Mathematics

Planning Guide

Grade 8 *Solving Equations*

Patterns and Relations (Variables and Equations)

Specific Outcome 2

This Planning Guide can be accessed online at:
http://www.learnalberta.ca/content/mepg8/html/pg8_solvingequations/index.html

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Planning Guide: *Grade 8 Solving Equations*

Strand: Patterns and Relations (Variables and Equations)
Specific Outcome: 2

This *Planning Guide* addresses the following outcomes from the Program of Studies:

Strand: Patterns and Relations (Variables and Equations)

Specific Outcomes: 2. Model and solve problems concretely, pictorially and symbolically, using linear equations of the form:
- $ax = b$
- $\frac{x}{a} = b,\ a \neq 0$
- $ax + b = c$
- $\frac{x}{a} + b = c,\ a \neq 0$
- $a(x + b) = c$

where a, b and c are integers.

Curriculum Focus

The changes to the curriculum targeted by this sample include:

- The general outcome focuses on understanding how to represent algebraic expressions in multiple ways; whereas the previous mathematics curriculum focused on the procedural solving and verifying of one-step and two-step linear equations.
- The specific outcome focuses on modelling and solving problems using various forms of linear equations concretely, pictorially and symbolically; whereas the previous curriculum focused on the solution and verification of one- and two-step, first degree equations. Linear equations of the form $x + a = b$ have been moved to Grade 7 and of the form $a(x + b) = c$ has been added to the Grade 8 curriculum.

What Is a Planning Guide?

Planning Guides are a tool for teachers to use in designing instruction and assessment that focuses on developing and deepening students' understanding of mathematical concepts. This tool is based on the process outlined in *Understanding by Design* by Grant Wiggins and Jay McTighe.

Planning Steps

The following steps will help you through the Planning Guide:

Step 1: Identify Outcomes to Address

Guiding Questions

- What do I want my students to learn?
- What can my students currently understand and do?
- What do I want my students to understand and be able to do based on the Big Ideas and specific outcomes in the program of studies?

Big Ideas

- Algebraic reasoning involves representing, generalizing and formalizing patterns and regularity in all aspects of mathematics. It is this type of reasoning that is at the heart of mathematics as a science of pattern and order (Van de Walle 2001, p. 384).
- Patterns are key factors in understanding mathematical concepts (Burns 2000, p. 112).
- Patterns can be created, recognized and extended. When this happens, making generalizations, seeing relationships and understanding the order and logic of mathematics will occur (Burns 2000, p. 112).
- Functions evolve from the investigation of patterns and this allows us to investigate results beyond the information at hand (Burns 2000, p. 112).
- Studying patterns, functions and algebra involves investigating numerical and geometrical patterns. These can be represented verbally and symbolically in several ways, including in tables, with symbols and graphically (Burns 2000, p. 112).
- Variables are symbols that take the place of numbers or range of numbers (Van de Walle 2001, p. 384).
- Equations and inequalities are used to express relationships between two quantities (Van de Walle 2001, p. 384).

Sequence of Outcomes from Program of Studies

See http://education.alberta.ca/teachers/core/math/programs.aspx for the complete program of studies.

Grade 7	Grade 8	Grade 9
Specific Outcomes	**Specific Outcomes**	**Specific Outcomes**
6. Model and solve, concretely, pictorially and symbolically, problems that can be represented by one-step linear equations of the form $x + a = b$, where a and b are integers. 7. Model and solve, concretely, pictorially and symbolically, problems that can be represented by linear equations of the form: • $ax + b = c$ • $ax = b$ • $\frac{x}{a} = b,\ a \neq 0$ where a, b and c are whole numbers.	2. Model and solve problems concretely, pictorially and symbolically, using linear equations of the form: • $ax = b$ • $\frac{x}{a} = b,\ a \neq 0$ • $ax + b = c$ • $\frac{x}{a} + b = c,\ a \neq 0$ • $a(x + b) = c$ where a, b and c are integers.	1. Generalize a pattern arising from a problem-solving context, using a linear equation, and verify by substitution. 2. Graph a linear relation, analyze the graph, and interpolate or extrapolate to solve problems. 3. Model and solve problems, using linear equations of the form: • $ax = b$ • $\frac{x}{a} = b,\ a \neq 0$ • $ax + b = c$ • $\frac{x}{a} + b = c,\ a \neq 0$ • $ax = b + cx$ • $a(x + b) = c$ • $ax + b = cx + d$ • $a(bx + c) = d(ex + f)$ • $\frac{x}{a} = b,\ x \neq 0$ where a, b, c, d, e and f are rational numbers.

