

# Planning Guide

## Grade 8 *Integers*

### Number Specific Outcome 7

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## Planning Guide: *Grade 8 Integers*

**Strand:** Number

**Specific Outcome:** 7

This *Planning Guide* addresses the following outcomes from the Program of Studies:

**Strand:** Number

**Specific Outcomes:** 7. Demonstrate an understanding of multiplication and division of integers, concretely, pictorially and symbolically.

### Curriculum Focus

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The changes to the curriculum targeted by this sample include:

- The general outcome focuses on developing number sense; whereas the previous mathematics curriculum focused on applying arithmetic operations on decimals and integers, and illustrating their use in solving problems.
- The specific outcome focuses on the understanding of multiplication and division of integers—concretely, pictorially and symbolically; whereas the previous mathematics curriculum focused on the addition, subtraction, multiplication and division of integers in Grade 7.

### What Is a Planning Guide?

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**Planning Guides** are a tool for teachers to use in designing instruction and assessment that focuses on developing and deepening students' understanding of mathematical concepts. This tool is based on the process outlined in *Understanding by Design* by Grant Wiggins and Jay McTighe.

### Planning Steps

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The following steps will help you through the Planning Guide:

- **Step 1: Identify Outcomes to Address** (p. 3)
- **Step 2: Determine Evidence of Student Learning** (p. 4)
- **Step 3: Plan for Instruction** (p. 5)
- **Step 4: Assess Student Learning** (p. 21)
- **Step 5: Follow-up on Assessment** (p. 27)

## Step 1: Identify Outcomes to Address

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### Guiding Questions

- What do I want my students to learn?
- What can my students currently understand and do?
- What do I want my students to understand and be able to do based on the Big Ideas and specific outcomes in the program of studies?

### Big Ideas

Integers:

- are a set of numbers that include the following: (... -3, -2, -1, 0, 1, 2, 3, ...)
- include positive and negative whole numbers
- add to number the idea of opposite, so that every number has both size and a positive or negative relationship to other numbers. A negative number is the opposite of the positive number of the same size (Van de Walle and Lovin 2006, p. 131)
- are successive numbers that differ by 1
- form a subset of the rational numbers
- represent the difference between two objects in a set
- are numbers that deal with direction and magnitude.

### Sequence of Outcomes from the Program of Studies

See <http://education.alberta.ca/teachers/core/math/programs.aspx> for the complete program of studies.

<b>Grade 7</b>	<b>Grade 8</b>	<b>Grade 9</b>
<b>Specific Outcomes</b>	<b>Specific Outcomes</b>	<b>Specific Outcomes</b>
6. Demonstrate an understanding of addition and subtraction of integers, concretely, pictorially and symbolically.	7. Demonstrate an understanding of multiplication and division of integers, concretely, pictorially and symbolically.	There are no directly related specific outcomes in Grade 9.

## Step 2: Determine Evidence of Student Learning

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### Guiding Questions

- What evidence will I look for to know that learning has occurred?
- What should students demonstrate to show their understanding of the mathematical concepts, skills and Big Ideas?

### Using Achievement Indicators

As you begin planning lessons and learning activities, keep in mind ongoing ways to monitor and assess student learning. One starting point for this planning is to consider the achievement indicators listed in the *Mathematics Kindergarten to Grade 9 Program of Studies with Achievement Indicators*. You may also generate your own indicators and use them to guide your observation of the students.

The following indicators may be used to determine whether or not students have met this specific outcome. Can students:

- identify the operation required to solve a given problem involving integers?
- provide a context that requires multiplying two integers?
- provide a context that requires dividing two integers?
- model the process of multiplying two integers using concrete materials or pictorial representations and record the process?
- model the process of dividing an integer by an integer using concrete materials or pictorial representations and record the process?
- generalize and apply a rule for determining the sign of the product and quotient of integers?
- solve a given problem involving the multiplication of integers (2-digit by 1-digit) without the use of technology?
- solve a given problem involving the division of integers (2-digit by 2-digit) with the use of technology?
- solve a given problem involving integers, taking into consideration order of operations?

Sample behaviours to look for related to these indicators are suggested for some of the activities listed in **Step 3, Section C: Choosing Learning Activities** (p. 8).

## Step 3: Plan for Instruction

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- What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should I use?
- How will I meet the diverse learning needs of my students?

### A. Assessing Prior Knowledge and Skills

Before introducing new material, consider ways to assess and build on students' knowledge and skills related to the multiplication of whole numbers and the understanding of integers as outlined in the Grade 7 curriculum. For example:

**Activity 1:** Have the students write what they know about multiplication. Use the think–pair–share strategy. This could be used again at the end of the lesson to show what they have learned during the lesson.

**Activity 2:** Give the students a multiplication fact; e.g.,  $8 \times 4$  or  $3 \times 6$ . Ask them to explain how they got the answer; e.g., The answer is \_\_\_\_\_. I know this because ...

**Activity 3:** Have the students solve the following problem:

Jane has invited eight friends to her birthday party. She would like to give each of her friends one large cupcake or two small cupcakes. A large cupcake costs \$2.50 and a small cupcake costs \$1.50. Which size of cupcake do you think Jane should buy? Justify your answer.

**Activity 4:** Find a set of integers for each of the following set of clues:

- a. odd numbers between  $-15$  and  $+8$  that are multiples of 3
- b. even numbers less than  $-10$
- c. greater than  $-25$  that are multiples of 2 and 5

**Activity 5:** Solve the following problem:

The temperature at 6:00 a.m. is  $-18^{\circ}\text{C}$  and at 9:00 a.m. is  $-25^{\circ}\text{C}$ . Place these temperatures on a number line and write a true sentence relating these temperatures.

