VW. c. From your sketch, what can you say for sure about the coordinates of U and U'?	3. The student is unable to reflect over side VW.	3. Student responds with a diagram that could look like this or the reverse.  He or she responds with an answer that the coordinates of U and U' have the same *x*-coordinate or that if U is a positive *y*-coordinate, the U' is a negative *y*-coordinate, and vice versa.

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4. Ask the students to: • put a check mark on shapes that are congruent to  • put an X on shapes congruent to  • shade in shapes that are congruent to 	4. If diagrams are numbered from left 1 to 11, student responds with a check mark in figures 1, 7 and 11; an *x* in figures 2, 4, 5, 9, 10; and has shaded in figures 3 and 6.	4. If diagrams are numbered from left 1 to 11, student responds with a check mark in figures 1 and 7; an *x* in figures 2, 5 and 9; and has shaded figures 3 and 6.

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B. Choosing Instructional Strategies

Consider the following general strategies for teaching the transformation of congruent polygons on a Cartesian plane:

- Build on students' knowledge of 2-D shapes from previous grade levels. This knowledge should include topics such as classification of shapes, symmetry of shapes and transformation of shapes.
- Provide hands-on activities.
- Encourage the use of manipulatives such as tracing paper and graph paper to explore the meaning of congruence of polygons.
- Have the students use a graphic organizer such as a Frayer Model to reflect on their learning of the congruence of polygons.
- Have the students share their ideas and justify their thinking by explaining why they used a particular strategy to make the transformation.

C. Choosing Learning Activities

The following learning activities are examples of activities that could be used to develop student understanding of the concepts identified in Step 1.

Sample Activities:

1. **Shape Short** (p. 11)
2. **Archimedes' Puzzle** (p. 14)
3. **Transformations** (p. 15)

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Sample Activity 1: Shape Sort

This activity adapted from John A. Van de Walle, LouAnn H. Lovin, *Teaching Student-Centered Mathematics: Grades 5–8,* 1e (p. 188). Published by Allyn and Bacon, Boston, MA. Copyright © 2006 by Pearson Education. Reprinted by permission of the publisher.
AND
From John A. Van de Walle, *Elementary and Middle School Mathematics: Teaching Developmentally* (p. 313), 4/e. Published by Allyn and Bacon, Boston, MA. Copyright © 2001 by Pearson Education. Reprinted by permission of the publisher.

Have the students work in groups of four with a set of 2-D shapes similar to those shown to complete the following activities:

- Students each randomly select two shapes and find something that is alike about their two shapes and something that is different. (Have them select their shapes before they know the task.)
- The group selects one shape at random and places it in the centre of the workspace. Their task is to find all other shapes that are like the target shape, but all according to the same rule. For example, if they say, "This one is like our shape because it has a curved side and a straight side," then all other shapes that they put into the collection must have these properties. Challenge them to do a second sort with the same target shape but a different property.
- Have the students share their sorting rules with the class and show examples. All students then draw a new shape that will also fit in the group according to the rule. Encourage them to write about their new shape and why it fits the rule.
- Do a "secret sort." Have a student create a small collection of about five shapes that fit a secret rule. Leave others that belong in your group in the pile. The other students try to find additional pieces that belong to the set and/or guess the secret rule. If time permits, allow another student to do a secret sort.
- During this exercise, teachers could reinforce mathematical terms such as the names of polygons, lines of symmetry, right angles, parallel lines, congruent figures, similar figures and so on.

Note: It is important that students choose how to do the sort and not the teacher.

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Examples of possible sorts:

Shapes with curved edges

Opposite sides "go the same way"—parallelograms

Three sides—triangles

Shapes with a "square corner"—right angle

These all "dent in"—concave

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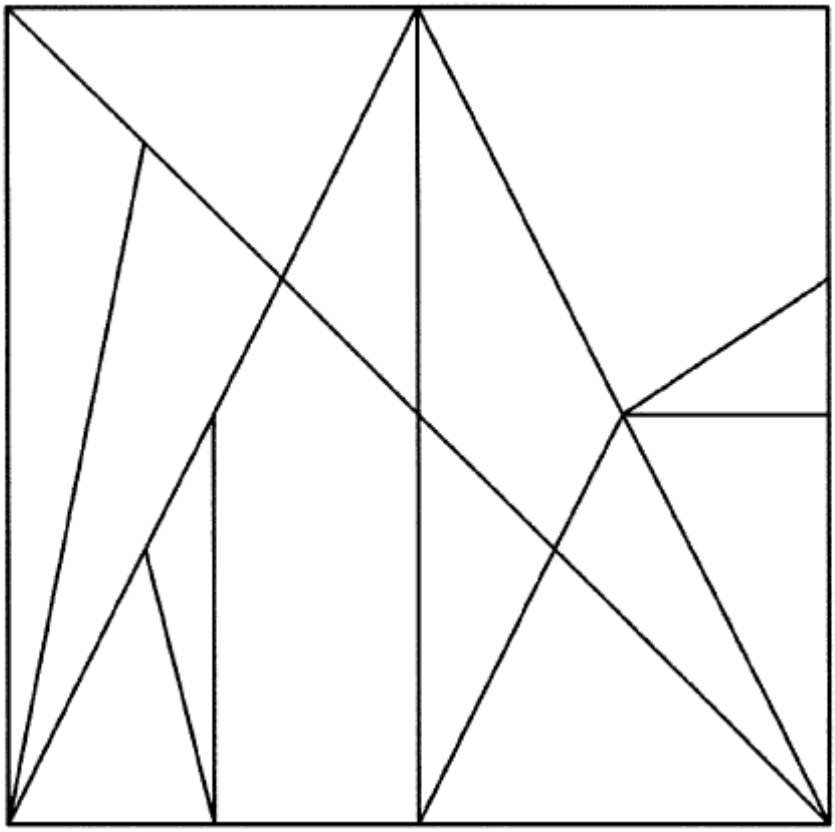
Sample Activity 2: Archimedes' Puzzle

(This activity adapted from National Council of Teachers of Mathematics, "Archimedes' Puzzle," *NCTM Illuminations*, 2000–2008, http://illuminations.nctm.org/LessonDetail.aspx?id=L720 (Accessed April 2008). Used with permission.)

The *Stomachion* is an ancient puzzle that is at least 2 200 years old. It consists of 14 pieces that can be cut from a 12 × 12 square. As with its cousin the tangram, the object of the *Stomachion* is to rearrange the pieces to form interesting shapes.



Instructional Suggestion:

- Give each group of students two copies of the *Stomachion.* Allow the students time to cut out the pieces from one copy. Have the students discuss the following questions amongst themselves:
  - What kinds of pieces appear in the *Stomachion?* (All of the pieces have 3, 4 or 5 sides, that is, they are triangles, quadrilaterals and pentagons.)
  - Are any of the pieces congruent to one another? How do you know? (Yes, there are two pairs of congruent triangles.)
  - From its original place in the *Stomachion*, how can the congruent figures be moved so that they would be on top of each other. (Encourage the use of the following concepts: translations [slides], reflections [flips] and rotations [turns].)
- Facilitate a discussion in which students share their answers to the questions.

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Sample Activity 3: Transformations

- Transformations includes a study of translations (slides), reflections (flips) and rotations (turns) and the study of symmetries.
- Students will need many sheets of graph paper for this section.
- Students could use different coloured pencils to do the transformations.
- Students could be given a graph paper transparency so that they could share their results in a class discussion.

a. Translations (slides)
   - Have the students draw a simple four-sided figure in the first quadrant of a Cartesian plane. Label the coordinates of the vertices ABCD. Suggest that students use values of *y* less than 7.
   - Ask the students to translate (slide) the figure up 2 units and to the right 7 units (indicating direction and length). Label the vertices and find the coordinates for the position of the figure A'B'C'D'.

   Note: A' is read as A prime and corresponds to vertice A; B' is read as B prime and corresponds to vertice B; C' is read as C prime and corresponds to vertice C; and D' is read as D prime and corresponds to vertice D. Figure A'B'C'D 'is congruent to Figure ABCD.
   - Have the students share their results.

b. Reflections (flips)
   - Have the students reflect (flip) figure A'B'C'D' (from above) over the line y = 8. Label the vertices of the new figure A"B"C"D" and find the coordinates of the vertices.

   Note: A" is read as A double prime and corresponds to vertice A'; B" is read as B double prime and corresponds to vertice B'; C" is read as C double prime and corresponds to vertice C', and D" is read a D double prime and corresponds to D". Figure A"B"C"D" is congruent to Figure A'B'C'D'.

c. Reflections (flips) and Rotations (turns)
   - On a new Cartesian plane, have the students draw another four-sided figure using the coordinates of the vertices: A (2,1), B (2,3), C (4,4) and D (4,0). Reflect figure ABCD around the line *y* = 6. Label the vertices of the new figure A'B'C'D'.
   - Have the students rotate figure A'B'C'D' 90 degrees clockwise around point (vertice) C. Label the new figure A"B"C"D" and find the coordinates of the vertices.
   - Have the students share their results.

   Note: If students experience difficulty with rotations, have them use tracing paper.