Step 2: Determine Evidence of Student Learning

Guiding Questions

- What evidence will I look for to know that learning has occurred?
- What should students demonstrate to show their understanding of the mathematical concepts, skills and Big Ideas?

Using Achievement Indicators

As you begin planning lessons and learning activities, keep in mind ongoing ways to monitor and assess student learning. One starting point for this planning is to consider the achievement indicators listed in the *Mathematics Kindergarten to Grade 9 Program of Studies with Achievement Indicators*. You may also generate your own indicators and use them to guide your observation of the students.

The following indicators may be used to determine whether or not students have met this specific outcome. Can students:

- model a given problem with a linear equation and solve the equation using concrete models; e.g., counters, integer tiles?
- verify the solution to a given linear equation using a variety of methods, including concrete materials, diagrams and substitution?
- draw a visual representation of the steps used to solve a given linear equation and record each step symbolically?
- solve a given linear equation symbolically?
- identify and correct an error in a given incorrect solution of a linear equation?
- apply the distributive property to solve a given linear equation; e.g., $2(x + 3) = 5$ is equivalent to $2x + 6 = 5$?
- solve a given problem using a linear equation and record the process?

Sample behaviours to look for related to these indicators are suggested for some of the activities listed in **Step 3, Section C: Choosing Learning Activities** (p. 11).

Step 3: Plan for Instruction

Guiding Questions

- What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should I use?
- How will I meet the diverse learning needs of my students?

A. Assessing Prior Knowledge and Skills

Before introducing new material, consider ways to assess and build on students' knowledge and skills related to representing algebraic expressions in multiple ways. For example:

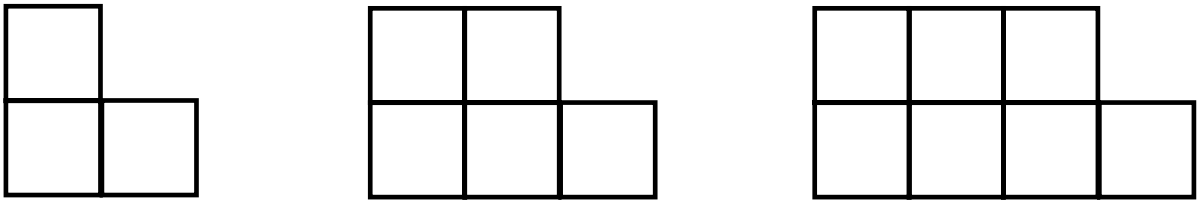
Activity 1:

There are several possible patterns in the following list of names:

Ann, Brad, Carol, Daniel, _____, _____

Describe two of the patterns you found.
What is a possible fifth term in the list of names? Why?

Activity 2:

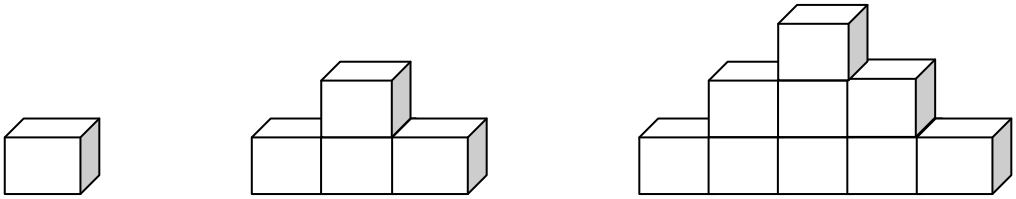


Construct a chart showing the term number and the increasing number of small squares in the pattern above.

How could you find the number of small squares in the seventh term?

For the pattern shown above, write an algebraic expression showing the total number of small squares, where t = the term number.