If a student appears to have difficulty with these tasks, consider further individual assessment, such as a structured interview, to determine the student's level of skill and understanding. See **Sample Structured Interview: Assessing Prior Knowledge and Skills** (p. 6).

## Sample Structured Interview: Assessing Prior Knowledge and Skills

Directions	Date:	
	Not Quite There	Ready to Apply
1. Have the student write what he or she knows about multiplication.	1. Student stumbles over words, makes comments such as "you just do it" and is unable to express multiplication in his or her own words.	1. Student clearly understands multiplication if he or she can explain it using the following ideas: repeated addition, matching sets of objects, can be related to the area of a rectangle or represented by a tree diagram.
2. Give the student a multiplication fact; e.g., $8 \times 4$ or $3 \times 6$ . Ask him or her to explain how he or she got the answer.	2. Student answers, "I just know that it is 32 or 18 or whatever." He or she is probably relying on multiplication facts rather than understanding the true meaning of multiplication.	2. Student answers 8 groups of 4 added together makes 32 or 3 groups of 6 added together makes 18 or relates the answers to any of the ideas expressed in question 1 or draws a diagram.
3. Jane has invited eight friends to her birthday party. She would like to give each of her friends one large cupcake or two small cupcakes. A large cupcake costs \$2.50 and a small cupcake costs \$1.50. Which size of cupcake do you think Jane should buy? Justify your answer.	3. Student multiplies $\$2.50 \times 8$ to get \$20 and $\$1.50 \times 8$ to get \$12 and does not provide an explanation for his or her answer.	3. Student multiplies $\$2.50 \times 8$ to get \$20 and $\$1.50 \times 8 \times 2$ to get \$24. Student chooses the big cupcake because it is cheaper. Alternatively, the student chooses the small cupcakes because her friends could have two different kinds or any other reasonable explanations so long as the math is correct.

<p>4. Find a set of integers for each of the following set of clues:</p> <p>a. Odd numbers between <math>-15</math> and <math>+8</math> that are multiples of 3.</p> <p>b. Even numbers less than <math>-10</math>.</p> <p>c. Greater than <math>-25</math> that are multiples of 2 and 5.</p>	<p>a. Student can place <math>-15</math> and <math>+8</math> on a number line but cannot identify multiples, or counts by threes backward from <math>+8</math>.</p> <p>b. Student places <math>-10</math> on number line but chooses numbers greater than <math>-10</math>.</p> <p>c. Student places <math>-25</math> on number line but does not understand the meaning of multiples.</p>	<p>a. Student answers <math>-12, -9, -6, -3, 3, 6</math>, indicating that a multiple is a certain number of groups of 3 added together; i.e., <math>-6</math> is made up of 2 groups of <math>-3</math> added together.</p> <p>b. Student answers <math>-12, -14, -16, -18, \dots</math></p> <p>c. Student answers <math>-20, -10, 10, 20, 30, \dots</math>, recognizing that these numbers are groups of 10.</p>
<p>5. The temperature at 6:00 a.m. is <math>-18^{\circ}\text{C}</math> and at 9:00 a.m. is <math>-25^{\circ}\text{C}</math>. Place these temperatures on a number line and write a true sentence relating these temperatures.</p>	<p>5. Student places the temperatures on a number line but is unable to write a statement.</p>	<p>5. Answers may vary. Student places temperatures on a number line and provides a statement relating the two temperatures, such as: "It was <math>8^{\circ}\text{C}</math> colder at 9:00 a.m. than it was at 6:00 a.m." OR "The temperature dropped <math>8^{\circ}\text{C}</math> over 3 hours this morning."</p>



## **B. Choosing Instructional Strategies**

Consider the following general strategies when planning lessons for the multiplication and division of integers:

- Use a number line and integer tiles to connect the concrete, pictorial and symbolic representations of the multiplication and division of integers.
- Use graphic organizers such as an anticipation/reaction guide, Frayer model, modified Frayer model and concept definition map.
- Use learning strategies such as the think–pair–share strategy.

## **C. Choosing Learning Activities**

The following learning activities are examples of activities that could be used to develop student understanding of the concepts identified in Step 1.

### **Sample Activities:**

1. **Anticipation/Reaction Guide** (p. 9)
2. **Multiplication of Integers** (p. 10)
3. **Compare Whole Numbers and Integer Multiplication** (p. 14)
4. **Word Sort** (p. 15)
5. **Division of Integers** (p. 17)
6. **Reinforcing Learning** (p. 20)

## Sample Activity 1: Anticipation/Reaction Guide

The anticipation/reaction guide is a strategy consisting of a set of carefully selected statements that serve as a pre-post inventory for a topic to be learned or a reading selection. This strategy accesses prior knowledge, sets a purpose for learning the topic or reading the selection and requires justification of the true or false statements.

### Anticipation/Reaction Guide

Directions: In the column labelled *before*, place a T if you believe the statement to be true. Place an F if you believe it is not true. After learning more about the topic, place either T or F in the *after* column.

Instruction Suggestion: Once the *before* column has been filled in by students, facilitate a discussion about students' choices. Try to encourage a class consensus before lesson begins. Revisit the chart at end of lesson.