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d. Combination of Transformations
   - Have the students work in pairs.
   - Have the students start with a new figure—a design of their choice—and perform a combination of transformations—at least two. For example a translation of up 6 and to the right 2 and then rotate around point (6,8).
   - Other combination transformation suggestions:
     – Translate up 3 units and move 2 units to the right, rotate clockwise around the point (6,6).
     – Reflect around the line $x = 3$ and rotate 180 degrees.
     – Reflect around the line $y = 6$, translate down 1 unit and to the right 1 unit and rotate clockwise 90 degrees.
     – If using a reflection, students might try to have one side of their image touch one side of the original design.
     – If using a rotation, students might try to have one vertex of the image touch the vertex of the original design.
   - Have the students share their results with other members of the class.

e. The exploration of translations, reflections and slides can also be done using dynamic geometry programs.

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Step 4: Assess Student Learning

Guiding Questions

- Look back at what you determined as acceptable evidence in Step 2.
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Sample Assessment Tasks

In addition to ongoing assessment throughout the lessons, consider the following sample activities to evaluate students' learning at key milestones. Suggestions are given for assessing all students as a class or in groups, individual students in need of further evaluation, and individual or groups of students in a variety of contexts.

A. Whole Class/Group Assessment

Have the students create a design using their choice(s) of transformations. Have the students explain how their designs were formed from the combination of transformations that they chose. This could be done in pairs or as individuals. Have the students share their results. This could be done as a take home assignment and/or used as an assessment tool for the outcome.

B. One-on-one Assessment

1. Quadrilateral ABCD has line coordinates A (1,2), B (1,6), C (4,7) and D (3,1). Reflect the quadrilateral around the line $x = 6$ and then translate the quadrilateral A'B'C'D' up 3 units and to the left 2 units. What are the coordinates of quadrilateral A"B"C"D"?
2. Figure A"B"C"D" has coordinates A" (10,9) B" (9,6), C" (7,4) and D" (6,5). Figure ABCD has coordinates A (12,11), B (13,14), C (15,16) and D (14,15). Describe a possible transformation from figure ABCD to figure A"B"C"D".

   Possible solutions: Figure ABCD was reflected around the line $y = 10$ and then reflected again around the line $x = 11$ OR Figure ABCD was rotated 180 degrees, clockwise or counter clockwise, around point (10,11).
3. Have the students demonstrate their understanding of transformations as follows:
   - Draw a figure of their choice in Quadrant I of the Cartesian plane.
   - Demonstrate a combination of transformations:
     - All three transformations—translation, reflection and rotation—must be demonstrated.
     - Each drawing must contain a combination of at least two different transformations.
     - All drawings must have vertices properly labelled and the coordinates of each vertice given.
     - The drawings must be accompanied with a written explanation justifying the end result of the combination of transformations.

   Students may wish to create a pattern with the transformations.

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C. Applied Learning

Provide opportunities for students to use the strategies learned related to transformations in practical situations and notice whether or not the strategies transfer.

Find an example of a translation, a reflection and a rotation in the real world. Examples could include the reflection of the mountains in Lake Louise, a Ferris wheel rotating around the centre of the wheel and an elevator sliding up and down in an office building.

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Step 5: Follow-up on Assessment

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction?

A. Addressing Gaps in Learning

Students experiencing difficulties with these activities may have to spend time with more hands-on activities involving basic translations, reflections and rotations.

B. Reinforcing and Extending Learning

Students who have achieved or exceeded the outcomes will benefit from ongoing opportunities to apply and extend their learning. These activities should support students in developing a deeper understanding of the concept and should not progress to the outcomes in subsequent grades.

Consider strategies such as:

- Go to http://illuminations.nctm.org/LessonDetail.aspx?id=L720 for more ideas using the *Stomachion.*
- Have the students research tessellations.

Line Symmetry
The study of line symmetry can be included in the study of transformations because a figure that has line symmetry can be folded on a line so that two parts of the figure match. The line of symmetry, then, is considered to be a line of reflection in which one part of the figure is reflected onto the other.

To demonstrate line symmetry:

a. Show the students examples and non-examples.
b. Have the students fold a piece of paper in half and cut out a shape of their choosing—the fold line will be the line of symmetry or have the students do the Dot Grid Line Symmetry activity as follows.

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Dot Grid Line Symmetry Activity

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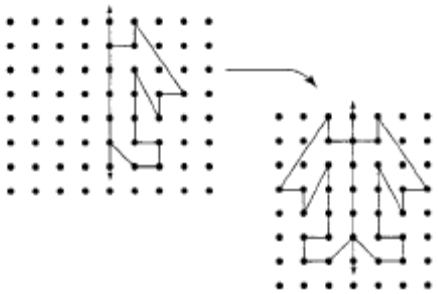
- Give the students either isometric or rectangular dot grid paper.
- Have the students draw a line through several dots—the line can be horizontal, vertical or skewed.
- Have the students draw a design on one side of the drawn line that touches the line in some way.
- The task is to make the mirror image of the design on the other side of the line.



The results could be checked with a mirror.

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