Activity 3: Describe how to build the fourth shape in the following series:



Activity 4: If c represents the number of cats in the schoolyard, what situation could each of the following represent?

- $c - 3$
- $\frac{c}{2}$

Activity 5: When you double Carole's age and add 10, you get her mother's age. Write a mathematical expression that shows the mother's age. Tell what your variable represents.

If a student appears to have difficulty with these tasks, consider further individual assessment, such as a structured interview, to determine the student's level of skill and understanding. See **Sample Structured Interview: Assessing Prior Knowledge and Skills** (p. 9).

Sample Structured Interview: Assessing Prior Knowledge and Skills

Directions	Date:	
	Not Quite There	**Ready to Apply**
1. There are several possible patterns in the following list of names: Ann, Brad, Carol, Daniel,_____, _____ Describe two of the patterns you found. What is a possible fifth term in the list of names? Why?	1. Student cannot justify his or her response or is only able to identify one pattern.	1. a. Student responds with two of the following possible answers: • Pattern is in alphabetical order. • The number of letters in each name increases by one each time. • The names alternate girl then boy then girl and so on. b. Eleanor, Emmalou, Eveline or Ellymae are suitable answers.
2. a. Construct a chart showing the term number and the increasing number of small squares as shown in the pattern in Step 3, Part A, Activity 2. b. How could you find the number of small squares in the seventh term? c. For the pattern shown in Step 3, Part A, Activity 2, write an algebraic expression showing the total number of small squares, where t = the term number.	2. Student responds with a correct chart but is unable to continue any further with the question.	2. a. Student responds with chart: Term Number | 1 | 2 | 3 | 4 Number of Small Squares | 3 | 5 | 7 | 9 b. Student responses could be: • Draw all the figures and count the square for Figure 7. • Extend the chart in a. • Create an algebraic expression as in part c and substitute seven for the variable. c. Student responds with the answer that the total number of squares is $2t + 1$, where t = term number.
3. Describe how to build the fourth shape as shown in the series in Step 3, Part A, Activity 3.	3. Student is unable to explain the pattern.	3. Student responds with an answer such as "Place the object in term three on a row of seven blocks" or "Add seven blocks underneath the object in term three."

4. If c represents the number of cats in the schoolyard, what situation could each of the following represent? • $c - 3$ • $\frac{c}{2}$	4. Student does not understand the operation happening to the number of cats.	4. a. Student responds with an answer such as "The number of cats in the schoolyard after three ran away." b. Student responds with an answer such as "The number of cats in the schoolyard if half of them ran away."
5. When you double Carole's age and add 10, you get her mother's age. Write a mathematical expression that shows the mother's age. Tell what your variable represents.	5. Student responds with an answer such as mother's age is $2m + 10$ and does not know what m represents.	5. Student responds with an answer such as "If m represents Carole's age, then her mother's age is $2m + 10$."

B. Choosing Instructional Strategies

Consider the following general strategies for teaching how to solve linear equations.

- Students need to be engaged in learning activities that will enhance their understanding of algebraic concepts.
- Students should be given many opportunities to make sense of the way that algebra uses symbols and letters to represent mathematical ideas. It is important for students to connect the differences and the similarities between algebraic notation and whole number operations.
- Students should practise procedures for evaluating and solving equations only after they have learned to make sense of equations, expressions and how and why they are used in algebra.
- The use of models such as balances and algebra tiles help students to visualize concepts. Visual representations build lasting and meaningful understandings in algebra.
- Students must be given opportunities to communicate their thinking with other students and with the teacher. Students learn by explaining their reasoning and by listening to the strategies of other students.
- Students develop an understanding of concepts through problem solving.

C. Choosing Learning Activities

The following learning activities are examples of activities that could be used to develop student understanding of the concepts identified in Step 1.

Sample Activities:

1. **Comparing Equations** (p. 12)
2. **Balances** (p. 13)
3. **Forward Thinking Operations** (p. 17)
4. **Solving Equations** (p. 19)

Sample Activity 1: Comparing Equations

This activity is designed to be a "starter." Starters are used to develop quick and easy warm-ups that focus the students at the start of a mathematics class. To fit the category of a "starter," activities will:

- appear simple enough that every student can give a response
- be thought provoking, stimulating to the imagination, mysterious, puzzling and evoke visualization
- lend themselves to a variety of approaches to solutions
- lend themselves to oral discussion and mental computations rather than pen and pencil calculations
- connect to important mathematic concepts
- expose some of the basic common knowledge/misconception that students bring.
- focus on making sense and understanding, not remembering rules and procedures.