#### TOPIC: INTEGERS

Before	Statements	After
	1. The difference between any two integers is always another integer.	
	2. The product of any two integers is always another integer.	
	3. The sum of two integers is less than either of the numbers being added.	
	4. The quotient of any two integers is always another integer.	
	5. Zero is an integer.	

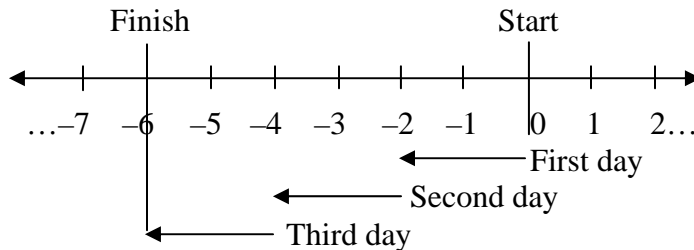
## Sample Activity 2: Multiplication of Integers

The following activities explore multiplication of integers using a number line, integer tiles and patterns.

**Note:** Multiplication by a positive first factor is thought of as repeated addition (as illustrated in Activities 2a and 2b). Multiplication by a negative first factor is thought of as repeated subtraction (as illustrated in Activities 2c and 2d) (Van de Walle and Lovin 2006, p. 145).

### a. Number line

You have no money and borrow \$2 each day for three days. What is your total debt at the end of the third day?



$$(+3) \times (-2) = (6)$$

"Positive 3 sets" of  $-2$  means to add 3 sets of  $-2$  to zero.

My total debt is  $-\$6$ .

### b. Integer tiles

White ( $-$ )       Red ( $+$ )

A debt of \$2 is shown by  $(-2)$ . The number of days is shown by  $(+3)$ .

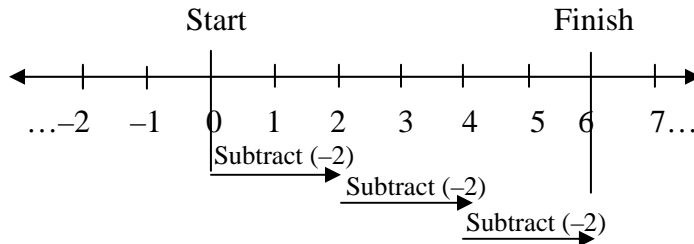
$$(+3) \times (-2) =$$

- $\rightarrow$  first day  $(-2)$
- $\rightarrow$  second day  $(-2)$
- $\rightarrow$  third day  $(-2)$

$$(+3) \times (-2) = (-6)$$

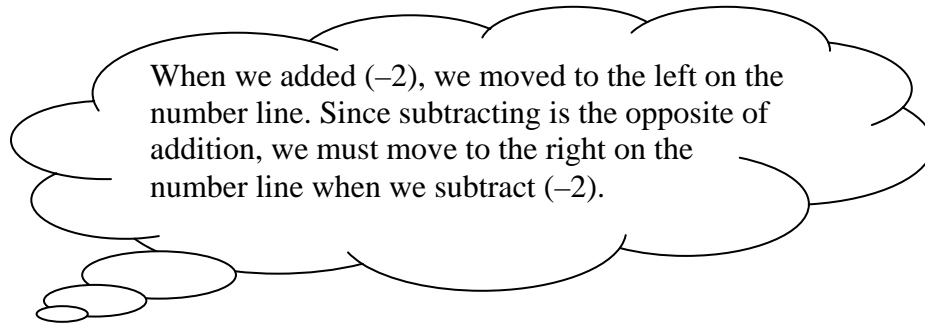
- c. This activity illustrates multiplication with a negative first factor, again with the use of a number line.

You remove 3 debts of \$2. What is your gain or loss by this action?



$$(-3) \times (-2) = (+6)$$

"Negative 3 sets" of  $-2$  means to subtract 3 sets of  $-2$  from zero.



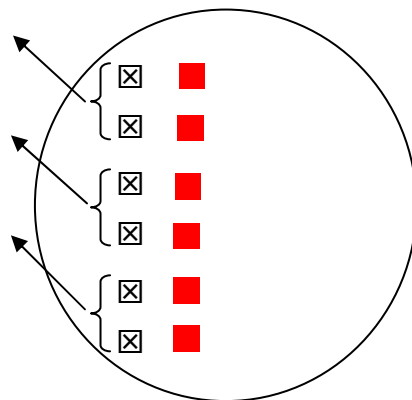
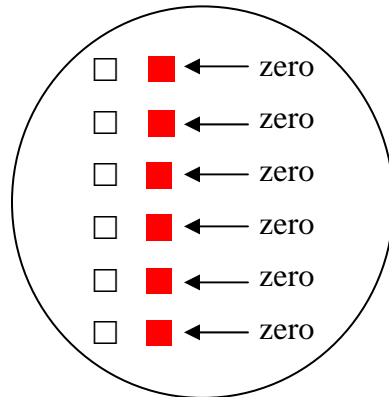
My gain is +\$6.

- d. This activity illustrates multiplication of a negative first factor using integer tiles and the Zero Principle.

We can also use integer tiles to solve the problem.