Look For …
Do students:
☐ have an understanding of what the equations mean rather than just trying to solve for an answer?

Starter 1:

$$n + 5 = 12$$
$$x + y = 10$$

How are these equations the same? How are they different?

Possible Responses
They both have variables.
They both have numbers.
They are both adding.
They both have equal signs.
One uses n and the other uses x, y
One has an answer and the other has many answers.

Starter 2:

$$n + 5 = 12$$
$$x + y = 10$$

Work with a partner to create a word problem for each question.

Teacher Prompt:
Encourage the students to use specific contexts like measurement, money or weights, or create a problem that might come from geometry.

Sample Activity 2: Balances

The balance activities provided here come from Wheatley and Abshire's work. These ideas are classroom tested and have been found to be very effective for engaging middle grade students.

The goal is to build mental images that enhance algebraic reasoning.

Mathematics instruction is most effective for students when they experience ideas in settings that are potentially meaningful. This allows them to build understanding, not just follow procedures. All students have experience with balance either in walking and riding or in actually using a balance.

Balance tasks must be interpreted. There is no operation sign provided, so the students have to decide how to act, what operation to perform and how to think about the task.

- The solutions to the balance questions can be expressed as additions or subtractions, as multiplication or division.
- The numbers are varied left to right so that students are encouraged to be flexible in their thinking. You can work backwards or forwards, making sense as you go.
- The role of the teacher is to bring students' attention to the connections between solutions and how operations can be reversed.

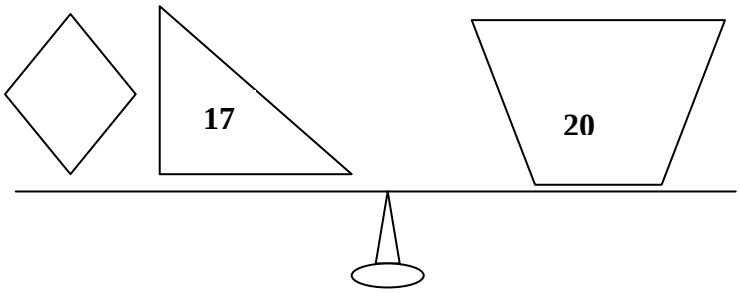
Materials:

- Balance
- Blocks for demonstrating balancing
- Practice pages (provided at the end of the activity)

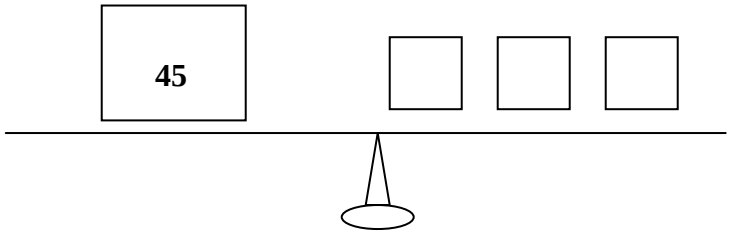
Instruction suggestions:

- Discuss balances and balancing. Use the balance to demonstrate. Have the students discuss how to balance items and how to bring the balance back to even when an object is removed from either side.
- Use the following balances as examples. Transfer to the overhead and direct a discussion with the students about solving balances.
- Hand out copies of the balance pages provided. Have the students work in pairs, and then bring the class together for a class discussion. Have the students explain and defend their solutions to the class.

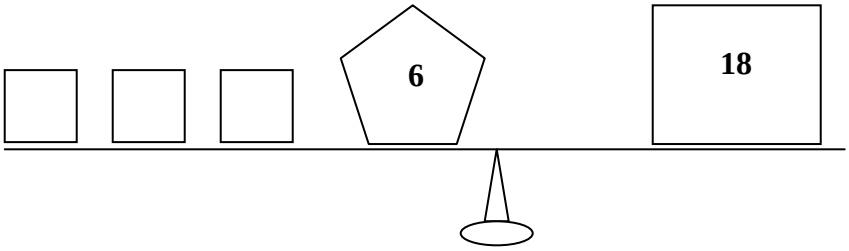
Balance # 1



Balance # 2



Balance # 3



Sample Dialogue for Balance # 1

Teacher: Talk to me about this balance.

Student: There is 17 and something on one side and 20 on the other. It is going to tip to the right.

Teacher: How do you know?

Student: 17 is less than 20, so it will tip to the right.

Teacher: How do we make it balance?

Student: Put 3 into the rhombus.

Teacher: How can we explain this picture in words?

Student: Something plus 17 equals 20.
To make 20 on both sides, you need to add a 3 to the left side.
What plus 17 gives you 20?

Teacher: How did you decide what number to put in the rhombus? What did you think about? Encourage the students to use the word replace for "put in the rhombus." Ask: "What can I replace the rhombus with?"

Student: I know 3 + 17.

- I thought about subtracting 20 – 17. The answer is what is missing.
- I know 20 – 3 = 17, so I knew it was 3.
- I counted back from 20 – 19, 18, 17. It's three.
- I started at 17 and counted forward to twenty. It's three.

Teacher: How could we illustrate each of these strategies as an equation?

? +17 = 20
? = 3

□ + 17 = 20
□ = 3

$x + 17 = 20$ $(x = 3)$

20 – 17 = ?

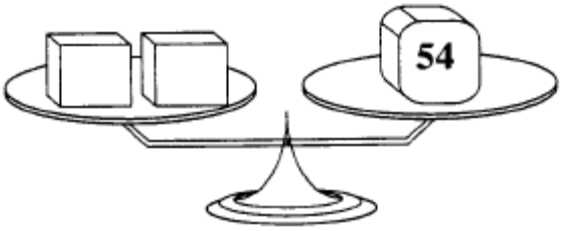
20 – something (x) = 17

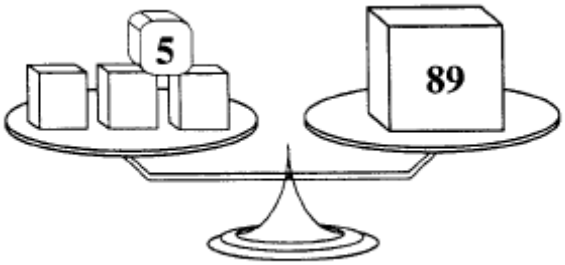
20 – □ = 17

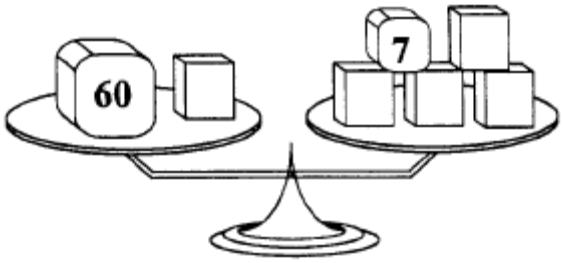
Teacher: Compare these two equations. Why do they both work? Which one looks like the balance?

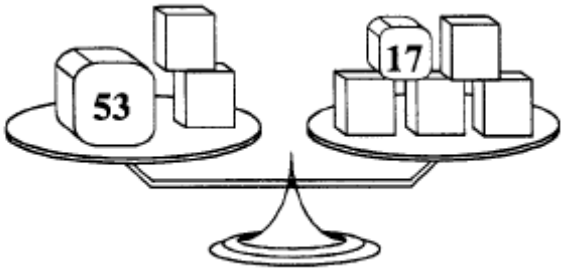
$x + 17 = 20$ $20 - 17 = x$

After solving the following balances, write an equation for each one. Boxes that are the same size and shape must have the same number in them. Think about: "What can I replace the empty boxes with?"