□ White (-)      ■ Red (+)

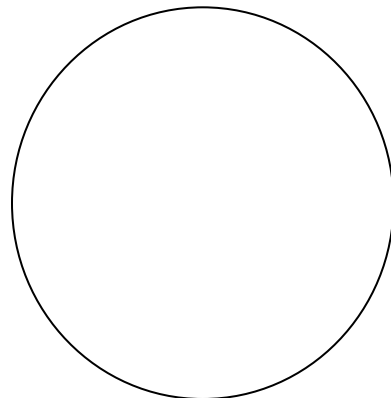
$(-3) \times (-2) = ?$  This can be read as "Remove 3 groups of  $(-2)$ ." We put zeros in the circle so that 3 groups of  $(-2)$  can be removed from the circle.



$$(-3) \times (-2) = (+6)$$

□ White (-)      ■ Red (+)

The circle below is a working space. Use integer tiles to solve each of the multiplication questions below.



$$(+2) \times (-3) = \underline{\quad}$$

$$(-2) \times (+3) = \underline{\quad}$$

$$(-2) \times (-3) = \underline{\quad}$$

e. Multiplying Integers Using Patterns

We can also learn how to multiply integers by using patterns. For example, we can use a pattern to solve the following problem.

You remove 3 debts of \$2. What is your gain or loss by this action?

This problem can be represented by the following:

$$(-3) \times (-2) = ?$$

We know:  $(+3) \times (-2) = (-6)$ .

We also know:  $(+2) \times (-2) = (-4)$ .

We also know:  $(+1) \times (-2) = (-2)$ .

We also know:  $(0) \times (-2) = 0$ .

Following a pattern,  $(-1) \times (-2) = (+2)$ .

Similarly,  $(-2) \times (-2) = (+4)$ .

Therefore,  $(-3) \times (-2) = (+6)$ .

My gain is +\$6.

**Note:** At some point in the discussion of these activities, students should discover that when integers with the same sign are multiplied together, the answer is positive. They should also discover that when integers with different signs are multiplied together, the answer is negative. Some suggested questions that you might ask include:

1. What pattern is being formed as we continue the multiplication?
2. Does the pattern continue in both directions?
3. What do you notice about the signs of the integers being multiplied and the answers?
4. What conclusions can you make about the multiplication of integers?

### Sample Activity 3: Compare Whole Number and Integer Multiplication

Have the students work in pairs or groups to identify the similarities and differences between the multiplication of whole numbers and the multiplication of integers.

Some similarities could include:

Multiplying whole numbers and multiplying integers:

- are operations using multiplication facts
- can be represented with manipulatives and diagrams
- include the use of the commutative and associative properties for multiplication
- are used to check division
- have the identity element of one.

Some differences could include:

- In multiplying whole numbers, the product is always greater than or equal to at least one of the numbers being multiplied; whereas the product of integers may be less than any of the integers being multiplied.
- The product of whole numbers is always a whole number; whereas the product of integers includes whole numbers as well as negative products.
- When multiplying whole numbers, only one product is equal to 1; whereas when multiplying integers two products are equal to 1.

## Sample Activity 4: Word Sort

### What is it?

Word sorts (Gillet and Temple 1982) help students recognize the semantic relationships among key concepts. In a closed sort, the teacher provides categories into which students assign the words. In an open sort, students create categories with appropriate labels and place the words in these created categories. Students justify their choice of categories and offer alternative ways to classify the words. Word sorts help the students develop a deeper understanding of key concepts by using complex reasoning skills of classification and deduction.

Adapted with permission from Mary Lee Barton and Clare Heidema, *Teaching Reading in Mathematics (2<sup>nd</sup> Edition): A Supplement to Teaching Reading in the Content Areas Teacher's Manual (2<sup>nd</sup> Edition)* (Aurora, CO: Mid-continent Research for Education and Learning (McREL), 2002), p. 86.

### When can it be used?

Word sorts can be used:

- before starting a unit in math by having the students sort words related to prerequisite concepts for that unit
- during the math unit as various concepts are developed and the relationship among concepts are explored
- after the math unit is completed for review and for assessment.



### **What thinking processes are used?**

As students complete the word sorts, they use many thinking processes, such as making connections among the concepts, determining importance by deciding on the categories, synthesizing by classifying the concepts into various categories in a number of different ways, and monitoring by critically analyzing and justifying the placement of the words into the categories.

### **How to use it:**

1. List the terms on index cards (one word per card), or write each term on a different slip of paper and place all the slips in an envelope.
2. Have the students work individually or in groups to sort the words into categories. Depending on the concepts and the students' level of understanding, the sorts can be closed or open. Model this process for students by thinking aloud as you sort the cards.
3. Have the students glue or tape their classified words with the appropriately labelled categories on chart paper to display for discussion with the whole class. Students communicate their understanding as they justify to the other students why they placed the words into specific categories.
4. Encourage the students to find more than one way to classify the vocabulary terms. Classifying and then reclassifying helps the students extend and refine their understanding of the concepts being studied.
5. Word sorts can be displayed in a number of ways, including semantic mapping, Venn diagrams, tree diagrams and semantic feature analysis charts.

Adapted with permission from Mary Lee Barton and Clare Heidema, *Teaching Reading in Mathematics (2<sup>nd</sup> Edition): A Supplement to Teaching Reading in the Content Areas Teacher's Manual (2<sup>nd</sup> Edition)* (Aurora, CO: Mid-continent Research for Education and Learning (McREL), 2002), p. 86.