Balance graphics reproduced with permission from Grayson H. Wheatley and George E. Abshire, *Developing Mathematical Fluency: Activities for Grades 5–8* (Tallahassee, FL: Mathematics Learning, 2002), pp. 254, 255. www.mathematicslearning.org

Sample Activity 3: Forward Thinking Operations

Forward Thinking Operations has been found extremely helpful for students as they learn to build an explicit awareness of the mathematical method being symbolized by the use of letters and numbers. Students who were taught to use it became proficient quickly in solving equations and outperformed students who tried to use the inverse operations and could not make sense when they had to do the same operation to both sides of the equation (Kieran 1988, 1992; Owens 1993).

Example 1:

_____ + 17 = 24

Teacher: Cover the blank with your hand and read the equation: "Something plus 17 equals 24. What number plus 17 equals 24?"
Possible Responses:
I just knew it.
I started at 17 and counted on.
I thought about the subtraction 24 – 17 equals what?
I thought about what plus 17 gives you 24.

Example 2:

$3 \times \square + 6 = 18$

Teacher: Cover the three times square with your hand and read: "A number plus 6 equals 18. What number do I add to 6 to give me 18? Something under my hand plus 6 = 18.

Possible Responses:
I know that 12 + 6 = 18.
I subtracted 18 – 6 = 12.
I thought 6 and counted to 18.

Teacher: My hand is covering 12.
Move your hand to cover the +6 and say:
3 times something equals 12.
3 times what number equals 12?

Teacher: Invite responses from the group. 4 time 3 = 12.
So the missing number is 4? How could we check to be sure?
Possible response: Put the four in and try the equation to see if it works.

Extension: At this point you could invite the students to work in partners to build two more sample equations similar to the two we just solved. If you do this with the students, some may challenge themselves with harder equations but most will stick with very simple ones.

After a few minutes, gather some samples on the board and consider how they are similar to the first two.

Example 3:

48 – 3 ◊ = 12

Teacher: What should I cover now?

Response: Cover the 3 and the rhombus.

Teacher: What should I ask?

Response: Ask: "Forty-eight minus something equals 12."

At this point, have the students work in pairs to find a solution. The solution must be verified. Then have the students create more sample equations similar to this one. Ask the students how this equation is different from the other two?

After a few minutes, gather some samples on the board and have the students share their ideas.

Sample Activity 4: Solving Equations

Have the students:

- work with partners to solve the following equations with algebra tiles
- model and draw them out
- verify their answers.

Students will take turns doing the following:

- Decide who will be the scribe and who will model the algebra tiles.
- The partner modelling with the tiles will tell the other person the steps to solve the equation. The scribe writes down the procedure algebraically.
- Partners will switch roles.

The equations to solve are:

$3x = 5 + 1$

$\frac{x}{3} = 4 + 2$

$3x = 4 - 1$

$2x = 3 + 1 - 4$

Step 4: Assess Student Learning

Guiding Questions

- Look back at what you determined as acceptable evidence in Step 2.
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

In addition to ongoing assessment throughout the lessons, consider the following sample activities to evaluate students' learning at key milestones. Suggestions are given for assessing all students as a class or in groups, individual students in need of further evaluation, and individual or groups of students in a variety of contexts.

A. Whole Class/Group Assessment

In small groups, have students create a one-page summary (could use Frayer model, mind map, chart or other graphic organizer of their choice) of all techniques they have used to solve equations. Students may choose to represent the solution to the same problem in a variety of ways.

B. One-on-one Assessment

Activity 1: Give the students a copy of the following:

These are equations:

$3 + 4 = 7$	$b + 3 = 19$
$W + 4 = 12 - w$	$7 \times 8 = 56$
$65 - 45 = x$	$2 = 6 - 4$
$3 + 4 = 5 + 2$	$6 = 6$

These are expressions:

$3 + 2$	5	4^5
x^2	$4y - 5$	7×3
$\frac{42}{5}$	$y^3z - 2y$	

Where do these go? How did you decide? How would you explain to someone the differences between expressions and equations?

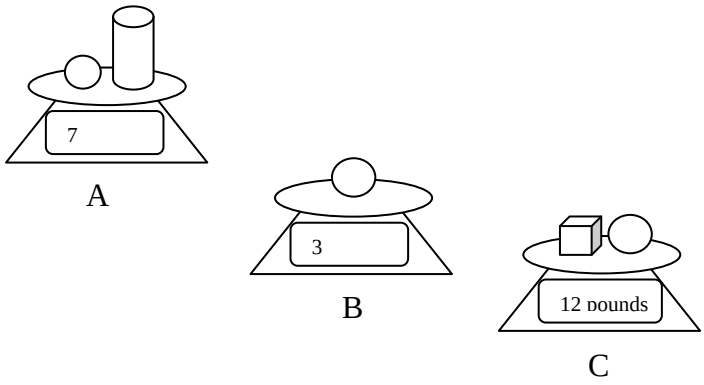
$6b - 4$ $3x = 28 - 5$ xy

$\frac{6}{y}$ n^2 $5x - 2y + 3$

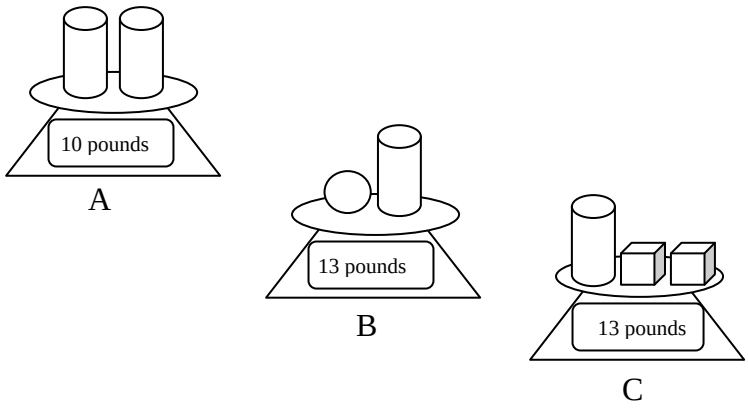
Activity 2: For the following problems, have the students:

- determine the weight of each object and
- tell how they solved the problem by recording the solution steps.

a.



b.



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Activity 3: A baker is packaging cookies in identical boxes. She has filled seven boxes with all but 5 cookies in another box. She has packaged 51 cookies. How many cookies are in a full box? Draw a picture for this problem. Write an algebraic equation for the problem.

Adapted with permission from Grayson H. Wheatley and George E. Abshire, *Developing Mathematical Fluency: Activities for Grades 5–8* (Tallahassee, FL: Mathematics Learning, 2002), p. 257. www.mathematicslearning.org.

C. Applied Learning

Provide opportunities for the students to use their algebraic reasoning in practical situations and notice whether or not the students can use symbols and letters to represent mathematical ideas, to communicate their thinking and to solve problems.

Activity: Go to http://illuminations.nctm.org/LessonDetail.aspx?id=L658 for a great lesson in which students can explore Fibonacci sequence, examine how the ratio of two consecutive Fibonacci numbers creates the Golden Ratio and identify real-life examples of the Golden Ratio.

In this activity, students will:

- represent, analyze and generalize a variety of patterns with tables, graphs and words
- model and solve contextual problems using various representations such as table, graphs and equations.

Step 5: Follow-up on Assessment

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction?

A. Addressing Gaps in Learning

Consider the following strategies to assist students who are having difficulty:

- Work one-on-one with a student as he/she completes a balance problem. Listen carefully to the language the student uses in order to determine where his/her misunderstanding is.
- Encourage the student to use algebra tiles and/or draw pictures in order to visually represent the problem.
- If a student has difficulty following an algebraic solution, work first to have the student verbally and/or visually represent the solution process. Then move to translating the process into the algebraic representation.

B. Reinforcing and Extending Learning

Students who have achieved or exceeded the outcomes will benefit from ongoing opportunities to apply and extend their learning. These activities should support students in developing a deeper understanding of the concept and should not progress to the outcomes in subsequent grades.

Activity 1: Go to: http://illuminations.nctm.org/LessonDetail.aspx?id=L263 for a lesson on magic squares.

Note to Teacher: In this activity, students will:

- represent, analyze and generalize a variety of patterns with tables, graphs, words and, when possible, symbolic rules
- develop meaning for integers and represent and compare quantities with them
- develop and use strategies to estimate the results of rational-number computations and judge the reasonableness of the results.

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