### **Suggestions for word sorts:**

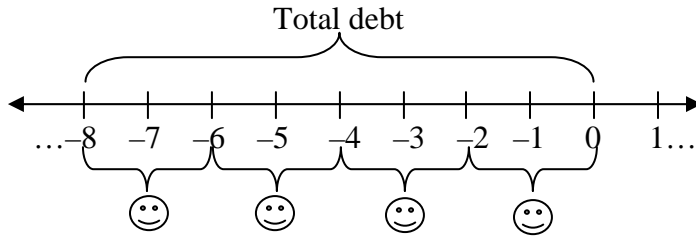
integers, addition, even numbers, odd numbers prime numbers, composite numbers, negative integer, positive numbers, whole numbers, subtraction, multiplication, division, 0,  $\frac{12}{2}$ , -100, comparing, ordering, thermometer, debts/credits, above/below sea level.

## Sample Activity 5: Division of Integers

The following activities are designed to help students develop an understanding of the division of integers.

### a. Division of Integers by Equal Sharing

A debt of \$8 is shared equally among 4 people. What is the amount of debt for each person?



$$(-8) \div (+4) = (-2)$$

This could also be written as  $(+4) \times (-2) = (-8)$ .

Each person has a debt of \$-2.

We can use integer tiles to show the division.

A debt of \$8 is shown by  $(-8)$ .

The number of people is shown by  $(+4)$ .

$$(-8) \div (+4) = (?) \text{ OR } (+4) \times (?) = (-8)$$

- First person has a debt of  $(-2)$ .
- Second person has a debt of  $(-2)$ .
- Third person has debt of  $(-2)$ .
- Fourth person has debt of  $(-2)$ .

$$(-8) \div (+4) = (-2) \text{ OR } (+4) \times (-2) = (-8)$$

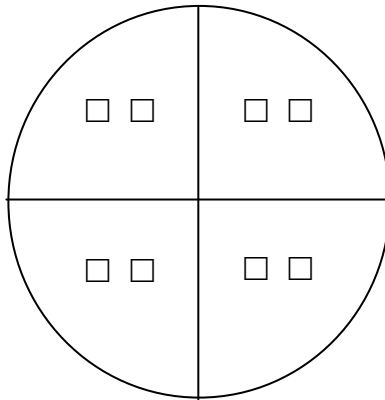
b. Dividing Integers by Equal Grouping

You accumulated a debt of \$8 by spending \$2 each day for how many days? (Note: You started with no money.)

□ White (-)      ■ Red (+)

We will use integer tiles to solve the problem.

The debt is shown by  $(-8)$ . The spending each day is shown by  $(-2)$ . We place 8 white tiles  $(-8)$  in the circle and put them into groups of  $(-2)$ , making 4 equal groups.



$$(-8) \div (-2) = (+4) \text{ OR } (+4) \times (-2) = (-8)$$

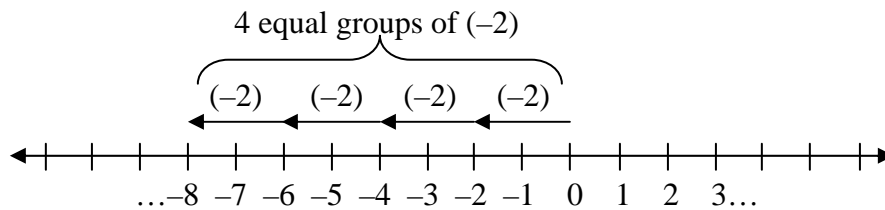
I spent \$2 a day for 4 days to accumulate the debt of \$8.

A number line can also be used to solve this equal grouping division problem; for example:

The debt is shown by  $(-8)$ .

The spending each day is shown by  $(-2)$ .

We show  $(-8)$  on the number line and divide the distance from 0 to  $(-8)$  into groups of  $(-2)$ , making 4 equal groups.



Four groups of  $(-2)$  are taken out of  $(-8)$ .

$$(-8) \div (-2) = (+4) \text{ OR } (+4) \times (-2) = (-8)$$

I spent \$2 a day for 4 days to accumulate the debt of \$8.

c. Dividing Integers Using Patterns

A debt of \$6 is shared between 2 people. What is the amount of debt for each person?

This problem can be represented by the following:

$$(-6) \div (+2) = ?$$

We know:  $(+6) \div (+2) = (+3)$ .

We also know:  $(+4) \div (+2) = (+2)$ .

We also know:  $(+2) \div (+2) = (+1)$ .

We know:  $(0) \div (+2) = 0$ .

Following a pattern,  $(-2) \div (+2) = (-1)$ .

Similarly,  $(-4) \div (+2) = (-2)$ .

Therefore,  $(-6) \div (+2) = (-3)$ .

**Each person has a debt of \$3.**

**Note:** At some point during these activities, students should discover that when integers with the same sign are divided, the answer is positive and that when integers of different signs are divided, the answer is negative.

## Sample Activity 6: Reinforcing Learning

Have the students explain or show their thinking as they solve each of the following problems.

1. Write the set of integers that are less than  $-5$  and are divisible by 4 and 3.
2. Four people share a debt of \$60 equally. What is each person's share of the debt?
3. The temperature is  $-12^{\circ}\text{C}$ . It drops 3 degrees every hour for 3 hours. What is the temperature at the end of the third hour?

## Step 4: Assess Student Learning

### Guiding Questions

- Look back at what you determined as acceptable evidence in Step 2.
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

In addition to ongoing assessment throughout the lessons, consider the following sample activities to evaluate students' learning at key milestones. Suggestions are given for assessing all students as a class or in groups, individual students in need of further evaluation, and individual or groups of students in a variety of contexts.

### A. Whole Class/Group Assessment

**Activity 1:** Have the students complete a Frayer model (an example has been provided) or a modified Frayer model for multiplication and/or division of integers. Students could choose to use one of the following types of Frayer models, choose to draw a chart of their own or students decide as groups the headings to be used and then fill in the chart individually.

<p><b>Definition</b></p> <p style="text-align: center;">Integers are a set of numbers that include the following: (... -3, -2, -1, 0, 1, 2, 3, ...)</p>	<p style="text-align: right;"><b>Facts</b></p> <ul style="list-style-type: none"> <li>• Include positive and negative whole numbers.</li> <li>• Successive integers differ by one.</li> <li>• Every negative number is the opposite of the positive number of the same size.</li> <li>• Represents the difference between two objects in a set.</li> <li>• Can be represented on a number line.</li> </ul>
<p><b>Examples</b></p> <p style="text-align: center;">-25</p> <p style="text-align: center;"><math>-\frac{6}{3}</math></p> <p style="text-align: center;">0</p> <p style="text-align: center;">8</p>	<p style="text-align: center;"><b>Integers</b></p> <p style="text-align: right;"><b>Non-examples</b></p> <p style="text-align: right;">-2.76</p> <p style="text-align: right;"><math>4\frac{1}{3}</math></p> <p style="text-align: right;">0.2</p> <p style="text-align: right;"><math>-\frac{4}{5}</math></p>

Format adapted from D. A. Frayer, W. C. Frederick and H. J. Klausmeier, *A Schema for Testing the Level of Concept Mastery* (Working Paper/Technical Report No. 16) (Madison, WI: Research and Development Center for Cognitive Learning, University of Wisconsin, 1969). Adapted with permission from the Wisconsin Center for Education Research, University of Wisconsin-Madison.

### Examples of Frayer Models

Essential Characteristics	Word	Nonessential Characteristics
Examples		Non-examples

Definition	Word	Characteristics
Examples		Non-examples

### Example of a Modified Frayer Model

<b>Definition</b>	<b>Examples</b>	<b>Non-examples</b>
<b>Visual and Numeric Representation</b>	<b>Word Problem</b>	

Framer Model formats adapted from D. A. Frayer, W. C. Frederick and H. J. Klausmeier, *A Schema for Testing the Level of Concept Mastery* (Working Paper/Technical Report No. 16) (Madison, WI: Research and Development Center for Cognitive Learning, University of Wisconsin, 1969). Adapted with permission from the Wisconsin Center for Education Research, University of Wisconsin-Madison.

## Activity 2: Integer Two-Ways

The three numbers in any row or column must form a correct multiplication sentence. The self-checking nature of these activities provides immediate feedback. Encourage the students to develop their own two-ways and have other students in the class complete them.

1.

$\otimes$		
2		-4
	-16	-32

2.

$\otimes$		
	-3	9
	-2	
	-6	

3.

$\otimes$		
-1		4
	7	-56

4.

$\otimes$		
-2		8
	-6	-24

5.

$\otimes$		
-3		9
		6
	-6	

6.

$\otimes$		
-2		4
	-4	-16

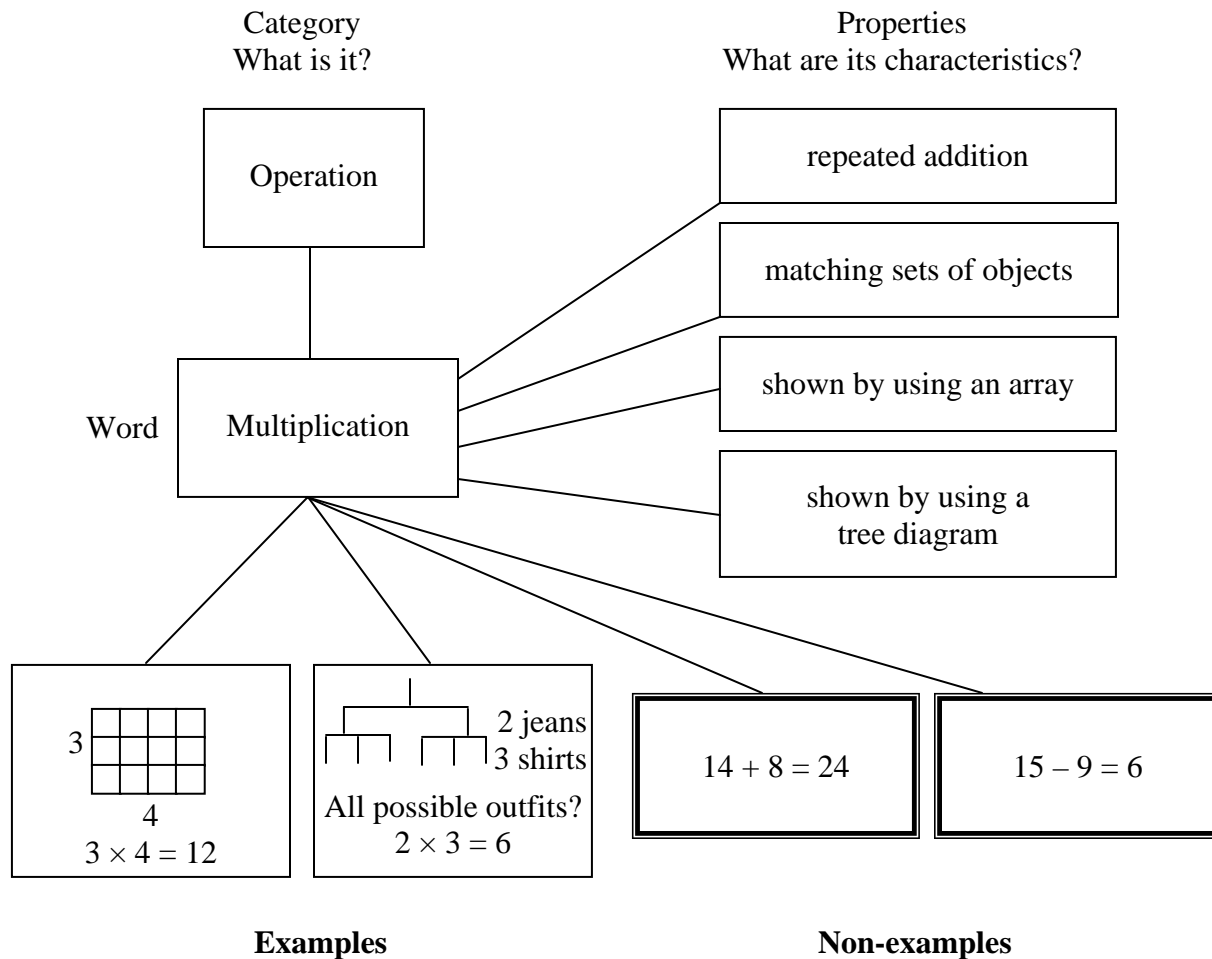
Problems reproduced with permission from Grayson H. Wheatley and George E. Abshire, *Developing Mathematical Fluency: Activities for Grades 5-8* (Tallahassee, FL: Mathematics Learning, 2002), p. 85.  
[www.mathematicslearning.org](http://www.mathematicslearning.org)



## B. One-on-one Assessment

**Activity 1:** Another graphic organizer that students might complete is a concept definition map. A concept definition map differs from the Frayer model in that students are reflecting on the concepts by writing in their own words the information that is required on the map. Work through an example with the students (see the example on multiplication provided) and then have students fill out their own. This could be an individual or a group activity. The word could be *multiplication, division or integers*.

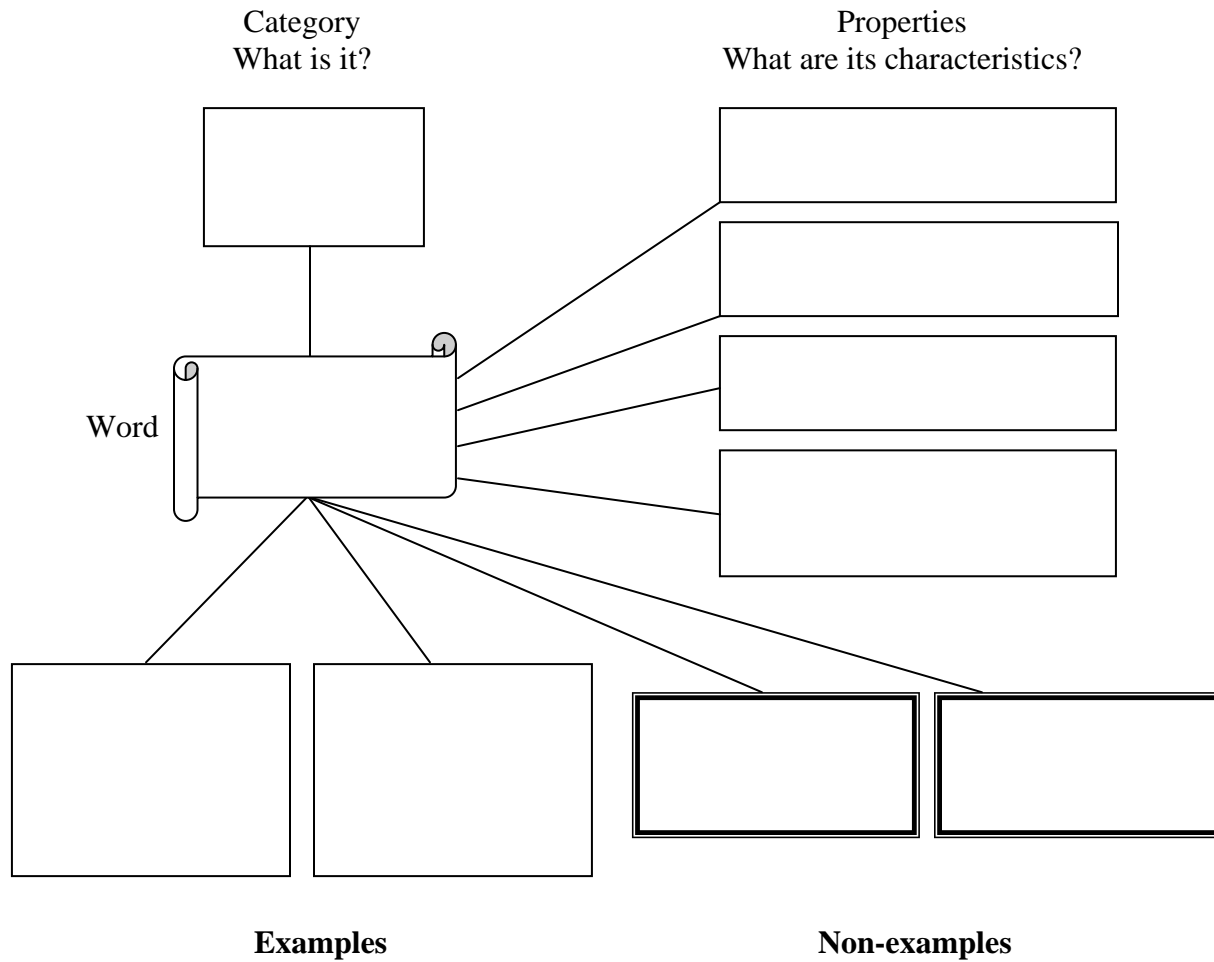
### Concept Definition Map



Format adapted from Robert M. Schwartz, "Learning to Learn Vocabulary in Content Area Textbooks," *Journal of Reading* 32, 2 (November 1988), p. 110, Example 1. Adapted with permission from International Reading Association.

# Concept Definition Map Blackline Master

## Concept Definition Map



Adapted from Robert M. Schwartz, "Learning to Learn Vocabulary in Content Area Textbooks," *Journal of Reading* 32, 2 (November 1988), p. 110, Example 1. Adapted with permission from International Reading Association.

**Activity 2:** Explain your thinking with words or diagrams as you solve each of the following problems.

1. Marcy has \$16 and spends \$3 per day. Johnny has \$20 and spends \$4 per day. Who will have more money or less debt at the end of 7 days?
2. Write the set of integers that are greater than  $-5$  and are divisible by 4 and 3.
3. Four people share a debt of \$56 (i.e.,  $-56$ ) equally. What is each person's share of the debt?

### C. Applied Learning

Provide opportunities for the students to use what they have learned about multiplication and division of integers in practical situations and notice whether or not the knowledge is being transferred.

**Activity:** Have the students play the game "Operation Integers." Consider having the students play just using the operations of multiplication and division.

Players: 2 to 4

Materials: A deck of cards (no face cards)

Description: Deal all the cards face down on the table. Black suits are positive and red suits are negative. Each player turns over two cards and decides whether to add, subtract, multiply or divide the two numbers on the cards. The player who has the greatest result wins all the cards that are face up.

Goal: The play continues until one person (the winner) has all the cards.

Variations:

- Use fewer cards or cards with only certain numbers.
- Use fewer operations.
- Turn over three or four cards instead of two cards for each player.
- The player who has the least sum, difference, product or quotient wins all the cards that are face up.
- Each player rolls two (or more) dice with integers on each face rather than using playing cards. The player with the greatest (or least) number resulting from the operations scores one point. The winner is the player with the most points.

## Step 5: Follow-up on Assessment

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### Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction?

### A. Addressing Gaps in Learning

- Use manipulatives to connect the concrete, pictorial and symbolic modes of multiplying and dividing integers.
- Relate the multiplying and dividing of integers to the multiplying and dividing of whole numbers.
- Use problem situations that are simple in nature.

### B. Reinforcing and Extending Learning

Students who have achieved or exceeded the outcomes will benefit from ongoing opportunities to apply and extend their learning. These activities should support students in developing a deeper understanding of the concept and should not progress to the outcomes in subsequent grades.

Consider strategies such as:

**Activity:** Write a multiplication/division story that follows two rules:

1. It must end in a question.
2. The question must be one that's possible to answer by multiplying or dividing integers. Solve the story problem in as many ways as you can.  
Exchange papers and solve each other's problems.

Adapted from Burns, Marilyn. *About Teaching Mathematics: A K–8 Resource, Second Edition*, p. 200. Copyright © 2000 by Math Solutions Publications. Adapted by permission. All rights reserved. (Note: This title is now in its third edition, copyright © 2007.)

## Bibliography

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- Alberta Education. *The Alberta K–9 Mathematics Program of Studies with Achievement Indicators*. Edmonton, AB: Alberta Education, 2007.
- Barton, Mary Lee and Clare Heidema. *Teaching Reading in Mathematics (2<sup>nd</sup> Edition): A Supplement to Teaching Reading in the Content Areas Teacher's Manual (2<sup>nd</sup> Edition)*. Aurora, CO: Mid-continent Research for Education and Learning (McREL), 2002.
- Burns, Marilyn. *About Teaching Mathematics: A K–8 Resource*. 2<sup>nd</sup> ed. Sausalito, CA: Math Solutions Publications, 2000.
- Frayser, D. A., W. C. Frederick and H. J. Klausmeier. *A Schema for Testing the Level of Concept Mastery* (Working Paper/Technical Report No. 16). Madison, WI: Research and Development Center for Cognitive Learning, University of Wisconsin, 1969.
- Gillet, Jean Wallace and Charles Temple. *Understanding Reading Problems: Assessment and Instruction*. Boston, MA: Little, Brown, 1982.
- National Council of Teachers of Mathematics. *Principles and Standards for School Mathematics*. Reston, VA: NCTM, 2000.
- Schwartz, Robert M. "Learning to Learn Vocabulary in Content Area Textbooks." *Journal of Reading* 32, 2 (November 1988), pp. 108–118.
- Van de Walle, John A. *Elementary and Middle School Mathematics: Teaching Developmentally*. 4<sup>th</sup> ed. Boston, MA: Addison Wesley Longman, Inc., 2001.
- Van de Walle, John A. and LouAnn H. Lovin. *Teaching Student-Centered Mathematics: Grades 5–8*. Boston, MA: Pearson Education, Inc., 2006.
- Wheatley, Grayson H. and George E. Abshire. *Developing Mathematical Fluency: Activities for Grades 5–8*. Tallahassee, FL: Mathematics Learning, 2002.